Optimization check of certain recent practical formulations for concrete creep

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Optimization algorithms that have recently become available in computer libraries revolutionize checking and identification of theoretical creep laws from test data. Much more detailed comparisons with creep test data are now feasible, and with little effort. Formulating the optimality condition in terms of a sum-of-squares objective function and expressing various positiveness constraints by quadratic substitutions, one can apply the Marquardt algorithm. In this manner, two recently proposed formulations are examined: (a) *the viscoelastic model with reduced time, and* (b) *the rate-of-flow method. It is shown that none of these formulations is capable of giving a satisfactory description of creep data which cover the full range of interest in creep durations and ages at loading, even though an acceptable agreement has previously been demonstrated for creep data of narrow time range. Previously it has been found by the same method that the recently proposed creep formulation for C.E.B. Recommendations suffers by the same limitations. That formulation and the two formulations examined herein share the underlying concept of separating the total creep strain in reversible creep and irreversible creep. The results demonstrate that this (theoretically unfounded) concept is contradicted by creep data of not too limited time range.*

OBJECTIVE

Powerful optimization techniques for nonlinear sum-of-squares problems have recently become available as standard subroutines in computer libraries. This has opened new possibilities that revolutionize checking and identification of various theoretical constitutive relations. Finding the optimum fit of given long-range creep data by a certain formulation proposed for use in structural analysis used to be a very tedious task and was, therefore, usually shunned and never practiced in full scope. However, with optimization subroutines this task has become relatively easy.

In a recent paper [1], optimization has been used to check the creep function of the form proposed for new C.E.B. Recommendations [13] and serving as basis for the "improved Dischinger method." It was found that this function is unable to represent adequately the experimental creep curves which cover the full range of interest in ages at loading and creep durations, even though previously a satisfactory agreement has been demonstrated for test data plotted in actual time scales for which only one order of magnitude of creep durations and one order of magnitude of ages at loading can be shown. Based on this result it was concluded that the proposed C.E.B. creep function should not be adopted by C.E.B. [1].

The aim of this study is to subject to a similar scrutiny two other recently proposed creep functions, namely, the viscoelastic formulation with reduced time from references [7] and [10] and the formulation of the rate-of-flow method from references [5] and [6]. It will be seen that their limitations are similar to those of the proposed C.E.B. creep function.

It is generally accepted that within the range of service loads the creep of concrete can be approximately treated according to the principle superposition in time. Then, the uniaxial creep law is completely characterized by the creep compliance function, $J(t, t')$, which represents the strain, ε, (creep strain plus instantaneous strain) at time t caused by unit stress acting since time t' [2]. The time is conveniently measured since the instant of set of concrete.

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Reprinted from Materials and Structures (RILEM, Paris), Vol. 9, 1976, pp. 91-96

VOL. 9 - N° 50 - MATÉRIAUX ET CONSTRUCTIONS

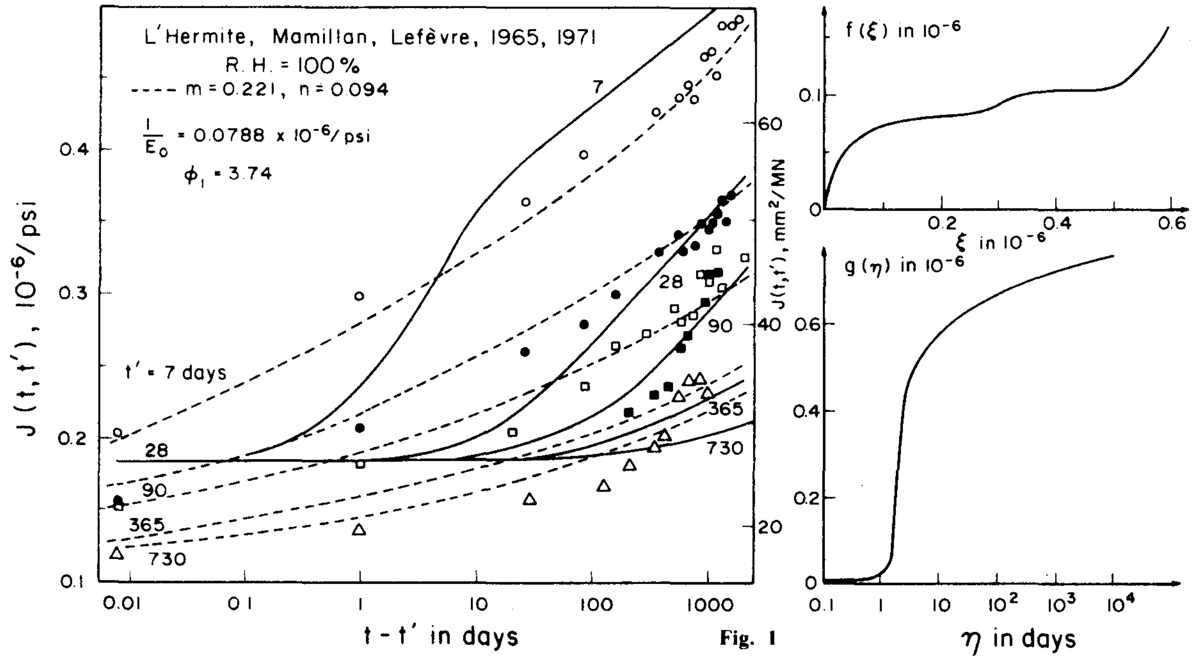

Fig. 1

Fig. 1-4. Comparisons of various creep data ([11], [12], [8], [9]) with the optimum fits by viscoelasticity with reduced time ([7], [10]). Equation (1) (solid lines). Also shown is the fit by double power law ([4], [1]) (dashed lines).

VISCOELASTIC FORMULATION WITH REDUCED TIME

For a general form of $J(t, t')$, as well as for various special forms which closely fit test data, the structural creep effects are difficult or even impossible to analyse exactly without the aid of an electronic computer, unless the approximate age-adjusted effective modulus method is used [3]. For this reason, a considerable effort has been devoted to finding such simplified forms of $J(t, t')$ which allow structural analysis to be done by hand calculations.

