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NOTES

COMMENT ON THE USE OF ROSS' HYPERBOLA AND RECENT COMPARISONS OF VARIOUS PRACTICAL CREEP PREDICTION MODELS[a]

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The question of evaluating and comparing various competing creep and shrinkage formulations for codes continues to be in the center oı interest. This note is motivated by the work of Hilsdorf and Müller, which was originally presented in the form of a limited circulation report [1] to the invited participants of ACI Rüsch Symposium held in Washington in 1979. This work has subsequently received wider circulation, was summarized in a proceedings paper [2], and attracted attention in discussions at numerous conferences. Analysis of this work may therefore be of broader interest.

Hilsdorf and Müller attempt to compare to the existing test data three comprehensive creep prediction models: (1) ACI Committee 209 Model [3] due to Branson; (2) CEB-FIP Model [4] due to Rüsch, Hilsdorf and Jungwirth [5]; (3) Bažant and Panula's (BP) Model [6]. Their conclusions on the relative merits of these three models are however at variance with those reached at Northwestern University [7, 6] and are much more favorable to the CEB-FIP Model. We will now show that this disagreement is due mainly to an inadequacy of one traditional, widely used method for extrapolating creep measurements. Furthermore, we will seize this opportunity to point out some other questionable aspects behind Hilsdorf and Müller's conclusions.

Range of Applicability of Ross' Hyperbola

Hilsdorf and Müller attempt a comparison of a type from which previous works refrained — they try to directly compare the test data with the so-called "final" values of creep. One problem with this approach of course is that, from all we know from tests, the creep of concrete does not attain a "final" value within the normal lifetimes of structures. For practical purposes this question is, though, unimportant. For models which theoretically give no bounded final value [6] we may call, e.g., the 50-year value the "final" value.

More serious is, however, the problem of experimental verification of the

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"final" value indicated by a creep prediction formula. Most creep tests lasted much less than 50 years. So, if one insists on verifying the "final" values alone one must first extrapolate the measurements. For this purpose, one of course needs to assume a certain creep formula in advance. It appears that such extrapolated final values strongly depend on the choice of the formula. So one does not really compare the creep prediction formula to the measured final values, but to predictions of another formula. Such comparisons are therefore biased, i.e., dependent on the choice of the extrapolation formula. For this reason the analyses in Refs. 6 and 7 refrained from attempting direct comparisons with any extrapolated final values, even though these values are of the greatest practical interest. The objective statistical approach is to compare the creep prediction formula only to the measured values, even if long-time measurements are unavailable.

Now consider the particular formula which Hilsdorf and Müller chose for extrapolating the measured data to infinite time — namely the Ross' hyperbola [8]

$$J(t,t') = \frac{1}{E(t')} + C, \qquad C = \frac{\bar{t}}{a + b\bar{t}}, \qquad (\bar{t} = t - t') \tag{1}$$

in which $J(t,t')$ = compliance function = strain produced at age t by a constant unit uniaxial stress applied at age t', C = specific creep, $E(t')$ = short-time elastic modulus at age t', $\bar{t}$ = stress duration; and a, b = coefficients which depend on t' as well as temperature, hygrometric conditions, and composition of concrete. Ross proposed plotting the measured values in the graph of $1/C$ versus $1/\bar{t}$, which should be a straight line of slope a and intercept b since Eq. (1) may be rewritten as $1/C = a(1/\bar{t}) + b$. Since for $\bar{t} \to \infty$, $\ell im\, C = 1/b$, the extrapolated final value is obtained by measuring the intercept b of the regression line. We illustrate it in Fig. 1, in which we also show the measured values from the creep tests of Hanson and Harboe [9, 10]. We do not show, however, all measured points, but only those which fall within the range $\bar{t}$ = 3 days to 60 days.

In the 1930's, when Ross did his work, no long-time creep measurements were available. All data existing at that time appeared to be in acceptable agreement with Eq. (1). However, when such measurements became available, it appeared that the hyperbolic expression does not agree at all with the measurements. This was pointed out by Wittmann [11] and was also shown for test data for a certain nuclear containment shell [12]. We demonstrate it here by plotting in Fig. 2 the full time range of Hanson and Harboe's data, as well as several other typical data from the literature [13-17]. We show the best straight line fits (regression lines, Eq. 2), and we plot the corresponding hyperbolas (Eq. 1) in Fig. 3. We see a huge disagreement with test data.

