Size Effect Hidden in Excessive Dead Load Factor

Zdeněk P. Bažant, F.ASCE,1 and Dan M. Frangopol, F.ASCE2

Abstract: The paper shows that the excessive value of the dead load factor in the ultimate load requirements of the current structural design code implies a size effect. The size effect implied, however, does not have a rational form; it cannot distinguish among various types of failure in which very different size effects apply. This size effect partly compensates for the absence of the actual size effect, primarily the size effect due to energy release, from the current code specifications. Therefore, it would be dangerous to reduce the dead load factor without simultaneously introducing size effect provisions into the code. The question of a possible reduction in the dead load factor cannot be separated from the question of size effect, and so the fracture experts and reliability experts must collaborate. Further it is shown that a possible size effect is hidden in a reliability-based code due to the fact that the reliability implied in the code increases with load factor without simultaneously introducing size effect provisions into the code. The question of a possible reduction in the dead load effect to the overall gravity load effect. The overreliability ratio defined in this study may be used to quantify the additional size effect that can be hidden in a reliability-based code.

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Introduction

Structural engineering statisticians in various countries have recently been suggesting that the dead load factor currently used in ultimate limit design is much too large compared to the actual statistical variability of dead loads, especially the own weight of structures, and could therefore be drastically reduced. However, contemplating such a reduction one must realize that an excessive load factor implies a size effect (as pointed out by Bažant 2000). The implied size effect would be removed if the dead load factor were reduced, and so a drastic reduction of the dead load factor would be dangerous if provisions for the size effects in brittle failures were not simultaneously introduced into the code.

The point to note is that the larger the structure, the higher is the percentage of the own weight contribution $D_z$ to the ultimate load $U$. Thus, if the load factor for the own weight is excessive, structures of large size are overdesigned from the viewpoint of plastic limit analysis—the theory underlying the current building codes. However, such an overdesign helps to counteract the neglect of the size effect in the current codes, which is inherent to plastic limit analysis concepts (and to all theories in which the failure is decided by criteria expressed solely in terms of stress and strain, ignoring the energy release rate effect) (Bažant 2000).

There are many instances of brittle failures of reinforced concrete structures exhibiting a large deterministic size effect that has been partially offset by an excessive safety factor. They include the diagonal shear and torsion of beams, punching shear of plates, anchor pullout, bar pullout, bar splice failure, failure of steel-concrete (or concrete-concrete) composite beams due to failing connectors, compression crushing failures of short and slender columns (tied, spiral, or confined in a steel tube), flexural pipe breaks, and bending failures of beams and frames due to compression crushing of concrete (typical of prestressed concrete). They also include all the situations where the strut-and-tie model (i.e., truss analogy) indicates the failure of a “compression strut” rather than steel bars.

The size effect in all these structures is caused by the energy release due to the stable growth of a large crack or cracking band before attaining the maximum load (Bažant 1984, 1993, 1999a,b; Bažant and Planas 1998; Bažant and Chen 1997). The cause of the size effect may be easily understood by noting that, since the strain energy stored in a structure increases roughly quadratically with its size, the energy release per unit advance of the cracking band or fracture increases roughly linearly with the structure size, while the rate of energy consumed by cracking or fracture remains approximately the same. The material strength at different points of the failure surface is mobilized almost simultaneously in small structures and far from simultaneously in large structures. Due to stress reduction caused by localized cracking or fracture, the material failure in large structures propagates through the cross section.

In plain concrete structures, such as gravity dams and arch dams, foundation plates and plinths, and retaining walls, a large deterministic size effect is also present. It is caused by stress redistribution, the extent of which is significant in a material with pronounced heterogeneity because a large zone of cracking develops before the maximum load is attained (this cracking zone represents the fracture process zone of a continuous fracture whose propagation normally begins at maximum load; see Bažant and Planas 1998).

In the case of very large plain concrete structures (e.g., arch dams, massive foundation plinths, and retaining walls), a significant size effect is also contributed by the randomness of material strength, as described by Weibull theory (Bažant and Chen 1997;
Bažant 1999a, Bažant and Novák 2000a,c). However, in reinforced concrete structures failing only after a large stable crack growth, the Weibull statistical size effect is insignificant (as far as the mean size effect is concerned; the standard deviation and statistical distribution of the size effect are, of course, always affected by material strength randomness).