One such function is of the form ([7], [10]):

$$J(t, t') = \frac{1}{E_0} + f[g(t) - g(t')] + g(t) - g(t') \qquad (1)$$

in which E_0 is a constant representing a typical value of the elastic modulus, and f and g are positive non-decreasing continuous functions of one variable. The advantage of this formulation lies in the fact that by introducing reduced time $\tau^* = g(t)$, J depends only on the difference $\tau^* - \tau^{*\prime}$ of reduced times, with the result that Laplace transformation can be applied in time τ^*, similarly to classical viscoelasticity. It has been demonstrated that equation (1) agrees satisfactorily with certain limited test data, but a comparison with the available measurements of creep curves of long duration and of a broad range of ages at loading has not been made.

According to the method of least squares, the objective of optimum fit of creep data may be formulated as [1]:

$$\Phi = \int_t \int_{t'} [J(t, t') - \bar{J}(t, t')]^2 \, d\, ln(t - t') \, d\, ln\, t' = \text{Min.} \quad (2)$$

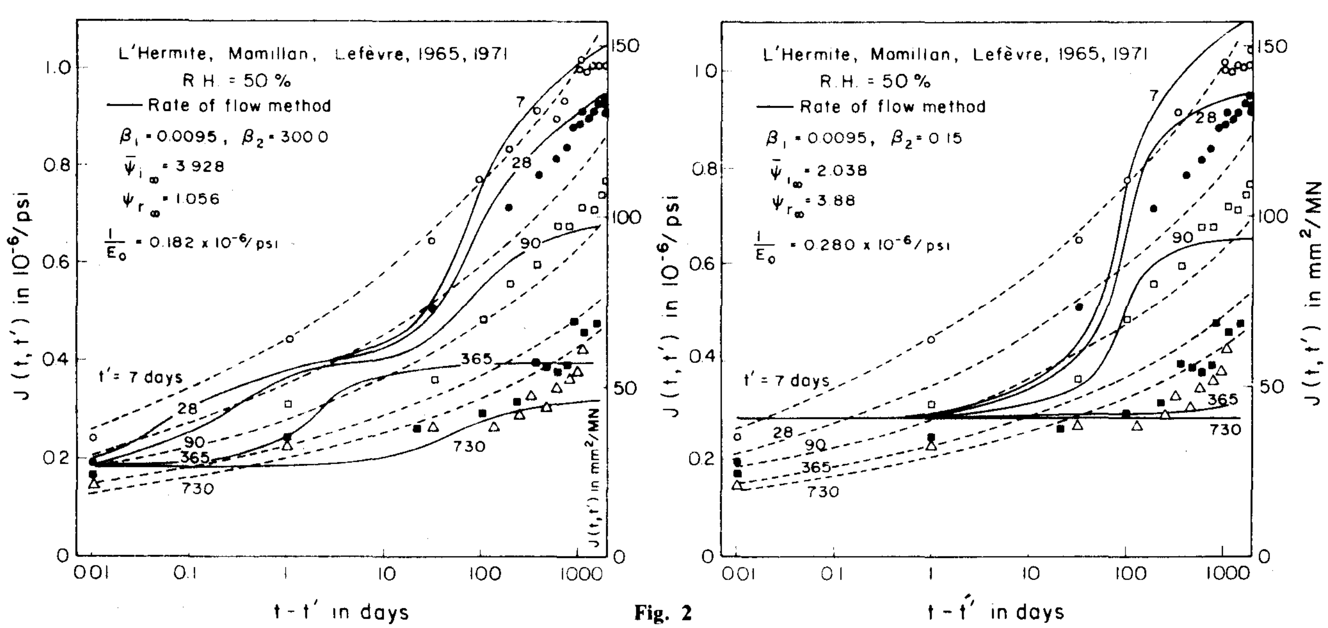

Fig. 2

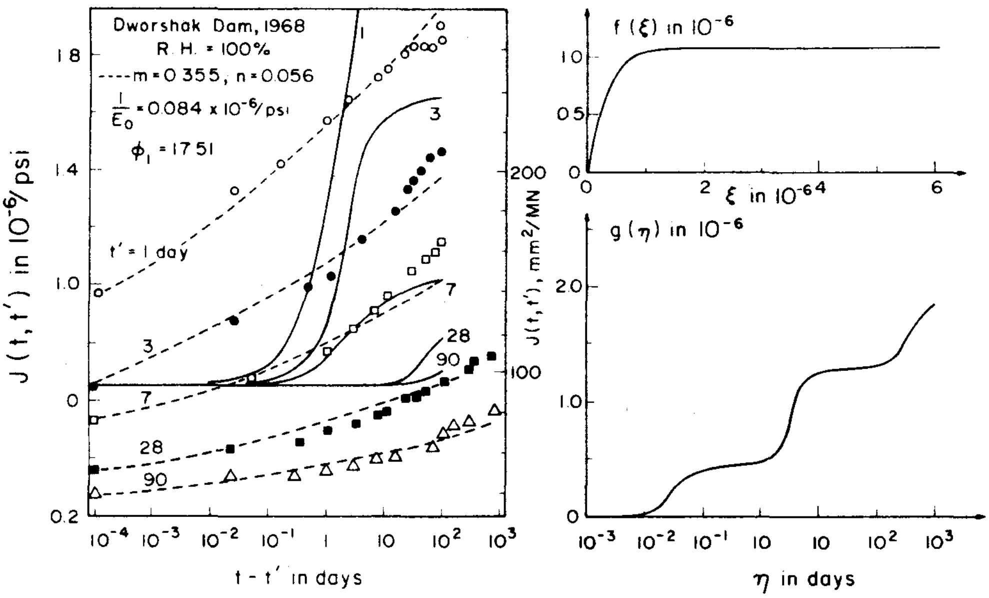

Fig. 3

in which $\bar{J}(t, t')$ is the measured creep function and $J(t, t')$ is the theoretical creep function. For numerical implementation, equation (2) may be approximated by the sum

$$\Phi = \sum_p w_p \sum_q [J(t_q, t'_p) - \bar{J}(t_q, t'_p)]^2 = \text{Min.,} \qquad (3)$$

where $t'_p (p=1, 2, \ldots, N_p)$ are the discrete values of the ages at load application; $t_q = t'_p + \bar{t}_q (q=1, 2, \ldots, N_q)$ in which $\bar{t}_q$ are the discrete values of the times elapsed from the instant of load application, t'_p; and w_p = chosen weights. When all values of both t'_p and t_q are almost uniformly spaced in $\log t'$- and $\log t$-scales, all weights w_q may be chosen as unity. For other cases, a rule for a suitable choice of w_p has been given in equation (4) of [1].

