Is the Sequential Travel Forecasting Paradigm Counterproductive?[1]

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Abstract: The sequential travel forecasting procedure is widely accepted without question by transportation planners, yet its origins are obscure, its effects on practice and research may well be negative, and by focusing attention on individual steps, it tends to impede overall progress in improving forecasting methods. Alternatives to the sequential procedure proposed by researchers over the past 30 years are examined, and recent advances are presented. A call for a new travel forecasting paradigm concludes the paper.

CE Database keywords: Travel demand; Forecasting; Traffic assignment; Modal split; Trip distribution models.

Introduction

For most urban transportation planners, whether they are practitioners or researchers, the sequential, or four-step, travel forecasting procedure is regarded as a cornerstone of our field. The sequential procedure is not just unquestioned; it has achieved the status of a universal truth, appearing as it does in all textbooks. Even so, one may wonder about the origins of this paradigm. Does it have any well-founded basis? Are there alternative views of how to predict travel choices in a congested transportation network? What should be done about the recent requirements for solving the four-step procedure with ‘‘feedback’’?

It is argued in this paper that the sequential travel forecasting procedure does not merit the untarnished reputation it enjoys. Moreover, it was not the first proposal for relating origin-destination choices to route choices and their associated link flows on the road network. Had the history of our field developed somewhat differently, the sequential procedure might well have been discarded

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long ago. Moreover, a recent breakthrough in the solution of the traffic assignment problem, when integrated into the larger travel-forecasting problem, offers a vastly improved and much more flexible approach to predicting future travel.

The objectives of this paper are to examine this issue, highlight the findings of recent research being published in academic research journals, and to discuss their implications for the future training and practice of urban transportation planners. This paper is also a call for change, for a revolution if you will, to reject the present grip of the four-step paradigm on travel forecasting and return to the *original* concept of an integrated relationship among travel choices and road network travel times and costs, as implemented in advanced travel forecasting algorithms.

Brief History of Sequential Procedure

From the viewpoint of practitioners, the sequential procedure is generally depicted as shown in Fig. 1, based on Ortúzar and Willumsen (2001). The four elements, or steps, are so familiar to practitioners that their description seems unnecessary. The dashed lines, however, bear some discussion. These lines represent the concept of ‘‘solving the sequential procedure with feedback.’’ Although this concept is vaguely defined, it is generally understood to mean that information from the solution of one sequential procedure is used in a subsequent solution. The objective of these iterations is to bring travel times and costs assumed in the trip distribution step into agreement with the results of the traffic assignment step. Such consistency can be achieved, but only if the feedback is performed in certain ways. See Boyce et al. (1994) for an initial exploration of this issue.

Origins of Sequential Procedure

The origins of the sequential procedure are obscure at best. The first *urban transportation study*, as professional practice was called then, was the Chicago Area Transportation Study (CATS). Volume I of the three-volume report (CATS 1959) contains a diagram showing the travel forecasting phase of the planning process as consisting of three steps: (1) Estimate trip generation; (2) Estimate trip distribution; and (3) Estimate future travel demand.

The concepts of modal split and traffic assignment were introduced in Volume II but without reference to a revised sequential procedure diagram. In the original CATS travel forecasting procedure implementation, trip origins and destinations were allocated to auto and transit modes before the trip distribution step. Then, auto trips from each origin zone were allocated to destinations using the intervening opportunities model and assigned to the road network using the tree-by-tree approach in an integrated manner (Schneider 1959).

Martin et al. (1961) prepared an early, and possibly the first, detailed review of urban travel forecasting methods. Included in this report is a large flowchart showing all aspects of data collection and reduction, plan formulation, and travel forecasting. The section of the flowchart entitled ‘‘Estimates of Future Travel’’ consists of the following steps: (1) Future trip generation; (2) Future modal split (trend analysis); (3) Interzonal transfers (growth factor methods, inter-area for-

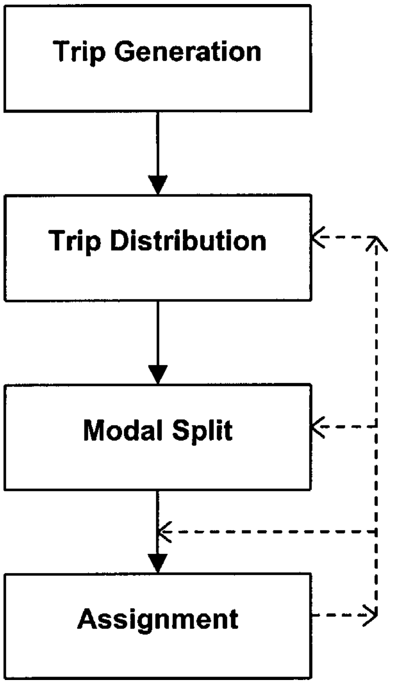

Fig. 1. Sequential procedure

mula methods); and (4) Assignment of interzonal transfers to transportation network (with or without capacity restraints).

