Materials and Structures, 1993, **26**, 567–574

Improved prediction model for time-dependent deformations of concrete: Part 7 — Short form of BP-KX model, statistics and extrapolation of short-time data

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For structures that do not have a high sensitivity to creep or for preliminary design of any structures, practising engineers demand a short formula for predicting the material creep properties. Such a formula is given in the present addendum to a previous six-part paper. It is based on optimal fitting of the previously published log–double power law to the formulae of the BP–KX model. A simple formula giving directly the compliance function rate is also presented. Finally, a simple method of improving the predictions on the basis of short-time measurements is described, and tables giving the statistics of the deviations of the prediction formulas of the simplified model and its short form from the data in the literature are presented.

1. INTRODUCTION

The simplified creep prediction model called the BP–KX model, presented in Part 6 of a previous six-part paper [1], is already quite simple and its evaluation normally takes much less time than the typical structural analysis for creep in which the model is used. However, the committees of engineering societies demand a formula that is not only simple, but also short. Therefore, such a short form will now be presented. It is based on least-squares optimal fitting of the formulae of the BP–KX model, which were previously justified by extensive comparisons with virtually all the existing, sufficiently documented test data from the literature, consisting of 347 different data series.

2. PROPOSED FORMULAE

The average compliance function for the cross-section of a long member, representing the sum of the instantaneous deformation, the basic creep and the additional creep due to drying, can be expressed as

$$J(t, t') = q_1 + C_0(t, t') + C_d(t, t', t_0) \qquad (1)$$

from which the creep coefficients result as

$$\varphi(t, t') = E(t')J(t, t') - 1,$$

and

$$C_0(t, t') = q_0 \ln\{1 + \psi_1[(t')^{-m} + \alpha](t - t')^n\} \qquad (2)$$

$$C_d(t, t', t_0) = q_5 k_h \varepsilon_{sh\infty}\left[\tanh\left(\frac{t - t_0}{2\tau_{sh}}\right)^{1/2} - \tanh\left(\frac{t' - t_0}{2\tau_{sh}}\right)^{1/2}\right]^{1/2} \qquad (3)$$

This expression contains three basic empirical parameters, q_1, q_0 and q_5, of dimensions 1/psi (1 psi (lb in^{-2}) = 6895 Pa). The term containing q_0 characterizes the basic creep compliance, and the term containing q_5 the drying creep compliance, i.e. the apparent additional creep due to drying. Furthermore, q_1 is a function of E_0 where E_0 is the asymptotic elastic modulus, which characterizes the strain for extremely short durations and is obtained by extrapolating the short-time creep measurements to zero time (infinitely fast loading). Function $C_0(t, t')$ is the basic creep compliance and $C_d(t, t', t_0)$ is the additional creep compliance due to drying taking place simultaneously with creep; t' = age at loading, t_0 = age at the start of drying, and t = time = current age of concrete (all in days). S and k_h are functions already defined for shrinkage (see Part 6 [1]) and $\varepsilon_{sh\infty}$ = final shrinkage strain for zero humidity and reference conditions.

The basic material parameters q_1, q_0 and q_5 appear in Equations 1–3 linearly, and so they can be easily determined from test data by linear regression. Wherever possible, parameters q_1, q_0 and q_5, or some of them, should be calibrated by fitting available test data for the particular concrete to be used, or at least a similar concrete used in a given geographic zone. Even the use of short-time data, with calibration of only one or two parameters among q_1, q_0 and q_5 (as described in Parts 1–3 [2–4]) is preferable to using no data at all.

In the absence of test data for concrete to be used for the planned structure, one may predict the values of q_1, q_0 and q_5 as follows:

$$q_1 = \frac{0.68 \times 10^6}{E_{28}} \qquad E_{28} = 57\,000(f'_c)^{1/2} \qquad (4)$$