To treat functions f and g as unknowns in the optimization process, they must be characterized by a set of discrete values $f_i = f(\xi_i)$ $(i=1, 2, \ldots, m)$ and $g_j = g(t_j)$ $(j=1, 2, \ldots, n)$, where t_j are chosen discrete values spaced uniformly in the logarithmic scale of t (age of concrete), and ξ_i are chosen evenly spaced discrete values of the argument $\xi = g(t) - g(t')$ in equation (1). For arguments ξ and t lying between two adjacent discrete values, i. e., between t_j and t_{j+1} or between ξ_i and ξ_{i+1}, interpolation is used to determine functions g and f. Beyond the maximum discrete values t_n or ξ_m of t_j or ξ_i, linear extrapolation from the last interval (t_{n-1}, t_n) or (ξ_{m-1}, ξ_m) is used. However, for good accuracy discrete values t_j and ξ_i should preferably be chosen so as to cover the full range, so that no extrapolation be needed. The interpolations and extrapolations are best programmed as linear in $\log t$ (rather than t) and in ξ.

To ensure that f and g be positive-valued non-decreasing functions and that E_0 be positive, it is expedient to introduce the quadratic substitutions :

$$\left.\begin{aligned} f_i &= x_1^2 + x_2^2 + \ldots + x_i^2 = \sum_{k=1}^{i} x_k^2, \\ g_j &= x_{m+1}^2 + x_{m+2}^2 + \ldots + x_{m+j}^2 = \sum_{k=m+1}^{m+j} x_k^2, \\ 1/E_0 &= x_N^2 \qquad (N = m+n+1). \end{aligned}\right\} \qquad (4)$$