**Design Code Based on Load Factors**

**Consequences of Excessive Dead Load Factor**

Let \( L \) and \( D \) be the internal forces caused by the live load and the dead load, and \( U \) the internal force caused by ultimate loads, i.e., the loads magnified by the load factors. Using the load factors currently prescribed by the building code (ACI Standard 318 1999), one has

\[
U = 1.4D + 1.7L
\]

Consider now that the dead load factor 1.4 is excessive and that a realistic value, justified by statistics of dead load, should be \( \mu_D \). Then the ratio of the required ultimate design value of the internal force to the realistic ultimate value, which may be called the overdesign ratio (Bažant 2000) is

\[
R = \frac{U_{\text{design}}}{U_{\text{real}}} = \frac{1.4D + 1.7L}{\mu_D D + 1.7L}
\]

The effect of \( \mu_D \) on the overdesign ratio is shown in Fig. 1. This effect increases with the contribution of the dead load to the total load.

Let us limit consideration to the dead loads caused by the own weight of structures, which, for example, dominate the design of large span bridges. For a bridge of a very large span, the dead load may represent 90% of the total load, and the live load 10%. In that case, the overdesign ratio is

\[
R = \frac{1.4 \times 0.9 + 1.7 \times 0.1}{\mu_D \times 0.9 + 1.7 \times 0.1}
\]

For small scale tests, which were used to calibrate the present code specifications, the own weight may be assumed to represent less than 2% of the total load. In that case, the overdesign ratio is

\[
R_0 = \frac{1.4 \times 0.02 + 1.7 \times 0.98}{\mu_D \times 0.02 + 1.7 \times 0.98}
\]

Although a precise value is debatable and should be determined by extensive statistics, it seems reasonable to assume that the own weight of a very large structure cannot be underestimated by more than 5%. This means that \( \mu_D = 1.05 \). So,

\[
R = 1.28, \quad R_0 \approx 1.00
\]

It follows that, compared to the tests used to calibrate the code, a structure of a very large span is overdesigned, according to the current theory, by about 28% (Bažant 2000). Such overdesign compensates for a size effect in the ratio 1.28. This is approximately the size effect for very large spans that is unintentionally hidden in the current code specifications. The 28% reduction is also roughly the maximum size effect that can be implied by the dead load safety factor currently used.

**Simple Example of Heavy Large-Span Girder**

Consider now geometrically similar bridge girders of different sizes \( d \), where \( d \) denotes the characteristic size (dimension) of the structure, to which all the other dimensions of the structure are proportional. For example, \( d \) may be taken as the depth of the girder at the support. For the sake of simplicity and as an extreme case, assume that all the dead load is the own weight of a large structure such as a bridge. The weight per unit length of the girder may be written as

\[
q = g_o \rho d^2
\]

Here \( \rho \) is the average weight of the material per unit volume of the structure, and \( g_o = \text{dimensionless factor characterizing the geometry of the cross section.} \]

The bending moment in the critical cross section is expressed as

\[
M = g_m q l^2
\]

where \( l = \text{span}, g_m = \text{dimensionless factor characterizing the structural system (for a simply supported beam or a pair of two cantilevers connected by a hinge, for instance, } g_m = 1/8 \). The nominal stress \( \sigma_{N_0} \) due to own weight, which may, for example, be defined as the critical stress \( \sigma \) due to own weight to be compared with material strength \( \sigma_o \), may be expressed as

\[
\sigma_{N_0} = g_o \rho d
\]

where \( g_o = \text{dimensionless geometry factor.} \]

For a homogeneous beam of a rectangular cross section with equal compressive and tensile yield limits is failing by plastic bending, then

\[
g_o = 4 g_m g_o (l/d)^2
\]

where \( l/d = \text{const when the size effect for geometrically similar structures is considered; for a concrete girder, unprestressed or prestressed, the expression is of course more involved, although Eq. (8) remains valid.} \]

The nominal stress due to the live load is

\[
\sigma_{N_L} = g_L L
\]

where \( g_L \) is again a dimensionless geometry factor.

