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Statistical Extrapolation of Shrinkage Data — Part II: Bayesian Updating




by Z. P. Bažant, J. K. Kim, F. H. Wittmann, and F. Alou

The statistical information previously available in the literature on concrete shrinkage is exploited to improve the extrapolation of short-time shrinkage measurements into long-time measurements. With this approach the long-time predictions can be significantly improved, compared to the predictions obtained by statistical regression from the measured data alone, as described in Part I of this study. The method of analysis consists of latin hypercube sampling of random parameters of the shrinkage formula for the prior, and in adjustments of the weights of the samples on the basis of the Bayes theorem. The shrinkage formula of the BP model, which is justified by diffusion theory, is used. This formula gives prior mean predictions that are rather close to the measured short-time data. Using this formula, good long-time predictions can be obtained even with measurements of only three-day duration.

The formulas from the ACI and CEB-FIP models are found unsuitable for Bayesian extrapolation. An improvement of the formula for predicting the final shrinkage strain from concrete composition is also presented.

Keywords: Bayes theorem; concretes; deformation; diffusion; drying; errors; extrapolation; regression analysis; **shrinkage; statistical analysis**; volume change.

In Part I of this study,[1] extrapolation of short-time shrinkage data was considered under the assumption that the correct shrinkage formula is known but nothing is known about the values of its parameters. However, from statistical studies of numerous data in the literature, we can deduce certain prior statistical information on the values of these parameters, and thereby improve the long-time extrapolation. The methodology to do that is worked out in this paper and examples are given. All the definitions and notations from Part I[1] are retained.

A similar problem was solved for concrete creep in a previous paper.[2] The solution was obtained analytically, in combination with numerical integration of a certain integral. The analytical solution was made possible by linearization of the creep law. For creep, the statistics based on transformed variables in which the creep law is linearized appeared to be quite realistic. This is not so for shrinkage, however, as was found in Part I.[1] Therefore, a different approach must be used for our problem. We will adopt the sampling approach — a novel approach in Bayesian prediction. In mathematical terms, this sampling approach was developed and presented at a recent conference,[3] and we begin by describing it.

BAYESIAN STATISTICAL PREDICTION VIA SAMPLING

Denote as Y_i ($i = 1, 2, \ldots I$) the long-time shrinkage strains (at times t_i) that we want to predict, and as X_m ($m = 1, 2, \ldots M$) the short-time shrinkage strains (at times t_m) that have been measured and that we want to use for improving (updating) the predictions of Y_i. The values of Y_i and X_m that may be predicted on the basis of the available prior information (without taking the present measurements X_m into account) will be denoted as Y_i' and X_m'. The values of Y_i' may be predicted as some known functions y_i of certain random parameters $\xi_1, \xi_2, \ldots \xi_N$; i.e., $Y_i' = y_i(\underset{\sim}{\xi})$ where $\underset{\sim}{\xi} = (\xi_1, \xi_2, \ldots \xi_N)$ = vector of random parameters. Predictions can also be obtained for the short-time strains represented by X_m'. The predicted values, which generally differ from the measured values, may be regarded as functions of $\underset{\sim}{\xi}$; $X_m' = x_m(\underset{\sim}{\xi})$.

Statistical evaluation of various test data from the literature has led to certain, albeit quite limited, information on the statistical properties of the random parameters $\xi_1, \xi_2, \ldots \xi_N$.[4-7] This may be regarded as the prior statistical information, characterized by the prior probability density distributions $f_n'(\xi_n)$ with means $\bar{\xi}_n$ and standard deviations s_n' ($n = 1, 2, \ldots N$). With this knowledge it is possible to predict the statistical distributions of $X_m' = x_m(\underset{\sim}{\xi})$ and compare them to the available measurements X_m. The objective is to use this comparison to improve (update) the statistical information on random parameters ξ_n. This improved statistical information may be characterized by updated, or

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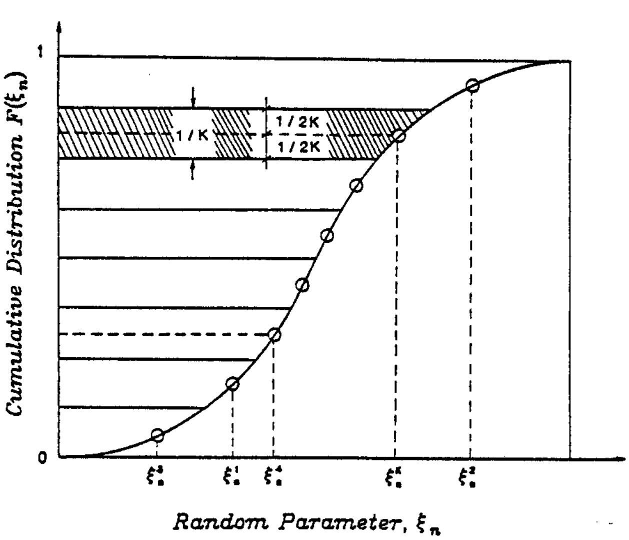

Fig. 1 — Partitioning of range of random variable into intervals

posterior, distributions $f_n''(\xi_n)$, from which the updated, or posterior, distributions $f_i^{Y''}(Y_i'')$ of the long-time strains Y_i may be obtained according to $y_i(\xi)$. The double primes are used to distinguish the posterior characteristics from the prior characteristics, which are labeled by single primes.