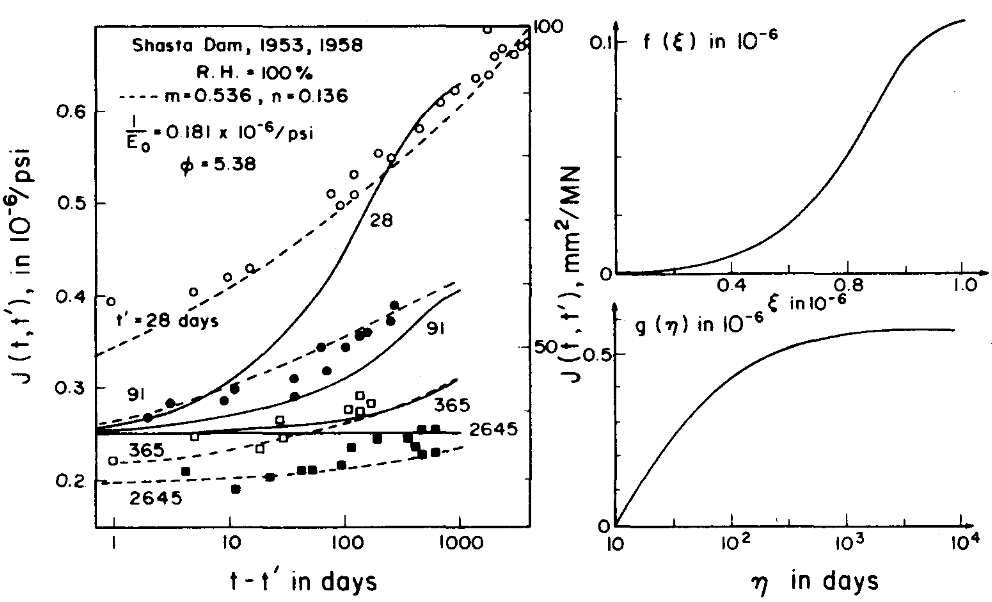

Fig. 4

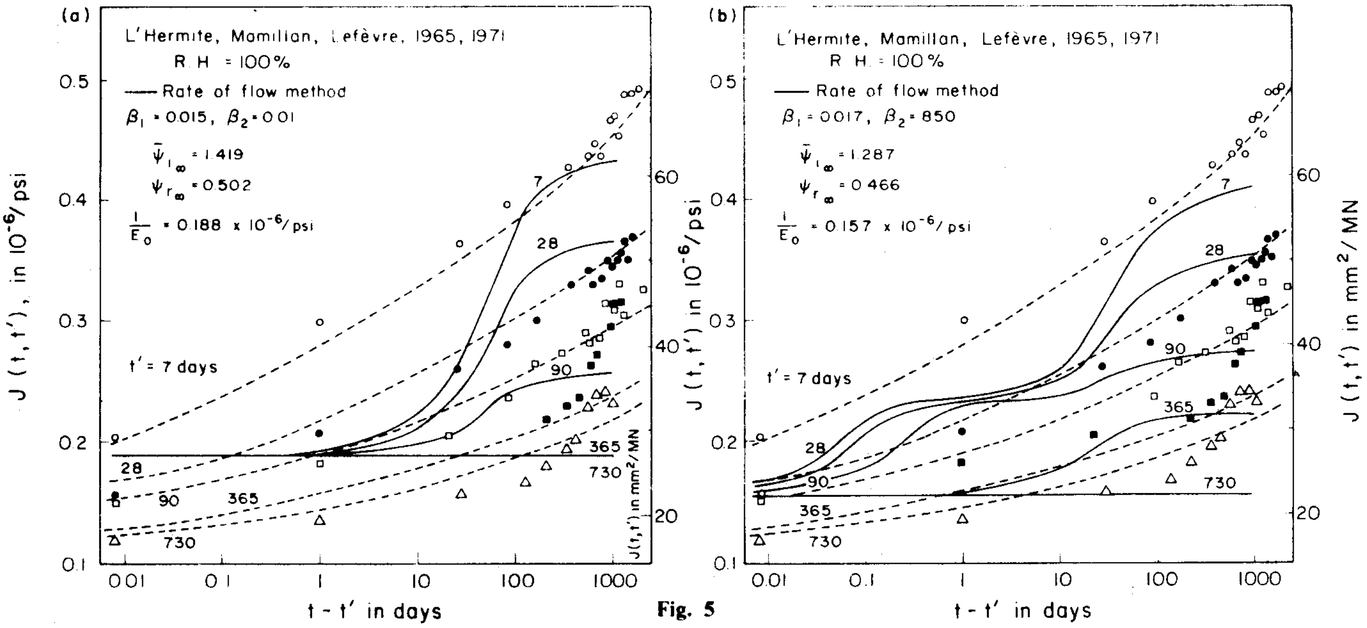

Fig. 5

Fig. 5-8. — Comparisons of various creep data ([11], [12], [8], [9]) with the optimum fits by rate-of-flow method ([5], [6]). Equations (7)-(9) (solid lines). Also shown is the fit by double power law ([4], [1]) (dashed lines).

For a given set of test data, expression Φ in equation (3) may be regarded as a function of unknowns x_1, $x_2, \ldots, x_N$ and the optimization problem may be stated as $\Phi(x_1, x_2, \ldots, x_N) = \text{Min}$. The solution of the optimization problem is facilitated by the fact that Φ represents a sum of squares, i. e.,

$$\Phi = \sum_{r=1}^{M} [F_r(x_1, x_2, \ldots, x_N)]^2 = \text{Min}. \tag{5}$$

in which

$$\left. \begin{aligned} F_r &= w_p [J(t_q, t'_p) - \bar{J}(t_q, t'_p)]; \\ r &= (p-1) N_q + q; \\ M &= N_q N_p. \end{aligned} \right\} \tag{6}$$

A very efficient method of solving the foregoing optimization problem is the Marquardt algorithm (for reference, *see* [1]). A standard library subroutine is available for this algorithm and the only programming to be done by the user is to write a subroutine for calculating functions F_r from any specified values of $x_1, \ldots, x_N$.

As compared with the optimization problem from reference [1], there is here one additional difficulty that must be properly tackled. It is the uncertainty about the range of the argument $\xi = g(t) - g(t')$ of function f in equation (1) (domain of definition). If the range from $\xi = 0$ to ξ_m (the last and largest ξ_i-value) is too large, i. e., much larger than the range of the ξ-values which the resulting optimum function $f(\xi)$ actually covers within the scope of given test data,