A feedback arrow points from step 4 to step 3. The terms used are defined in the report; many have passed out of use during the past 40 years.

In these early reports, there is a clear reference to the four-step, sequential travel forecasting procedure, comprised of separate models connected by a procedural flowchart describing a rather mechanical view of travel forecasting. As transportation engineering courses in universities were implemented, the sequential procedure also took on the role of a pedagogical device.

Curiously, in the early travel forecasting literature authored by practitioners, there is little discussion of the underlying, more general problem of travel demand, or choice, and its relation to congested road travel times. As shown later in this paper, the four-step procedure may now be seen to be a rather crude solution procedure, or heuristic, proposed in an attempt to solve the unstated, underlying problem of the relationship of origin-destination, mode, and route choices to network costs.

Views of Early Practitioners

A review of early manuals and reports describing travel forecasting software reveals a prevailing attitude that travel forecasting should be simple, a view that continues to this day. Little concern is expressed about the true underlying complexity of urban travel phenomena, such as the structure of choices of destination, mode, submode, or routes, or their relation to urban activities. For this reason, it seems, early manuals and computer programs prepared by the Bureau of Public Roads and the Urban Mass Transportation Administration organized the models and programs into four steps. A relatively late example that draws on these earlier developments is the report by Spear (1977). Patriksson (1994, Chapter 1) presents a definitive review of early models and their application.

A major software development effort, the *Urban Transportation Planning System* (UTPS) for the IBM mainframe computer, was jointly undertaken during the 1970s by the Federal Highway Administration and the Urban Mass Transportation Administration (U.S. DOT 1977) under the leadership of Robert B. Dial. Although this effort introduced many innovations, the basic framework of the sequential procedure remained in tact, as expressed by modules related to trip distribution (UMODEL), modal split (ULOGIT), and traffic assignment (UROAD). This development was terminated about 1981; the source code developed by contractors to U.S. DOT was ported to early versions of the IBM personal computer (PC), resulting in the software system *MINUTP*. Hence, this early commercial software system not only retained the sequential procedure as its organizing concept, but also carried over many limitations and shortcomings of UTPS, such as the use of integer arithmetic for origin-destination and link flows and attempted shortcuts in the implementation of the traffic assignment algorithm.

An extension of the sequential procedure occurred as a result of the enactment of the Intermodal Surface Transportation Efficiency Act of 1991, which required the sequential procedure to be solved with ''feedback.'' As explained by Garrett and Wachs (1996, p. 199), the application of the four-step procedure was deemed inadequate because of the inconsistency of travel times and costs among the trip distribution, mode split and traffic assignment steps. Little guidance or understanding of the difficulty of achieving improved consistency was offered, however. Anecdotal evidence suggests that ''feedback'' is performed in a variety of ways with little understanding of the iterative process or expectation of why improved consistency should be achieved.

Implications for Early Research

During the period, 1965–1980, academic researchers began to investigate improved model formulations pertaining to the individual steps of the sequential procedure. For example, Wilson (1970) proposed the entropy-maximizing formulation of the trip distribution model and related modal choice and residential location models. Working in a completely different theoretical paradigm, McFadden (1974) derived the logit mode choice model based on random utility theory. Williams (1977) produced his seminal treatment of nested logit models and their properties. Several researchers during the period 1969–1973 realized that a method for solving the quadratic programming problem (Frank and Wolfe 1956) could be applied to solve the traffic assignment problem. This realization depended on an understanding that the traffic assignment problem could be formulated as a convex optimization problem, a point that had eluded practitioners from the outset of the field in the late 1950s until its introduction into UTPS and related software in the late 1970s (Boyce et al. 1979).

What all of this research has in common is its focus on individual models, or steps, of the sequential procedure. The fact that the overall travel forecasting problem was segmented into steps meant that the energies of these researchers were also largely focused on these individual steps. Few individuals looked beyond a specific step to the larger problem. Even those who had an understanding of the larger framework, described later, may have been discouraged from doing so by the very different modeling styles and approaches of the different steps.

During the 1970s, trip distribution and modal split became the domain of entropy maximizing and random utility modelers. These competing approaches share the methods of convex optimization, but the context is matrices of trips or travel flows with certain stochastic properties. In contrast, traffic assignment is fundamentally a deterministic network problem, again formulated with convex optimization methods. Few researchers from network research, primarily from the field of operations research, seemed comfortable with the approach and assumptions of random utility or entropy maximization, and vice versa. However, a very few individuals did succeed in exploring both sides of these problems. In the next section, their contributions to travel forecasting are examined; these may be seen as a more holistic approach than the conventional sequential procedure.