In previous Bayesian studies of creep,[2,8] $f_n''(\bar{\xi}_n)$ and $F_i^{Y''}(Y_i'')$ for long-time creep strains have been obtained

Table 1 — Example of interval numbers of random parameters ξ_n sampled for individual computer runs (for K = 8)

Run	ξ_1	ξ_2	ξ_3	ξ_4	ξ_5	ξ_6	ξ_7	ξ_8
1	2	4	1	7	8	7	5	4
2	6	8	5	2	3	2	3	5
3	1	5	4	5	6	1	8	6
4	5	6	2	6	2	4	7	1
5	3	2	7	3	7	8	2	2
6	4	7	3	8	5	3	6	7
7	8	1	8	1	1	5	1	3
8	7	3	6	4	4	6	4	8

analytically for the case when the creep formula is linearized in certain transformed variables, and when there are only two random parameters ξ_1 and ξ_2. These simplifications are insufficient for the prediction of long-time shrinkage, for which generally the functions $y_i(\xi)$ are highly nonlinear and more than two random parameters are involved. In such a case, an analytical solution, such as that in References 2 and 8, would present considerable difficulties; thus, a sampling approach is preferable.

An efficient sampling approach is the latin hypercube sampling. In this method, the known distribution $f_i'(\xi_n)$ of each input parameter ξ_n is partitioned into K intervals $\Delta\xi_n^{(k)}$ (strata) of equal probability $1/K$ (k = 1, . . . K). The subdivision may be obtained according to the cumulative probability distribution $F(\xi_n)$ as illustrated in Fig. 1. The number of random parameter samples is chosen in latin hypercube sampling to be the same as the number of computer runs to be made, and from each interval the random parameter value is sampled exactly once, i.e., it is used in one and only one computer run. The sampled value need not be taken randomly within the interval, but may be taken at the centroid of the interval (Fig. 1).

The random selection of the intervals $\Delta\xi_n^{(k)}$ to be sampled for a particular computer run can be carried out as follows. With each random parameter one may associate a sequence of integers representing a random permutation of the integers 1, 2, . . . K. For eight intervals (K = 8), such random permutations are illustrated in the columns of Table 1. A different random permutation is used for each column (each ξ_n). Thus, each interval number appears in each column of Table 1 exactly once. The numbers of the intervals used in the individual latin hypercube samples are then represented by the lines in Table 1. For each such a sample $\xi_n^{(k)}$ (n = 1, . . . N), one can then calculate the shrinkage strain values $X_m'^{(k)}$ that correspond to the measured short-time shrinkage strains X_m, and also the corresponding long-time shrinkage strains $Y_i'^{(k)}$. From these results, one can then obtain the mean prior prediction $\bar{X}_m'$ for the measured strains X_m, and also the mean prediction $\bar{Y}_i'$ of the long-time strains, as well as the corresponding standard deviations

$$\bar{X}'_m = \frac{1}{K}\sum_{k=1}^{K} X'^{(k)}_m,$$
$$s^{X'}_m = \left[\frac{1}{K}\sum_{k=1}^{K}(X'^{(k)}_m - \bar{X}'_m)^2\right]^{1/2} \tag{1}$$
$$(m = 1, 2, \ldots M)$$

$$\bar{Y}'_i = \frac{1}{K}\sum_{k=1}^{K} Y'^{(k)}_i,$$
$$s^{Y'}_i = \left[\frac{1}{K}\sum_{k=1}^{K}(Y'^{(k)}_i - \bar{Y}'_i)^2\right]^{1/2} \tag{2}$$
$$(i = 1, 2, \ldots I)$$

According to Reference 3, the posterior (updated) probabilities of values $Y^{(k)}_i$ may now be determined as follows. Since one calculated value $Y^{(k)}_i$ is obtained for each random sample $\underset{\sim}{\xi}^{(k)} = (\xi^{(k)}_1, \ldots \xi^{(k)}_N)$, i.e., $Y^{(k)}_i = y_i(\underset{\sim}{\xi}^{(k)})$, the posterior probability of $Y^{(k)}_i$ is the same as the posterior probability of $\underset{\sim}{\xi}^{(k)}$, i.e., $P''(Y^{(k)}_i) = P''(\underset{\sim}{\xi}^{(k)})$. While the prior probabilities of all latin hypercube samples $\underset{\sim}{\xi}^{(k)}$ and of the corresponding $X^{(k)}_m$ and $Y^{(k)}_i$ are equal, the posterior ones are not. The posterior probability is a conditional probability, the condition being that the values $X_1, \ldots X_M$ have been observed; i.e., $P''(X^{(k)}_m) = P''(Y^{(k)}_i) = P''(\underset{\sim}{\xi}^{(k)}) = P(\underset{\sim}{\xi}^{(k)}|\underset{\sim}{X})$ where $\underset{\sim}{X} = (X_1, X_2, \ldots X_M)$ = vector of the measured shrinkage strains on which the updating is based. According to the Bayes theorem[2,9-16]

$$P''(X^{(k)}_m) = P''(Y^{(k)}_i) = P(\underset{\sim}{\xi}^{(k)}|\underset{\sim}{X}) = c_1 L(\underset{\sim}{X}|\underset{\sim}{\xi}^{(k)})\, P'(\underset{\sim}{\xi}^{(k)}) \tag{3}$$

The function $L(\underset{\sim}{X}|\underset{\sim}{\xi}^{(k)}) = L(X_1, X_2, \ldots X_M|\xi^{(k)}_1, \xi^{(k)}_2, \ldots \xi^{(k)}_N)$ is called the likelihood function; it represents the relative joint conditional probability of observing the measured values $X_1, X_2, \ldots X_M$ under the condition that the parameter vector $\underset{\sim}{\xi}$ coincides with the kth sample $\underset{\sim}{\xi}^{(k)} = (\xi^{(k)}_1, \ldots \xi^{(k)}_N)$. The prior probability of each sample is

$$P'(\underset{\sim}{\xi}^{(k)}) = P'(X'^{(k)}_m) = P'(Y'^{(k)}_i) = 1/K = \text{constant} \tag{4}$$

and c_1 is a normalizing constant to be determined from the condition

$$\sum_{k=1}^{K} P''(Y^{(k)}_i) = 1 \tag{5}$$

As is usually the case in Bayesian estimation, the most difficult task is the determination of the likelihood function, representing the relative joint probability of observing the measured shrinkage values $X_1, X_2, \ldots X_M$. These values are certainly to some extent correlated, but since we have at present no information on the correlation of the successive values of creep or shrinkage, we will assume for the sake of simplicity that the values $X_1, X_2, \ldots X_M$ are statistically independent, i.e., uncorrelated. Obviously, the longer the time interval, the weaker the correlation of the shrinkage values at the beginning and at the end of the interval, and so the assumption of statistical independence should be acceptable if the measurements are spaced sufficiently sparsely in time.