$$q_0 = 0.88 w^{1.58}(\log_{10} f'_c)^{-4.18} \qquad (5)$$

$$\psi_1 = 9.32 \quad \alpha = 0.016 \quad m = 0.75 \quad n = 0.32 \qquad (6)$$

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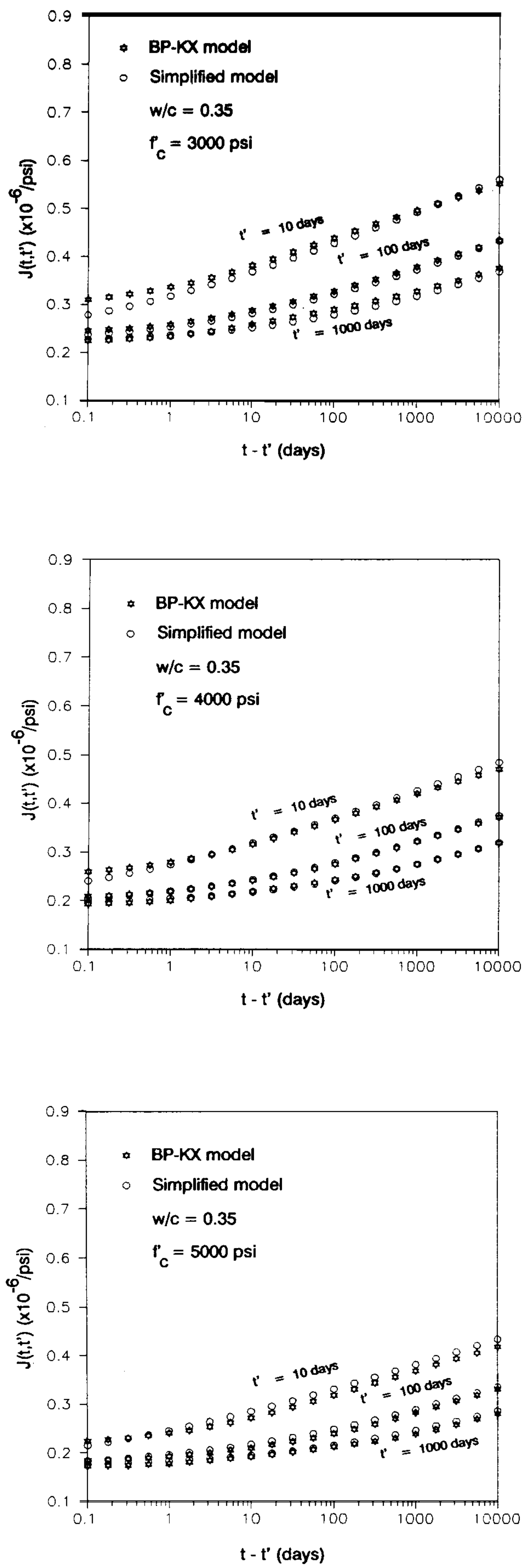


Fig. 1 Comparisons of the present model with the BP–KX model for $w/c = 0.35$.

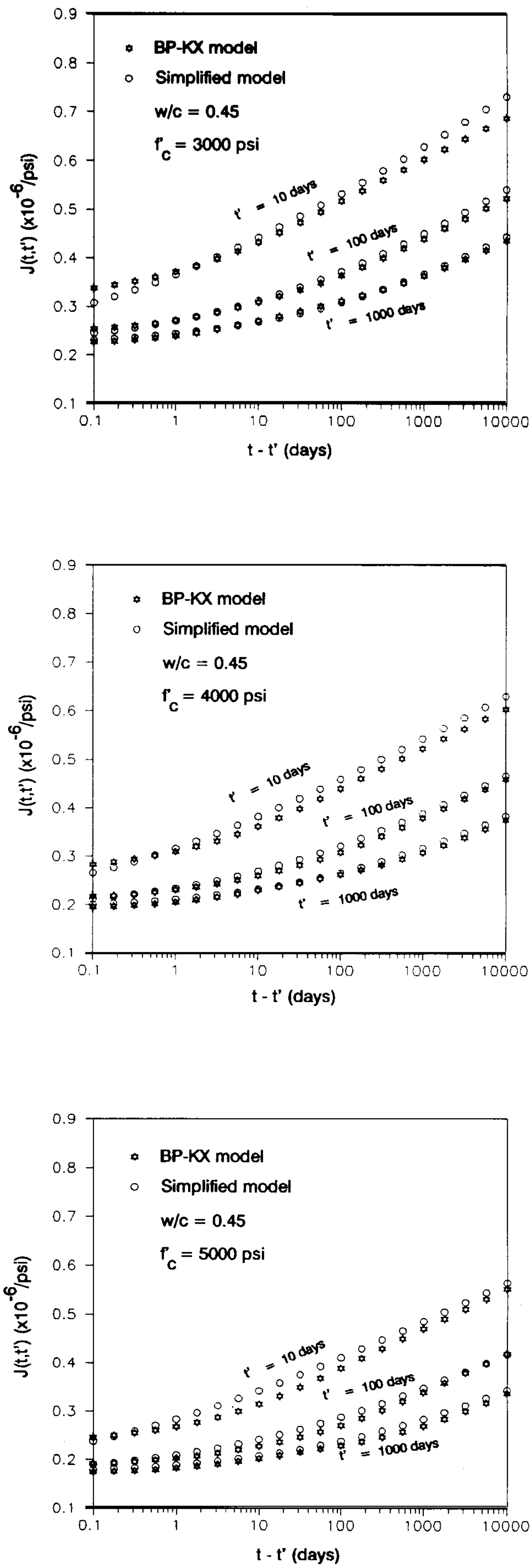


Fig. 2 Comparisons of the present model with the BP–KX model for $w/c = 0.45$.