Integrated Models of Travel Forecasting

Basic Concepts

In order to consider a completely different approach to travel forecasting, readers are asked to put aside temporarily the concept of the sequential procedure and to consider the problem anew. This was the situation faced by Martin Beckmann and his coauthors, C. Bart McGuire and Christopher B. Winsten, in the early 1950s when they formulated the first model of travel demand and route choice (Beckmann et al. 1956).

Let us begin with the behavioral assumptions.

1. The flow of persons per hour traveling from an origin to a destination (demand) decreases in relation to increases in travel time and cost of that journey on the road network;
2. The routes of travel selected from each origin to each destination have equal travel times and costs, and no unused route has a lower travel time and cost; and
3. Route travel times and costs reflect total vehicle flows on the links of the road network, resulting from these origin-destination-route choices.

We may refer to these assumptions as describing a *network equilibrium*. The first assumption is a general statement of the travel demand relationship associated with all travel forecasting methods. The second assumption is generally associated with the name of Wardrop (1952) but was independently proposed by Beckmann (C. B. McGuire, personal interview, 2000). The third assumption relates to the interrelationship between road generalized costs and origin-destination demand. To be clear, *travel time and cost* refer to a weighted linear combination of travel time and monetary cost, which is now defined as *generalized cost*.

Suppose that a single mathematical problem could be posed whose *solution* corresponds precisely to these three assumptions. Beckmann et al. (1956) proposed the formulation of this mathematical problem but did not succeed in finding a procedure to solve it computationally, except for small examples. In the authors' own words, ''Demand refers to trips, and capacity refers to flows on roads. The connecting link is found in the distribution of trips over the network according to the principle that traffic follows shortest routes in terms of average cost. The idea of equilibrium in a network can be described as follows. The prevailing demand for transportation . . . gives rise to traffic conditions that will maintain that same demand'' (Beckmann et al. 1956, p. 59).

Following Beckmann's original formulation of this *integrated model* of travel demand and route choice as a constrained minimization problem, it has been posed in several different ways. Moreover, several solution methods, or algorithms, have been proposed and evaluated for solving the problem. The details of how this is done need not concern us here. The interested reader may wish to consult Boyce and Daskin (1997) or Oppenheim (1995) for comprehensive statements of two different, but similar, approaches to formulating and solving this problem. A more rigorous treatment is given by Patriksson (1994).

It is ironic that Beckmann et al. (1956) performed their highly innovative study at the Cowles Commission for Research in Economics at the University of Chicago during 1951–1955, just prior to the initiation of the Chicago Area Transportation Study in 1955. Evidently, staff members at CATS were never aware of their book, despite its extensive distribution in three printings, and Beckmann was certainly not aware of CATS (M. Beckmann, personal interview, 1998). Had a connection of this rather theoretical study with the practical but highly scientific effort at CATS been achieved, the field might have evolved rather differently.

As the sequential paradigm took hold during the decade following the publication of Beckmann's formulation, the notion of an integrated model seems to have been lost. One researcher, J. D. Murchland, did speculate about the concept of combining the steps of the sequential procedure and by 1970 had a working computer code (Murchland 1966, 1970; J. Murchland, personal interview, 1970). In turn, he influenced S. P. Evans, a mathematically talented graduate student at University College London, to pursue these ideas in her PhD thesis. Evans (1973, 1976) formulated a ''combined'' model of trip distribution and traffic assignment as a constrained optimization problem, analyzed its relation to Beckmann's formulation, and proposed an algorithm for its solution that is proven to converge to the desired equilibrium solution, the achievement that had eluded Beckmann earlier.

Florian et al. (1975) also proposed a solution algorithm for the same combined model (Florian and Nguyen 1978). Frank (1978) demonstrated, for a small network, that the partial linearization algorithm of Evans was clearly superior to the full linearization approach of Florian et al. (1975). Dafermos and Nagurney (1984) subsequently formulated models of travel demand and route choice using a more general mathematical construct, the variational inequality problem. An analysis of the relationship of these and other formulations is found in Boyce (1998). Hence, beginning in the late 1960s with the studies by Dafermos, Murchland, and Evans, and continuing through the mid-1980s with the efforts of Florian, Nguyen, LeBlanc, and Dafermos and Nagurney, there was a small but sustained research effort addressing integrated models of travel demand and route choice. Except for the PhD thesis of Frank (1978) and the research of LeBlanc and Farhangian (1981), there were few computational studies of these combined models.