A better approach would be to assume statistical independence for the successive increments of shrinkage rather than for their total values. This would, however, lead to the problem of Bayesian estimation for a nonstationary stochastic process with independent increments, which is a much more difficult problem to handle than the present Bayesian problem in the setting of statistical regression. That problem is difficult enough for practical application even outside the Bayesian context.[17]

According to the simplifying assumption of statistical independence, we have

$$L(\underset{\sim}{X}|\underset{\sim}{\xi}^{(k)}) = L(X_1, X_2, \ldots X_M|\underset{\sim}{\xi}^{(k)}) = \prod_{m=1}^{M} f^X_m(X_m|\underset{\sim}{\xi}^{(k)}) \tag{6}$$

The distribution $f^X_m(X_m|\underset{\sim}{\xi}^{(k)})$ represents the density distribution of the conditional probability to obtain any value X_m under the condition that the random parameter values are $\xi^{(k)}_1, \ldots \xi^{(k)}_N$. Note that this probability density is not the prior probability of X_m and cannot be taken the same as $f^{X'}_m(X'^{(k)}_m)$ for the prior, which characterizes the statistical scatter of the properties of all kinds of concretes in general. Rather, $f^X_m(X_m|\underset{\sim}{\xi}^{(k)})$ should be interpreted simply as a characteristic of the statistical scatter of the properties of one particular concrete. Generally, the standard deviation s^X_m of $f^X_m(X_m|\underset{\sim}{\xi}^{(k)})$ will be smaller than that of the prior $f^{X'}_m(X'^{(k)}_m)$. The fact that the probability described by f^X_m is conditional to $\underset{\sim}{\xi}^{(k)}$ means that it refers to one particular concrete, for which $\underset{\sim}{\xi}^{(k)}$ are essentially fixed and known.

Substituting Eq. (6) and (4) into Eq. (3) and (2), we obtain the result

$$P''(Y^{(k)}_i) = P''(\underset{\sim}{\xi}^{(k)}) = c_0 p_k,$$
$$p_k = \prod_{m=1}^{M} f^X_m(X_m|\underset{\sim}{\xi}^{(k)}) \tag{7}$$

in which $c_0 = c_1/K$ = constant. This constant may be determined from the normalizing condition [Eq. (7)]; $c_0 = (\Sigma\, p_k)^{-1}$.

The foregoing relations hold generally true for any probability distribution. To obtain the prior probability density distribution $f^{X'}_m(X'^{(k)}_m)$, one may use many samples $\underset{\sim}{\xi}^{(k)}$ to generate many values $X'^{(k)}_m$ ($k = 1, \ldots K$), then construct the histogram of these values and fit to it a suitably chosen distribution function. To obtain the posterior probability density distributions $f^{X''}_m(X''_m)$ and $f^{Y''}_i(Y''_i)$, one needs to construct the weighted his-

Table 2 — Mean values and coefficients of variation (c.v.) used for BP model

D, mm	Random parameters							
	ϵ_{sh_∞} (in 10^{-6})		h		τ_{sh}		r	
	mean	c.v.	mean	c.v.	mean	c.v.	mean	c.v.
83	1146	0.08	0.65	0.05	92	0.3	1	0.1
160	1127	0.03	0.65	0.05	341	0.3	1	0.1
300	1122	0.01	0.65	0.05	1200	0.3	1	0.1

tograms of values $p_k X''^{(k)}_m$ and $p_k Y''^{(k)}_i$ calculated for all samples $\underline{\xi}^{(k)}$, and then again fit to these histograms a suitably chosen distribution function. The posterior (updated) means and standard deviations of the predictions of X''_m and Y''_i are obtained as

$$\bar{X}''_m = \frac{1}{\sum_k p_k} \sum_k p_k X'^{(k)}_m, \qquad s^{X''}_m = \left[\frac{1}{\sum_k p_k} \sum_k p_k \left(X'^{(k)}_m - \bar{X}''_m\right)^2 \right]^{1/2} \tag{8}$$

$$\bar{Y}''_i = \frac{1}{\sum_k p_k} \sum_k p_k Y'^{(k)}_i, \qquad s^{Y''}_i = \left[\frac{1}{\sum_k p_k} \sum_k p_k \left(Y'^{(k)}_i - \bar{Y}''_i\right)^2 \right]^{1/2} \tag{9}$$

From now on we assume that all distributions are normal (Gaussian). Then the probability densities are

$$f^{X'}_m (X'_m) = \frac{1}{s^{X'}_m \sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{X'^{(k)}_m - \bar{X}'_m}{s^{X'}_m}\right)^2 \right] \tag{10}$$

$$f^{X}_m (X_m | \underline{\xi}^{(k)}) = \frac{1}{s^{X}_m \sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{X_m - X'^{(k)}_m}{s^{X}_m}\right)^2 \right] \tag{11}$$

where $X'^{(k)}_m$ is the value calculated for the kth parameter sample $\underline{\xi}^{(k)}$. According to Eq. (7)

$$p_k = \exp\left[-\sum_{m=1}^{M} \frac{1}{2}\left(\frac{X_m - X'^{(k)}_m}{s^{X}_m}\right)^2 \right] \tag{12}$$

in which the multiplicative constant was dropped, since only the relative values of p_k affect the result. The posterior probability density distributions are

$$f^{X''}_m (X''_m) = \frac{1}{s^{X''}_m \sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{X_m - \bar{X}''_m}{s^{X''}_m}\right)^2 \right] \tag{13}$$

$$f^{Y''}_i (Y''_i) = \frac{1}{s^{Y''}_i \sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{Y_i - \bar{Y}''_i}{s^{Y''}_i}\right)^2 \right] \tag{14}$$

where $\bar{X}''_m$, $s^{X''}_m$, $\bar{Y}''_i$, and $s^{Y''}_i$ are obtained from Eq. (8) and (9).

