

INTERNAL STRESSES IN REINFORCED PLASTICS CAUSED BY SHRINKAGE OF RESIN AND THERMAL DEFORMATION

ZDENĚK P. BAŽANT*)

Viscoelastic analysis of internal stresses caused by shrinkage of resin at polymerization on the basis of a time-variant Boltzmann's standard model for the creep of resin. Reasons for small value of internal stresses due to shrinkage. Consequences for the rate of increase of mechanical parameters during polymerization in comparison with the rate of shrinkage. Behaviour of glass fibres at shrinkage. Internal stresses produced by thermal deformations and an estimation of their values.

Although the problem of internal stresses in reinforced plastics by shrinkage of resin at polymerization has been dealt with by various authors, there is up to this time still no uniform opinion on their actual values and their influence on strength and life time. In view of a relatively great shrinkage of unreinforced pure resin — e. g. linear shrinkage about — 1,6 % for polyester (volume shrinkage about — 6 %) — and of a much smaller value of linear shrinkage of reinforced plastics — e. g. about — 0,05 % for fibreglass polyester laminate reinforced by fabric (representing 50 % of weight), some authors have stated the internal stress in a resin to be of considerably great values, more than about 100 kp/cm². This supposition has been supported by the experiments of Mc GARRY [1], [2] who measured directly the stresses in glass balls situated in a resin during polymerization. If these results were true, however, and not influenced by some secondary effects, the internal stresses due to shrinkage would substantially affect the strength and life time and would have to be considered in the analysis.

In the present paper, an analysis is submitted which shows that the internal stresses caused by shrinkage in reinforced plastics must be rather small, whereas the thermal stresses can reach greater values. There will be made a theoretical analysis of fibreglass plastics, in which there will be necessary to take the creep of resin into account at which all mechanical parameters of resin must be considered as time variable. So far the problem has been solved only for a time variable module of elasticity [4], while an analysis respecting the creep of resin has not yet been done. Simultaneously some interesting results for mechanical properties of resin and for the behaviour of glass fibres during the polymerization will be obtained. In addition to this, a viscoelastic analysis of internal stresses caused by thermal deformations will be made.

*) ZDENĚK P. BAŽANT, Ing., CSc., Building Research Institute of the Technical University Prague (Stavební ústav ČVUT v Praze, Solinova 7, Praha 6-Dejvice, Czechoslovakia).

ACTA POLYTECHNICA
Práce Českého vysokého učení technického v Praze
I, 1, 1967

Vydává České vysoké učení technické v Praze ve Státním pedagogickém nakladatelství v Praze. Vychází v šesti řadách: řada I — stavební, řada II — strojní, řada III — elektrotechnická, řada IV — technicko-teoretická, řada V — společenské vědy, řada VI — všeobecná. Každá řada vychází čtyřikrát ročně.

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Adresa redakce: Acta Polytechnica, Elektrotechnická fakulta ČVUT v Praze, Technická ul. 2, Praha 6-Dejvice

ACTA POLYTECHNICA
I, 1, 1967

Издатель: Политехнический институт в Праге. Выходит в шести сериях: серия 1-я — строительная, серия 2-я — машиностроительная, серия 3-я — электротехническая, серия 4-я — теоретическая, серия 5-я — общественные науки, серия 6-я — общая. Каждая серия выходит 4 раза в год.

Herausgeber: Technische Hochschule Prag. Erscheint in sechs Reihen: Reihe I — Bauwesen, Reihe II — Maschinenbau, Reihe III — Elektrotechnische, Reihe IV — Theoretische, Reihe V — Gesellschaftswissenschaften, Reihe VI — Allgemeine.

Published by the Technical University of Prague in six series: Series I — Civil Engineering, Series II — Mechanical Engineering, Series III — Electrical Engineering, Series IV — Theoretical, Series V — Social Sciences, Series VI — General.

Publiés par l'Ecole Polytechnique de Prague en six séries: Série I — Génie Civil, Série II — Constructions Mécaniques, Série III — Electrotechnique, Série IV — Théorique, Série V — Sciences sociales, Série VI — Générale.

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Адрес редакции — Schriftleitung — Editorial Office — Adresse de la rédaction: Acta Polytechnica, Elektrotechnická fakulta ČVUT v Praze, Technická ul. 2, Praha 6-Dejvice, ČSSR.

1. Discussion of some experiments with resin concretes

Certain simple experiments with resin concretes have been performed by SKUPIN [3] at the Building Research Institute in Prague which indicate the internal stresses caused by shrinkage to be very small — about 1 kp/cm² at maximum. In these experiments there has been measured and mutually compared the linear shrinkage of two polyester resin concretes in which the filler has been realized once by small glass balls, the second time by small cubes of foamed PVC with a dynamic elasticity module of about 50 kp/cm², the ratio of volume of resin to that of filler being in both cases 1 : 3. The total linear shrinkage observed for concrete with foamed PVC filler was about — 0,20 %, for concrete with glass balls about — 0,05 %. What is the most considerable is that the difference between these two values is very small even if the elasticity module of foamed PVC at small deformation is about 50 kp/cm² and is then negligible with respect to the module of glass balls, the foamed PVC filler being thus practically equivalent with air bubbles. We may state that the stress in the resin at concrete with foamed PVC filler is much smaller than the value $3 \cdot 0,20 \% \cdot 50 = 0,3$ kp/cm², the reduction by creep of resin being disregarded. The shrinkage observed, from the instant at polymerization when the state of resin changes from a plastic to a rigid one, was $-0,3 \cdot 10^{-5}$ or $-1,9 \cdot 10^{-5}$, respectively. Hence, we estimate the difference of internal stress in resin between these two cases to be, at maximum, $(1,9 - 0,3) 10^{-5} \cdot 33\,000 = 0,53$ kp/cm² where 33 000 kp/cm² is the elasticity module of a hardened resin. The total stress in a resin of concrete with glass filler is then obtained : $0,3 + 0,53 \approx 1$ kp/cm² at maximum.