Implementation, Estimation, and Validation

In collaboration with L. J. LeBlanc and R. W. Eash and several PhD students, the author initiated studies to implement, estimate, and validate combined models of origin-destination, mode, and auto route choice, beginning in 1980 and continuing for the next 20 years. Most of these studies were performed with data provided by the Chicago Area Transportation Study. Initially, aggregated zone systems and road networks were used, but since 1997, these studies have been performed on the same zone system and road network used by CATS in its long-range transportation planning activities (1,790 zones; 39,000 links).

Studies during the 1980s demonstrated the feasibility of applying the algorithm of Evans to solve large-scale problems (Boyce et al. 1983, 1985). An historical retrospective of model formulations was also published (Boyce et al. 1988). First attempts to estimate the parameters of the combined model in a way that is self-consistent with the model's solution were completed in the early 1990s (Boyce et al. 1992; Boyce and Zhang 1997, 1998). Finally, a larger study of the estimation and validation of a multiclass, multimodal combined model was completed in 2000 (Boyce and Bar-Gera, submitted for publication).

Concurrently, Lam and Huang formulated the first multiclass combined model and estimated its parameter values on Hong Kong data (Lam and Huang 1992, 1994; Huang and Lam 1992). The resulting three-class model was compared with the model used for transportation planning in the Hong Kong region. In other lines of combined model research, Safwat formulated models in which trip generation is integrated into a single-class formulation (Safwat and Magnanti 1988), and Abrahamsson and Lundqvist (1999) examined ways to formulate and estimate simultaneous and nested travel choice functions embedded in a combined model. Finally, de Cea, and Fernández (2001) and their professional collaborators implemented combined models for 450 zones and a network of 8,000 links with 13 user classes, 3 trip purposes, and 11 transport modes for Santiago, Chile; these models predict 439 trip matrices using a relatively small PC. Similar models have been applied in Valparaiso and Concepción and several other urban areas in Chile. Additional technical details concerning the formulation and solution of these combined models may be found in Florian et al. (2002).

New Travel Forecasting Paradigm

Boyce and Daskin (1997) were perhaps the first to present a new pedagogical approach to the study of integrated models of travel choice. In contrast to the

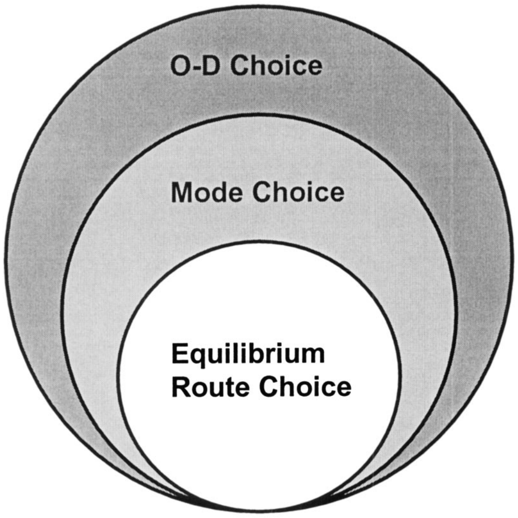

Fig. 2. Diagram of new travel forecasting paradigm

sequential paradigm, they began their exposition of an integrated model with the traffic assignment problem with fixed demand. Having expounded upon the properties of this model, they relaxed the assumption of fixed demand, first with regard to mode choice and then with regard to origin-destination choice. In this way, they showed that a family of models could be formulated and solved, working outwards from the traffic assignment problem that lies at the *core* of the equilibrium route choice concept representing road congestion. A diagram expressing this new paradigm is shown in Fig. 2.

In this paradigm, the formulation of an integrated model begins with the core problem of equilibrium, or user-optimal, route choice with flow dependent travel costs. To expand the problem formulation, other choices are added by relaxing assumptions concerning exogenous travel flows. Dispersion and side constraints are added to make flows more behaviorally and empirically realistic, resulting in one internally consistent model. Given this formulation, convergent algorithms can be devised to solve the model to meaningful levels of accuracy. Values of model parameters that are internally self-consistent with the model solution can also be estimated.

Several implications of an integrated, internally consistent travel and route choice model are apparent. First, the comparison of alternative scenarios is enhanced by the consistency of the model solutions and by opportunities to solve the model to finer levels of convergence than formerly possible, as described later. The existence of this refined representation and solution provides a new impetus for improving data and network representations. As this entire solution can be validated as a single, consistent forecast, it is more useful for evaluating air quality and energy implications of future travel; likewise, the forecasts provide necessary inputs for management of transportation operations.

Despite the recent advances in the formulation and solution of these models, much remains to be accomplished. The training required to contribute to this field is substantially more advanced than the training presently received by many practitioners. This is not *rocket science*. No, it is *harder*! Researchers, software engineers, and practitioners will require more rigorous training in constrained optimization and equilibrium methods to contribute to this more advanced field. New textbooks must be prepared, including appropriate mathematical requisites. The current generation of practitioners may need to be retrained to solve these new models or perform related duties.