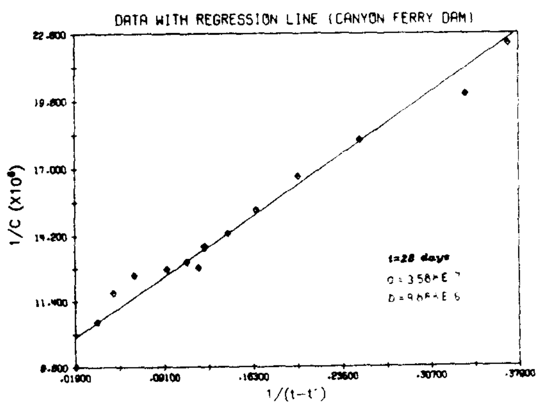

Fig. 1 Regression Line for Ross' Hybola Illustrated with Creep Test Data of Hanson and Harboe [9, 10] of Limited Time Range.

Note that even when the errors of Ross' hyperbola are not too conspicuous in the plot of $1/C$ versus $1/\bar{t}$, they may be blatant in the plot of $J(t,t')$ versus $\log \bar{t}$. The inverse scales $1/C$ and $1/\bar{t}$ introduce a bias in that they obscure the errors for long times by crowding together the points for large C and large $\bar{t}$ (see points near the origin in Fig. 2; a small change in their ordinate represents a large change in C). Thus, the practice of showing only the plot of $1/C$

versus 1/E may be misleading.

Another difficulty is due to the need to subtract the elastic deformation $1/E(t')$ from $Jt(t,t')$ before one can plot the measured points. The value of $1/E(t')$ is rather ambiguous; it makes a great difference whether we consider the short-term deformation as the deformation for a load duration of 2 hours, or 5 min., or 1 sec. For the plots in Fig. 3, $1/E(t')$ has been evaluated from the double power law as $J(t,t')$ for $t = t' + 0.1$ day (since 0.1 day gives the best agreement with the ACI value for E). However, from the statistical viewpoint, one should preferably determine 1/E as the value which minimizes the sum of squared deviations from the straight line, but that often yields (1/E) values which correspond to unreasonable load durations.

Based on their use of Ross' hyperbola, Hilsdorf and Müller (p. 30 of Ref. 1) find that the long-time predictions of the BP model (taking $\bar{t} = 50$ years as the "final" value) are greatly in error, much more so than those of the CEB-FIP model. We must recognize however that their finding is a consequence of the fact that the formulas for the last two models are bounded, i.e., have a final value, while the double power law used in the BP model is unbounded. What is in error is the premise (i.e., the use of Ross' hyperbola). One needs to first assume a bounded formula to get a "final" value, and then one concludes that the bounded formulas agree and the unbounded ones disagree with this "final" value. This is an example of a circular argument.

It should also be realized that the limitations just described do not refer exclusively to Ross' hyperbola. They are characteristic of many attempts to obtain a "final" value of creep on the basis of various bounded formulas.

Selection and Scope of Test Data

Further disagreement in conclusions of the two parallel studies [1, 2, 6, 7] are caused by great differences in the scopes of the test data used. The comparisons at Northwestern University [7, 6] made from a computerized data bank involved 80 test series (over 800 response curves), nearly all that is found in the literature, while those of Hilsdorf and Müller involved selected 7 test series.

Some test data compare with a given theory more favorably than others. As known from the theory of statistical sampling, the statistical parameters of a limited selection of data approximate those of the complete set of data only if the selection is random. Thus, unless done by casting dice, the decision to select for comparisons with the theory certain test series and omit others inevitably involves subjective bias. The effect on the evaluation can be large. Ref. 7 shows an example where comparison of a shrinkage formula with 12 different test series yielded coefficient of variation 31.7% while the use of eight of those test series (still apparently a large body of experimental evidence) yielded 12.6%.

Significant differences further exist in the breadth of the time range of various test data series used in these investigations. Contrary to the work at Northwestern University, Refs. [1] and [2] involve few test data for high ages at loading (over one year) or low ages at loading (under 28 days), for long creep durations (over one year), for relative humidities outside the 50% to 70% range, for sealed specimens. The test data for creep at various temperatures were altogether omitted. This limitation of the range of test data used for comparisons was partly necessary because the CEB-FIP model covers many fewer influencing factors than does the BP model.

