Spring 2022 CEE-ME426-2: Nonlinear Finite Element II with Reduced Order Methods by Deep Learning Discrete Calculus

Instructor: Professor Wing Kam Liu (w-liu@northwestern.edu) and Co-Founder of HIDENN-AI, LLC
Guest Lecturers: Professor Mark Fleming (mark.fleming@northwestern.edu, and CTO of Fusion Engineering), Dr. Ye Lu (ye.lu@northwestern.edu)
Days and Times: Tuesday-Thursday 9:30 am – 10:50 am; TECH M128 zoom link (To be announced)
Office hours: Professor Wing Kam Liu: By appointment
Graders/Computer Instructors: Sourav Saha, Jiachen Guo
Contributors: Chanwook Park, Alberto Ciampaglia, Xiaoyu Xie

The course teaches students about deep learning inspired finite element methods and algorithms. Deep Learning Discrete Calculus (DLDC) is the latest development in the field of discrete calculus-based computational driven by deep learning. This course covers two complementary focus areas: (1) *Non-linear DLDC materials modeling* by the finite element and reduced-order multiscale modeling and simulation; (2) *DLDC approaches* for the analysis and design of complex material systems. Applications include soft matter and biomaterials, multiscale materials system, topology optimization, and large deformation analysis for manufacturing and extreme dynamics.

Who should take this course: This course will be of interest to students with a background in the following subjects: aerospace, applied mathematics, biomedical, civil, structural and mechanical engineering, and materials science.

Module 1 (3 weeks): **Finite Element Artificial Intelligent Solvers with Deep Learning Discrete Calculus (DLDC)**

- Introduction of 1D Deep-learning Neural Network (DNN)
- Structure for DLDC for Numerical Differentiation
- Derivation of 1D finite element interpolation using DNN
- Introduction of three operators: addition, multiplication, and inverse (divide) operators
- Principle of minimum potential energy and FEM mesh adaptivity
- B-spline and Reproducing Kernel Particle Method (RKPM)
- Numerical integration using DLDC

Module 2 (4 weeks): **Discover Linear and Non-Linear Material Laws by Deep Learning Methods**

Introduction to Deep Learning (DL) and its application to the discovery of general nonlinear elastic and elastic-plastic material laws without explicitly knowing the mathematical functional forms using stress and strain invariants.

- Stress and strain invariants
- Material laws: hyperelasticity and plasticity
- Mechanistic material law modeling using 1D data (MAP123 algorithm)
- Data-driven discovery of material laws using sparse identification
- Introduction to Digital Image Correlation (DIC)
- Discovering material laws from experiments using DIC

Module 3 (3 weeks): **Multiscale Reduced Order DLDC Method: Micromechanics with Modules 1 and 2**

- Eshelby's problem
- Concentration and Interaction tensors
- Derivation of Lipmann-Schwinger equation
- Introduction to Self-consistent Clustering Analysis (SCA)
- DLDC approximation of SCA and validation using DIC

Projects: Students must complete a final project. The proposal is due by the 4^{th} week.
Homework: Homework assignments for each module related to the subject material will be given.
Reading Assignments: Three sets of reading assignments are to be completed with a comprehensive written summary of the suggested articles.

Grading: Homework (3×10%) + Reports of papers reading (3×10%) + Final Project (40%)

Textbooks:

1. Belytschko T, Liu WK, Moran B, Elkhodary K, Nonlinear Finite Element for Continua and Structures, John Wiley & Sons LTD, 2nd Edition 2013.
2. Liu, W.K., Gan, Z. and Fleming, M., 2021. Mechanistic Data Science for STEM Education and Applications. Springer.
3. Optional: Hughes, T.J., 2012. *The finite element method: linear static and dynamic finite element analysis*. Courier Corporation.