2. Stress-strain relation for time-variant creep

For the relation of normal stress σ_x and normal strain ϵ_x at creep of resin we will introduce the Boltzmann's (or standard) rheological model according to Fig. 1, in which the spring constants E_p , E_v and the viscosity constant η_p in

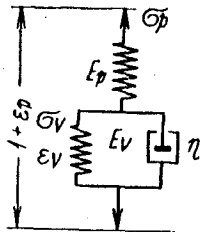


Fig. 1. Boltzmann's rheological model for creep of resin.

contradistinction to the usual theory of viscoelasticity, would be considered as time variable [5] during the polymerization process, i. e. dependent on the degree of polymerization. Denoting by σ_v and ϵ_v the stress and strain in the dash pot, Fig. 1, and by t the time from the beginning of polymerization, the stress strain relations for individual model elements are

$$(1) \quad \dot{\sigma}_x = E_p(\dot{\epsilon}_x - \dot{\epsilon}_v), \quad \dot{\sigma}_x - \dot{\sigma}_v = E_v \dot{\epsilon}_v, \\ \sigma_v = \eta_p \dot{\epsilon}_v$$

where the derivatives with respect to time are denoted by points. Eliminating σ_v and ϵ_v from these equations, there is obtained, for the creep law of resin, the following linear differential equation of second order with time variable coefficients:

$$(2) \quad \ddot{\epsilon}_x(t) + a(t)\dot{\epsilon}_x(t) = \frac{1}{E_p} [\ddot{\sigma}_x(t) + b(t)\dot{\sigma}_x(t)]$$

in which

$$(3) \quad a(t) = \frac{E_v + \dot{\eta}_t}{\eta_t}, \quad b(t) = a(t) + \frac{E_p}{\eta(t)} - \frac{\dot{E}_p}{E_p}$$

This creep law is linear and the principle of superposition applies. It will be convenient to write it in an operator form:

$$(4) \quad \epsilon_x(t) = E_p^{-1} \sigma_x(t)$$

in which E_p^{-1} is the differential operator for creep:

$$(5) \quad E_p^{-1} = \frac{1}{E_p} \frac{\frac{d^2}{dt^2} + b(t) \frac{d}{dt}}{\frac{d^2}{dt^2} + a(t) \frac{d}{dt}}$$

The creep curves for constant stresses applied in different times are represented on Fig. 2.

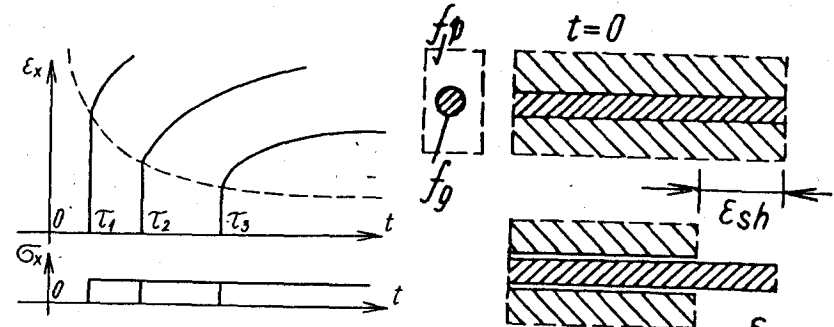


Fig. 2. Creep curves of resin at constant stress applied in different times of polymerization.

Fig. 3. Scheme for the analysis of shrinkage effect of resin.

3. Viscoelastic analysis of stress caused by shrinkage in fibreglass plastics

Now, let us consider a plate of fibreglass plastics reinforced, for the sake of simplicity, only in one direction x by glass fibres, Fig. 3, which will be meanwhile considered as perfectly straight. The easiest way to derive the differential equa-

tion of the problem is to use the analogy of time — variant creep with elasticity [5], which is based on the formal analogy of the creep law (4) with the Hooke's law. We will start, therefore, from the equations of problem for the case of elastic resin.

Let us denote by f_g or f_p the cross-section of glass fibres or that of resin corresponding to unity length of plate and by $\varepsilon_{sh}(t)$ the relative shrinkage in time t from the beginning of polymerization. The stress in glass fibres is $\sigma_{x_g}(t) = E_g \varepsilon_x(t)$. Assuming that the plate of laminate possesses the possibility of free shortening, the average stress σ_{x_p} in the resin results from the equilibrium condition:

$$(6) \quad \sigma_{x_p} = -\sigma_{x_g} \frac{f_g}{f_p}.$$

Assuming further that the plane cross-sections remain plane and that the fibres are uniformly distributed in the section, the deformational condition requires equal strain in fibres and resin, i. e. for an elastic resin $\varepsilon_x = \varepsilon_{sh} + \sigma_{x_p}/E_p$. Hence, with respect to (6), we obtain

$$(7) \quad \varepsilon_x \left(1 + \frac{E_g f_g}{E_p f_p}\right) = \varepsilon_{sh}$$

For a viscoelastic resin the corresponding equation is obtained, in replacement of $1/E_p$ by the operator (5), i. e.

$$(8) \quad \varepsilon_x(t) + \frac{E_g f_g}{E_p f_t} \frac{d^2}{dt^2} + \frac{b(t)}{E_p f_t} \frac{d}{dt} \varepsilon_x(t) = \varepsilon_{sh}(t).$$

After modification we receive

$$(9) \quad \ddot{\varepsilon}_x(t) + g(t) \dot{\varepsilon}_x(t) = h(t)$$

where

$$(10) \quad g(t) = \frac{a(t) + b(t)/n_t}{1 + 1/n_t}, \quad h(t) = \frac{\dot{\varepsilon}_{sh}(t) + a(t)\varepsilon_{sh}(t)}{1 + 1/n_t}, \quad n_t = \frac{E_p f_p}{E_g f_g}.$$

This is the final differential equation of the problem. The initial condition for the strain and the rate at the beginning of polymerization ($t = 0$) are $\varepsilon_x(0) = 0$, $\dot{\varepsilon}_x(0) = 0$, because $E_p f_t(0) = E_p f_t(0) = \eta_t(0) = 0$.