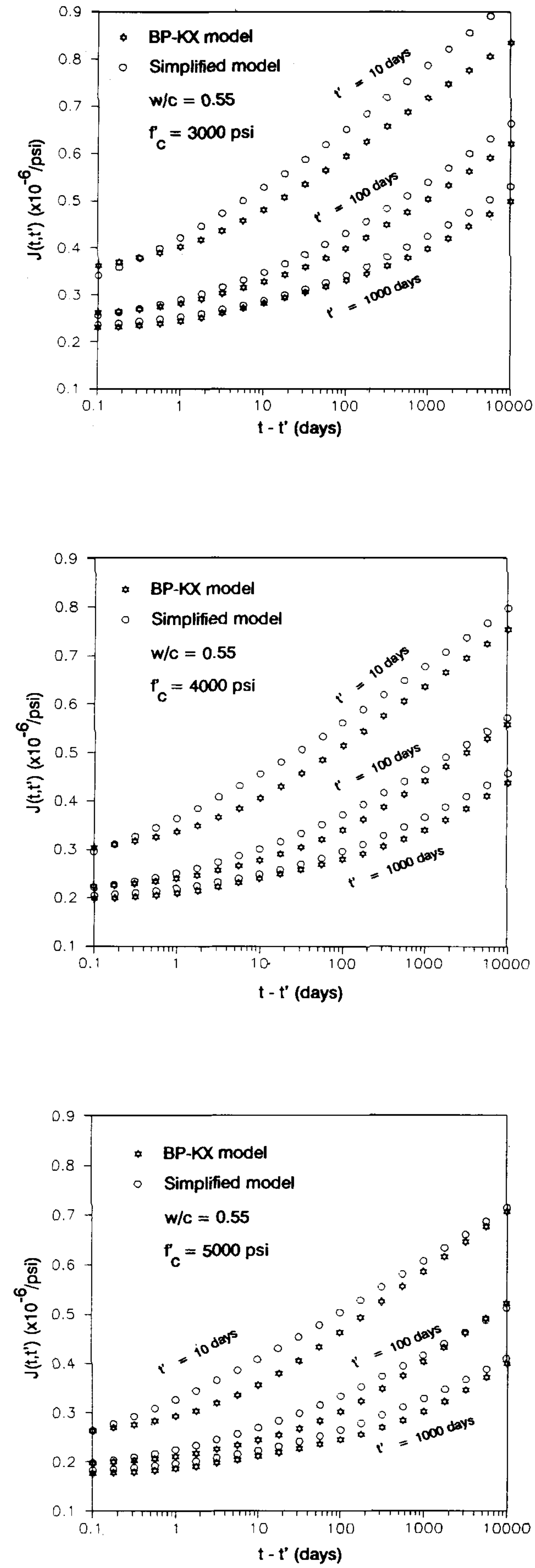


Fig. 3 Comparisons of the present model with the BP–KX model for $w/c = 0.55$.

For drying creep

$$q_5 = 40(f'_c)^{-1/2} \tag{7}$$

Here f'_c is the average 28-day standard cylinder strength in psi; E_{28} (also in psi) is the conventional elastic modulus at 28 days (which is taken here according to the well-known ACI formula); w = specific water content of concrete mix in lb ft^{-3} (1 lb ft^{-3} = 16.02 kg m^{-3}). Note that $w = (w/c)c$ where w/c = water/cement ratio by weight and c = specific cement content.

3. COMPARISON WITH BP–KX FORMULAE AND PARAMETER RANGES

Figs 1–3 show comparisons with the formulae of the BP–KX model from Part 6 [1] for three typical values of the water/cement ratio and standard cylinder strength, with the following parameters being fixed: $c = 300$ kg m^{-3}, $s/c = 2$, $g/c = 3$ where s and g are the sand and gravel contents, respectively.

The parameters in Equations 4–6 have been adjusted so as to give, in these figures, the optimal least-squares fits of the BP–KX formulae, while the formula for q_5 is taken from the BP–KX model.

4. USE OF THE MODEL IN COMPUTER PROGRAMS

Although in computer programs one should always prefer the full prediction model, there might be instances where the present simple form is desired to be used. Efficient computer programs for concrete creep are written on the basis of a rate-type form of the creep law associated with either the Kelvin chain model or the Maxwell chain model [5]. In that case, it is convenient to convert or approximate the log–double power law involved in the present model to the approximate equivalent formulae for the full BP–KX model. This conversion may be approximately effected by replacing $C_0(t, t')$ given by Equation 2 with the sum of the terms involving q_2, q_3 and q_4 in the BP–KX model [1] in which $q_2 = q_0(q_2/q_0)$, $q_3 = q_0(q_3/q_0)$, $q_4 = q_0(q_4/q_0)$ and the ratios are taken according to the formulae for q_2, q_3 and q_4 given in the BP–KX model. Then the formulae given by Bažant and Prasannan [5] may be used to obtain the equivalent rate-type model based on the Kelvin chain with ageing.

5. BETTER ALTERNATIVE FOR STEP-BY-STEP COMPUTER ANALYSIS

The basic creep formulation resulting from the solidification theory has a much simpler form if one specifies the compliance rate $\dot{J}(t, t') = \partial J(t, t')/\partial t$ instead of the compliance function itself. For computer solutions this is actually more convenient, for two reasons: (1) the step-by-step algorithms for creep structural analysis are based directly on the expression for $\dot{J}(t, t')$, and (2) the expansion into Dirichlet series and conversion of the stress–strain relation to a rate-type form is best done directly on the basis of $\dot{J}(t, t')$. The compliance rate for basic creep, $\dot{C}_0(t, t') = \partial C_0(t, t')/\partial t$ has the following simple form:

$$\dot{C}_0(t, t') = \frac{n(q_2 t^{-m} + q_3)}{(t - t') + (t - t')^{1-n}} + \frac{q_4}{t}$$

$$m = 0.5, n = 0.1 \quad (8)$$

in which t and t' must be given in days; q_2, q_3 and q_4 are three empirical parameters (constants) of the dimensions 1/psi (1 psi = 6896 Pa). The terms containing q_2, q_3 and q_4 represent the rates of the ageing viscoelastic compliance, of the non-ageing viscoelastic compliance and of the flow compliance, respectively. The rate of drying creep compliance is obtained by differentiating Equation 3. The reason for using in Equation 1 the approximation by log-double-power law is that the integral of Equation 8 cannot be expressed in a closed form.