Now, noting that, in Eq. (2) for the ultimate state, the unfactored dead load contribution to the ultimate internal force is proportional to \( g_o \rho d \) and the unfactored live load contribution is proportional to \( g_L L \), one concludes that the size effect implied by the excessive dead load factor is approximately

\[
\frac{\sigma_N}{\sigma_{N_0}} = \frac{1}{R} = \frac{\mu_D \rho d + b}{1.4 \rho d + b}, \quad a = g_o \rho, \quad b = 1.7 g_L L
\]

where \( a \) and \( b \) are constants if geometrically similar structures are considered.
Inadequacies of Excessive Dead Load Factor as a Substitute for Size Effect

1. Equation (11) is similar to the formula for a quasibrittle size effect in structures failing due to a crack with a finite residual cohesive stress (Bažant and Chen 1997), but the expression would have to appear under a square root. The size effect given by this expression is too strong when \( d \) is close to the value \( b/1.4a \), and too weak when it is not (it can even be stronger than \( d^{-1/2} \)), which is in fact thermodynamically impossible; see Bažant (1993).

2. Equation (11) implies a finite residual strength for \( d \rightarrow \infty \). However, for most situations, a finite residual strength (which corresponds to a finite stress transmitted across the critical crack) cannot be justified, and even if it could the large size limit of Eq. (11) is too large.

3. The hidden size effect implied by the current codes is the same for brittle failures (such as the diagonal shear failure), which do exhibit a size effect, and ductile failures (such as the bending failure due to tensile steel yielding), which do not.

4. Even for brittle failures alone, the own weight is very poorly correlated to the brittleness number which controls the size effect. For example, a very tall column or pier might not be protected by the excessive dead load factor because the own weight might cause no significant bending moment; yet brittle failure due to compression crushing in flexure, which exhibits a size effect, may be caused mainly by horizontal loads such as wind or earthquake, for which the load factor is not excessive. As another example, the Sleipner oil platform (which sank because of a shear failure of the reinforced concrete plate in its tricell) was not protected either because the load was caused by water pressure whose safety factor is not deemed to be excessive. When a differential settlement or shrinkage contributes to brittle failure of a large structure, the own weight and size effect might not be correlated at all.

5. For some brittle failures, 28% as the maximum capacity reduction due to size effect is much too small, for others it is excessive.

6. Equation (11) can never substitute for the Weibull statistical size effect, which is important for the bending of unreinforced cross sections thicker than about 1 m. For example, for fracture a 7 m thick arch dam due to flexure in a horizontal plane, the statistical size effect reduces the flexural strength to about 50% of that observed in standard laboratory tests (see Bažant and Novák 2000b, c) (this implies that, in the case of the Malpasset Dam, the tolerable abutment displacement was only about 50% of that deduced from such laboratory tests without consideration of size effect).

7. Since prestressed concrete structures are generally lighter than un prestressed ones, the size effect implied for them by the code is generally smaller. Yet prestressed concrete structures are more brittle than un prestressed ones and thus generally exhibit stronger size effects.

On the average, though, having in the code at least the hidden size effect implied by the excessive dead load factor is nevertheless far better than having no size effect at all. Without it, many more catastrophic failures of very large structures would probably have occurred in the past.

Remark: In regard to the foregoing point 4, there are many other examples of catastrophes where the size effect must have been a significant contributing factor even though it was not recognized in the official investigations (Bažant 1999b; Bažant and Novák 2001). They include the following: the toppling of the Han–Shin freeway viaduct during the Kobe earthquake in 1995; the collapse of the Cypress Viaduct on Nimitz Freeway in Oakland during the Loma Prieta earthquake in 1989 (e.g., Levy and Salvadori 1992); the collapse of the St. Francis dam near Los Angeles in 1928 (Pattison 1998); the collapse of Malpasset arch dam in French Maritime Alps in 1954 (e.g., Levy and Salvadori 1992; Bažant and Novák 2000c); and the collapse of the Schoharic Creek Bridge on the New York Thruway in 1987 (Swenson and Ingraffea 1991). For a thorough report on the sinking of the Sleipner platform during a submergence test in a Norwegian fjord in 1991, see Jacobsen and Rosendahl (1994); unfortunately, though, the inevitable role of the size effect (despite having been pointed out after the catastrophe by the first author as a consultant to the designer, Det Norske Veritas) was not mentioned.