PRIOR UNCERTAINTY OF MATERIAL PARAMETERS

BP model

The shrinkage formula for this model as given by Eq. (1) and (2) of Reference 1, $\epsilon = \epsilon_\infty [1 + (\tau_{sh}/\hat{t})^r]^{-1/2r}$ with $\epsilon_\infty = \epsilon_{sh_\infty}(1 - h^3)$, in which ϵ_{sh_∞} and r are material parameters ($r = 1$); τ_{sh} is the shrinkage square halftime which is a constant parameter for one type of specimen of the same age at drying start (but is proportional to the diameter-square if the cylinder size is varied); h is the environmental relative humidity (maintained as constant as possible, in laboratory tests); and $\hat{t}$ is the duration of drying. We consider, more generally, ϵ_{sh_∞} as the basic parameter, while in previous regression analysis,[1] parameter ϵ_∞, which includes the humidity effect, was considered to be basic.

By analyzing a vast number of test data from literature, the coefficient of variation of the deviations of measured shrinkage strains ϵ from the BP shrinkage formula was determined[4] as 16.5 percent. From this we have to estimate the coefficients of variation of parameters ϵ_{sh_∞}, τ_{sh}, h, and possibly also r. The basic information for this purpose is assembled in the works by Madsen and Bažant[6] and Bažant and Liu.[18]

In these works, the shrinkage halftime τ_{sh} is considered as a function of a number of parameters: cement content c; water-cement ratio w/c; ratios of aggregate and sand to cement, a/c and s/c; gravel-sand ratio g/s; design standard compression strength f'_c, temperature, etc. The coefficients of variation for these random material parameters are determined as well. Using random simulation by latin hypercube sampling of all these random parameters, we obtain the coefficient of variation for τ_{sh} as roughly between 0.25 and 0.32, and we choose the value 0.3 for further calculations. For the coefficient of variation of the environmental relative humidity h, the value 0.2 was considered previously for weather fluctuations,[4] but for the controlled environment of the laboratory we have 0.05. As for parameter r, no meaningful information can be extracted for the prior from the data in the literature; as a pure guess we assume its coefficient of variation to be 0.1.

With all this information, the coefficient of variation of ϵ_{sh_∞} can be obtained by random simulation using latin hypercube sampling. We carry out all the calculations separately for each specimen size. In Table 2, the means and coefficients of variation of the prior obtained for the four random parameters in the BP shrinkage formula are listed separately for each cylinder diameter D.

ACI model

Based on the test data from the literature, the coefficient of variation for the deviations of the shrinkage strain from the ACI formula [Eq. (4) in Reference 1] was determined to be 52.5 percent. No separate statistical information exists for the coefficient τ_0 in this formula, for which the mean $\tau_0 = 35$ days is used, although this parameter is known to be quite uncertain. As a pure guess, we assume the coefficient of variation

Table 3 — Mean values and coefficients of variation (c.v.) used for ACI model

D, mm	Random parameters			
	ϵ_∞ (in 10^{-6})		τ_0	
	mean	c.v.	mean	c.v.
83	585	0.48	35	0.25
160	584	0.48	35	0.25
300	511	0.50	35	0.25

Table 4 — Mean values and coefficients of variation (c.v.) used for CEB-FIP model

D, mm	Random parameters	
	ϵ_∞ (in 10^{-6})	
	mean	c.v.
83	411	0.717
160	361	0.717
300	313	0.717