It may be noted that the foregoing expression for $\dot{C}_0(t, t')$ satisfies the requirement of non-divergence of creep curves for different ages at loading. The approximate expressions for $J(t, t')$ in Equations 5–8 of Part 2 [3] and Equations 11–14 of Part 6 [1] satisfy this requirement only approximately, but the violations of non-divergence are very small. For the log-double-power law in Equation 2 the violations are not so small, but still acceptable for practical purposes. For other models (ACI, CEB), they are significant.

The material parameters $q_1, q_2, \ldots, q_5$ all appear in Equations 1, 3 and 8 linearly, and so they can be easily determined from test data by linear regression. Wherever possible, parameters $q_1, \ldots, q_5$ should be calibrated by fitting the test data available for the particular concrete to be used, or at least a similar concrete used in a given geographic zone. Even the use of short-time data, with calibration of only one or two parameters among $q_1, \ldots, q_5$ (as described in Parts 1 to 3 [2–4]) is preferable to using no data at all.

In the absence of short-time measurements on the given concrete, one may predict the values of $q_1, \ldots, q_5$ from the concrete strength and composition using Equations 9–12 of Part 2 [3].

6. IMPROVEMENT OF PREDICTION BY LINEAR REGRESSION OF SHORT-TIME DATA

The greatest source of uncertainty of the creep and shrinkage prediction model is the dependence of material parameters on the composition of concrete. This uncertainty can often be greatly reduced by carrying out short-time measurements of the creep and shrinkage of the given concrete [6] and adjusting the values of the parameters q_1 and q_0 in Equations 1–3 accordingly, instead of determining them from Equations 4 and 5. In contrast to other models, including the original BP model, the solidification theory, which is the basis of the present model, has the advantage that the adjusted values of the parameters q_1 and q_0 can be easily obtained by linear regression of the short-time test data. Measurements of short-time data are particularly important for special concretes, such as high-strength concrete, because for them the available formulae for taking the composition of concrete into account may involve larger errors.

To illustrate the procedure, consider the drying creep data of Rostasy *et al.* [8], for which the prediction of the present short form is not too good [4]. Let us pretend we know only the data for the first 2 days of creep duration, which are shown in Fig. 4 by the solid circles. We want to use these data points to determine parameters q_1 and q_0. For this purpose, we rewrite Equation 1 as

$$J(t, t') = q_1 + q_0 F(t, t', t_0) \quad (9)$$

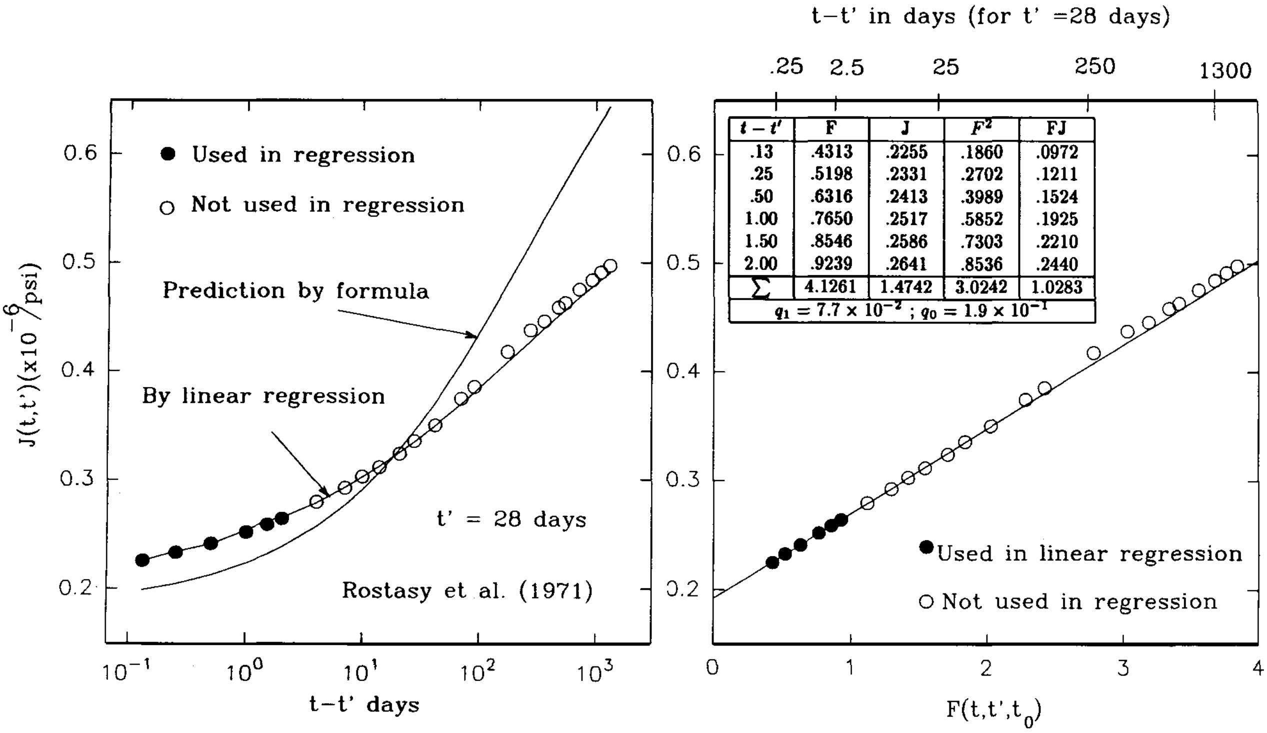
where

$$F(t, t', t_0) = \ln[1 + \psi_1(t'^{-m} + \alpha)(t - t')^n] + \frac{q_5}{q_0} k_h \varepsilon_{sh\infty} \left[\tanh\left(\frac{t - t_0}{2\tau_{sh}}\right)^{1/2} - \tanh\left(\frac{t' - t_0}{2\tau_{sh}}\right)^{1/2} \right]^{1/2} \quad (10)$$