Examples of Recent Research Advances

In order to illustrate recent research accomplishments and advantages of the integrated model approach, two examples are briefly presented from detailed research papers presently in press. An example from our validation studies of a two-class, multimodal origin destination (OD), mode and auto route choice model for the morning peak period of the Chicago region is presented first (Boyce and Bar-Gera 2001, 2003). The travel classes are home-to-work and other, which are approximately equal in size in terms of the total person trips per hour. The model represents 1,790 zones covering over 10,000 km^2 (4,000 mil^2) using 39,000 road links, the zone system and network used by practitioners in the Chicago region. Transit travel times and fares are exogenous to the model.

The model was solved by the Evans algorithm with the single execution of a computer program to a relative gap of 0.1%. The relative gap is a measure of the convergence of the solution procedure; a relative gap of 0.1% means that the difference between the objective function and a lower bound available from the optimization formulation is less than 0.1% of the lower bound. As the relative gap approaches zero, the true equilibrium solution is approached, but in a practical sense, this value is never achieved.

The model was estimated with data from a 1990 household travel survey (HHTS) undertaken by CATS. Using the estimated parameter values, the predicted regional mean travel times for auto and transit and the predicted regional mode split agree with the same values calculated from the observed survey choices using the zone-to-zone predicted auto travel times. Such an estimation is termed *selfconsistent*. Auto and transit travel times reported by the survey respondents were not used as they cannot be regarded as reliable. Model predictions for the home-to-work class were validated with the 1990 census transportation planning package (CTTP).

Current travel forecasting practice, as represented by CATS and other large planning agencies, is to perform three solutions of the sequential procedure by averaging link flows and trip tables of successive solutions. Typically, the sequential procedure is repeated three times, a relatively time-consuming task. In contrast, if the Evans algorithm is solved to a relative gap of 0.1%, the trip tables are

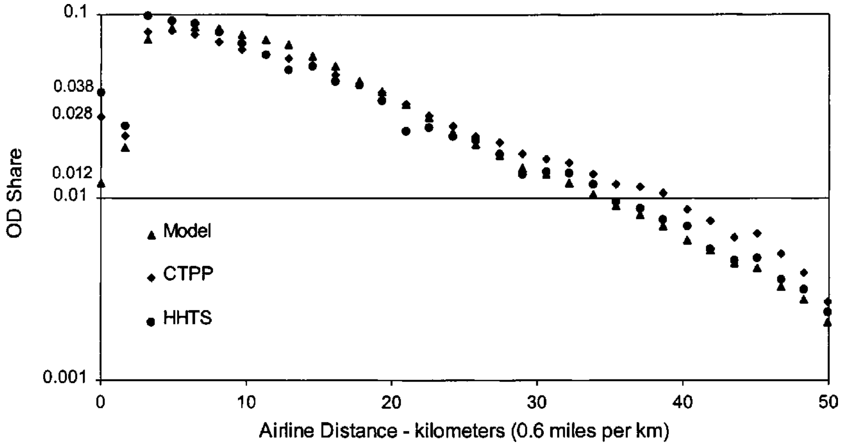

Fig. 3. Origin-destination share of home-to-work travel by airline distance

computed and assigned about 40 times; however, the number of underlying all-or-nothing assignments is similar for both methods.

Fig. 3 shows the correspondence of predicted origin-destination shares of total travel by both auto and transit to comparable the 1990 journey-to-work CTPP data and the CATS 1990 household travel survey, each aggregated to intervals of 1.6 km (1 mile) airline distances between origins and destinations. The zero-distance interval represents intrazonal travel. These intrazonal flows could have been adjusted to correspond to the observed flows by tuning the assumed intrazonal travel times and distances. Such tuning would have further improved the goodness-of-fit, which is clearly very good even without such tuning.

Fig. 4 shows the share of travelers choosing transit, aggregated as above by origin-destinations pairs to intervals of 1.6 km (1 mile). The general trend of the observed modal choice is predicted well by the model, especially for flows up to 30 km. As with the OD shares, the intrazonal mode shares could have been brought into agreement by tuning the assumed travel times and distances. Prediction of flows beyond 30 km is less impressive, but the data are also quite irregular.