The creep influencing factors included in both models are: (1) age at loading, (2) ambient humidity or sealing, (3) size, (4) strength of concrete, (5) cement type, (6) water-cement ratio (consistency). The BP model also includes: (7) shape, (8) age at the start of drying, (9) drying duration (for creep), (10) delay of loading after drying, (11) temperature effect on creep rate, (12) simultaneous effect of temperature and drying, (13) temperature effect on aging rate, (14) age at the start of heating, (15) amplitude of stress cycling (if any), (16) mean stress in case of cycling, (17) nonlinearity at high stress, (18) cement content, (19) unit weight of concrete, (20)

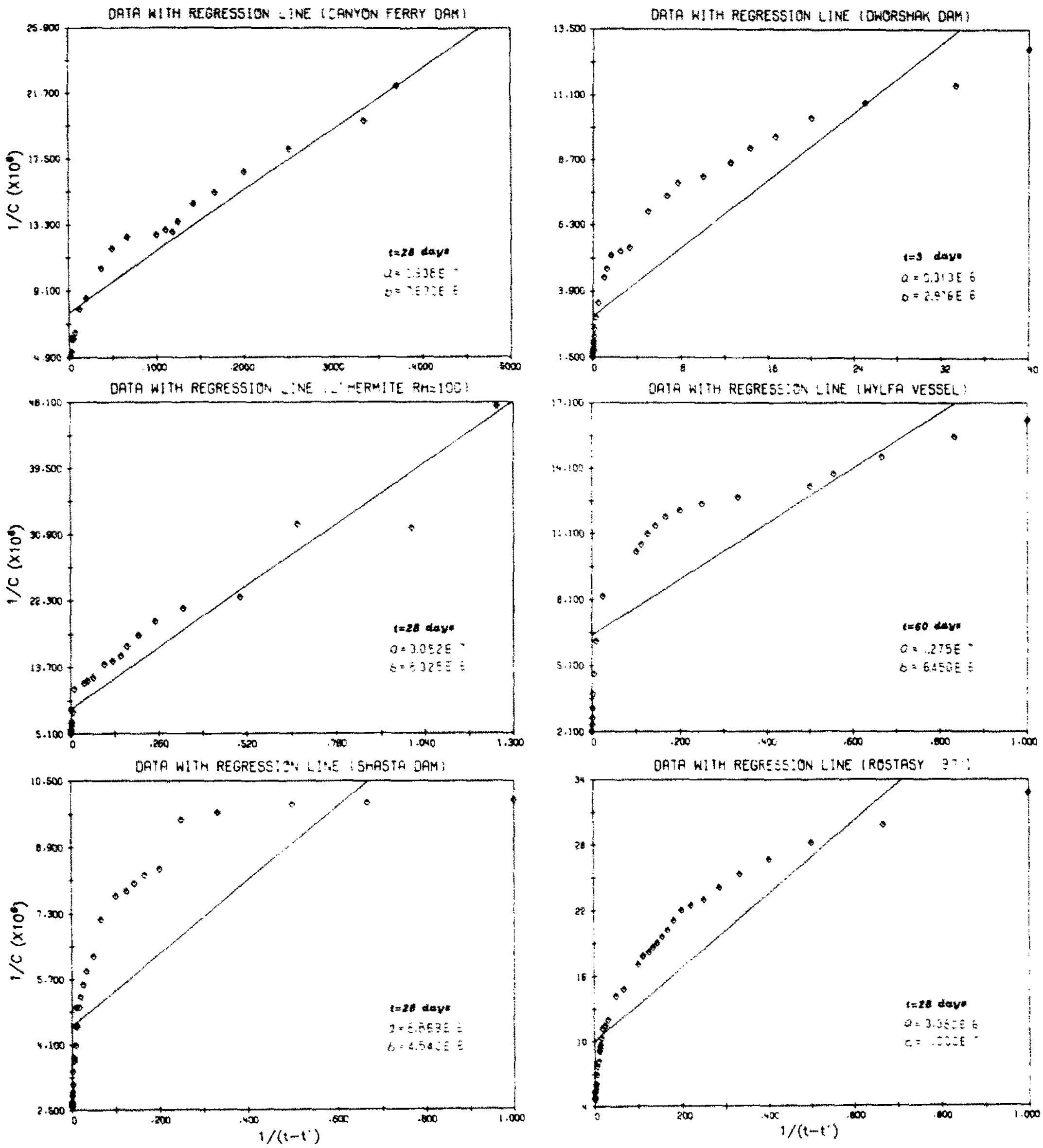

Fig. 2 Regression Lines for Ross' Hyperbolas Applied to Various Creep Test Data from the Literature (Canyon Ferry and Shasta Dams [9, 10], Dworshak Dam [13], L'Hermite [14], Wylfa Vessel [15, 16], Rostasy [17]).

sand-aggregate ratio, (21) aggregate-cement ratio. A model [9] which covers all these 21 factors is of course at a disadvantage compared to one which covers only 6 factors if the test data for comparison cover only these 6 factors. Thus the CEB-FIP Model and the BP Model are not really comparable.

Conclusions

1. Ross' hyperbola can describe creep only within one to two orders of magnitude of creep duration and is invalid for extrapolations to long times.