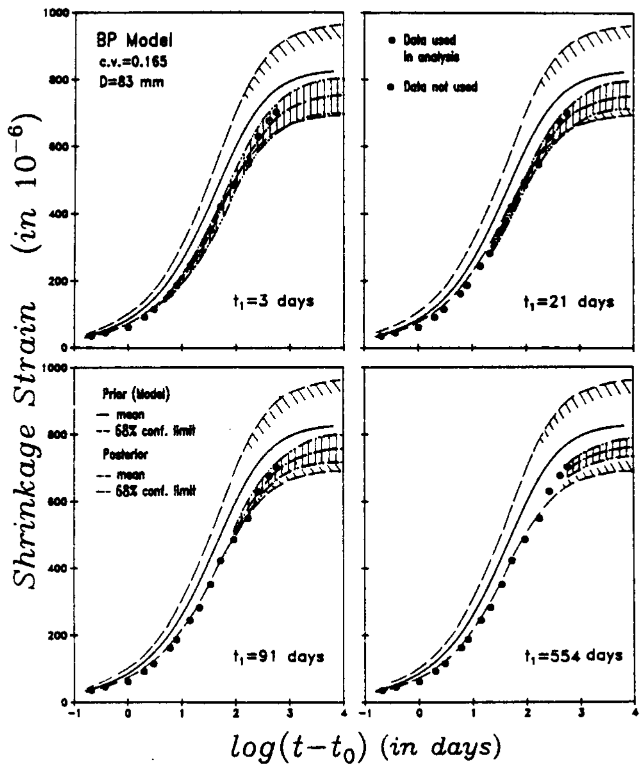

Fig. 2(a) — Bayesian analysis by BP model for D = 83 *mm (3.27 in.)*

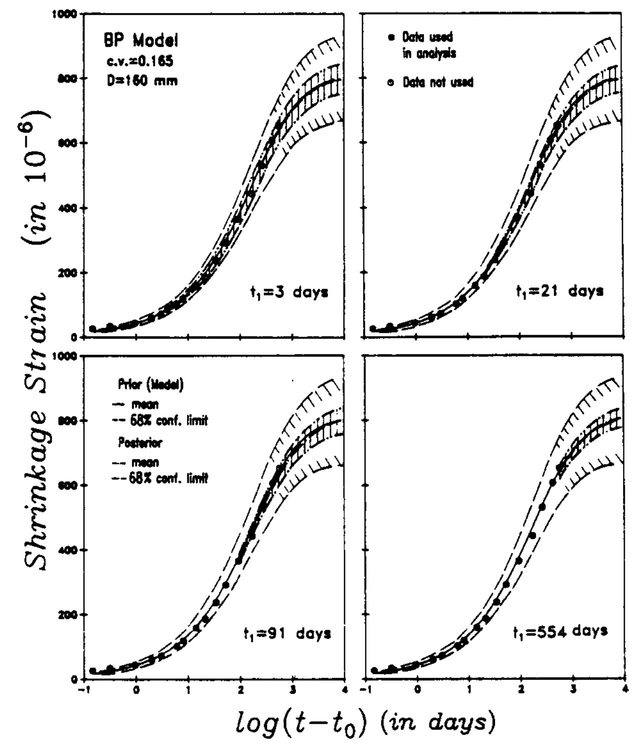

Fig. 2(b) — Bayesian analysis by BP model for D = *160 mm (6.30 in.)*

of τ_0 to be 0.25. The coefficient of variation of ϵ_∞ can then be obtained from latin hypercube sampling in the same manner as before. We try various values of the coefficient of variation of ϵ_∞ and, by considering two random parameters ϵ_∞ and τ_0, calculate the corresponding coefficient of variation of shrinkage strain ϵ_{sh}, and then select the case that gives overall, for all specimen sizes, the value of the coefficient of variation of ϵ_{sh} that is closest to 52.5 percent. In this manner, we obtain for the random material parameters in the ACI model the values in Table 3.

CEB-FIP model

By analyzing the test data in the literature, the coefficient of variation for the deviations of the shrinkage strain from the formula was determined as 71.7 percent.[4] In this model, the function $\beta_s(t)$ has been defined by graphs, and thus it is impossible to identify any random parameters for this function. Therefore, we can consider for this model only one random parameter, the final shrinkage ϵ_∞, which must then have the coefficient of variation 71.7 percent. The statistics of the parameters of this model are listed in Table 4.

BAYESIAN PREDICTIONS FOR THE PRESENT TEST SERIES

Using the shrinkage measurements carried out at Swiss Federal Institute of Technology in Lausanne,[1,19] we apply the present procedure to generate Bayesian predictions of the mean and of the confidence limits for future shrinkage strains, assuming that the test data are known only up to time t_1. Various values of t_1 were considered, and the predictions were compared with the subsequently measured data. The results are shown in Fig. 2, 3, and 4 for the various prediction models — BP, ACI, and CEB-FIP, and various specimen sizes. The curves of the mean are plotted, as are those of the mean plus-minus the standard deviation, which represent the 68.4 percent confidence limits (i.e., limits such that 15.8 percent of the data points would fall above the band, and 15.8 percent below the band). We do not show the 95 percent confidence limits used in the preceding paper on regression because, for the ACI and CEB-FIP models, these limits are so wide that the lower one would go into negative shrinkage values in some cases (Fig. 3 and 4). We show the mean and the confidence limits both for the prior and for the posterior