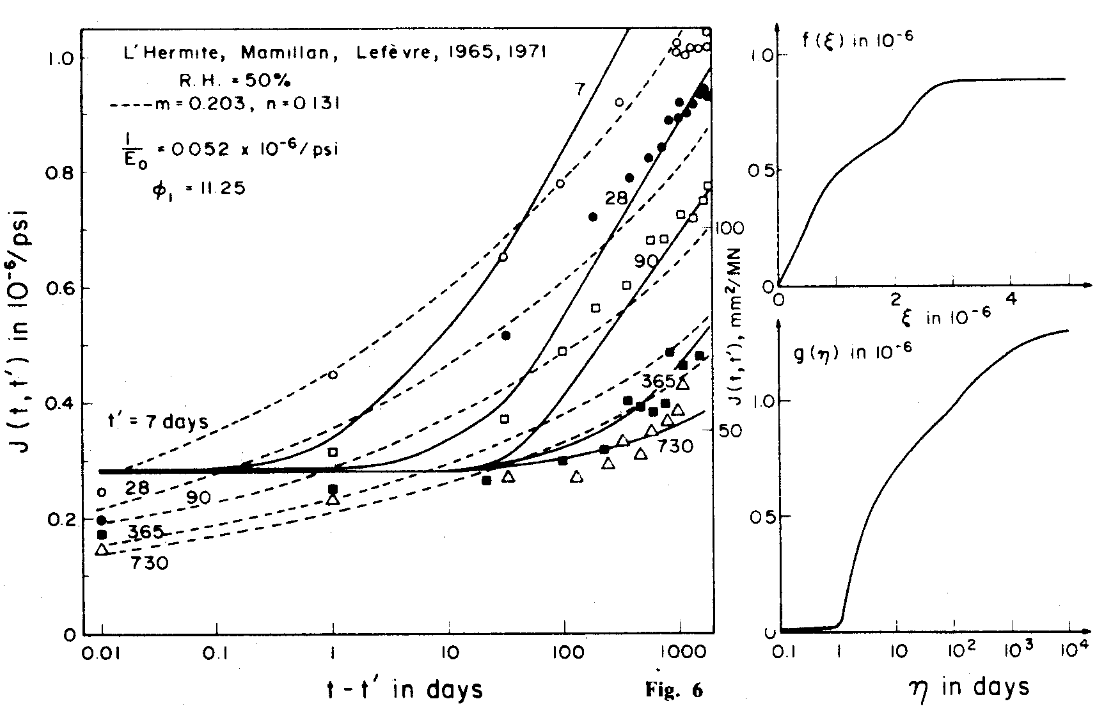

Fig. 6

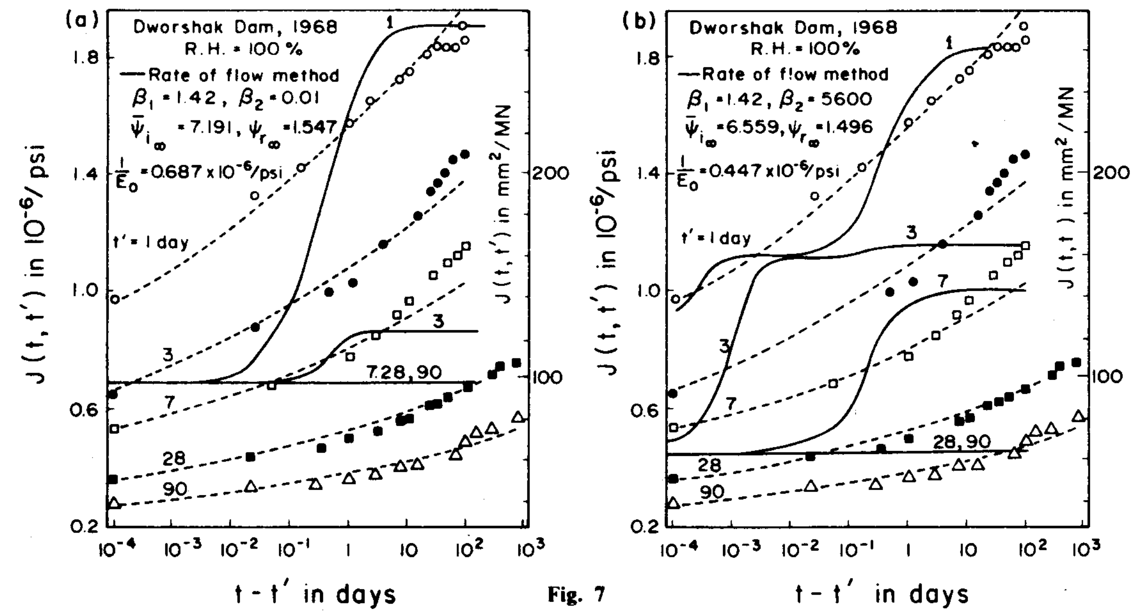

Fig. 7

then $f_m = f(\xi_m)$ has no effect on the value of Φ, and so the f_m-value is indeterminate and the optimization problem lacks unique solution. In such a case the optimization subroutine fails to converge and the analyst must exercice judgment in reducing the ξ_m-value along with the spacing of all ξ_i values. On the other hand, if the analyst goes to the other extreme, choosing the range too narrow (i. e., ξ_m too small), then the minimum values of f used in the time range of test data are based on extrapolation from the largest two discrete values, f_{m-1} and f_m. This is much less accurate than interpolation and the analyst must increase ξ_m along with the spacing of all ξ_i, so as to avoid extrapolation. A message signaling the use of extrapolation along with the value of $\xi - \xi_m$ should be incorporated into the program. Due to the complication just described, the optimization program must be run several times on a trial-and-error basis, until the proper choice of ξ_i is ascertained.

Furthermore, it is necessary to take into account the fact that addition of an arbitrary constant to function g has no effect on $J(t, t')$ because only the difference $g(t) - g(t')$ appears in equation (1). Thus, one of the discrete values g_i should be chosen. In fact, it must be chosen or else the optimization problem would not have a unique solution and the optimization algorithm would fail to converge. The chosen value may be the value of $g(t)$ at $t = 0$ and the choice $g(0) = 0$ is most natural. This value must be considered in writing the subroutine for functions F_r but may not be included among the unknown discrete values g_j; i. e., $\xi_1 > 0$, $g_1 > 0$.

A program for the foregoing problem is listed in the Appendix as an example for the reader who might wish to approach a similar problem by optimization. Comments within the program should provide sufficient information on its use.

The optimum fits obtained with equation (1) are shown in figures 1-4 by solid lines for the most extensive creep data available in the literature ([11], [12], [8], [9]).

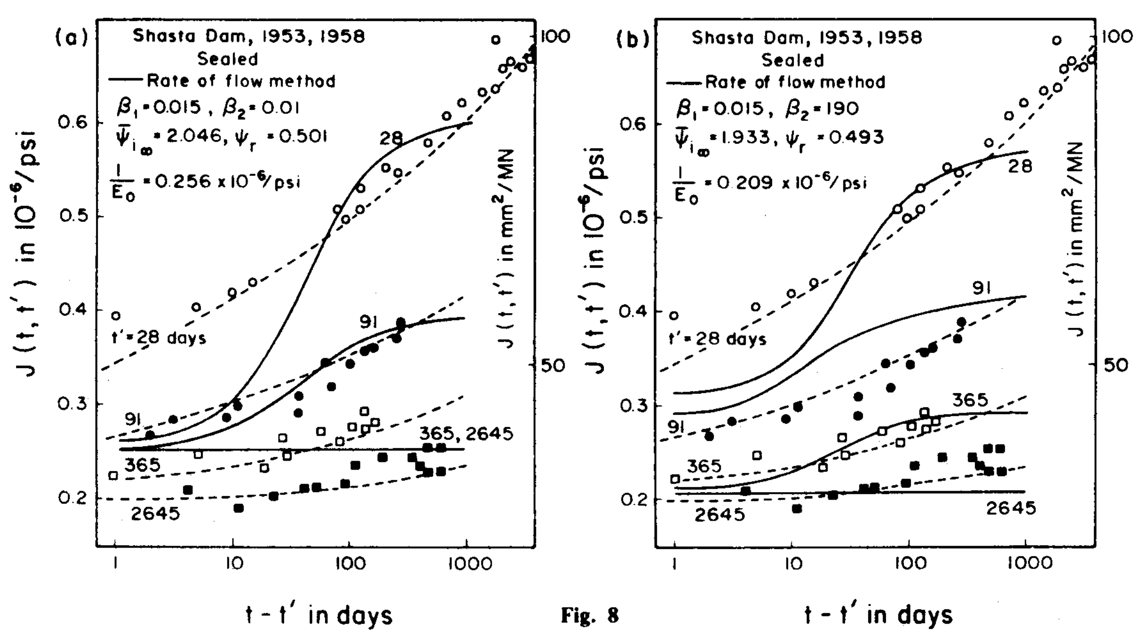

Fig. 8

It is seen that the agreement with test data is unsatisfactory, even though previously a satisfactory agreement has been demonstrated for test data of relatively narrow time range. For comparison, the dashed lines indicate the optimum fits by the double power law proposed in reference [4] (m, n, φ_1, E_0 as indicated are its parameters).

In conclusion, the applicability of the viscoelastic formulation with reduced time based on (1) is rather limited.