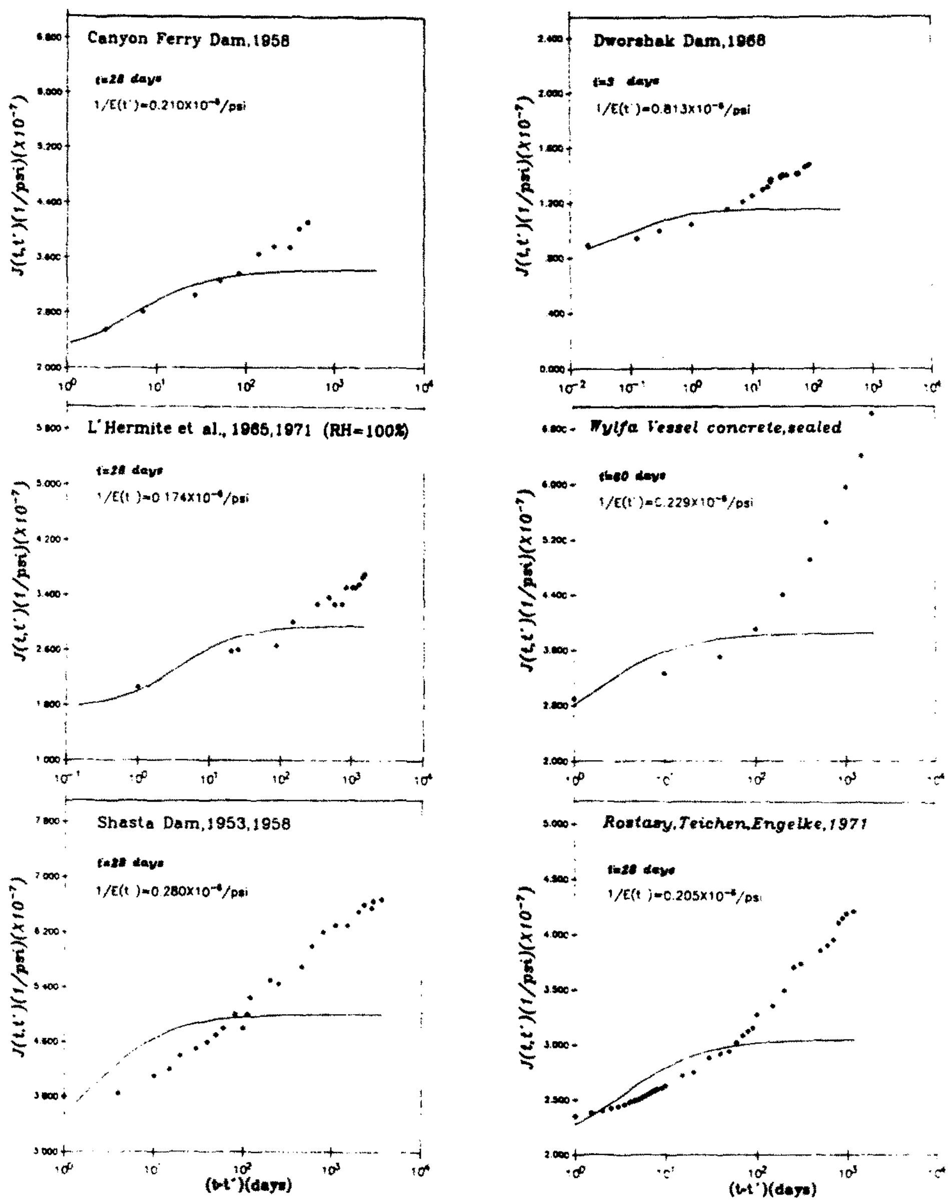

Fig. 3 Ross' Hyperbolas Corresponding to the Regression Lines from Fig. 2

Due to the use of inverse scales, the plot obscures large errors in creep strains for long-times. It is incorrect to use Ross' hyperbola to determine the "final" creep value.

2. Müller and Hilsdorf's comparison of the "final" creep values predicted by various models is invalid.

3. The selection as well as the scope of the test data used by them does not permit an unbiased comparison between the CEB-FIP and BP models.

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DISCUSSIONS

A DISCUSSION OF THE NOTE "COMMENTS ON THE USE OF ROSS' HYPERBOLA AND RECENT COMPARISONS OF VARIOUS PRACTICAL CREEP PREDICTION MODELS" BY Z.P. BAZANT AND J.C. CHERN*

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Regarding Dr. Bažant's and Chern's note we would like to make the following comments.

1. The reader of the note must have the impression that our evaluation comprises only three prediction methods and that it is based solely upon comparison of "final" values of creep.

   However, in addition to the methods mentioned by the authors the following procedures were included: German Prestressed Concrete Code DIN 4227 /20/, CEB/FIP recommendation 1970 /21/ and a method of the British Concrete Society /22/.