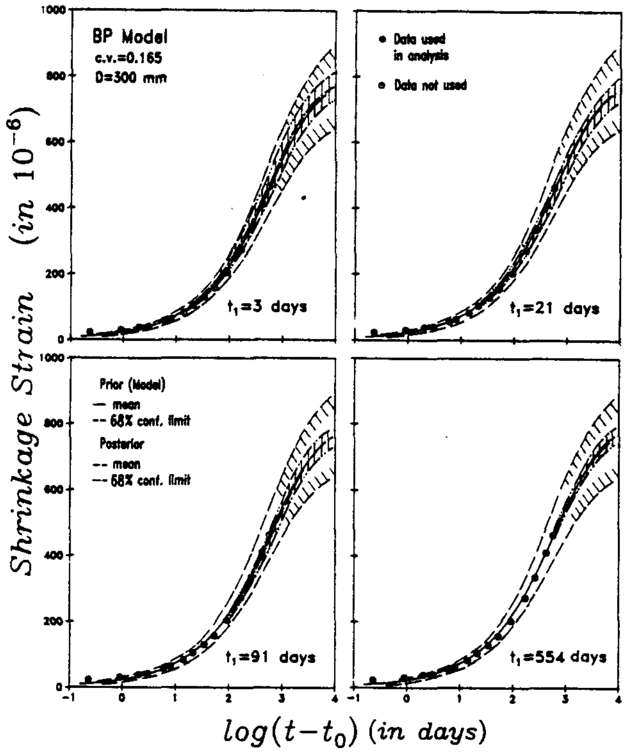

Fig. 2(c) — Bayesian analysis by BP model for D = *300 mm (11.81 in.)*

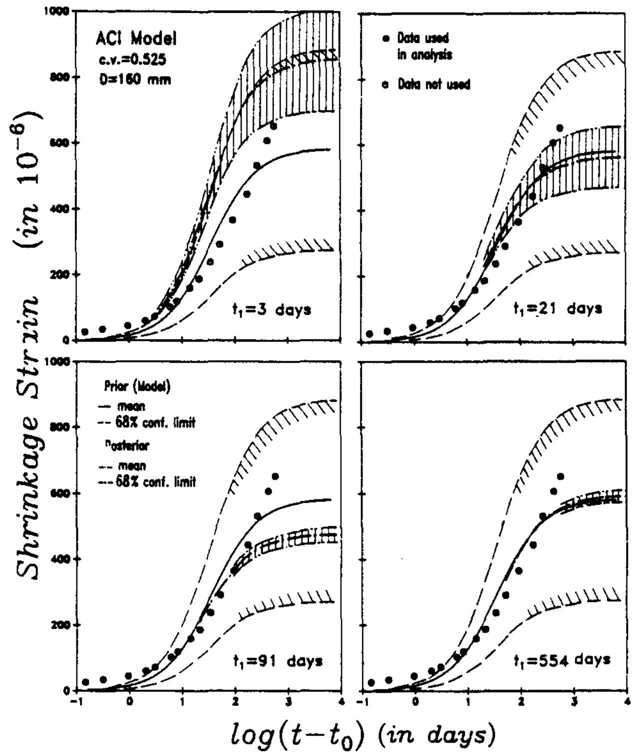

Fig. 3(b) — Bayesian analysis by ACI model for D = *160 mm (6.30 in.)*

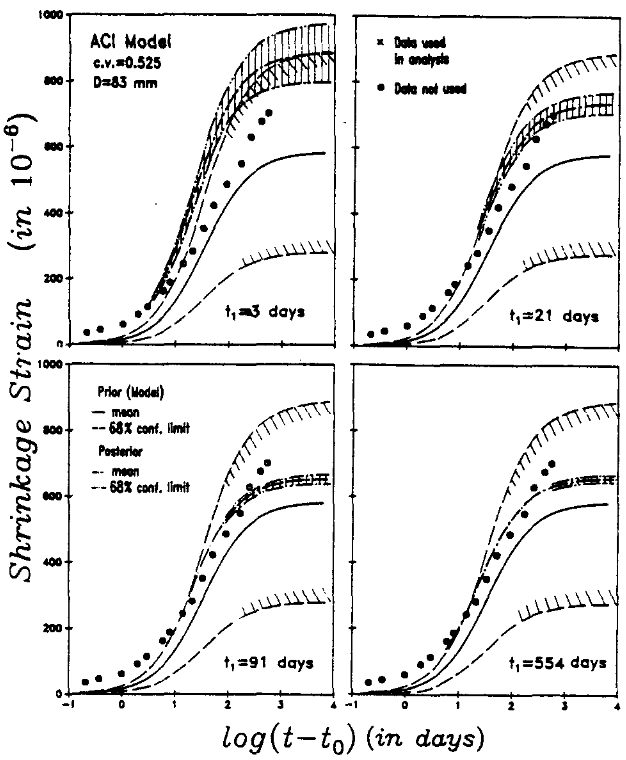

Fig. 3(a) — Bayesian analysis by ACI model for D = *83 mm (3.27 in.)*

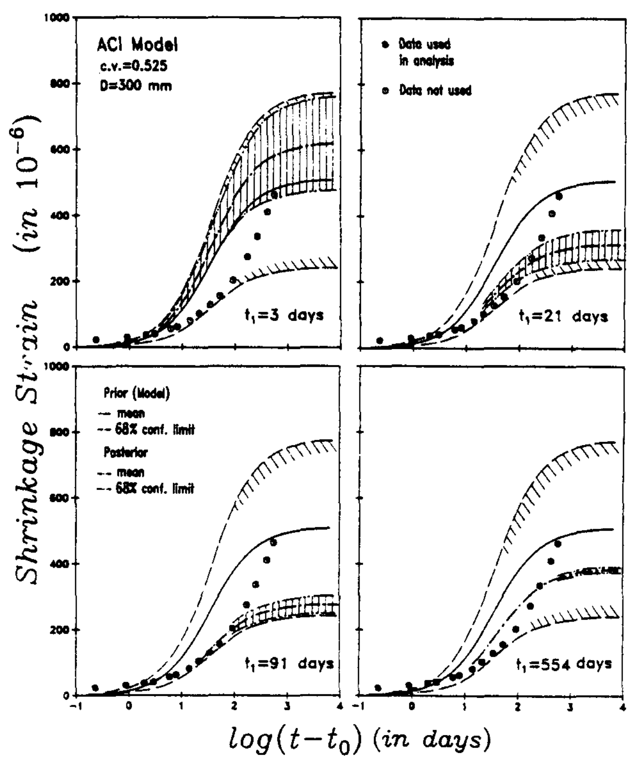

Fig. 3(c) — Bayesian analysis by ACI model for D = *300 mm (11.81 in.)*

obtained by Bayesian prediction (the band of posterior confidence limits is cross-hatched). All the results shown were calculated using 32 intervals for latin hypercube sampling for each random parameter; 16 intervals were also tried, but gave significantly different predictions, while for 64 and 96 intervals the predictions differed little. Since repeated runs for the latin hypercube samples did not yield the same results and scattered up to about 15 percent from the average, the entire analysis procedure to obtain the posterior predictions was repeated 10 times, and the average of the 10 runs has been plotted in the figures.