FORMULATION IN TERMS OF RATE-OF-FLOW METHOD

The rate-of-flow method has been considered in slightly varied forms since its incerption more than a decade ago. In its latest form ([5], [6]), it is defined by the creep function:

$$J(t, t') = \frac{1}{E_0}\left[1 + \psi_i(\bar{t}, t') + \psi_r(\bar{t}, t')\right], (\bar{t} = t - t') \quad (7)$$

in which

$$\psi_i(\bar{t}, t') = \psi_{i_*} e^{-\beta_1 t'}(1 - e^{-\beta_1 \bar{t}}), \quad (8)$$

$$\psi_r(\bar{t}, t') = \psi_{r_*}(1 - e^{-\beta_2 \Psi_i(\bar{t}, t')}), \quad (9)$$

where E_0, Ψ_{i_*}, Ψ_{r_*}, β_1 and β_2 are five constants. Functions $\Psi_i(\bar{t}, t')$ and $\Psi_r(\bar{t}, t')$ are intended to describe the irreversible component and the reversible component of creep. However, it should be kept in mind that, because of aging in concrete, a separation of the total creep strain into such components is not justified by thermodynamics and should be regarded strictly as an assumption made for convenience; *see* [1].

The objective of optimum fit of creep data has been again expressed in the form of equation (2), which was approximated [equation (5)] by a discrete sum of squares of functions F_r which are defined by equation (6) and depend on five variables $X_1 = 1/E_0$, $X_2 = \Psi_{i_*}/E_0$, $X_3 = \Psi_{r_*}/E_0$, $X_4 = \beta_1$, $X_5 = \beta_2$. Functions F_r have been programmed and made available to a library subroutine for Marquardt optimization algorithm. However, it appeared that this algorithm was not converging properly with respect to the unknowns β_1 and β_2. This is not too surprising because optimization for exponent coefficients in sums of exponentials (Dirichlet series) is generally a difficult problem which frequently leads to numerical instability or other troubles. In the present case, at a certain optimization stage the available optimization subroutine made too large a change in β_1 or β_2, such that $e^{\beta_1 \bar{t}}$ or $e^{\beta_2 \Psi_i}$ became essentially zero and the subsequent smaller changes in β_1 and β_2 did not have any appreciable effect on the value of these exponentials. A simple remedy was achieved, however, by considering only $X_1 = 1/E_0$, $X_2 = \Psi_{i_*}/E_0$ and $X_3 = \Psi_{r_*}/E_0$ as unknowns in the optimization algorithm and fixing the values of exponent coefficients β_1 and β_2 in advance. The optimization program was then run for various combinations of β_1 and β_2 chosen by trial and error, and the sum-of-squares function Φ was evaluated for each case. A plot of Φ versus β_1 and β_2 was then constructed, from which the overall optimum was determined.

The optimum fits obtained are shown by the solid lines in figures 5 (*a*), 6 (*a*), 7 (*a*), 8 (*b*) for the same test data ([11], [12], [8], [9]) as before. It is seen that the agreement is unsatisfactory. The steps in the shape of the curves are due to the fact that exponential curves plotted in log-scale have a significant slope only within a single decade and that the obtained values of β_1 and β_2 differ too much. It has also been tried to obtain optimum fits under the condition that steps in the creep curves may not occur. For this purpose it was necessary to restrict coefficient β_2 to sufficiently small values. The fits obtained under this condition are shown by the solid lines in figures 5 (*b*), 6 (*b*), 7 (*b*), 8 (*b*) and it is seen that the misfit is worse than for unrestricted β_2. For comparison, the optimum fits by the double power law from reference [4] are also indicated.

Consequently, the applicability of the rate-of-flow method is also rather limited.

CONCLUSIONS

1. Marquardt algorithm for nonlinear sum-of-squares optimization problems is an effective, simple and easy way of checking the applicability of various formulations for creep.

2. The applicability of both the viscoelastic formulation with reduced time and the rate-of-flow method is rather limited. Satisfactory description of test data is impossible when the data cover a broader range of creep durations and of ages at loading.

3. The disagreement with test data is of about the same nature and degree as that demonstrated previously [1] for the creep function proposed [13] for C.E.B. Recommendations and serving as basis for the "extended Dischinger method" [13].

REMARK ON SEPARABILITY OF REVERSIBLE CREEP

The present two formulations share with the formulation proposed for C.E.B. Recommendations the basic assumption that the total creep strain can be separated into reversible and irreversible components. Theoretically this is in general unfounded for aging materials, except for infinitesimal strain increments. The present results (along with those from reference [1]) demonstrate that this separation is contradicted by available creep data of not too limited time range.

ACKNOWLEDGMENT

Thanks are due to U.S. National Science Foundation for supporting this research under Grant GK-26030.