The integration of the new origin-based algorithm of Bar-Gera (2002) into a single-class model of origin-destination, mode, and route choice is shown in Fig. 5, together with comparable results for the Evans algorithm and for the sequential procedure with feedback. As can be seen, the relative gap achieved by the origin-based algorithm is much smaller than for the other two procedures, reaching a value below $10^{-9}\%$, a level of interest primarily to researchers. More important from a practitioner's standpoint, the sequential procedure with feedback only approaches a relative gap of 1% after more than 50 iterations and over 10% after 3 iterations. The Evans algorithm also converges slowly after the first 10 iterations and eventually achieves a relative gap of 0.02%. In contrast, the origin-based algorithm quickly surpasses the solution of the other two methods and

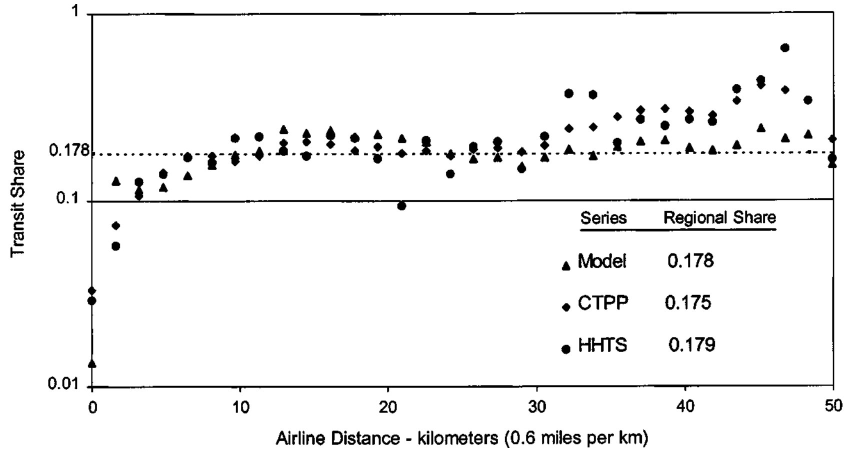

Fig. 4. Transit share of home-to-work travel by airline distance

continues to decrease until machine accuracy is reached. These computations were performed on a Compac Alpha model DS20E computer with a CPU speed of 666 MHz and 256 MB RAM, a computer slower than the PCs now typically used by practitioners. Additional details on the synthesis of this recent breakthrough in solving the traffic assignment problem with combined models of travel and route choice may be found in the work by Bar-Gera and Boyce (2002).

Solving travel forecasting models to internally consistent solutions to a level of convergence of 0.01% relative gap or less is important for two reasons. First,

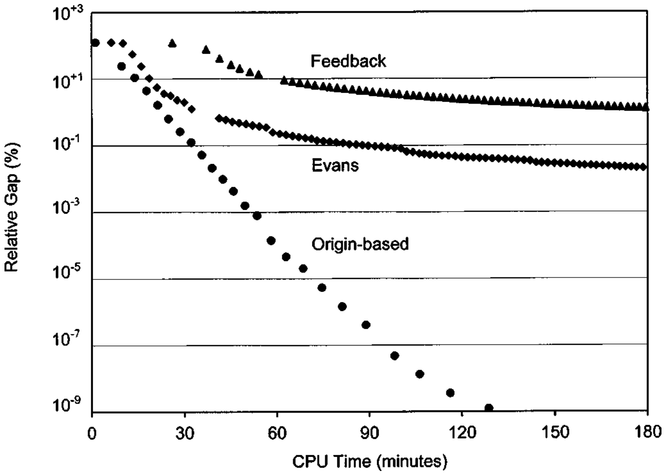

Fig. 5. Relative gap versus computation time for three solution procedures

experiments conducted to date have shown that stability of link flows on moderately congested road networks is generally only achieved at this level (Boyce et al. 2002; Ralevic-Dekic 2001). Some widely used travel forecasting software is unable to achieve such stable solutions. Second, the general objective of travel forecasting is to compare scenarios concerning future transportation investments and development policies. Such comparisons are meaningful only if the differences in the results are large compared to the level of model convergence. Otherwise, the observed differences may simply reflect the inaccuracy of the forecasts.

Conclusions

Through the arguments and perspective presented earlier in this paper, it is shown that the widely accepted sequential procedure is actually counterproductive. Its tendency to focus attention on individual steps, together with the ill-defined ‘‘feedback’’ requirement, has for some time actually slowed the adoption of innovations in travel forecasting methods.

The experience of the 50-year history of urban travel forecasting strongly indicates that meaningful advances over the sequential procedure require a *revolutionary* approach, not the evolutionary, piecemeal improvements to individual steps introduced in the past. The integrated or combined model approach on which our field was actually founded before the advent of urban transportation studies offers the basis for such a revolution. To achieve the advances inherent in this approach and to reap the benefits of the recent breakthrough in solving the traffic assignment problem, we must move forward now, taking care to retrain instructors and practitioners to understand and use them. It is encouraging that professionals in Chile have implemented and applied very detailed travel forecasting models based on this approach.

An ongoing, coordinated effort is required to achieve the advanced level of travel forecasting practice envisaged in this paper. The key to achieving this vision is the professional practitioner community itself. If the community does not insist on having these advances available, then the necessary investments by technical and knowledge providers (software developers, textbook writers, instructors, and retraining programs) will not be forthcoming. Our field is at a crossroads in this respect. Although much work remains to be done, I am optimistic that the promise of both past and recent research advances will be realized.