where the ratio q_5/q_0 is evaluated according to Equations 5 and 7 from the composition of the given concrete. If the data agreed with our prediction model exactly, the plot of $J(t, t')$ versus $F(t, t', t_0)$ (Fig. 4) would have to be a single straight line for all t, t' and t_0. Therefore, the vertical deviations of the data points from this straight line represent errors, which are regarded as random and are to be minimised by regression. So we consider the plot of the known (measured) values $Y = J(t, t')$ (up to $t - t' = 2$ days) versus the corresponding values of $X = F(t, t', t_0)$ calculated from Equation 10, and use the method of least squares to pass through these points the regression line $Y = AX + B$. Then the slope A and the Y-intercept B of this line give the values of q_1 and q_0 that are optimum in the sense of the least-squares method; $A = q_0$ and $B = q_1$.

The linear regression calculations and the resulting improved values of q_1 and q_0 are shown in the table in Fig. 4. They are made according to the well-known normal equations $q_0 = [n \sum (F_i J_i) - (\sum F_i)(\sum J_i)] \times [n \sum (F_i^2) - (\sum F_i)^2]^{-1}$ and $q_1 = \bar{J} - q_0 \bar{F}$ derived by minimising the sum of squared vertical deviations from the data points; subscripts $i = 1, 2, \ldots, n$ label the known data points, n is their total number, $F = F(t, t', t_0)$, $J = J(t, t')$, $\bar{J}$ = mean of all the measured $J(t, t')$-values, and $\bar{F}$ = mean value of all the corresponding F. Comparing the creep predicted by regression with the subsequently measured data (empty circles) in Fig. 4, we see that the improvement of long-time predictions achieved by short-time measurements is in this example very significant.

In the preceding example, we had only one measured creep curve. But the method can of course be applied even when there are several short-time creep curves for different t', for example t' = 10 days and 30 days. Then all



$t-t'$	F	J	F^2	FJ
.13	.4313	.2255	.1860	.0972
.25	.5198	.2331	.2702	.1211
.50	.6316	.2413	.3989	.1524
1.00	.7650	.2517	.5852	.1925
1.50	.8546	.2586	.7303	.2210
2.00	.9239	.2641	.8536	.2440
$\sum$	4.1261	1.4742	3.0242	1.0283
$q_1 = 7.7 \times 10^{-2}$; $q_0 = 1.9 \times 10^{-1}$				

Fig. 4 Example of improvement of creep prediction by means of linear regression of short-time data shown by the solid points (the subsequent empty points have been ignored in the regression).

the values measured during the initial periods of both tests, for example 30-day periods, are used in the calculations of $F(t, t', t_0)$.

An experimentalist planning creep measurements should note that the use of short-time data for improving the predictions is more successful if the creep measurements begin at very short times after loading, and if the shrinkage measurements also begin within a very short time after stripping of the mould (preferably within a few seconds after loading or stripping). The reason is that the creep curves have been found to be smooth through the entire range from 0.001 s to 30 years, and the initial shrinkage curves have been shown to follow a simple law, being initially proportional to $\hat{t}^{1/2}$. If, in our example of a short-time creep test, the first reading were taken only 1 day after loading, as is often done, we would have in Fig. 4 only the last three solid points, and obviously from these three points it would be impossible to determine the slope of the regression line with any certainty. To get an equally significant prediction improvement, it would then be necessary to measure creep up to at least 20 days of creep. As for the shrinkage test, it is rendered useless if the first reading is not taken immediately after stripping.

In the case that the time range of the measured data is so short that it is impossible to determine both q_0 and q_1, one must give up on adjusting the slope, i.e., one must take $q_0 = 1$ and determine, by matching of the measured data, only one parameter, q_1. This can be achieved by choosing q_1 so that the centroid of the measured ε_{sh}-values would coincide with the centroid of the corresponding predicted values.