   In our evaluation the main emphasis was placed upon comparison of predicted creep functions with experimental data. The trends regarding the reliability of predicted creep functions were similar to those obtained for "final" creep values.

2. In /1, 2/ the problems of determining final creep values have been pointed out. In the present context it is not practical to continue the discussion on the formal existence of a "final" creep value. Creep may well continue forever, but if the annual creep increment is small compared with live load strains it is sufficient and useful for design purposes to assume that a terminal value has been reached. This value may be a true final value or a value which is likely to occur after a certain period of loading such as 30 or 50 years.

   Unfortunately, no published creep data for periods of loading exceeding 24 years are available. Thus we have to resort to extrapolation procedures. Parabolic functions such as those suggested by Shank or Bažant et al tend to overestimate creep

* CCR 12, 527 (1982).

after long periods of loading whereas the Ross hyperbola underestimates creep if the data base does not cover a sufficiently long period of loading /19/. Uncertainties exist for our extrapolation as well as for the values after long durations of loading as predicted i.e. extrapolated by the BP-method.

3. In their presentation of the Ross method, the authors deduce the following linearisation of the Ross hyperbola

$$\frac{1}{C} = a \cdot \frac{1}{\bar{t}} + b \qquad (2)$$

which differs considerably from the linearisation which is de facto proposed by Ross in his paper /8/:

$$\frac{\bar{t}}{C} = a + b \cdot \bar{t} \qquad (3)$$

One may easily show that for data points ($\bar{t}$/C) which do not follow exactly a hyperbolic function of the Ross type a linear regression results in different values for the coefficients a and b depending on the use of either eq. (2) or eq. (3).

It can also be shown that the linearisation of eq. (2) puts weight on data points after short durations of loading, whereas eq. (3) puts weight on data points after long durations of loading.

In our analysis we used eq. (3) and followed Ross' advice to give more weight to experimental data after longer durations of loading when estimating the coefficient b of eq. 1 in the original note, because "a curve which would average the observed data would give a false limiting value" /8/.

This approach has been followed by many researchers who have used the Ross method to estimate "final" creep values, but not to predict creep-time relationships.

In his original paper Ross evaluated the accuracy of the method on the basis of creep experiments with a duration of loading of 2680 days i.e. more than 7 years.

4. Regarding our data selection the authors state that our data base "involves few test data for high ages at loading (over one year) or low ages at loading (under 28 days), for long creep durations (over one year), for relative humidities outside the 50 % to 70 % range, for sealed specimens".

Our comparison reported in /2/ is based upon 15 references and 102 experiments. In our most recent evaluation (1982) which yielded results very similar to those reported in /2/, a total of 146 experiments from 18 references which include 46 types of structural concrete (/2/ : 29 types) has been used. The experiments evaluated met the requirements put forward in /18/. All of these experiments had durations of loading longer than 1 year and as an average of 1300 days. In most experiments the relative humidity ranged between 50 and 70 percent at room temperature because this range is of particular practical significance. Nevertheless, 34 experiments (/2/ : 18 experiments) had been included with a relative humidity larger than 70 percent. Furthermore, 58 experiments (/2/ : 39 experiments) had an age at loading lower than 28 days and 13 experiments (/2/ : 8 experiments) had an age at loading equal to or larger than one year.

5. Report /1/ contained all information on the data used in the evaluation. However, this report is not available and received only limited circulation. In our published paper /2/ the complete data base could not be included because of space limitations. However, it has been made and will be made available upon request (see /2/).

6. It has not been questioned that for the BP -method over 800 creep curves have been used. However, a significant number of those cover parameters outside the range of structural concrete. Many of them do not meet other requirements put forward in /18/. After a thorough search of the creep literature we are rather certain that most of the available creep data unquestionably suitable for an evaluation of creep prediction methods for structural concrete are now included in our data bank.

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REPLY TO HILSDORF AND MÜLLER'S DISCUSSION OF "COMMENTS ON THE USE OF ROSS' HYPERBOLA AND RECENT COMPARISONS OF VARIOUS PRACTICAL CREEP PREDICTION MODELS"*

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Hilsdorf and Müller's detailed discussion is deeply appreciated. They raise several interesting points which call for further analysis.

The discussers claim that a plot of $\bar{t}/C$ versus $\bar{t}$ (Eq. 3) is preferable to the plot of $1/C$ versus $1/\bar{t}$ (Eq. 2). This is not true, for three reasons:

1) As one can verify by numerical examples, both plots yield essentially the same values of a and b.

