CEMENT and CONCRETE RESEARCH. Vol. 8, pp. 381-386, 1978. Pergamon Press, Inc.
Printed in the United States.

NOTE

A NOTE ON AMELIORATION OF THE CREEP FUNCTION FOR "IMPROVED DISCHINGER METHOD"

Zdeněk P. Bažant[1] and Liisa Panula[2]
Department of Civil Engineering
Northwestern University, Evanston, Illinois 60201

In Annex e of the new Model Code of C.E.B. (Comité Européen du Béton), a creep function of the following form will be allowed to be used for concrete (see Eq.e.3 on p. 249 of [1] or Eq.e.4 on p. 233 of [2]):

$$J(t,t_0) = \frac{1}{E_c(t_0)} + \frac{\phi(t,t_0)}{E_{c_{28}}} \tag{1}$$

with

$$\phi(t,t_0) = \phi_d \beta_d(t-t_0) + \phi_f[\beta_f(t) - \beta_f(t_0)]. \tag{2}$$

Here $J(t,t_0)$ = strain at time t due to a unit stress acting since time t_0 (creep function), $\phi(t,t_0)$ = creep coefficient, ϕ_d = 0.4 = coefficient of the so-called "delayed elasticity," ϕ_f = coefficient of the so-called "delayed plasticity" (flow, irreversible creep), β_d = function of time $(t-t_0)$ given by a graph, $E_{c_{28}}$ = elastic modulus of concrete at the age of 28 days, and $E_c(t_0)$ = elastic modulus of concrete at age t_0.

The foregoing creep function is based on the assumption that both the shape of the creep curve and the effect of age at loading can be determined from one and the same time function, $\beta_d(t)$, which is a very crude simplification of the reality [3]. For $E_c(t)$ = const. = E_{28} and coefficient values indicated in Ref. 1 and 2 (taking coefficient λ from [1] for sealed specimens as 29), comparisons of Eq. (2) with the experimental curves of creep of specimens loaded at widely different ages are indicated by the solid lines in Figs. 1f-j, and 2f-j. These figures include all of the well-documented data available in the literature on the age effect in creep (see Refs. 6-11). For the actual, time-variable elastic modulus $E_c(t)$, the comparisons are shown in these figures by dashed lines. Because C.E.B. does not indicate the age-dependence of the elastic modulus, the ACI Committee 209 recommendation [5] has been used to calculate strength f_c' at time t, from which the elastic

[1]Professor of Civil Engineering
[2]Graduate Research Assistant

modulus $E_c(t)$ as well as $E_{c_{28}}$ has been evaluated using the C.E.B. formula (f_c' in MPa):

$$E_c(t_0) = 9500\ \sqrt[3]{f_c'(t_0)}, \qquad f_c'(t_0) = f_{c_{28}}' \frac{t_0}{4 + 0.85t_0} . \tag{3}$$

It should be noted, however, that the comparison with test data, as apparent from the figures, is inferior to that attainable with other forms of the creep function, including the previous C.E.B. creep function (1970). Nevertheless, the use of Eq. (2) will be allowed by the next edition of C.E.B. recommendations, for reasons other than experimental comparisons. Therefore, any possible amelioration within the capability of the creep function in the form of Eq. (2) is highly desirable. Indeed, some degree of amelioration is possible.

In their previous discussion, Rüsch et al. reduced the disagreement with test data by shifting the creep curves vertically, the curves for young ages at loading being shifted upwards and the curves for old ages at loading being shifted downwards (see Fig. 1 on p. 633 of Vol. 5, Ref. 4). Such vertical shifts are equivalent to replacing the actual elastic modulus $E_c(t_0)$ by some other fictitious modulus values. These values, as implied by the vertical shifts of Rüsch et al., were indicated in Fig. 10 on p. 122 of Vol. 7, Ref. 3.

Following up this idea, it is now proposed to embody the vertically shifted creep curves of Rüsch et al. [4] in the C.E.B. formulation. The actual elastic modulus $E_c(t_0)$ will be replaced by modulus $E_{ic}(t_0)$ called initial creep modulus, i.e.,

$$J(t,t_0) = \frac{1}{E_{ic}(t_0)} + \frac{\phi(t,t_0)}{E_{c_{28}}} . \tag{4}$$

Modulus $E_{ic}(t_0)$ is a fictitious quantity which has physically nothing in common with the actual elastic modulus; it merely serves to set the initial value of the creep curves, such that optimum fit for longer creep durations which matter most be obtained.