Design Code Based on Reliability Concept

Size Effect Hidden in Reliability-Based Code

Let us now examine the hidden size effect from the probabilistic viewpoint of a code based on reliability concepts (Cornell 1969; Ang and Tang 1984; Ellingwood 1996; Ellingwood et al., 1980; Frangopol 1999; Frangopol et al. 1998; Ghosn and Frangopol 1999; Nowak 1995). The overdesign ratio, \( R \), can be interpreted from a probabilistic viewpoint as the ratio \( r \) of the real (i.e., actual) reliability index, \( \beta_{\text{real}} \), to the design reliability index (i.e., the reliability index of the structure associated with the main descriptors of random variables implied in a reliability-based code), \( \beta_{\text{design}} \). Therefore, the overreliability ratio \( r \) is defined as follows:

\[
r = \frac{\beta_{\text{real}}}{\beta_{\text{design}}} 
\]

where

\[
\beta_{\text{design}} = \frac{E(R) - [E(D) + E(L)]}{\sqrt{\sigma^2(R) + \sigma^2(D) + \sigma^2(L)}} 
\]

Here \( E(D) \) = mean dead load effect, \( E(L) \) = mean live load effect, \( E(R) \) = mean resistance, \( \sigma(D) \) = standard deviation of the dead load effect, \( \sigma(R) \) = standard deviation of the live load effect, and \( \sigma(L) \) = standard deviation of the resistance.

Although a precise value of \( \beta_{\text{real}} \) should be determined by reliability experts, it seems reasonable to assume two cases:

\[
\beta_{\text{real}}^{(1)} = \frac{E(R) - [\alpha_1 E(D) + E(L)]}{\sqrt{\sigma^2(R) + \alpha_1^2 \sigma^2(D) + \sigma^2(L)}} 
\]

\[
\beta_{\text{real}}^{(2)} = \frac{E(R) - [\alpha_2 E(D) + E(L)]}{\sqrt{\sigma^2(R) + \alpha_2^2 \sigma^2(D) + \sigma^2(L)}} 
\]

In these two equations, it is assumed that the dead load \( D \) specified in a reliability-based code is excessive and that a realistic value, justified by dead load statistics, should be \( \alpha_1 D \) in the first equation and \( \alpha_2 D \) in the second (\( \alpha_1 < 1, \alpha_2 < 1 \)).

Note that \( \alpha_1 \) in Eq. (14) affects only \( E(D) \), while \( \alpha_2 \) in Eq. (15) affects both \( E(D) \) and \( \sigma(D) \). Since a code has to be conservative, both \( \alpha_1 \) and \( \alpha_2 \) must be less than 1 (i.e., \( \alpha_1 < 1 \) and \( \alpha_2 < 1 \)). For typical values of the coefficient of variation of \( D \) and \( L \), \( V(D) = 0.05 \) and \( V(L) = 0.20 \). Fig. 2 shows that the dispersion in the total load effect, \( \sigma^2(D + L) \), decreases with \( E(D) \), but the contribution of \( E(D) \) to this dispersion increases. Due to this fact, as indicated in Figs. 3 and 4, the influence of \( V(L) \) on \( \sigma^2(D + L) \) and \( \beta_{\text{design}} \) respectively, decreases with \( E(D) \) and is negligible for very high values of \( E(D) \), say \( E(D) > 0.90 \).
It is interesting to note that a possible size effect can be hidden in a code based on the reliability index concept since $\beta_{\text{design}}$ is increasing with the contribution of the dead load effect to the total load (see Fig. 4).

The effects of $\alpha_1$ and $\alpha_2$ on the overreliability ratio $r$ are shown in Figs. 5(a and b), respectively. The maximum values of $r$ are indicated in Table 1. As expected, $\alpha_1$ has more influence than $\alpha_2$. However, for the range of values assumed, the difference between these two influences is negligible (less than 2.3%; see Table 1).

