Title no. 106-S02

Does Strength Test Satisfying Code Requirement for Nominal Strength Justify Ignoring Size Effect in Shear?

by Zdeněk P. Bažant and Qiang Yu

A recent University of Toronto test of a 925 mm (36.4 in.) deep beam without stirrups showed a shear strength v_c that is only slightly below the value $v_c = 2\sqrt{f_c'}$ required by ACI 318-08, and comfortably above the value $\phi \times 2\sqrt{f_c'}$ (where $\phi = 0.75$ is the strength reduction factor, and v_c and f_c' are in psi). On that basis, and in view of the safety provisions of the Code, it is often thought that the current shear strength provisions for beams up to 0.2 m (8 in.) deep, which neglects the size effect, are safe for beams up to 1 m (40 in.) deep. This is not true, however, for it must be expected that if numerous tests of 1 m (40 in.) deep beams with different shear spans and steel ratios, made of different concretes and under different hygro-thermal conditions, could be carried out, the beam strength would exhibit a similar statistical scatter, with approximately the same coefficient of variation (CoV), as the strength of beams up to 0.2 m (8 in.) deep, for which there are numerous test results in the database. Based on this expected scatter, it is shown that neglecting the size effect for beams up to 1 m (40 in.) deep is likely to increase the expected frequency of failures from approximately 1 in a million to approximately 1 in a thousand when the beam depth increases from 0.2 to 1 m (8 to 40 in.).

Keywords: deep beams; shear strength; size effect; stirrups.

INTRODUCTION

Although the basic theory of size effect in the shear failure of reinforced concrete beams was formulated more than two decades ago and experimental evidence has become overwhelming,[1,2] the ACI 318 Code has not adopted size effect provisions for beams of depths d up to 0.6 m (24 in.) and even 1 m (40 in.). In support, a recent experiment (Specimen BN100 at the University of Toronto[3,4]) was invoked, in which the strength of such a beam was almost equal to the nominal strength required by the Code and was much larger than the strength obtained after applying the understrength (or strength reduction) factor ϕ. The purpose of this paper is to show that such suggestions are unjustified and could likely lead to statistically dangerous designs with insufficient safety margins.

RESEARCH SIGNIFICANCE

The understanding of failure probability is essential for improving the design provisions for shear failure of reinforced concrete. The importance of this problem is demonstrated by a number of disasters in which the size effect in shear failure has recently been shown to have played a role. If the size effect is ignored or not predicted correctly, the failure probability becomes higher than what the risk analysis experts consider as acceptable. Because of a trend to larger structures, this is an issue of paramount significance for concrete engineering.

HOW TO INTERPRET DATABASE FOR SIZE EFFECT IN BEAM SHEAR

The size effect for beams without stirrups was experimentally demonstrated by Kani[5] for beams with an effective depth d up to 1.1 m (43 in.), and by Iguro et al.[6] and Shioya and Akiyama[7] for depths up to 3.0 m (118 in.). A very systematic size effect for beams (of the highest brittleness number so far) was demonstrated by tests of reduced-scale beams at Northwestern University.[8] Recently, University of Toronto tests[3,4] of three-point-bend beams without stirrups that were approximately geometrically similar and had depths ranging from 0.11 to 1.89 m (4.3 to 74.4 in.), extended the experimental evidence of size effect and showed that the strength of the largest test beam was 53% less than the nominal strength according to ACI 318-08.[9] To guard against such a situation, Section 11.4 of ACI 318-08[9] severely penalizes any beams without stirrups more than 254 mm (10 in.) deep by reducing the shear strength limit from $v_c = 2\sqrt{f_c'}$ (where v_c and f_c' are in psi) to $v_c = \sqrt{f_c'}$ (in effect, this implies a size effect factor of 2) (refer to Section 11.4.6.1 in ACI 318-08[9]).