The eq. (8) may be easily solved by quadratures. The equation is solved firstly as a first order equation for $\dot{\varepsilon}_x$ with variable coefficients, e. g. by the method of variation of constants. Its solution is following:

$$(11) \quad \varepsilon_x(t) = \int_0^t \dot{\varepsilon}_x(\tau) d\tau, \quad \dot{\varepsilon}_x(t) = e^{-F(t)} \left(\dot{\varepsilon}_x(0) + \int_0^t h(\tau) e^{F(\tau)} d\tau \right), \quad F(t) = \int_0^t g(\tau) d\tau.$$

The internal stress in glass fibres then has the value

$$(12) \quad \sigma_{x_g}(t) = E_g \varepsilon_x(t).$$

The internal stress in resin is given by the eq. (6).

For numerical analysis it is necessary to assume certain mathematical expressions for the time course of E_p , E_p , η_t . At the beginning of polymerization the resin represents a liquidity with elasticity, delayed elasticity and viscosity which is many times smaller than a hardened resin, i. e. practically zero. With respect to the fact that for $t \rightarrow \infty$ E_p , E_p and η_t must tend to the values for a hardened resin, we introduce

$$(13) \quad E_{p_t} = E_p(1 - e^{-t/\tau_0}), \quad E_{V_t} = E_V(1 - e^{-t/\tau_1}), \quad \eta_t = \eta(1 - e^{-t/\tau_2})$$

where τ_0 , τ_1 , τ_2 are the limit final constant values describing the creep of hardened polymerized resin, Fig. 4. Assuming the course of creep strain of hardened resin at constant unity stress applied in time τ in the exponential form (27) later introduced, there holds $\eta = \tau_p E_p / E_V = \frac{1}{E_{p_\infty}} - \frac{1}{E_p}$ [6]. The time course of shrinkage at polymerization may be expressed in the form, Fig. 4,

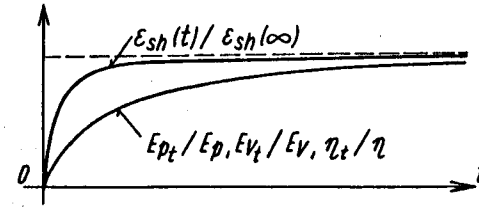


Fig. 4. Course of shrinkage and increase of mechanical parameters of resin during polymerization.

$$(14) \quad \varepsilon_{sh}(t) = \varepsilon_{sh\infty} (1 - e^{-\frac{t}{\tau_{sh}}}).$$

For the sake of simplicity let us meanwhile assume that $\tau_0 = \tau_1 = \tau_2 = \tau_{sh}$, i. e. that the model parameters are increasing proportionally with the shrinkage. For the constants E_V and η there is necessary to introduce the values corresponding to the retardation time $\tau_p \doteq \tau_{sh}$. Substituting a new independent variable

$$(15) \quad \xi(t) = 1 - e^{-\frac{t}{\tau_p}}$$

for t , we receive finally the solution (11) in the form

$$(16) \quad \dot{\varepsilon}_x(t) = \frac{n \varepsilon_{sh\infty}}{\tau_{sh}} \left(\frac{1+n\xi}{1-\xi} \right)^{-k} \int_0^\xi \left(\frac{1+n\xi'}{1-\xi'} \right)^{k-1} d\xi', \quad n = \frac{E_p f_p}{E_g f_g}.$$

The integral in this expression may be transformed to a binomial integral which, for a rational number k , may be again transformed to an integral of a rational function. Practically most suitable, however, is the way of numerical integration, at which the only difficulty is the infinite value of integrand for $\xi \rightarrow 1$.

We have numerically computed the final value of shrinkage of a fibre glass

laminate according to (16), considering $E_p = 33\,000 \text{ kp/cm}^2$, $E_{p\infty} = \frac{1}{5} E_p$, $E_g = 700\,000 \text{ kp/cm}^2$, $\varepsilon_{sh\infty} = -1,6 \%$, $f_p/f_g = 7$ (polyester laminate with 25 % of weight of glass in x direction) while the value of τ_{sh} there is not necessary to know. There has been obtained $\varepsilon_{sh\infty} = -0,051 \%$. Then the final internal stress of glass fibres is $\sigma_{x_p}(\infty) = -0,051 \% \cdot 700\,000 = -360 \text{ kp/cm}^2$ and of resin $\sigma_{x_p}(\infty) = 360/7 = 51 \text{ kp/cm}^2$.

If a laminate, reinforced in two perpendicular directions, would be considered, the resulting value with respect to the Poisson's ration of resin would be still somewhat greater.

The obtained value $-0,051 \%$ of shrinkage is in agreement with the results of tests, the values of internal stresses, however, do not agree at all with our previous considerations, based on shrinkage values observed at resin concrete with rather compressive filler.

Therefore, this agreement is more accidental than basic and the assumptions about the time course of E_{p_t} , E_{p_t} , η_t , particularly the assumption $\tau_0 = \tau_1 = \tau_2 = \tau_{sh}$, must have been incorrect. To achieve an agreement it is necessary to make another assumption.

4. Simplified analysis of shrinkage

The foregoing solution of the eq. (8) is relatively involved. Let us try to make an approximate analysis based on the fact that the observed values of shrinkage of glass reinforced laminates are many times smaller than that of pure resin and substitute then $\dot{\varepsilon}_x(t) \doteq 0$. Obviously, we obtain thus somewhat greater internal stresses than the actual ones in a laminate and we determine simultaneously an upper limit of internal stress which can be caused by shrinkage in a resin with prevented shortening.