Eqs. (12)–(15) characterize “overreliability” in the sense of the Cornell reliability index (Cornell 1969; Madsen et al. 1986), which is accurate only for a noncorrelated Gaussian distribution of the resistance and action of the load. It represents the ratio of the mean value and standard deviation of the safety margin (resistance minus the action of the load), in other words, the inverse of the coefficient of variation of the safety margin.

Overreliability could also be characterized in the sense of Hasofer-Lind reliability index (Hasofer and Lind 1974), which is a more general concept characterizing the smallest distance between the origin and the points lying on the failure surface (design points) in the standardized space of random variables. This

![Fig. 2. Dispersion in total load versus mean dead load](image2)

![Fig. 3. Effect of coefficient of variation of live load on dispersion in total load](image3)

![Fig. 4. Effect of coefficient of variation of live load on reliability index](image4)

![Fig. 5. Effects of (a) $\alpha_1$ and (b) $\alpha_2$ on overreliability ratio](image5)

**Table 1. Overreliability Ratio for $E(D) = 1$**

<table>
<thead>
<tr>
<th>Overreliability ratio</th>
<th>1.00</th>
<th>0.95</th>
<th>0.90</th>
<th>0.85</th>
<th>0.80</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_1 = r(\alpha_1)$</td>
<td>1.00</td>
<td>1.156</td>
<td>1.234</td>
<td>1.314</td>
<td></td>
</tr>
<tr>
<td>$r_2 = r(\alpha_2)$</td>
<td>1.00</td>
<td>1.143</td>
<td>1.214</td>
<td>1.285</td>
<td></td>
</tr>
</tbody>
</table>
AASHTO LRFD Bridge Design Specifications

The design of bridges is governed by the Design Specifications of the American Association of State Highway and Transportation Officials (AASHTO) and follows the Load Resistance Factor Design (LRFD), which is a statistical enhancement of the plastic limit design and thus does not capture the brittle (fracture mechanics aspects) of structural behavior, especially the deterministic size effect in brittle failures due to energy release and the stress redistribution engendered by a large fracture process zone. The total factored load prescribed in the AASHTO LRFD Bridge Design Specifications (1994) is

\[ Q = \eta \sum_{i} \gamma_i Q_i \]  

(16)

where \( \eta \) = load modifier accounting for ductility, redundancy, and operational importance; \( Q_i \) = given loads \((i = 1,2,3,...)\), and \( \gamma_i \) = corresponding load factors.

There are five strength limit states to be checked, depending on the type of load combination to be considered as follows:

- **Strength I**: Basic load combination characterizing the normal vehicular use of the bridge without load.
- **Strength II**: Load combination, without wind, relating to the use of the bridge by owner specified special design vehicles, or by evaluation permit vehicles, or both.
- **Strength III**: Load combination relating to the bridge exposed to wind velocity exceeding 90 km/hr.
- **Strength IV**: Load combination relating to very high ratios of the dead load effect to the live load.
- **Strength V**: Load combination relating to the normal vehicular use of the bridge with wind of 90 km/h velocity.

The standard calibration process for the strength limit states prescribed in AASHTO LRFD Bridge Design Specifications (1994) consisted of trying out various combinations of load and resistance factors on a number of bridges and their components. Combinations that yield a reliability index close to the target value of 3.5 were selected. However, as indicated in the commentary (AASHTO 1994), the calibration process has been carried out for bridges with spans not exceeding 60 m (Nowak 1995).

One of the strength limit states considered in the specifications (i.e., Strength IV) characterizes the case of very high ratios of the dead load effect to the live load effect.

However, for large-span bridges, the ratio of the dead load effect to the live load effect is rather high, and could result in a set of resistance factors different from those found acceptable for short-span and medium-span bridges. It was for this reason that the load combination IV was introduced into the code. It appears that this load combination will govern when the ratio of the force effects of the dead load and the live load exceeds about 7.0.