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References

Abrahamsson, T., and Lundqvist, L. (1999). ‘‘Formulation and estimation of combined network equilibrium models with applications to Stockholm.’’ *Transp. Sci.,* 33, 80–100.

Bar-Gera, H. (2002). ''Origin-based algorithm for the traffic assignment problem.'' *Transp. Sci.*, 36(4).

Bar-Gera, H., and Boyce, D. (2003). ''Origin-based algorithms for combined travel forecasting models.'' *Transp. Res., Part B*, in press.

Beckmann, M., McGuire, C. B., and Winsten, C. B. (1956). *Studies in the economics of transportation*, Yale University Press, New Haven, Conn.

Boyce, D. (1998). ''Long-term advances in the state of the art of travel forecasting methods.'' *Equilibrium and advanced transportation modelling*, P. Marcotte and S. Nguyen, eds., Kluwer, Dordrecht, The Netherlands, Chap. 4, 73–86.

Boyce, D., and Bar-Gera, H. (2001). ''Network equilibrium models of travel choices with multiple classes.'' *Regional science perspectives in economic analysis*, M. L. Lahr and R. E. Miller, eds., Elsevier Science, New York, Amsterdam, 85–98.

Boyce, D., and Bar-Gera, H. (2003). ''Validation of urban travel forecasting models combining origin-destination, mode and route choices.'' *J. Regional Sci.*, in press.

Boyce, D. E., Chon, K. S., Ferris, M. E., Lee, Y. J., Lin, K. T., and Eash, R. W. (1985). ''Implementation and evaluation of combined models of urban travel and location on a sketch planning network.'' *Final Rep.*, Univ. of Illinois, Urbana-Champaign and Chicago Area Transportation Study, Chicago.

Boyce, D. E., and Daskin, M. S. (1997). ''Urban transportation.'' *Design and operation of civil and environmental engineering systems*, C. ReVelle, and A. McGarity, eds., Wiley, New York, Chap. 7, 277–341.

Boyce, D. E., Eash, R. W., and Janson, B. N. (1979). ''Equilibrium trip assignment: Advantages and implications for practice.'' *Transp. Res. Rec. 728*, Transportation Research Board, Washington, D.C., 1–8.

Boyce, D. E., LeBlanc, L. J., and Chon, K. S. (1988). ''Network equilibrium models of urban location and travel choices: A retrospective survey.'' *J. Regional Sci.*, 28, 159–183.

Boyce, D. E., LeBlanc, L. J., Chon, K. S., Lee, Y. J., and Lin, K. T. (1983). ''Implementation and computational issues for combined models of location, destination, mode and route choice.'' *Environ. Plan. A*, 15, 1219–1230.

Boyce, D. E., Lupa, M. R., and Zhang, Y. (1994). ''Introducing 'feedback' into four-step travel forecasting procedure vs. equilibrium solution of combined model.'' *Transp. Res. Rec. 1443*, Transportation Research Board, Washington, D.C., 65–74.

Boyce, D., Ralevic-Dekic, B., and Bar-Gera, H. (2002). ''Convergence of traffic assignments: How much is enough?'' *J. Transp. Eng.*, in press.

Boyce, D. E., Tatineni, M., and Zhang, Y. (1992). ''Scenario analyses for the Chicago region with a sketch planning model of origin-destination, mode and route choice.'' *Final Rep. to the Illinois Dept. of Transportation*, Urban Transportation Center, Univ. of Illinois, Chicago.

Boyce, D. E., and Zhang, Y. (1997). ''Calibrating a combined model of trip distribution, modal split, and traffic assignment.'' *Transp. Res. Rec., 1607*, Transportation Research Board, Washington, D.C., 1–5.

Boyce, D., and Zhang, Y. (1998). ''Parameter estimation for combined travel choice models.'' *Network infrastructure and the urban environment*, L. Lundqvist, L.-G. Mattsson, and T. J. Kim, eds., Springer, Berlin, Chap. 10, 177–193.

Chicago Area Transportation Study. (1959). *Final Rep.*, Vol. I, Chicago.

Dafermos, S., and Nagurney, A. (1984). ''Stability and sensitivity analysis for the general network equilibrium-travel choice model.'' *Proc., 9th Int. Symp. on Transportation and Traffic Theory*, VNU Press, Utrecht, The Netherlands, 217–231.

de Cea, J., and Fernández, J. E. (2001). ''ESTRAUS: A simultaneous equilibrium model to analyze and evaluate multimodal urban transportation systems with multiple user classes.'' Preprint Paper No. 4307, *9th World Conf. on Transport Research*, Seoul, Korea, World Conf. on Transport Research Society, Lyou, France.