For an elastic resin there would be valid $\sigma_{x_p}/E_p = -\varepsilon_{sh}$. Replacing here $1/E_p$ with the creep operator E_p^{-1} (Equ.(5)), we receive the equation

$$(17) \quad \ddot{\sigma}_{x_p}(t) + b(t) \dot{\sigma}_{x_p}(t) = -E_{p_t}(t) [\dot{\varepsilon}_{sh}(t) + a(t) \varepsilon_{sh}(t)].$$

Let us now assume, more generally, that only the constants τ_0 , τ_1 and τ_2 are equal, whereas $\tau_p = \tau_{sh}$ is of another value, Fig. 4. According to (13), the eq. (17) then takes the form

$$(18) \quad \ddot{\sigma}_{x_p} + \frac{E_p}{\tau_0 E_p} \dot{\sigma}_{x_p} = -\frac{E_p \varepsilon_{sh\infty}}{\tau_{sh}^2} \left[e^{-\left(\frac{1}{\tau_0} + \frac{1}{\tau_{sh}}\right)t} + \left(\frac{\tau_{sh}}{\tau_0} - 1\right) e^{-\frac{t}{\tau_{sh}}} \right]$$

This is a linear differential equation with constants coefficients; The initial conditions are $\sigma_{x_p}(0) = 0$ and $\dot{\sigma}_{x_p}(0) = 0$ and the solution is as follows:

$$(19) \quad \sigma_{x_p}(t) = \sigma_{x_p}(\infty) - A e^{-\frac{E_p t}{E_{p\infty} \tau_0}} - B e^{-\frac{t}{\tau_{sh}}} - C e^{-\left(\frac{1}{\tau_0} + \frac{1}{\tau_{sh}}\right)t}$$

where

$$(20) \quad C = -\frac{E_{p\infty} \varepsilon_{sh}}{\left(\frac{\tau_{sh}}{\tau_0} + 1\right) \left[\frac{\tau_{sh}}{\tau_0} - \frac{E_{p\infty}}{E_p} \left(\frac{\tau_{sh}}{\tau_0} + 1\right) \right]}$$

$$B = \frac{E_{p\infty} \varepsilon_{sh\infty} \left(-\frac{\tau_{sh}}{\tau_0} + 1\right)}{\frac{\tau_{sh}}{\tau_0} - \frac{E_{p\infty}}{E_p}}, \quad A = -\frac{E_{p\infty}}{E_p} \left[B \frac{\tau_0}{\tau_{sh}} + C \left(1 + \frac{\tau_0}{\tau_{sh}}\right) \right]$$

From the condition $\sigma_{x_p}(0) = 0$ it follows further $\sigma_{x_p}(\infty) = A + B + C$ or, after modification,

$$(21) \quad \sigma_{x_p}(\infty) = -\frac{E_{p\infty} \varepsilon_{sh\infty}}{1 + \frac{\tau_0}{\tau_{sh}}}$$

If we should assume $\tau_0 \doteq \tau_{sh}$ as for the eqs. (15) and (16), we should receive from (21) the internal stress $\sigma_{x_p}(\infty) = \frac{1}{2} \cdot 6\,600 \cdot 1,6 \% = 53 \text{ kp/cm}^2$.

It is seen that the result of this approximate analysis eq (17) – (21) is practically the same as from the more exact equation (8), so that this approximate analysis is well acceptable. The corresponding internal stress in glass fibres is then calculated from the equilibrium equation (6) and the shrinkage of laminate is

$$(22) \quad \varepsilon_x(t) = \frac{\sigma_{x_g}(t)}{E_g} = -\frac{f_p \sigma_{x_p}(t)}{f_g E_g}$$

5. Discussion of shrinkage effects

From the assumption $\tau_0 = \tau_{sh}$ we came to an incorrectly great value of internal stress. Therefore, we will now invert our question and ask what must be the value of τ_0 and τ_{sh} in order that the value of internal stress would agree with an initially mentioned experiment. For this purpose we will use only the approximate formula (21). In order that $\sigma_{x_p}(\infty)$ would have the value of about 1 kp/cm^2 , it is necessary, as it follows from (21), that there must hold

$$(23) \quad \tau_0 \doteq 30 \tau_{sh}.$$

Thus an interesting result is received – the mechanical parameters of the resin, such as elasticity module, viscosity constant, etc. increase during the polymerization process much more slowly or much more later than the increase of shrinkage or, intuitively spoken, a substantial part of shrinkage of a resin is effected still in the initial period of polymerization when the resin is still rather „liquid“, so that no large stresses can be produced, and that in the later period,

when the resin becomes rigid, the increase of shrinkage is already rather small, being effected sooner.

If the final values of shrinkage corresponding to the internal stress $\sigma_{x_p}(\infty) = 1 \text{ kp/cm}^2$ is determined from the formula (21), there would be obtained for $E_g = 700\,000 \text{ kp/cm}^2$ that $\varepsilon_{sh}(\infty) = -0,001\%$. The actually observed values are, however, about $-0,05\%$ and, in order that this value would be obtained, the actual effective module E_g of reinforcement must possess the value about $15\,000 \text{ kp/cm}^2$.

This disagreement is, however, easily explainable if we are aware that, with respect to the foregoing result, the relatively more liquid than rigid resin in the primary period of great shrinkage is not sufficiently rigid to prevent the thin glass fibres to buckle transversely at increase of compressive stress caused by shrinkage in them, Fig. 5. This phenomenon is also supported by the fact that

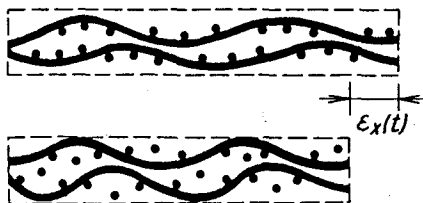


Fig. 5. Waving and buckling of glass fibres during polymerization.

the glass fibres in the reinforcing fabric are not straight, but have an initial wave form. Therefore the effective module \bar{E}_g of glass fabric during the polymerization shrinkage can have the obtained value $\bar{E}_g = 15\,000 \text{ kp/cm}^2$ which is 50-times smaller than the module of straight fibres. Partially, this reduction of the effective module of glass reinforcement is also caused by the influence of transverse fibres which have been disregarded in the previous analysis.