The dead load of a bridge consists of two main components: \( DC \), i.e., the dead load of structural components and nonstructural attachments, and \( DW \), i.e., the dead load of wearing surfaces and utilities. Likewise, the vehicular load consists of \( LL \), i.e., the vehicular live load, and \( IM \), i.e., the vehicular dynamic load allowance. If only the dead and vehicular loads are considered, the load combinations associated with strength limit states I and IV are as follows:

- **Strength I**: \( 1.25DC + 1.25DW + 1.75(LL + IM) \)  

Strength IV: \( 1.5DC + 1.5DW \)  

(17)  

(18)

If \( DC/(LL + IM) > 7 \), then, according to LRFD, the limit state IV dominates. In that case, the overdesign ratio is

\[ R = \frac{1.5DC + 1.5DW}{\mu_{DC} + \mu_{DW}} \]  

(19)

where \( \mu_{DC} \) and \( \mu_{DW} \) are the dead load multipliers. As already mentioned, it seems reasonable to assume that the own weight of a large span bridge cannot be underestimated by more than 5%. Therefore, \( \mu_{DC} = 1.05 \), at most. However, it is more difficult to predict \( \mu_{DW} \).

Fig. 6 shows that the overdesign ratio \( R \) increases with both the contribution of the own weight to the total dead load and the decrease in \( \mu_{DW} \). Debatable though a precise value is, it seems reasonable to assume that \( R \) lies approximately in the interval:

\[ 1.30 \leq R \leq 1.40 \]  

(20)

Overreliability Ratio According to AASHTO LRFD Specifications

Fig. 7 represents the overreliability ratio \( r \) as a function of the mean own weight, \( E(\DC) \), by assuming a target reliability level of 3.5, i.e., \( \beta_{\text{design}} = 3.5 \). The factor \( \alpha \) in Fig. 7 is identical to \( \alpha_2 \) in
Synergistic efforts. Only collaboration can lead to rational design in size effect. Cracking and fracture is the only way to capture such differences with size effect are available, a realistic computer simulation of reality, which drops from $2.32 \times 10^{-4}$ to $8.84 \times 10^{-5}$.

Overdesign in Combining Realistic Computer Simulations with Excessive Dead Load Factor

If the load capacity of large concrete structures is calculated by finite elements, and if a realistic nonlocal model for distributed cracking or a realistic fracture mechanics model for discrete cracks are used, then the deterministic size effects due to energy release and stress redistribution caused by a large fracture process zone are captured automatically. Normally these finite element results are subjected to the same safety factors as the crude approximations based on the existing code formulas lacking size effect. This practice, however, generates systematic overdesign and overreliability of very large structures.

Realistic finite element analysis of the aforementioned kind ought to be combined with a realistic dead load factor, or with a realistic reliability approach to dead loads.

This of course does not mean that such finite element analysis of large and sensitive structures would be useless. To capture the differences in size effect among different types of failure, such analysis is indispensable. The size effect hidden in the current codes is excessive for safety against some types of failures (failures due to tensile yielding of reinforcing bars), and insufficient for some other failures (slab punching, diagonal shear, torsion, etc.). Aside from those cases for which explicit design formulas with size effect are available, a realistic computer simulation of cracking and fracture is the only way to capture such differences in size effect.

Consequently, finite element specialists and structural reliability experts must not work in isolation. They should engage in synergistic efforts. Only collaboration can lead to rational design procedures.

Conclusions

1. The excessive value of the dead load factor in the ultimate load requirements of the current structural design code implies a size effect.
2. The size effect implied, however, does not have a rational form. It cannot distinguish among various types of failure in which very different size effects apply. For some types, the implied size affect is excessive, for others insufficient.
3. It would be dangerous to reduce the dead load factor without simultaneously introducing size effect provisions into the code.
4. The question of a possible reduction in the dead load factor cannot be separated from the question of size effect.
5. The reliability experts and fracture experts must collaborate in efforts to remedy the code deficiencies.
6. In a reliability-based code, a size effect may be hidden due to the fact that the reliability implied in the code increases with the contribution of the dead load to the overall gravity load effect.
7. The overreliability ratio proposed here may be used to quantify the additional size effect that can be hidden in a reliability-based code.
8. Finite element analysis with realistic cracking and fracture models automatically captures the size effect, but when it is combined with the currently prescribed load factors it generates systematic overdesign and overreliability of many large and heavy structures. A much lower dead load factor, justified by statistics, should be used in conjunction with such realistic computer analysis.
9. To develop a rational procedure for a design-based computer analysis of structures, the researchers in finite element analysis of concrete structures and in structural reliability must work in synergy.

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