2) It is not true that determination of the "final" creep value from the plot of $1/C$ versus $1/\bar{t}$ gives too little weight to the long time creep data and too much weight to the short-time creep data. The opposite appears to be true. The y-intercept (i.e., point $1/\bar{t} \to 0$) is very close (in the horizontal direction) to the points for high $\bar{t}$ (Fig. 4) and is, therefore, influenced by an error, e_1, at points for large $\bar{t}$ (small $1/\bar{t}$) much more than by an error, e_2, at points for small $\bar{t}$ (large $1/\bar{t}$) which lie far from the intercept (see Fig. 4).

3) The plot of $\bar{t}/C$ versus $\bar{t}$ (Eq. 3) may be misleading since it gives an impression that the error is less than it actually is (compare Fig. 7a-d with Fig. 7e-h discussed later). The reason is that this plot, unlike the other one, does not become a horizontal line in the special limit case when C does not vary with time ($C \approx$ const.); rather, it reduces to a plot of $\bar{t}$ versus $\bar{t}$, i.e., a straight line of slope 1. Therefore, when C increases with time, a large part of the variation in the plot of $\bar{t}/C$ versus $\bar{t}$ is of deterministic nature ($\bar{t}$ as a function of $\bar{t}$) and is not due to a variation of C. Thus, the reason that the plot of $\bar{t}/C$ versus $\bar{t}$ appears to give a better fit is that it superimposes upon the random scatter of creep strain as a function of time a deterministic dependence of $\bar{t}$ versus $\bar{t}$, thereby hiding the misfit of the creep formula and creating an illusion of a good agreement (such as that apparent from Fig. 7a-d below).

The plot which matters most for comparing a creep prediction formula with test data is the plot of $J(t,t')$ versus $\log(t-t')$. Such plots were shown in Fig. 3, and from these plots (as well as Fig. 7) it is clear that, regardless of which plot is used for linear regression, the shape of Ross'

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hyperbola does not agree at all with the available long-time creep data and represents just about the worst possible choice among the previously proposed formulas. Although the use of any empirical formula introduces some degree of bias, the power law, $J(t,t') = a + b(t-t')^{1/8}$ would have been clearly preferable to carry out the extrapolations (which could be accomplished by plotting $J(t,t')$ versus $(t-t')^{1/8}$. The statistics that Hilsdorf and Müller [1, 2] obtained would then change substantially.

The discussers try to argue against the power law. They say that it tends "to overestimate creep after long periods of loading". However, from their phrase "functions such as those suggested by Shank or Bažant et al." it seems that they might be unaware of an important difference between the original form of the power law, as suggested by Straub and Shank, and the new form, as suggested by Bažant et al., in the double power law [23]. Long-time creep is considerably overestimated by the original form in which the power function is not applied to the total creep strain $C(t,t')$ (per unit stress), but only to that part of the creep strain $C_1(t,t')$ that accumulates after an initial short-time loading of approximately 1 hour duration (Figs. 5,6) [23]. These two parts are defined by $C(t,t') = J(t,t') - 1/E_0$ (see Refs. 23, 24) $C_1(t,t') = J(t,t') - 1/E$ where E = conventional elastic modulus and E_0 = instantaneous (true) elastic modulus which corresponds to loading applied at infinitely high rate; E_0 is close to the usual dynamic modulus, and is obtained as the left-hand side horizontal asymptote in the plot of $J(t,t')$ versus $\log (t-t')$ (Fig. 6). The so-called short-time strain $1/E$ contains much creep strain, usually over 30% of $1/E$ value (Fig. 5). Exclusion of this creep strain from the original form of power law greatly reduces the range of applicability. The fact that the left-hand side asymptotic value $1/E$ of the power curve $(t-t')^n$ in Fig. 8b is placed too high forces one to give the power curve a large curvature, i.e., use a higher exponent n, in order to fit the short-time creep data. Exponent n here comes to be about 1/3, while the correct exponent obtained with the correct left-hand side asymptotic value $1/E_0$ (Fig. 6) is about 1/8. The excessively large curvature causes the original form of the power law to pass high above the creep data for longer creep durations (Fig. 6). It was for this reason that power law was judged in older works to be inapplicable to long-time creep. Now it is well known, however, that the power law works quite well (and far better than Ross' hyperbola) even for very large creep durations provided that all short-time creep strain is included in the power law [23]. (The power law is not perfect, of course, and improvements appear to be possible - one is the log-double power law, presently under study by J. C. Chern at Northwestern University).