On the other hand, the fact, that the effective module of the fibre glass fabric during polymerization must be of much smaller value than $700\,000 \text{ kp/cm}^2$, indicates, according to eq. (22), that the internal stress in resin must be of a value much smaller than 51 kp/cm^2 in order to agree with the value $-0,05\%$ of shrinkage, even if the considerations resulting from primarily mentioned experiment have not been taken into account.

6. Thermal stresses

In contradistinction to the effect of shrinkage, the thermal deformations of reinforced plastics with a hardened resin are of greater importance. The coefficient of thermal deformation for glass is $\alpha_g = 6 \cdot 10^{-6}/^\circ\text{C}$. For polyester resin there was found [4] $\alpha_p = 50 \cdot 10^{-6}/^\circ\text{C}$ at the temperature -20°C and $\alpha_p = 120 \cdot 10^{-6}/^\circ\text{C}$ at 40°C , between which a linear interpolation is possible. The module of polyester has a constant value of $33\,000 \text{ kp/cm}^2$ below 20°C and beyond it decreases linearly up to 80°C when it has the value of about 4000 kp/cm^2 [4].

In order to calculate the internal stresses, let us first consider, as before, a laminate which is reinforced only in one direction x by perfectly straight and parallel fibres. Let this laminate at the instant $t = 0$ be subjected to a stepwise sudden change ΔT of temperature T , Fig. 6. Obviously there holds

$$(24) \quad \sigma_{x_g} = E_g(\varepsilon_x - \alpha_g \Delta T), \quad \alpha_{x_p} = \bar{E}_p(\varepsilon_x(0) - \bar{\alpha}_p \Delta T)$$

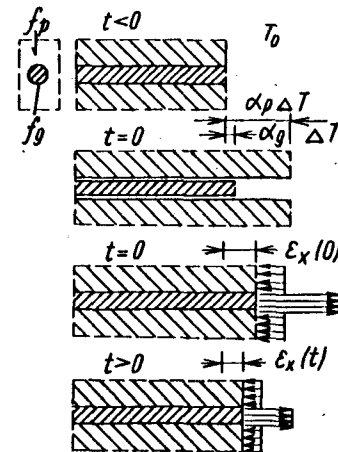


Fig. 6. Scheme for the analysis of the effect of temperature change.

where \bar{E}_p and $\bar{\alpha}_p$ represent the average module of resin and the average coefficient of the thermal deformation at the temperature change ΔT . Since \bar{E}_p and $\bar{\alpha}_p$ vary linearly in ΔT , the values \bar{E}_p and $\bar{\alpha}_p$ apply for the temperature $T_0 + \frac{1}{2}\Delta T$.

The expression (24) for σ_{x_g} is valid at an arbitrary time $t \geq 0$, the expression for $\sigma_{x_p}(0)$ applies only at the instant $t = 0$, as the later values of $\sigma_{x_p}(t)$ are affected by creep of resin. Substituting the relations (24) into the equilibrium condition (6) there follows

$$(25) \quad \frac{\varepsilon_x(0)}{\Delta T} = \frac{\bar{\alpha}_p \bar{E}_p f_p + \alpha_g E_g f_g}{\bar{E}_p f_p + E_g f_g} = \bar{\alpha}$$

where $\bar{\alpha}$ represents the average short-time coefficient of thermal deformation of the laminate for a given sudden change of temperature ΔT . By elimination of $\varepsilon_x(0)$ and $\sigma_{x_p}(0)$ from (24) and (6) the initial value of internal stress $\sigma_{x_p}(0)$ in a resin is obtained as follows:

$$(26) \quad \sigma_{x_p}(0) = \frac{\bar{E}_p(\alpha_g - \bar{\alpha}_p)\Delta T}{1 + \bar{E}_p f_p / E_g f_g}$$

when the resin becomes rigid, the increase of shrinkage is already rather small, being effected sooner.

If the final values of shrinkage corresponding to the internal stress $\sigma_{x_p}(\infty) = 1 \text{ kp/cm}^2$ is determined from the formula (21), there would be obtained for $E_g = 700\,000 \text{ kp/cm}^2$ that $\epsilon_{sh}(\infty) = -0,001 \%$. The actually observed values are, however, about $-0,05 \%$ and, in order that this value would be obtained, the actual effective module E_g of reinforcement must possess the value about $15\,000 \text{ kp/cm}^2$.

This disagreement is, however, easily explainable if we are aware that, with respect to the foregoing result, the relatively more liquid that rigid resin in the primary period of great shrinkage is not sufficiently rigid to prevent the thin glass fibres to buckle transversely at increase of compressive stress caused by shrinkage in them, Fig. 5. This phenomenon is also supported by the fact that

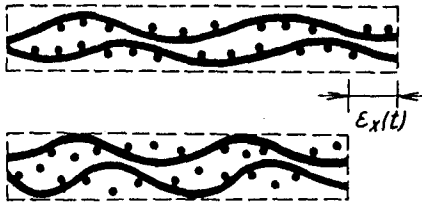


Fig. 5. Waving and buckling of glass fibres during polymerization.

the glass fibres in the reinforcing fabric are not straight, but have an initial wave form. Therefore the effective module \bar{E}_g of glass fabric during the polymerization shrinkage can have the obtained value $\bar{E}_g = 15\,000 \text{ kp/cm}^2$ which is 50-times smaller than the module of straight fibres. Partially, this reduction of the effective module of glass reinforcement is also caused by the influence of transverse fibres which have been disregarded in the previous analysis.