Appendix. Optimization program for creep function.

```
      PROGRAM OPTIM(INPUT,OUTPUT)
C OPTIMIZES THE FIT OF CREEP DATA BY CREEP FUNCTION BASED ON VISCO-
C ELASTICITY WITH TIME REDUCED FOR AGING.
C   INPUT**   NF,NG=NO. OF POINTS DEFINING FUNCTIONS F AND G.
C NT=NO. OF POINTS ON CREEP CURVE.  NCURVE=NO. OF CREEP CURVES
C OR NO. OF AGES AT LOADING.  AGE(I)=DISCRETE VALUES OF THE AGES
C AT LOAD APPLICATION.  TBAR(I)=DISCRETE VALUES OF THE TIMES
C ELAPSED FROM THE INSTANT OF LOAD APPLICATION.  DELG(I)=CHOSEN
C DISCRETE VALUES OF THE ARGUMENT OF FUNCTION F AT WHICH F IS
C CHARACTERIZED BY DISCRETE VALUES.  FF(I)=F(G(T)-G(TP)), WHERE
C T=TIME=CURRENT AGE.  TP=AGE AT LOAD APPLICATION AND AGE(I).
C SUBROUTINE ZXMARQ FOR MARQUARDT ALGORITHM FOR FINDING THE MINIMUM
C OF THE SUM OF SQUARES OF FUNCTIONS DIF MUST BE SUPPLIED.
C   OUTPUT**   SUMF,SUMG AND E01=OPTIMIZED DISCRETE VALUES OF
C FUNCTIONS F(G(T)-G(TP)), G(TP) AND OF 1/E0.  CREEP(I)=OPTIMIZED
C VALUES OF CREEP FUNCTION.  DIFF(I)=DIFFERENCES(DEVIATIONS) OF
C OPTIMUM FIT FROM CREEP DATA.
      DIMENSION X(17),DIFF(32),FF(8),GG(8),WA(221),A(8),RAGE(8)
      COMMON /A/AGE(8),AGEL(8),RAGEL(8),TBAR(8).CRDATA(32),DELG(8)
      COMMON /B/NF,NG,NU,NT,NG1,XMAXG,XMING
      DATA CRDATA/.35,.4125,.4875,.6125,   .25,.2875,.34,.408,
     1 .225,.23,.27,.33,    .1825,.19,.2125,.24/,  NT/4/,
     2 NF/4/,  NG/4/,  NCURVE/4/,  E01/.06/,  AGE/28.,91.,365.,2645./,
     3 TBAR/1.,10.,100.,1000./,  RAGE/10.,100.,1000.,10000./,
     4 FF/.0,.006,.05,.11/,  GG/.0.,4,.56,.56/,  DELG/.0,.3,.6,.9/
     5 ,ITMAX/40/,  EPS/0.001/,  NSIG/2/,  XMAXG/0.0/,  XMING/1.E30/
      EXTERNAL DIF
      PRINT 30
   30 FORMAT(18X 2HNF 8X 2HNG  8X 2HNT  4X 6HNCURVE /)
      PRINT 31,NF,NG,NT,NCURVE
   31 FORMAT(10X,4I10)
      DO 1 I=1,NCURVE
    1 AGEL(I)=ALOG10(AGE(I))
      DO 2 I=1,NG
    2 RAGEL(I)=ALOG10(RAGE(I))
      PRINT 33
   33 FORMAT(// 12X 22HASSUMED INITIAL VALUES /)
      PRINT 34,E01
   34 FORMAT(/ 12X 6H1/E0 = F9.5)
      PRINT 35
   35 FORMAT(// 12X 13HF(G(T)-G(TP)) /)
      PRINT 36,(FF(I),I=1,NF)
   36 FORMAT(10X,F15.5)
      PRINT 37
   37 FORMAT(// 20X 5HG(TP) /)
      PRINT 36,(GG(I),I=1,NG)
C COMPUTE SQUARE ROOTS OF INCREMENTS OF ASSUMED FUNCTIONS F AND G
C AND STORE THE RESULT IN X(I).
      CALL XINCR(NF,FF,X)
      CALL XINCR(NG,GG,A)
      DO 3 I=1,NG
      J=NG+I
    3 X(J)=A(I)
      NP=NT*NCURVE
      NU=NF+NG+1
      NG1=NF+1
C       NP=NO. OF ALL DATA POINTS IN WHICH SQUARE DEVIATION IS
C       EVALUATED.  NU=NO. OF UNKNOWN PARAMETERS.  NG1=FIRST
C       LOCATION OF FUNCTION G IN VECTOR X(I).
      X(NU)=SQRT(E01)
C OPTIMIZATION BY MARQUARDT ALGORITHM.  ITMAX=THE MAXIMUM ALLOWABLE
C NO. OF ITERATIONS PER ROOT.   EPS=1ST STOPPING CRITERION - A MINIMUM
C VALUE IS ACCEPTED IF ABS(DIF) .LE. EPS.  NSIG=2ND STOPPING CRITERION
C - A MINIMUM VALUE IS ACCEPTED IF TWO SUCCESSIVE APPROXIMATES TO A
C GIVEN VALUE AGREE IN THE FIRST NSIG DIGITS.   WA(NU)=A VECTOR WORK
C AREA OF LENGTH NU(NU+1)/2+4*NU, WHERE NU=NO. OF ELEMENTS OF VECTOR
C X(I).  IER=ERROR PARAMETER(OUTPUT).  MARQUARDT ALGORITHM
C AUTOMATICALLY CALLS FUNCTION DIF. X(I)=OUTPUT=OPTIMIZED VALUES.
C DIFF(I)=ASSOCIATED DEVIATIONS FROM CREEP DATA. ITMAX=NO. OF
C ITERATIONS USED.
      CALL ZXMARQ(DIF,EPS,NSIG,NP,NU,1.0,X,DIFF,ITMAX,WA,IER)
      IF (XMAXG.LE.DELG(NF-1)) PRINT 38
   38 FORMAT(/ 10X 45HRANGE OF ARGUMENT OF FUNCTION DELG(I) IS TOO
     133HLARGE.  TRY NEW VALUES OF DELG(I) /)
      IF (XMING.GE.DELG(2)) PRINT 39
   39 FORMAT(10X 47HDELG(2) IS TOO SMALL. TRY NEW VALUES OF DELG(I) /)
      PRINT 40,ITMAX
   40 FORMAT(/ 10X 7HITMAX = I4 /)
      PRINT 41
   41 FORMAT(// 12X 16HOPTIMIZED VALUES /)
      E01=X(NU)**2
      PRINT 34,E01
       CALCULATE THE SUM OF SQUARES OF DEVIATIONS FROM DATA.
      PRINT 35
      SUMF=0.0
      DO 4 I=1,NF
      SUMF=SUMF+X(I)**2
    4 PRINT 36, SUMF
      PRINT 37
      SUMG=0.0
      DO 5 I=1,NG
      SUMG=SUMG+X(NF+1)**2
    5 PRINT 36,SUMG
C       CALCULATE CREEP CURVES FOR OPTIMUM PARAMETERS X(I)
C       OBTAINED.
      K=0
      DO 6 J=1,NP,NT
      K=K+1
      KK=0
      PRINT 42,RAGE(K)
   42 FORMAT(/// 10X 5HAGE = F9.2,6H  DAYS)
      PRINT 43
   43 FORMAT(/ 21X 4HTBAR 10X 5HCREEP 5X 10HCREEP DATA 5X
     1 10HDIFFERENCE /)
      J1=J+NT-1
      DO 6 I=J,J1
      KK=KK+1
      CREEP=CRDATA(I)+DIFF(I)
    6 PRINT 44,KK,TBAR(KK),CREEP,CRDATA(I),DIFF(I)
   44 FORMAT(I10,F15.1,3F15.5)
      END

      SUBROUTINE XINCR(N,F,X)
C COMPUTES SQUARE ROOTS X(I) OF INCREMENTS OF FUNCTIONS F OR G.
C THE USE OF X(I) WILL ASSURE THAT THE FUNCTIONS ARE POSITIVE-
C VALUED MONOTONICALLY INCREASING FUNCTIONS.
      DIMENSION F(9),X(9)
      N1=N-1
      SUM=0.0
      X(1)=SQRT(F(1))
      DO 1 I=1,N1
      I1=I+1
      SUM=SUM+X(I)**2
    1 X(I1)=SQRT(F(I1)-SUM)
      RETURN
      END

      FUNCTION XINTPN(XX,Y,X,N1,N)
C INTERPOLATES FUNCTION VALUE XINTPN FOR SPECIFIED ARGUMENT XX,
C USING SPECIFIED DISCRETE FUNCTION VALUES Y(I) AT ARGUMENT X(I).
C INTERPOLATION IS LINEAR IN X(I).  N1,N=FIRST AND LAST LOCATION
C OF THE INTERPOLATION FUNCTION IN VECTOR X(I).  XX=SPECIFIED
C DISCRETE VALUES OF X(I).
      DIMENSION Y(17),X(8)
C       SEARCH FOR THE INTERVAL WHERE XX IS LOCATED.
      DO 1 K=2,N
      B=X(K)-XX
      IF (B) 1,2,2
    1 CONTINUE
    2 K1=K-1
C       THE INTERVAL WHERE XX IS LOCATED IS (K1,K).  NOW FIND
C       FUNCTION VALUES AT K1 AND K BY SUMMING INCREMENTAL
C       SQUARES.  FOR FUNCTION G, USE K=K+NF, K1=K1+NF.
      A=XX-X(K1)
      IF (N1 .EQ. 1) GO TO 4
      K=K+N
      K1=K1+N
    4 CONTINUE
      SUM=0.0
      DO 3 L=N1,K1
    3 SUM=SUM+Y(L)*Y(L)
      SUM1=SUM+Y(K)*Y(K)
C       FIND FUNCTION VALUE BY INTERPOLATION.  (SAME FORMULA ALSO
C       WORKS FOR EXTRAPOLATION
      XINTPN=(SUM*B+SUM1*A) /(A+B)
      RETURN
      END

      FUNCTION DIF(Y,I)
C COMPUTES THE DIFFERENCE BETWEEN ASSUMED FUNCTION AND MEASURED
C FUNCTION.  DISCRETE VALUES OF F(G(T)-G(TP)), G(TP) AND 1/E0 ARE
C ALL STORED IN VECTOR Y.  DIMENSION OF Y(I) MUST BE GREATER
C THAN NF+NG+1.
      DIMENSION Y(17)
      COMMON /A/AGE(8), AGEL(8),RAGEL(8),TBAR(8),CRDATA(32),DELG(8)
      COMMON /B/NF,NG,NU,NT,NG1,XMAXG,XMING
      IC1=(I-1)/NT
      IC=IC1+1
      IT=I-IC1*NT
C       IC=NO. OF THE CURVES ON WHICH THE POINTS LIE.  IT=NO. OF
C       THE POINTS ON EACH CURVE.
      TLOG=ALOG10(AGE(IC)+TBAR(IT))
      GT=XINTPN(TLOG,Y,RAGEL,NG1,NG)
      GTP=XINTPN(AGEL(IC),Y,RAGEL,NG1,NG)
C       GT AND GTP ARE INTERPOLATED VALUES OF FUNCTION GG(TP)
      DG=GT-GTP
      XF=XINTPN(DG,Y,DELG,1,NF)
C       XF=INTERPOLATED VALUE OF FUNCTION F(G(T)-G(TP))
      DIF=Y(NU)+XF+DG-CRDATA(I)
      IF (DG .GT. DELG(NF) .OR.  DG .LT. DELG(1)) PRINT 2,DG
    2 FORMAT(/ 10X 6HDELG =F9.5,29H OUT OF RANGE,TRY NEW VALUES
     1 29H OF DELG TO STAY WITHIN RANGE /)
      IF (DG.GT.XMAXG) XMAXG=DG
      IF (DG.LT.XMING) XMING=DG
      RETURN
      END
```