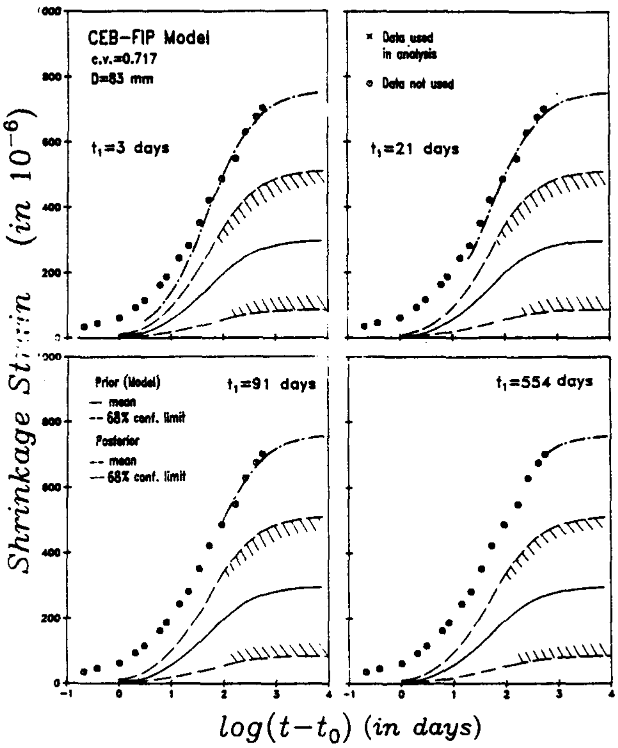

Fig. 4(a) — Bayesian analysis by CEB-FIP model for D *= 83 mm (3.27 in.)*

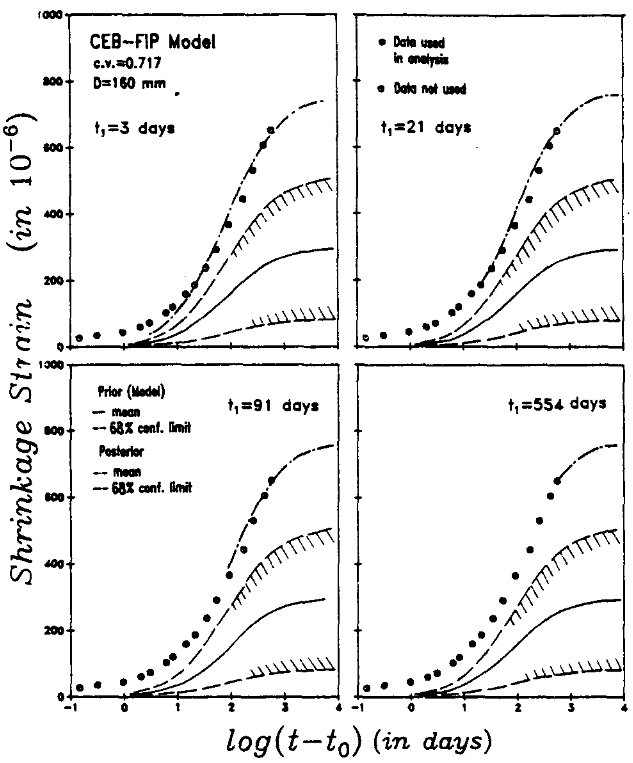

Fig. 4(b) — Bayesian analysis by CEB-FIP model for D *= 160 mm (6.30 in.)*

Compared to the predictions by statistical regression in the preceding paper,[1] the present Bayesian approach yields considerably better predictions; they better agree with the subsequently measured data not used for the prediction, and they have a considerably narrower scatter band. Fig. 2(a) through (c) show that even 3 days of measurement of shrinkage yield good long-time predictions, as far as the present data can show (of course, the final shrinkage value has not yet been approached in these tests, and our conclusions should be reexamined after this infomation becomes available).

The results predicted for the ACI and CEB-FIP models in Fig. 3 and 4 are conspicuously worse, which confirms that a good, physically based model is required for extrapolation of short-time data (whether Bayesian or just regression).

At the same time, Fig. 4(a) through (c) revealed some deceptive results: the predicted posterior scatter band is very narrow, yet far from the measured data. This reveals a certain inadequacy in the present method of analysis that may occur in several cases. We consider this question next.

LIMITATION OF APPLICABILITY OF THE PRESENT METHOD

The problem is that a poor model, such as that of ACI or CEB-FIP, may give a prior prediction that is relatively far from the measured short-time data. In such a case, the probability of observing the measured values is, according to the prior, very small. Consequently, hardly any latin hypercube samples based on the prior are likely to fall within the scatter band of the measured data, which means that the likelihoods in Eq. (6) are all almost 0. To obtain a prior prediction that would fall within the scatter band of measured data would require extremely many samples. For such a situation, the region for which the likelihood distribution

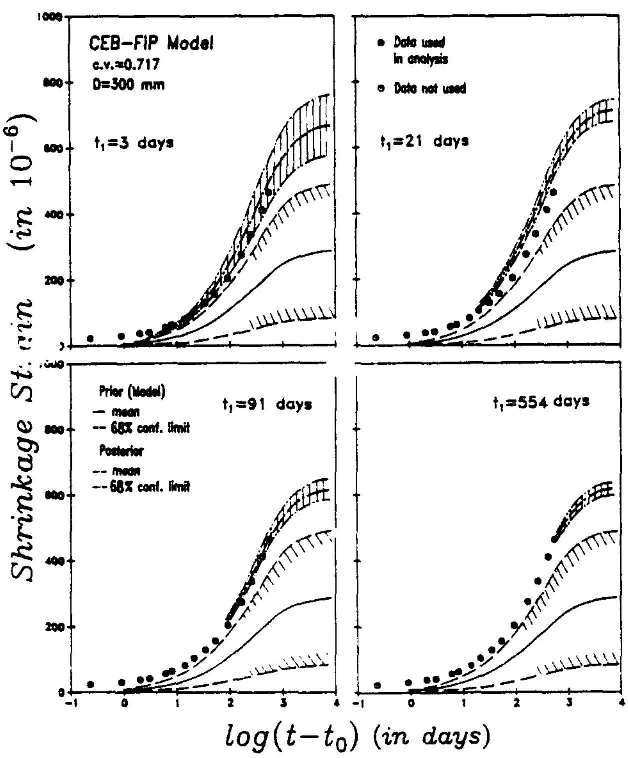

Fig. 4(c) — Bayesian analysis by CEB-FIP model for D *= 300 mm (11.81 in.)*

$f_m^{X'}$ does not have negligible values represents only a tiny island in the N-dimensional space of random parameters $\xi_1, \ldots \xi_N$.