On the other hand, the fact, that the effective module of the fibre glass fabric during polymerization must be of much smaller value than $700\,000 \text{ kp/cm}^2$, indicates, according to eq. (22), that the internal stress in resin must be of a value much smaller than 51 kp/cm^2 in order to agree with the value $-0,05 \%$ of shrinkage, even if the considerations resulting from primarily mentioned experiment have not been taken into account.

6. Thermal stresses

In contradistinction to the effect of shrinkage, the thermal deformations of reinforced plastics with a hardened resin are of greater importance. The coefficient of thermal deformation for glass is $\alpha_g = 6 \cdot 10^{-6}/^\circ\text{C}$. For polyester resin there was found [4] $\alpha_p = 50 \cdot 10^{-6}/^\circ\text{C}$ at the temperature -20°C and $\alpha_p = 120 \cdot 10^{-6}/^\circ\text{C}$ at 40°C , between which a linear interpolation is possible. The module of polyester has a constant value of $33\,000 \text{ kp/cm}^2$ below 20°C and beyond it decreases linearly up to 80°C when it has the value of about 4000 kp/cm^2 [4].

In order to calculate the internal stresses, let us first consider, as before, a laminate which is reinforced only in one direction x by perfectly straight and parallel fibres. Let this laminate at the instant $t = 0$ be subjected to a stepwise sudden change ΔT of temperature T , Fig. 6. Obviously there holds

$$(24) \quad \sigma_{x_p} = E_g(\epsilon_x - \alpha_g \Delta T), \quad \alpha_{x_p} = \bar{E}_p(\epsilon_x(0) - \bar{\alpha}_p \Delta T)$$

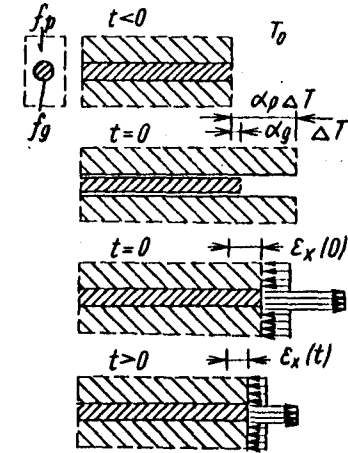


Fig. 6. Scheme for the analysis of the effect of temperature change.

where \bar{E}_p and $\bar{\alpha}_p$ represent the average module of resin and the average coefficient of the thermal deformation at the temperature change ΔT . Since \bar{E}_p and $\bar{\alpha}_p$ vary linearly in ΔT , the values \bar{E}_p and $\bar{\alpha}_p$ apply for the temperature $T_0 + \frac{1}{2}\Delta T$.

The expression (24) for σ_{x_p} is valid at an arbitrary time $t \geq 0$, the expression for $\sigma_{x_p}(0)$ applies only at the instant $t = 0$, as the later values of $\sigma_{x_p}(t)$ are affected by creep of resin. Substituting the relations (24) into the equilibrium condition (6) there follows

$$(25) \quad \frac{\epsilon_x(0)}{\Delta T} = \frac{\bar{\alpha}_p \bar{E}_p f_p + \alpha_g E_g f_g}{\bar{E}_p f_p + E_g f_g} = \bar{\alpha}$$

where $\bar{\alpha}$ represents the average short-time coefficient of thermal deformation of the laminate for a given sudden change of temperature ΔT . By elimination of $\epsilon_x(0)$ and $\sigma_{x_p}(0)$ from (24) and (6) the initial value of internal stress $\sigma_{x_p}(0)$ in a resin is obtained as follows:

(26)

$$\sigma_{x_p}(0) = \frac{\bar{E}_p(\alpha_g - \bar{\alpha}_p)\Delta T}{1 + \bar{E}_p f_p / E_g f_g}$$

The creep law of a hardened resin may be interpreted by a Boltzmann's model, Fig. 1, the parameters of which depend only on the temperature [6]. Assuming a stepwise sudden change ΔT of temperature there will therefore suffice to use the relations for a time invariant model, i. e. with time-invariable mechanical parameters. If the time course of creep strain of hardened resin at constant unity stress applied in time τ is introduced by an exponential:

$$(27) \quad \left(\frac{1}{E_{p\infty}} - \frac{1}{E_p} \right) \left(1 - e^{-\frac{t-\tau}{\tau_p}} \right)$$

in which $E_{p\infty}$ is the module of final deformation at constant stress and τ_p the retardation time, the parameters of model are $\eta = \tau_p E_p$, $1/E_V = 1/E_{p\infty} - 1/E_p$. The values of E_p , $E_{p\infty}$, τ_p depend on the temperature T . The differential equation for creep at variable stress and strain according to the principle of superposition then is

$$(28) \quad \varepsilon_x = E_p \sigma_x, \quad E_p = \frac{1 + \tau_p \frac{\partial}{\partial t}}{\frac{1}{E_{p\infty}} + \frac{\tau_p}{E_p} \frac{\partial}{\partial t}}$$

The differential equation for internal stresses at presence of creep may be most simply derived by use of the elastic-visco-elastic analogy from the equation for elastic deformations. If E_p and $\sigma_{x_p}(0)$ in the eq. (26) is interchanged with E_p (eq. (28)) and σ_{x_p} , which is possible, since the derivative in the numerator of E_p applies for stresses and in the nominator for deformations (while in the eq. (25) this replacement would not be possible), we arrive, after modifications and substitution of $\sigma_{x_p}(0)$ from (26), at the following differential equation of first order:

$$(29) \quad \tau_p \frac{\partial \sigma_{x_p}(t)}{\partial t} + \frac{1 + E_g f_g / E_{p\infty} f_p}{1 + E_g f_g / E_p f_p} \sigma_{x_p}(t) = \sigma_{x_p}(0).$$

In contradistinction to (25) and (26), the module E_p , as well as $E_{p\infty}$ and τ_p apply here for the steady temperature $T_0 + \Delta T$ after the initial change.