Let us now examine practical use of discussers' Eq. 3 to extrapolate to 50 years some very consistent and careful creep measurements, such as those

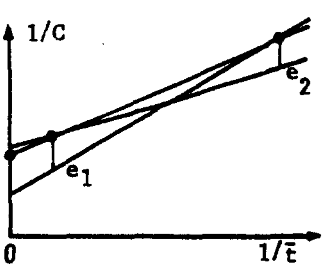

Fig. 4

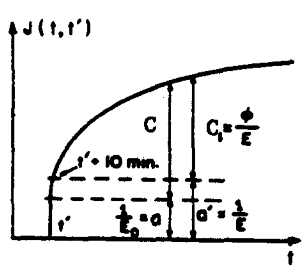

Fig. 5

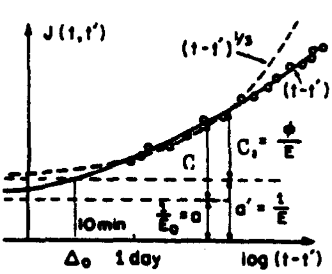

Fig. 6

by Rostasy, et al. [17] (Fig. 7), the duration of which is $\bar{t}$ = 3.7 years. The regression line obtained according to the discussers' method is shown in Fig. 7, and the corresponding Ross' hyperbola in Fig. 7. According to Ross' advice emphasized by the discussers, this hyperbola is made to fit closely the terminal segment of the measured data curve, as seen from Fig. 7g. Suppose now that the measurements terminate either at $\bar{t}$ = 1 month or at t = 6 months, instead of 3.7 years. Applying the discussers' method to such limited data and ignoring the data points beyond 1 month or 6 months, respectively, one obtains the regression lines shown in Fig. 7a, b, with the corresponding Ross' hyperbolas shown in Fig. 7e, f (and coefficients a, b of Eq. 3 listed in Figs. 7a-d).

If the discussers' method (Eq. 3) were valid, the Ross' hyperbolas in these three figures would have to yield essentially the same value at $\bar{t}$ = 18260 days = 50 years. They do not, and the discrepancies are huge. Extrapolation of the full 3.7-year data yields a 50 year value that is 2.63 times larger than the value obtained by extrapolation of the 1 month data. The long-time extrapolations drastically change with the duration of measurements, regardless of the manner in which Ross' hyperbola is applied. Therefore, Ross' hyperbola does not appear to be an acceptable approach even when discussers' Eq. 3 is used.

For comparison, Fig. 7 also shows extrapolations with the best formula that the writers presently know (it is called the log-double power law, and represents a gradual transition from the double power law for short and medium times to a logarithmic law for very long times). With this formula, the 50-year extrapolations obtained from the data terminating at 3.7 years, 6 months and 1 month do not differ from each other by more than 9%. With the double power law, the consistency of extrapolations is not much worse. Fig. 7d, h also shows the least square fits of the complete data. The parameters of the log-double power law and the coefficients of variation ω are also listed in Fig. 7e-h.

The discussers offer some justifications for having omitted many of the existing test data from their study. In the writers' opinion such omissions inevitably introduce subjective bias (which seems to have worked in favor of CEB-FIP Model in this case), and are unjustified. If some careful measurements by reputable experimentalists cover, e.g., only a 6 month duration but include, e.g., rather different ages at loading, or different humidity conditions, or different sizes, or different temperatures, or static and pulsating loads, etc., they are relevant and ought to be included. Even if some good data include, e.g., only one-month load duration, and if the creep prediction formula comes, e.g., 100% above this short-time curve, the error ought to be counted in the overall comparison. There exist well documented statistical examples demonstrating how subjective omissions from the data base, i.e., those not made by chance (e.g., by casting a dice), can falsely reduce the coefficient of variation of errors [7].

The discussers further state that in "most experiments" (used by them to calibrate their formulas) "the relative humidity ranged between 50 and 70% at room temperature because this range is of particular practical significance" This premise is not true, however, because it ignores the fact that the humidity effect is very different for different thicknesses D of the cross section, as known from tests as well as theoretical analysis by diffusion theory [23]. A 6 inch (15cm) diameter cylinder in a drying environment loses moisture at about the same rate as a 5 inch (12.5cm) thick slab, but about 4-times faster than a 10 inch (25cm) thick slab, and about 36-times faster than a 30 inch (75cm) thick slab (this fact is not adequately reflected in the CEB-FIP Model Code). Slabs of these thicknesses are quite typical for