Consequently, the present sampling approach to Bayesian analysis fails if the prior scatter band is far from the scatter band of the measured short-time data, which is likely to occur if the model used for the prior is poor. This underscores the need to use a very good model for Bayesian prediction, with the best possible physical justification. On the other hand, the benefit from the use of such a model is not great when the design is made without any measurements for the given concrete, because in such a case a huge uncertainty due to the effects of concrete composition on shrinkage is inevitably superimposed on the uncertainty of the model.

CONCLUSIONS

1. The recently developed Bayesian analysis[3] that is based on latin hypercube sampling of the random parameters of the prior and on adjustments of the weights of the samples according to Bayes' theorem, is an efficient approach to the extrapolation of short-time shrinkage data into long times, provided that the measured data do not lie at the margins of the scatter band of the prior. If the measured data lie outside the 90 percent confidence band of the prior, the method fails.

2. Compared to the extrapolation of short-time measurements by statistical regression, Bayesian extrapolation can provide significantly better long-time predictions. Good predictions appear to be possible even for measurements with only a three-day duration.

3. The BP formula is useful for Bayesian extrapolation of short-time shrinkage data; however, the ACI and CEB-FIP formulas are not.

4. The measured shrinkage values are rather close to the predictions of the BP model.

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APPENDIX—PREDICTION OF FINAL SHRINKAGE FROM CONCRETE COMPOSITION

Although the formula for ϵ_∞ originally given in the BP model [Eq. (1) through (10) in Reference 4] appears to give satisfactory results, it seems unnecessarily complicated. A simpler and apparently equally good formula is

$$\epsilon_\infty = \epsilon_p (1 - \nu_a), \qquad \epsilon_p = \epsilon_c \nu_w, \qquad \epsilon_c = k_c (\nu_w / \nu_c) \qquad \text{(A1)}$$

in which k_c is supposed to be a constant; ν_a, ν_c, and ν_w represent the volume fractions of aggregate, cement, and water in the concrete mix; and $\nu_a + \nu_c + \nu_w = \nu_v = 1$ where ν_v is the volume fraction of voids due to air in the concrete mix. Eq. (15) is similar to a previous formula of Alou[20], which represents a linearized form of Eq. (A1) applicable for water-cement ratios between 0.4 and 0.55 (by weight).

Although Eq. (A1) has been derived empirically, there exist some theoretical arguments in its favor. To describe the composite nature of concrete, one may assume the simple rheological model in Fig. 5, in which ϵ_p represents the shrinkage of cement paste, the element of stiffness R_1 represents the restraint exerted upon shrinkage of the cement paste by the aggregate of volume fraction v_a, and the element of stiffness R_2 represents the compliance due to the volume of cement paste $v_t = v_c + v_w$ and voids v_v. Expressing these stiffnesses as $R_1 = C_a v_a$ and $R_2 = C_a (1 - v_a)$, in which the same stiffness constant C_a is assumed for both elements, and noting that for the rheological model in Fig. 5 the overall shrinkage is given by the relation $\epsilon = \epsilon_p / [1 + (R_1/R_2)]$, we find that this rheological model yields the first relation in Eq. (A1). Furthermore, the second relation in Eq. (A1) (in which ϵ_c may be regarded as the shrinkage of hydrated cement grains) may be also obtained on the basis of this rheological model if one assumes $R_1 = C_w (1 - v_w)$ and $R_2 = C_w v_w$, similarly as before. Finally, for the shrinkage of the hydrated cement grains it is logical to assume that it is proportional to the fraction of cement grains that has been hydrated, which in turn may be assumed to be proportional to the water-cement ratio, v_w/v_c; from this the last relation in Eq. (A1) ensues.

Note that Eq. (A1) introduces indirectly the effects of various influencing factors that are not seen to appear in this relation. For example, the second relation in Eq. (A1) seems to neglect the effect of the aggregate content on the shrinkage of cement paste, but this is not so since v_w is the volume fraction of water within concrete, which is smaller for a higher aggregate fraction. Also, Eq. (A1) seems to neglect (contrary to the relation in Reference 4) the ratio of fine to crude aggregate (sand to gravel). Again, this is not so, because a change in the sand-gravel ratio requires a change in the water-cement ratio so as to maintain proper workability of the concrete mix. In this manner, the effect of the sand-gravel ratio is brought in by means of the effect of the water-cement ratio. Contrary to Reference 4, the strength of concrete does not appear in Eq. (A1); however, it does have an indirect effect because the strength of concrete is a function of the water-cement ratio and the cement and aggregate fractions.

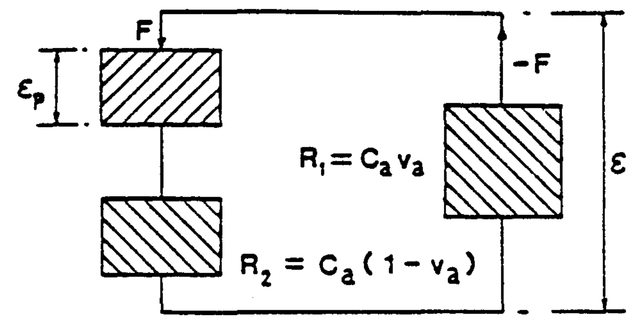

Fig. 5 — Rheological model for interaction of aggregate, cement, and water as a composite [Eq. (A1)]