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RÉSUMÉ

Examen au moyen de l'optimisation de quelques formulations pratiques récemment proposées pour le fluage du béton. — *La vérification et l'identification des lois théoriques de fluage à partir des résultats d'essai se trouvent renouvelées par les algorithmes d'optimisation depuis peu disponibles dans les informathèques. On peut, à présent, opérer des comparaisons bien détaillées avec les résultats d'essais de fluage, et cela sans trop de peine. En formulant les conditions d'optimalité dans les termes d'une fonction objective de somme des carrés, et en exprimant par des substitutions quadratiques des contraintes de différentes valeurs positives, on peut appliquer l'algorithme de Marquardt. C'est de cette façon qu'on étudie ici deux formulations récemment proposées :* (a) *le modèle viscoélastique avec réduction du temps, et* (b) *la méthode de la « vitesse de déformation plastique ».*

On sait qu'aucune de ces formulations ne peut bien rendre compte des résultats d'essai de fluage pour toute l'étendue des valeurs intéressantes de durée de fluage et d'âge de chargement, bien qu'on ait précédemment obtenu une certaine concordance avec les résultats d'essai dans un intervalle restreint. Cette même méthode a fait déjà révéler que la formulation du fluage récemment proposée par les Recommandations du C.E.B. était entachée des mêmes limitations. Celle-ci, ainsi que les deux formulations qu'on étudie ici ont en commun le même concept sous-jacent de séparation de la déformation totale de fluage en fluage réversible et fluage irréversible. Nos résultats démontrent que ce concept est en contradiction avec les résultats de fluage obtenus dans un intervalle de temps qui ne soit pas trop limité.