Formally, however, $E_{ic}(t_0)$ may be treated in structural creep analysis as an elastic modulus, and so Eqs. (1) and (2) retain the same form. Thus, all formulas based on them remain applicable, i.e., introduction of E_{ic} would not preclude the use of the traditional Dischinger-type methods of creep structural analysis, such as the improved Dischinger method (which has been the main motive for taking over Eq. (2) from German DIN specifications). As far as the variation of elastic modulus is concerned, Rüsch et al. stated [4] that it does not make the application of the improved Dischinger method any more difficult. The fact that E_{ic} does not represent the actual elastic modulus (as defined by codes) is immaterial, for two reasons: (a) E is a conventional value anyhow, and it itself includes about 30% of true creep strain; (b) The subdivision of the total strain between the elastic and creep components of the creep function is irrelevant for structural analysis, as long as the total strain, $J(t,t_0)$, is correct within the period of interest.

Forcing the ratio $E_{ic}(t_0)/E_{c_{28}}$ to be the same for all data sets, the optimum fits of experimental creep curves, with the proposed amelioration, are shown by the solid lines in Figs. 1a-e, and 2a-e. It is seen that the disagreement with test data is indeed appreciably reduced, although the formulation still remains to be distinctly inferior to certain other formulations. Note also that the improvement is not limited to small ages of concrete. The expression for E_{ic} which corresponds to these optimum fits of