The shrinkage predictions, too, can be improved on the basis of short-time measurements. This problem, however, is not as easy because the regression is not linear. Nevertheless, the following simple procedure is often possible. In the short-form model for shrinkage, which is the same as the simplified model from Part 6 [1], we ignore Equations 5–7 of [1] giving the effect of concrete composition and introduce into Equations 4, 1 and 2 parameters q_5 and q_6 as follows:

$$\tau_{sh} = 0.033 q_6 D^2 \qquad \varepsilon_{sh}(t, t_0) = q_7 \varepsilon_c \tag{11}$$

So we have

$$q_7 = \frac{\varepsilon_{sh}(t, t_0)}{\varepsilon_c} \qquad \varepsilon_c = k_h \tanh\left[\frac{t - t_0}{\tau_{sh}}\right]^{1/2} \tag{12}$$

(note that if our model described the test data exactly, we would have $q_6 = q_7 = 1$). Then we select a sequence of trial values for q_6, preferably a geometric progression such as $q_6 = 0.10, 0.18, 0.32, 0.56, 1.0, 1.8, 3.2, 5.6, 10$ (for which $q_{6,i+1} = 10^{1/4} q_{6,i}$, $i = 1, 2, \ldots$). Because ideally $q_7 \varepsilon_c - \varepsilon_{sh} = 0$, we want to minimize for each q_6-value the sum $S = \sum_i \Delta_i^2 = \sum_i (q_7 \varepsilon_{ci} - \varepsilon_{shi})^2$ where $i = 1, \ldots, N$ labels the measured points and the corresponding predicted values. From the minimising condition $\partial S / \partial q_7 = 0$ we get $q_7 = (\sum_i \varepsilon_{shi} \varepsilon_{ci}) / (\sum_i \varepsilon_{ci}^2)$. For this value of q_7 we then calculate the value of S, and repeat it for all the other q_6 values. Then we select the q_6-value for which S is minimum. There are, however, two caveats to mention:

- The foregoing procedure can work only if the first deformation measurement has been taken right after the stripping of the mould (within a few seconds). If it has been taken much later, a significant part of the initial shrinkage has been missed. This then means that the apparent initial shrinkage measurements do not

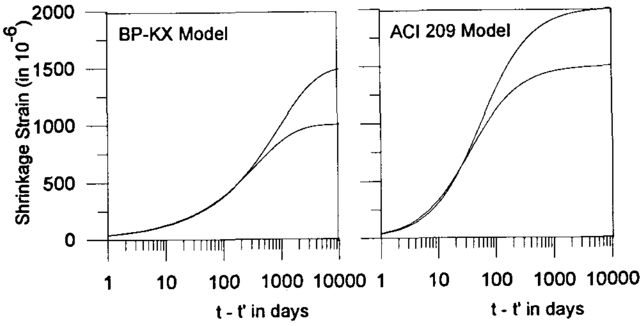


Fig. 5 Example of shrinkage curves giving nearly the same initial shrinkage but very different final values (left: $\varepsilon = 1500\tanh(\hat{t}/1500)^{1/2}$ and $1000\tanh(\hat{t}/600)^{1/2}$; right: $\varepsilon = 2000\hat{t}/(55+\hat{t})$ and $1500\hat{t}/(35+\hat{t})$; $\hat{t} = t - t_0$, and t' here stands for t_0).

evolve in proportion to $\hat{t}^{1/2}$, which means that there exists no simple law to extrapolate such distorted data.

- If the time range of the measured short-time shrinkage data is not sufficiently long, the problem of determining parameter q_7 which characterizes the final shrinkage value is ill-posed, that is, very different q_7 can give almost equally good fits of short-time data. To overcome this problem, it is necessary that the data range would reach to drying times for which the slope of the shrinkage curve in the logarithmic time scale begins to decrease. Otherwise, instead of attempting to calculate q_7, one must fix $q_7 = 1$ and select only the optimum q_6 value.

The second caveat is illustrated in Fig. 5. If the data do not reach significantly beyond the point of crossing of the two curves, both final values seen in the figure agree with the data equally well (except if the data scatter were much smaller than the spread of the two curves before the crossing point, which of course cannot be achieved in practice). The problem occurs not only for the shrinkage formula of the BP–KX model (Fig. 5, left) but also for all other shrinkage formulas, even those which do not give a very good shape of shrinkage curves, for example Ross' hyperbola used in the ACI 209 model (Fig. 5, right). For cylinders of 6 in (15 cm) diameter, reliable determination of the final value would thus normally require shrinkage tests of about 3 years duration, which is of course unacceptable for a designer. One may nevertheless exploit the fact that the shrinkage halftime decreases with the square of specimen diameter and an increase of temperature accelerates shrinkage. In this manner, using cylinders of 2 in diameter (if the aggregate size does not exceed about 0.75 in), one could reduce the aforementioned time to about 50 days.

The problem can be alleviated by a Bayesian statistical approach, exploiting prior knowledge of long-time data for similar concretes; but this can help only partly. Using this approach, one can reduce the duration of short-time shrinkage tests to about one-tenth of the aforementioned derivations, but with the penalty of a rather sophisticated probabilistic analysis exploiting other existing data; see the review in [9].

7. ERROR STATISTICS OF SIMPLIFIED BP–KX MODEL AND OF ITS SHORT FORM

In Parts 1–3 [2–4], the coefficients of variation ω of the vertical deviations of the full BP–KX prediction model from the available test data have been presented. Their values ought to be used in design for estimating the errors of prediction. This is important because the design should not be based on the predicted mean values of creep and shrinkage effects in structures, but on their confidence limits with a certain specified probability of not being exceeded (such as 95%).