There is obvious that for $t \rightarrow \infty$ a steady state takes place, i. e. $\lim_{t \rightarrow \infty} \partial \sigma_{x_p} / \partial t = 0$. Limiting the eq. (26), we find an expression for the final value of internal stress

$$(30) \quad \sigma_{x_p}(\infty) = \frac{1 + E_g f_g / E_p f_p}{1 + E_g f_g / E_{p\infty} f_p} \sigma_{x_p}(0).$$

The final stress $\sigma_{x_p}(\infty)$ in glass fibres may be determined from (6) and the final strain $\varepsilon_x(\infty)$ and the corresponding coefficient $\bar{\alpha}_\infty$ from the first equation (24).

7. Discussion of thermal effects

For the purpose of numerical discussion let us consider a laminate with 25 % of weight of glass fibres in the direction x , at which $f_p/f_g = 7$. For $\alpha_p =$

$= 98 \cdot 10^{-6}$ at $T = 20^\circ \text{C}$ and $E_p = 35\,000 \text{ kp/cm}^2$, we determine from (25) $n = 3,03$ and $\bar{\alpha} = 29 \cdot 10^{-6} \text{ }^\circ\text{C}$. For $\alpha_p = 120 \cdot 10^{-6} \text{ }^\circ\text{C}$ at 40°C and $E_p = 25\,000 \text{ kp/cm}^2$ we ascertain $\bar{\alpha} = 29 \cdot 10^{-6} \text{ }^\circ\text{C}$. For a laminate reinforced by a fibre glass fabric with 25 % of weight of glass in each direction, there has been observed at experiments $\bar{\alpha} = 12 \cdot 10^{-6} \text{ }^\circ\text{C}$, constantly from -20°C to 40°C [4]. The difference of the computed and observed values is, however, not surprising if we are aware that the transverse fibres of a fabric reduce essentially the thermal deformation of a resin and enlarge the effective module E_p . Assuming at 20°C zero internal stresses in a laminate, then the average stresses in fibres and in a resin, corresponding to the observed value $\bar{\alpha} = 12 \cdot 10^{-6} \text{ }^\circ\text{C}$ at the temperature -10°C according to (24) and (6) are $\sigma_{x_g} = 7 \cdot 10^5 (12 - 6) \cdot 10^{-6} (-10 - 20) = -126 \text{ kp/cm}^2$ and $\sigma_{x_p} = 126/7 = 18 \text{ kp/cm}^2$. If the temperature is increased from 20°C to 50°C , we determine the same values with an opposite sign which is not surprising because the module E_p reduces while α_p raises at an increase of temperature beyond 20°C . Assuming $E_p = 33\,000 \text{ kp/cm}^2$, $E_{p\infty} = \frac{1}{2} E_p$ these initial values are reduced by creep at the low temperature according to (30) approximately to $0,676 \sigma_{x_p}(0)$, at the high temperatures still more. We can thus conclude that the average internal thermal stress in a resin of fibre glass plastics is about 10 kp/cm^2 . The maximum local stress concentration is, however, still higher. Just at the surface of transversal fibres, the internal stress in resin at a change of temperature from 10°C to -10°C (average $\bar{\alpha}_p = 76 \cdot 10^{-6} \text{ }^\circ\text{C}$) can reach the value $(76 - 6) \cdot 10^{-6} \cdot 33\,000 = 70 \text{ kp/cm}^2$.

8. Conclusions

1. The theoretical viscoelastic analysis based on timevariant Boltzmann's model for creep of resin has shown that the internal stresses in fibre glass reinforced plastics do not reach considerable values, as it has been often supposed, but are rather small and agree with the value about 1 kp/cm^2 obtained by discussion of certain experiments. Therefore, in the analysis of strength and of life time, these stresses need not to be accounted for.

2. The rate of increase of the module of elasticity, viscosity constant etc. of a resin during the polymerization process is many times smaller than the rate of shrinkage. The shrinkage of a resin is affected from the most part already in the initial period of polymerization when the resin is still more liquid than rigid. This is an intuitive explanation of a small value of internal stresses caused by shrinkage.

3. The glass fibres subjected at shrinkage to compressive stress buckle considerably in transverse direction, the form of their waves becomes higher, particularly in the initial period of polymerization.

4. Internal stresses produced by thermal deformations are of greater importance. Assuming a change of temperature 30°C they have the average values about 10 kp/cm^2 in a resin, the local stress concentrations reaching about 70 kp/cm^2 . They are reduced with time by creep.

References

- [1] MC Garry, F. J.: ASTM Bulletin, 1959, No. 235, p. 63-68.
- [2] MC Garry, F. J., Chambers, R. E.: 14th Technical Conf. SPI, 1959, Section 12-B.
- [3] Skupin, L.: Resin Concretes (in Czech); Research Report No. 42/64, Building Research Institute TU Prague, 1964; see also: Rheological properties of resin concretes and their binder (in Czech), to be published in Stavivo, Prague.
- [4] Skupin, L.: Glass reinforced plastics, II (in Czech), Research report No. 15/63, Building Research Institute TU Prague, 1963.
- [5] Bažant Z. P.: Electric analogues for creep of concrete structures, Symp. Experimental Methods of investigating stress and strain in structures, Prague, Oct. 1965; see also Creep of concrete in structural analysis (in Czech), Prague, State Publ. Techn. lit., 1966.
- [6] Bažant Z. P.: Ein neues Modell zur Beschreibung der rheologischen Eigenschaften von glasfaserverstärkten Plasten, I. Int. Tagung über GFK und Epoxydharze, Institut für Kunststoffe DAW, Berlin, March 1965, paper No. E-3; see also: Plaste und Kautschuk, Leipzig 1965, No 10-11 and Research Report No 50-64 of the Building Research Institute TU Prague, 1964.