Evans, S. P. (1973). ''Some applications of optimisation theory in transport planning.'' PhD thesis, Traffic Studies, University College London, London.

Evans, S. P. (1976). ''Derivation and analysis of some models for combining trip distribution and assignment.'' *Transp. Res.,* 10, 37–57.

Florian, M., and Nguyen, S. (1978). ''A combined trip distribution, modal split and trip assignment model.'' *Transp. Res.,* 12, 241–246.

Florian, M., Nguyen, S., and Ferland, J. (1975). ''On the combined distribution-assignment of traffic.'' *Transp. Sci.,* 9, 43–53.

Florian, M., Wu, J. H., and He, S. (2002). ''A multi-class multi-mode variable demand network equilibrium model with hierarchical logit structures.'' *Transportation and network analysis: Current trends*, M. Gendreau, and P. Marcotte, eds., Kluwer, Dordrecht, The Netherlands, Chap. 8, 119–133.

Frank, C. (1978). ''A study of alternative approaches to combined trip distribution-assignment modeling.'' PhD thesis, Regional Science, Univ. of Pennsylvania, Philadelphia.

Frank, M., and Wolfe, P. (1956). ''An algorithm for quadratic programming.'' *Naval Res. Logistics Quart.,* 3, 95–110.

Garrett, M., and Wachs, M. (1996). *Transportation planning on trial*, Sage, Thousand Oaks, Calif.

Huang, H.-J., and Lam, W. H. K. (1992). ''Modified Evans' algorithms for solving the combined trip distribution and assignment problem.'' *Transp. Res.*, 26B, 325–337.

Lam, W. H. K., and Huang, H.-J. (1992). ''A combined trip distribution and assignment model for multiple user classes.'' *Transp. Res.*, 26B, 275–287.

Lam, W. H. K., and Huang, H.-J. (1994). ''Comparison of results of two models of transportation demand in Hong Kong: CDAM and a version of microTRIPS.'' *J. Adv. Transp.,* 28, 107–126.

LeBlanc, L. J., and Farhangian, K. (1981). ''Efficient algorithms for solving elastic demand traffic assignment problems and mode-split assignment problems.'' *Transp. Sci.,* 15, 306–317.

Martin, B. V., Memmott, F. W., III, and Bone, A. J. (1961). ''Principles and techniques of predicting future demand for urban area transportation.'' *Research Rep. No. 38*, M.I.T., Cambridge, Mass.

McFadden, D. (1974). ''Conditional logit analysis of qualitative choice behavior.'' *Frontiers of econometrics*, Academic, San Diego, 105–142.

Murchland, J. D. (1966). ''Some remarks on the gravity model of trip distribution, and an equivalent maximization formulation.'' *LSE-TNT-38*, Transport Network Theory Unit, London School of Economics, London.

Murchland, J. D. (1970). ''Road network traffic distribution in equilibrium.'' *Mathematical models in the social sciences*, 8, Anton Hain Verlag, Meisenheim am Glan, Germany, 145–183 (German translation).

Oppenheim, N. (1995). *Urban travel demand modeling*, Wiley, New York.

Ortúzar, J. D., and Willumsen, L. G. (2001). *Modelling transport*, 3rd Ed., Wiley, New York.

Patriksson, M. (1994). *The traffic assignment problem: Models and methods*, VSP, Utrecht, The Netherlands.

Ralevic-Dekic, B. (2001). ''Convergence of link flow solutions by origin-based and link-based traffic assignment algorithms.'' MS thesis, Civil Engineering, Univ. of Illinois at Chicago, Chicago.

Safwat, K. N. A., and Magnanti, T. L. (1988). ''A combined trip generation, trip distribution, modal split, and trip assignment model.'' *Transp. Sci.,* 18, 14–30.

Schneider, M. (1959). ‘‘Gravity models and trip distribution theory.’’ *Papers of the Regional Science Association*, 5, 51–56.

Spear, B. D. (1977). ‘‘Applications of new travel demand forecasting techniques to transportation planning.’’ Federal Highway Administration, U.S. Department of Transportation, Washington, D.C.

U.S. Department of Transportation. (1977). *Urban transportation planning system reference manual*, Urban Mass Transportation Administration, Washington, D.C.

Wardrop, J. G. (1952). ‘‘Some theoretical aspects of road traffic research.’’ *Proc., Institution of Civil Engineers, Part II*, 1, 325–378.

Williams, H. C. W. L. (1977). ‘‘On the formation of travel demand models and economic evaluation measures of user benefit.’’ *Environment and Planning A*, 9, 285–344.

Wilson, A. G. (1970). *Entropy in urban and regional modelling*, Pion, London.