The simplified BP–KX model from Part 6 [1] and the present short form of this model have also been compared to the data used in Parts 1–3 [2–4], which cover essentially all the available sufficiently documented data that exist in the literature. The comparisons for shrinkage, basic creep and drying creep are given in Table 1 where $\bar{\omega}$ and $\tilde{\omega}$ are the coefficients of variation of the vertical deviations from the various available data sets for the simplified BP–KX model from Part 6 and of the present short-form model, respectively. The last line of each table gives the overall coefficients of variation, defined as $\omega_{all} = [(\sum_k \omega_k^2)/N]^{1/2}$ where N is the number of data sets considered and ω_k ($k = 1, \ldots, N$) are the coefficients of variation for the individual data sets listed (for the references, see Parts 1–3 [2–4]).

Note that for the simplified BP–KX model the overall coefficients of variation are larger, albeit not much larger, than those listed in Parts 1–3 [2–4] for the full BP–KX model. Those for the present short form are still larger, albeit again not much larger. Further note that if, among the 20 data sets used for shrinkage, only the 12 most favourable data sets were selected, the $\tilde{\omega}_{all}$ value would be reduced from 41.7% to 23.3%. Likewise, if among the 15 data sets used for basic creep only the 7 most favourable data sets were selected, the $\tilde{\omega}_{all}$ value would be reduced from 23.1% (for the short form) to 13.4%. These observations, which are similar to those discussed in more detail in [6], document the dangerous deception hidden in the selective use of test data. Presenting a comparison with 12 or 7 data sets looks like plenty, yet it can be gravely misleading, unless the data to be used were chosen by casting dice.

The foregoing statistics (Table 1) considered all the data points that were reported in the literature sources cited in Parts 1–3 [2–4]. Naturally, it may be expected that a reduction in the coefficient of variation of errors could be achieved by limiting the ranges of parameters to the typical values encountered in practice. Only a mild range limitation, which has been proposed in Part 6 [1] and corrected by the Errata at the end of this paper, has been considered in these statistics:

For shrinkage:

$$3 \le t_0 \le 40 \text{ days} \qquad (13)$$

For creep (basic and drying):

$$3 \le t_0 \le 40 \text{ days} \quad 3 \le t' \le 365 \text{ days} \quad 1 \le g/s \le 3.5 \qquad (14)$$

where $t' \ge t_0$ and g/s is the gravel/sand ratio by weight.

Table 1 Coefficients of variation ω (as percentages) of the deviations of the model predictions from the basic creep test data available in the literature, considering both the full range of parameters and the restricted range defined by Equations 12 and 13; $\bar{\omega}$ refers to the simplified BP–KX model from Part 6 [1], and $\tilde{\omega}$ to the present short form

All Data		
Test data	$\bar{\omega}$	$\tilde{\omega}$
Keeton	24.0	15.0
Kommendant *et al.*	5.1	7.0
L'Hermite *et al.*	48.3	52.8
Rostasy *et al.*	9.1	11.0
Troxell *et al.*	9.3	16.0
York *et al.*	11.7	12.2
McDonald	23.0	24.1
Maity and Meyers	16.5	25.7
Mossiossian and Gamble	18.0	17.0
Hansen and Harboe *et al.* (Ross Dam)	17.5	34.6
Browne *et al.* (Wylfa Vessel)	32.0	31.2
Hansen and Harboe *et al.* (Shasta Dam)	27.9	18.4
Brooks and Wainwright	8.7	20.0
Pirtz (Dworshak Dam)	26.7	37.1
Hansen and Harboe *et al.* (Canyon Ferry Dam)	31.6	24.1
Russel and Burg (Water Tower Place)	17.4	15.5
$\bar{\omega}_{all}$ or $\tilde{\omega}_{all}$	23.1	25.3
Restricted Parameter Range		
Test data	$\bar{\omega}$	$\tilde{\omega}$
Keeton	24.0	15.0
Kommendant *et al.*	5.1	7.0
L'Hermite *et al.*	35.2	39.9
Rostasy *et al.*	9.1	11.0
Troxell *et al.*	9.3	16.0
York *et al.*	11.3	12.2
McDonald	23.0	24.1
Hansen and Harboe *et al.* (Ross Dam)	16.5	32.6
Browne *et al.* (Wylfa Vessel)	29.1	36.1
Pirtz (Dworshak Dam)	26.7	24.1
Russel and Burg (Water Tower Place)	17.4	15.5
$\bar{\omega}_{all}$ or $\tilde{\omega}_{all}$	20.9	23.7

$\bar{\omega}$: Part 6; $\tilde{\omega}$: Part 7.

The values of $\bar{\omega}$ for the simplified BP–KX model from Part 6 [1] and its short form from the present Part 7 have been recalculated for all the data sets excluding all the points outside the aforementioned range. The resulting overall coefficients of variation ω_{all} for such a limited range are listed in Table 2. As can be seen, they are smaller than those for the full range of data, but not much smaller. This has been so for various other mild range limitations, and therefore it appears that by modest restrictions of the range of parameters one cannot reduce the errors of the prediction model significantly.