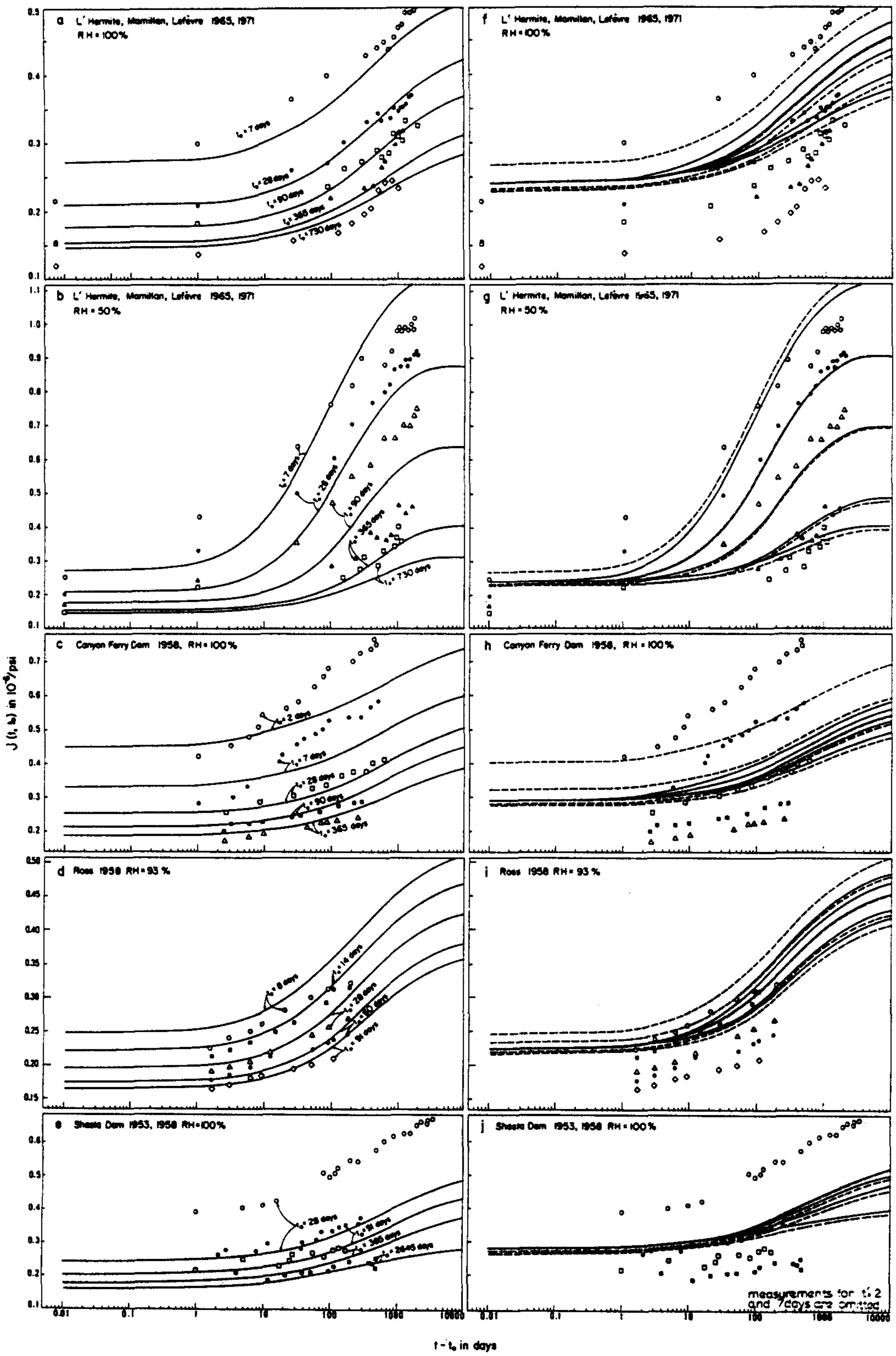

FIG 1. Comparisons with Creep Test Data Available in the Literature

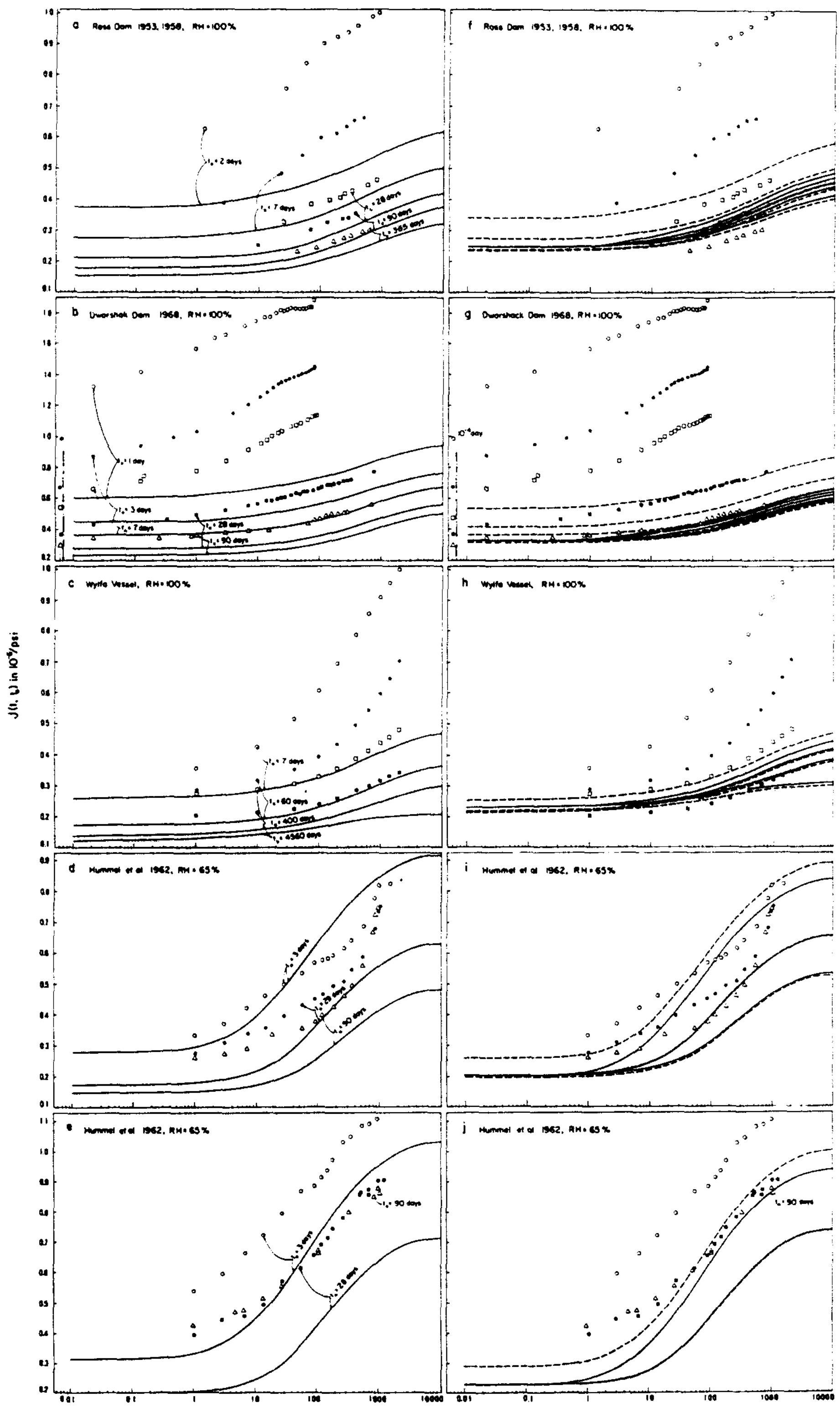

FIG. 2 Further Comparisons with Creep Data

test data is given by

$$\frac{1}{E_{ic}(t_0)} = \frac{0.46}{E_{c_{28}}}\left[1 + 0.85\left(\frac{28}{t_0}\right)^{0.4}\right] \tag{5}$$

where $E_{c_{28}}$ is the elastic modulus at the age of 28 days, as defined by C.E.B. [1,2]. Fig. 3 shows a plot of $E_{ic}(t_0)$ in comparison to the curve of actual modulus $E_c(t_0)$ according to ACI Committee 209 recommendation [5]. The values of $E_{ic}(t_0)$ are also very close to those employed by Rüsch et al. in Fig. 1 on p. 633 of Vol. 5, Ref. 4. It is now proposed that Eq. 5 be introduced in Annex e of the new edition of C.E.B. Model Code.