	Topics	Readings & Assignments
Week 1 (Module 1) Mar 29 (TU) & 31 (TH)	• Introduction of 1D **De**ep-learning **N**eural **N**etwork (DeNN) • Concept of **D**eep **L**earning **D**iscrete **C**alculus (DLDC) • Overview for Module 2 and Module 3 • Concept of Interpolation: Lagrange Polynomial • Numerical Differentiation with Interpolation • Structure for DLDC for Numerical Differentiation	• Papers in Module 1 • HW1 (with 2 options): analyze and predict the velocity, acceleration using DLDC (2 weeks to finish)
Week 2 (Module 1) Apr 5 (TU) & 7 (TH)	• Linear Finite Element Shape function with Neural Network • Concept of interpolation with Neural Network • Basic Operations: Addition, Multiplication, Division • Higher-order shape functions	• Chapters 1 and 3 from Mechanistic Data Science Book
Week 3 (Module 1) Apr 12 (TU) & 14 (TH)	• Principle of minimum potential energy • FEM mesh adaptivity • B-spline method • Reproducing Kernel Particle Method (RKPM) • Numerical integration using DLDC	Submission of Project Proposal
Week 4 (Module 2) Apr 19 (TU) & 21 (TH)	• Stress and strain invariants • Introduction to constitutive modeling • Elasticity and hyperelasticity • Elastoplasticity	Chapter 5 of "Nonlinear Finite Elements for Continua and Structures," Ted Belytschko, Wing Kam Liu, Brian Moran, and Khalil Elkhodary, Second Edition, John Wiley & Sons, Ltd, December 2013.
Week 5 (Module 2) Apr 26 (TU) & 28 (TH)	• Mechanistic data-driven modeling of constitutive laws from low-dimensional data • MAP123 for hyperelasticity • MAP123 for elastoplasticity	Papers [1] and [2] in Module 2 Selected sections of Chapters 3, 5 and 6 of Mechanistic Data Science book
Week 6 (Module 2) May 3 (TU) & 5 (TH)	• Mechanistic data-driven constitutive law discovery • Mechanistic data-driven stress update algorithm • Return mapping method for stress update • Stress update using DLDC	**HW2:** Discovering Drucker-Prager plasticity law using a mechanistic data-driven method
Week 7 (Module 2) May 10 (TU) & 12 (TH)	• Introduction to digital image correlation (DIC) • Discovering material laws from DIC data using DLDC	Papers [3] and [4] in Module 2
Week 8 (Module 3) May 17 (TU) & 19 (TH)	• Eshelby's problem • Micromechanics • Concentration and Interaction tensors	Papers in Module 3 **HW3.** Solving for stress in a single inclusion problem using DLDC reduced order method
Week 9 (Module 3) May 24 (TU) & 26 (TH)	• Lipmann-Schwinger equation • Introduction to reduced-order models such as SCA (self-consistent clustering analysis)	

Week 10 (Module 3) May 31 (TU) June 2 (TH)	• DLDC approximation of SCA	

Reading Assignments Module 1:

[1] Saha et al., (2021). Hierarchical Deep Learning Neural Network (HiDeNN): An artificial intelligence (AI) framework for computational science and engineering. *Computer Methods in Applied Mechanics and Engineering*, *373*, 113452.
[2] Zhang et al., (2021). Hierarchical deep-learning neural networks: finite elements and beyond. Computational Mechanics, 67(1), 207-230.

Reading Assignments Module 2:

[1] Tang et al., "MAP123: A data-driven approach to use 1D data for 3D nonlinear elastic materials modeling," Comput. Methods Appl. Mech. Engrg. 357 (2019) 112587.
[2] Tang et al., Map123-EP: A mechanistic-based data-driven approach for numerical elastoplastic analysis, Computer Methods in Applied Mechanics and Engineering 364 (2020) 112955.
[3] Yang, H., Qiu, H., Xiang, Q., Tang, S., & Guo, X. (2020). Exploring elastoplastic constitutive law of microstructured materials through artificial neural network—A mechanistic-based data-driven approach. Journal of Applied Mechanics, 87(9), 091005.
[4] Daoping Liu, Hang Yang, K.I. Elkhodary, Shan Tang, Wing Kam Liu, Xu Guo, Mechanistically informed data-driven modeling of cyclic plasticity via artificial neural networks, Computer Methods in Applied Mechanics and Engineering, Volume 393, 2022, 114766

Reading Assignment Module 3:

[1] Liu, Z., Bessa, M. A., & Liu, W. K. (2016). Self-consistent clustering analysis: an efficient multi-scale scheme for inelastic heterogeneous materials. Computer Methods in Applied Mechanics and Engineering, 306, 319-341.
[2] Liu, Z., Fleming, M., & Liu, W. K. (2018). Microstructural material database for self-consistent clustering analysis of elastoplastic strain softening materials. Computer Methods in Applied Mechanics and Engineering, 330, 547-577.
Optional
[3] Schneider, M. (2019). On the mathematical foundations of the self-consistent clustering analysis for non-linear materials at small strains. Computer Methods in Applied Mechanics and Engineering, 354, 783-801.

Policies

The following links detail university-wide policies that apply to this course. Please read them. Beyond these policies, please remember:

Your mental health is more important than coursework. Ask for help whenever you need it.
If you are unsure how to interpret the academic integrity policies, please ask. We can help you avoid violating policies; we cannot un-report a suspected violation.
We are committed to fostering a safe and welcoming environment for all. The Office of Equity links below include several resources for confidential help or formal reporting. If you need help navigating these resources, please ask.
Syllabus Statements for the following topics are found here: https://www.registrar.northwestern.edu/faculty-staff/syllabi.html

Academic Integrity
Accessibility
COVID-19 Testing Compliance
Diversity, Equity, and Inclusion
Prohibition of Recording of Class Sessions by Students
Support for Wellness and Mental Health
Guidance on Class Recordings
COVID-19 Classroom Expectations Statement
Exceptions to Class Modality
Policies for Accessibility: https://www.northwestern.edu/accessibility/index.html.

Policies for Title IX and Sexual Misconduct: https://www.northwestern.edu/sexual-misconduct/title-ix/index.html.

Additional policies from the Office of Institutional Equity: https://www.northwestern.edu/equity/policies-procedures/policies/index.html