The manner in which the ω values in Table 1 have been

Table 2 Same as Table 1 but for shrinkage and drying creep

SHRINKAGE	
Test data	$\bar{\omega}$
Hummel *et al.*	30.1
Rüsch *et al.* (1)	31.6
Wesche *et al.*	33.4
Rüsch *et al.* (2)	39.9
Wischers and Dahms	28.6
Hansen and Mattock	25.5
Keeton	56.5
Troxell *et al.*	73.5
Achl and Stökl	53.9
Stökl	35.1
L'Hermite *et al.*	88.6
York *et al.*	33.0
Hilsdorf	27.7
L'Hermite and Mamillan	58.4
Wallo *et al.*	19.7
Lambotte and Mommens	37.3
Weigler and Karl	34.1
Wittman *et al.*	17.5
Ngab *et al.*	12.1
McDonald	14.3
Russel and Burg (Water Tower Place)	30.9
$\bar{\omega}_{all}$	41.7

DRYING CREEP All Data		
Test data	$\bar{\omega}$	$\tilde{\omega}$
Hansen and Mattock	65.6	99.8
Keeton	18.4	10.3
Troxell *et al.*	8.4	10.9
L'Hermite *et al.*	17.9	11.3
Rostasy *et al.*	16.0	21.1
York *et al.*	31.0	48.9
McDonald	37.0	50.5
Hummel	25.1	30.5
L'Hermite and Mamillan	37.7	27.3
Mossiossian and Gamble	8.0	9.1
Maity and Meyers	8.5	64.5
Russel and Burg (Water Tower Place)	18.7	16.2
$\bar{\omega}_{all}$ or $\tilde{\omega}_{all}$	29.1	42.7
Restricted Parameter Range		
Test data	$\bar{\omega}$	$\tilde{\omega}$
Keeton	18.4	10.3
Troxell *et al.*	8.4	10.9
L'Hermite *et al.*	17.9	11.0
Rostasy *et al.*	16.0	21.1
York *et al.*	31.0	48.9
McDonald	37.0	50.5
Hummel	28.9	30.5
L'Hermite and Mamillan	37.7	27.3
Russel and Burg (Water Tower Place)	14.9	15.1
$\bar{\omega}_{all}$ or $\tilde{\omega}_{all}$	25.4	29.1

$\bar{\omega}$: Part 6; $\tilde{\omega}$: Part 7.

calculated has been essentially the same as in [7], in which the original (1978–79) BP model was presented. The sampling method was described in detail in [7]. Optimally, the measurements should be taken at time intervals that are constant in the logarithmic scale of the creep or shrinkage duration [7], i.e., in $\log(t - t')$ or $\log(t - t_0)$ scales, because what matters is the percentage change of the duration rather than the actual change. For example, the differences between the creep values at 10 and 20 days, or those at 1000 and 2000 days, have about the same significance because their ratios are the same, namely 2. But the difference between the creep readings taken at 1000 and 1010 days is insignificant, most of it being due to random errors of the measuring device and test control. At the same time, the creep values at these two times are much more strongly correlated than those at 10 and 20 days, or at 1000 and 2000 days. Consequently, by taking readings at both 1000 and 1010 days one in effect doubles the weight of the reading, compared to that taken at 10 days or at 20 days.

To avoid the subjective bias implied in such an improper spacing of the data points in time, each measured time curve used in the 1978–79 BP model had first been smoothed by hand and the data points to be used for statistics were then placed on the curves at equal intervals in log-time [7]. The hand smoothing also approximately eliminated the random scatter of strain measurements, which ought to be excluded from structural analysis because the response of structures does not depend on the error of measurements [7].

In the present statistical calculations, a slightly different but approximately equivalent approach has been adopted. The log-time scale has been divided into decades and the data points within each decade have been considered as one group. Then the data points in each group have been assigned a weight inversely proportional to the number of points in that group. By virtue of this approach, the statistics in Table 1 have been made virtually free of the subjective bias due to the experimentalist's choice of the reading times. They have been compensated for the subjective bias caused by crowding the readings in one part of the logarithmic time scale and taking sparse readings in another part.

ACKNOWLEDGEMENTS

This work was partially supported by the NSF Science and Technology Center for Advanced Cement-Based Materials at Northwestern University.

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ERRATA FOR PARTS 2, 3 AND 6 [1, 3, 4]

1. In Part 2 [3], p. 411, Equation 12, replace exponent −0.46 by 0.46.
2. In Part 2 [3], p. 410, Equation 9,

$$q_1 = \frac{10^6}{E_0} = \frac{10^6}{1.5E_\infty} \text{ in } 10^{-6} \text{ psi}$$

3. In Part 2 [3], p. 410, lines 5 and 8 below Equation 1, replace t by t'.
4. In Part 3 [4], p. 219, the left-hand side of Equation 5 should be $\varepsilon_{sh\infty}$ rather than ε_{sh}.
5. In Part 6 [1], p. 220, the parameter ranges in Equations 20 and 21 should be the same as in Equations 13 and 14 of the present Part 7.

RESUME

Modèle amélioré de prédiction des déformations du béton en fonction du temps: Part 7 – Forme simplifiées du modèle BP–KX

Dans le cas de structures qui ne montrent pas une forte sensibilité au fluage, ou bien pour le calcul préliminaire de quelques structures que ce soient, les ingénieurs ont besoin d'une formulation simplifiée qui leur permette d'anticiper les propriétés de fluage du matériau. Le complément qu'on donne ici à l'article en six parties précédemment publié propose cette formulation qui repose sur un ajustement optimal de la loi de double puissance logarithmique que l'on connaît déjà aux formules du modèle BP–KX. Une formule simple pour la vitesse de la compliance est aussi présentée. Finalement, on décrit une méthode simple pour améliorer la prévision à longue durée à partir de mesures de courte durée et on donne les statistiques des erreurs des formules présentées.