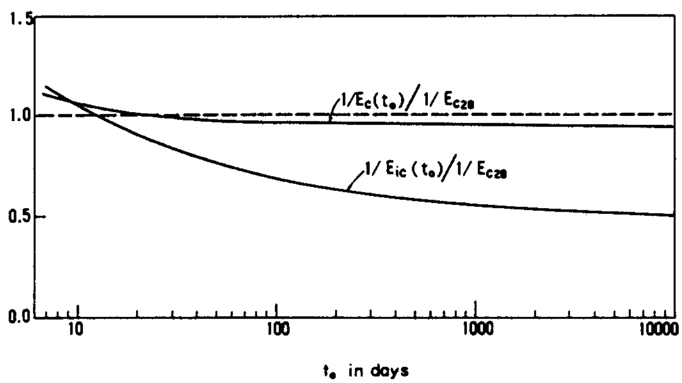

FIG. 3 Variation of Elastic Modulus and Initial Creep Modulus

Concluding Remark

It must be emphasized that the present proposal does not imply an endorsement of the creep function in the form of Eq. (2). However, given the fact that Eq. (2) is now adopted by C.E.B. and will for some time form part of C.E.B. Model Code, a constructive attitude must be taken and the formulation must be optimized at least within the imposed limitations. The proposal is made strictly in this spirit, while the previous criticism [3] remains valid.

References

1. C.E.B. (Comité Européen du Béton), 3rd Draft of Model Code for Concrete Structures, Bulletin d'Information No. 117-F, Paris, Dec. 1976.

2. C.E.B. (Comité Européen du Béton), Contribution to 19th Plenary Meeting, Bulletin d'Information No. 120, Sept. 1977.

3. Bažant, Z. P., Osman, E., "On the Choice of Creep Function for Standard Recommendations on Practical Analysis of Structures," Cement and Concrete Research 5,(1975), 129-138; with discussions and replies 5 (1975), 631-641 6 (1976) 149-157, 7 (1977) 111-130.

4. Rüsch, H., Jungwirth, D., Hilsdorf, H., Discussions of Ref. 3, Cement and Concrete Research 5 (1975), 631-641, 7 (1977) 111-130.

5. ACI Committee 209, Subcom. II (Chairman D. E. Branson), "Prediction of creep, shrinkage, and temperature effects in concrete structures" in Designing for Effects of Creep, Shrinkage and Temperature, American Concrete Institute Special Publication, No. 27, Detroit, 1971, pp. 51-93.

6. Hanson, J. A., "A ten-year study of creep properties of concrete," Concrete Laboratory, Report No. Sp-38, U.S. Department of the Interior, Bureau of Reclamation, Denver, Colorado, July 1953.

7. Harboe, E. M. et al., "A comparison of the instantaneous and the sustained modulus of elasticity of concrete," Concrete Laboratory Report No. C-854, Div. of Engng. Laboratories, U.S. Dept. of the Interior, Bureau of Reclamation, Denver, Colorado, March 1958.

8. Hummel, A., Wesche, K., Brand, W., "Der Einfluss der Zementart, des Wasser-Zement-Verhältnisses und des Belastungsalters auf das Kriechen von Beton," Deutscher Ausschuss für Stahlbeton Heft 146, W. Ernst & Sohn, Berlin 1962.

9. L'Hermite, R. G., Mamillan, M., Lefèvre, C., "Nouveaux résultats de recherches sur la déformation et la rupture du béton," Annales de l'Institut Technique du Bâtiment et des Travaux Publics, Vol. 18, No. 207-208, 1965, pp. 323-360 (see also International Conference on the Structure of Concrete, Cement and Concrete Association. London, 1968, pp. 423-433).

10. Pirtz, D., "Creep characteristics of mass concrete for Dworshak Dam," Report No. 65-2, Structural Engineering Laboratory, University of California, Berkeley, October 1968.

11. Ross, A. D., "Creep of concrete under variable stress," American Concrete Institute Journal, Vol. 54, 1958, pp. 739-758.