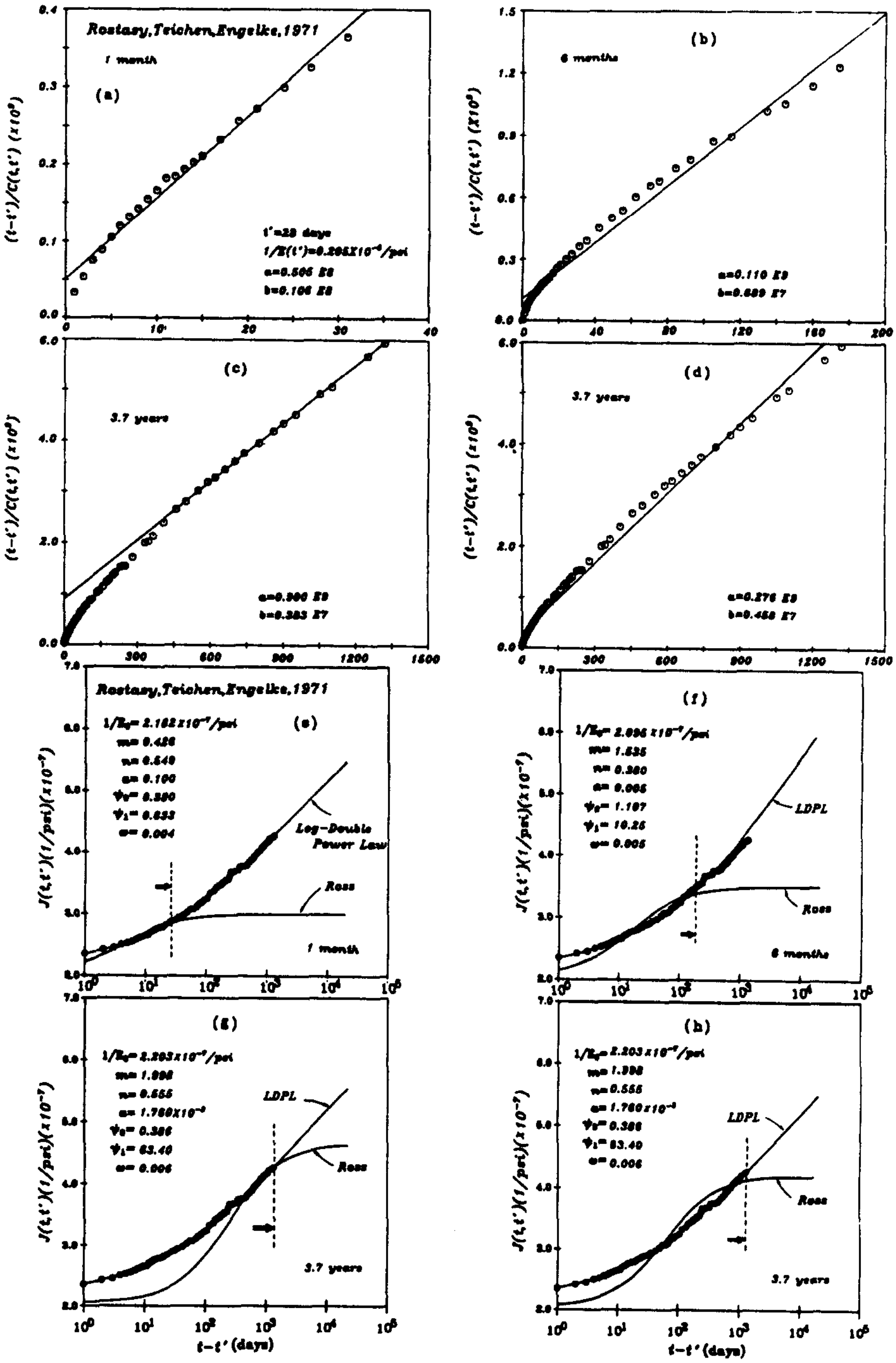
Rostasy, Teichen, Engelke, 1971
(a)
(b)
(c)
(d)
(e)
(f)
(g)
(h)
1 month
6 months
3.7 years
Log-Double Power Law
LDPL
Ross
t-t' (days)

Fig. 7

structures to which the CEB-FIP Model Code is intended to apply, e.g., the critical cross sections of large span bridges. From the mean drying rate (rate of loss of water) of a member of any thickness D at a certain environmental humidity h one can easily determine an equivalent environmental humidity h_{eq} which would give about the same creep for a 6 inch cylinder (see, e.g., Fig. 2 in Ref. 25). Thus, one can find that a 10 inch thick slab and a 30 inch thick slab exposed to h = 65% creep, over a long time period, about the same as a 6 inch diameter cylinder exposed to h_{eq} = 77% and h_{eq} = 90%, respectively. For a 90% relative humidity, the creep of standard 6 inch cylinders is much closer to the creep of a sealed specimen than to the creep of a cylinder exposed to a 65% relative humidity, and for 77% the creep is roughly the average of these two cases. Thus, unless good creep data were available for very thick specimens (which is not the case), the discussers should not omit from their comparisons the creep data for high humidities and sealed specimens, even if they intend the CEB formulation to be used only for non-massive structures, such as large span bridges.

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