SUMMARY

In the present paper a theoretical viscoelastic analysis of the internal stresses in reinforced plastics, especially in fibreglass plastics and also resin concretes, caused by the shrinkage at polymerization, as well as by thermal deformations, has been done. For the stress-strain relation at creep of resin during polymerization a time variant Boltzmann's (standard) model, the elements of which have time variable spring constant or viscosity constant, has been used. Formulae for calculating the internal stresses and shrinkage of reinforced plastics have been derived.

This analysis, as well as certain result of previous experiments with resin concretes, have shown that the internal stresses in resin are rather small, about 1 kp/cm², and need therefore not to be taken into consideration in the calculation of strength and life time. Simultaneously it was found that the rate of increase of elasticity module, of viscosity constant etc. of a resin during polymerization is many times smaller than the rate of shrinkage, i. e. the increase of mechanical parameters is effected from the most part later than shrinkage. In consequence of compressive stress the glass fibres buckle considerably in the transverse direction, especially in the initial period of shrinkage.

In contradistinction to this, the thermal stresses are of greater values, about 10 kp/cm² for a change of temperature 30 °C, with local stress concentrations up to 70 kp/cm². Formulae for their effect with respect to creep are presented.

VNITŘNÍ PNUTÍ VE VYZTUŽENÝCH PLASTICKÝCH HMOTÁCH, ZPŮSOBENÉ SMRŠŤOVÁNÍM PRYSKYŘICE A TEPelnými DEFORMACEMI

(Résumé)

Ing. ZDENĚK P. BAŽANT, CSc.
Stavební ústav ČVUT v Praze

V článku je provedeno teoretické viskoelastické řešení vnitřních pnutí ve vyztužených plastických hmotách, zvláště ve skelných laminátech a též plastbetonech, jež jsou způsobena polymerizačním smršťováním a tepelnými deformacemi. Pro vztah napětí a deformace při dotvarování pryskyřice během polymerace je uvažován Boltzmannův (normální) model, jehož prvky mají časově proměnné proužkové konstanty a konstanty viskozity. Jsou odvozeny vzorce pro výpočet vnitřních pnutí a smrštění plastických hmot.

Tento rozbor, stejně jako výsledky jistých starších pokusů s plastbetony, ukázal, že vnitřní pnutí v pryskyřici jsou značně malá, asi 1 kp/cm², a nemusí být proto uvažována při určení pevnosti a životní doby. Zároveň bylo zjištěno, že rychlost vzrůstu modulu pružnosti, konstanty viskozity atd. u pryskyřice během polymerace je mnohonásobně menší než rychlost smršťování, tj. vzrůst

mechanických parametrů se děje později než smršťování. V důsledku tlakových napětí vlákna vybočují v přísném směru, zvláště v počátečním stadiu smršťování.

Na rozdíl od toho tepelná napětí mají značně větší hodnoty, kolem 10 kp/cm² při změně teploty 30 °C, s lokálními koncentracemi napětí až do 70 kp/cm². Jsou odvozeny vzorce pro jejich vliv s ohledem na dotvarování.

- Obr. 1. Boltzmannův rheologický model pro dotvarování pryskyřice.
- Obr. 2. Křivky dotvarování pryskyřice při konstantním napětí, zavedené v různých okamžicích polymerizace.
- Obr. 3. Schéma pro výpočet účinku smrštění pryskyřice.
- Obr. 4. Průběh smrštění a vzrůst mechanických konstant pryskyřice během polymerizace.
- Obr. 5. Vyznění a vybočování skelných vláken během polymerizace.
- Obr. 6. Schéma pro výpočet účinků změny teploty.

ВНУТРЕННИЕ НАПРЯЖЕНИЯ В АРМИРОВАННЫХ ПЛАСТМАССАХ, ВЫЗЫВАЕМЫЕ УСАДКОЙ СМОЛЫ И ТЕМПЕРАТУРНЫМИ ДЕФОРМАЦИЯМИ

(Резюме)

ЭДЕНЕК П. БАЖАНТ*)

В статье приводится теоретическое вискоэластическое решение задачи внутренних напряжений в армированных пластмассах, в частности, в стеклянных слоистых пластинах, а также в пласт бетонах, вызываемых полимеризационной усадкой и температурными деформациями. Для выражения связи между напряжением и деформацией при усадке смолы в течение полимеризации применяется (нормальная) модель Больцмана, элементы которой имеют переменные во времени постоянные упругости и постоянные вязкости. Были выведены формулы для расчета внутренних напряжений и усадки пластмасс.

Настоящий анализ, равно как и результаты определенных более старых испытаний с пласт бетонами показал, что внутренние напряжения в смоле весьма малы примерно 1 кп/см² и поэтому при определении прочности и долговечности ими можно пренебречь. Одновременно было установлено, что скорость возрастания модуля упругости, константы вязкости и т. д. для смолы в течение полимеризации во много раз меньше скорости усадки, т. е. что возрастание механических параметров осуществляется позже усадки. В результате напряжений при сжатии они обнаруживают значительный боковой прогиб в поперечном направлении, в частности, в начальной стадии усадки.

В отличие от этого температурные напряжения характеризуются гораздо большими величинами примерно 10 кп/см² при изменении температуры 30 °C с местными концентрациями напряжения вплоть до 70 кп/см². Были выведены формулы для определения их влияния с учетом усадки.

Тексты и рисунки

- Рис. 1. Логическая модель Больцмана для усадки смолы.
- Рис. 2. Кривые усадки смолы при постоянном напряжении, прилагаемом в различные моменты полимеризации.
- Рис. 3. Схема расчета влияния усадки смолы.
- Рис. 4. Изменения усадки и возрастание механических констант смолы в течение полимеризации.
- Рис. 5. Волнистость и боковой прогиб стеклянных волокон в течение полимеризации.
- Рис. 6. Схема расчета влияния изменения температуры.

*) Инж. ZDENĚK P. BAŽANT канд. наук, Исследовательский строительный институт Политехнического института в Праге, (Stavební ústav ČVUT v Praze, Soltnova 7, Praha 6-Dejvice, СССР).