In one test series at the University of Toronto,[3,4] a single beam was tested for each size; see the diamonds in Fig. 1(a), where d is the effective beam depth (from top face to the centroid of longitudinal reinforcement at the bottom) and $v_c = V_u/b_w d$ is the nominal shear strength measured (V_u is the applied shear force and b_w is the beam width). The figure also shows the horizontal line of $v_c = 2\sqrt{f_c'}$, which represents the nominal strength, that is, the design shear strength, which must exceed the effect of design loads multiplied by their load factors and divided by the understrength factor ϕ for shear, which is 0.75 according to ACI 318-08.[9] The load factor is in this figure considered as 1.6, which applies to the live load (refer to a following comment on the combinations of live and dead loads).

Note in Fig. 1(a) that all the data points (plotted as diamonds) except the last one, that is, all those up to the depth of 1 m (40 in.), lie above the horizontal line of $v_c = \phi \times 2\sqrt{f_c'}$, where $2\sqrt{f_c'}$ is the nominal strength required by the standard ACI 318-08.[9] Based on this observation, it has often been suggested that the size effect need not be taken into account for beam depths up to 1 m (40 in.) and that any considerations of size effect might simply be avoided by banning beams without stirrups having a depth over 1 m (40 in.). If the full picture is considered, however, it transpires that this suggestion is imprudent, in several respects (note that f_c' is taken as 70% of the required average compressive strength f_{cr}' from standard tests, which approximately corresponds to ACI 318-08,[9] Section 5.3.2.2; the fact that the 70% reduction must be

ACI Structural Journal, V. 106, No. 1, January-February 2009.
MS No. S-2006-479.R2 received August 22, 2007, and reviewed under Institute publication policies. Pertinent discussion including author's closure, if any, will be published in the November-December 2009 *ACI Structural Journal* if the discussion is received by July 1, 2009.

Zdeněk P. Bažant *FACI, is the McCormick Institute Professor and W.P. Murphy Professor of Civil Engineering and Materials Science at Northwestern University, Evanston, IL. He is a Registered Structural Engineer in Illinois. He was the founding Chair and is a member of Joint ACI-ASCE Committee 446, Fracture Mechanics of Concrete, and a member of ACI Committees 209, Creep and Shrinkage in Concrete; 348, Structural Safety; and Joint ACI-ASCE Committees 334, Concrete Shell Design and Construction; 445, Shear and Torsion; and 447, Finite Element Analysis of Reinforced Concrete Structures.*

Qiang Yu *is a Postdoctoral Research Associate at Northwestern University. He received his BS from Tongji University, Shanghai, China; his MS from Tsinghua University, Beijing, China; and his PhD from Northwestern University. His research interests include fracture behavior and size effect in concrete structures.*

considered in failure probability analysis was established in detail by Bažant and Yu[10]).

First it should be pointed out that, according to the theory now generally accepted in the fracture mechanics community,[11] beams with stirrups must also suffer from size effect, albeit to a lesser extent. But this issue will better be relegated to a separate study.

Second, several kinds of theoretical arguments based on quasibrittle fracture mechanics have shown that the size effect is significant for beam depths from 100 mm (4 in.) up. This is evidenced by the trend of the data in Fig. 1(a), as well as those reported by other researchers.[3-8,12,13] This point will also be left aside because it is analyzed in depth elsewhere.[1,2,14]

Third, and regardless of the previous points, this reasoning is flawed statistically. The uncertainty in shear of beams of varying sizes cannot be treated as a problem of simple population statistics. Rather, it is a problem of statistical regression because the data exhibit a statistical trend. This is analyzed in the following.

WHAT IS STATISTICAL DISTRIBUTION OF SHEAR STRENGTH OF SMALL BEAMS?

While the probability density distribution (pdf) of strength scatter due to material randomness has recently been theoretically established for quasibrittle failures at crack initiation (Type I[15-17]), for those occurring after large stable crack growth (Types II or III[18,19]) it still remains unknown. Because the latter is the case, the choice of the pdf type must be empirical. But even if the pdf of scatter originating from material randomness were known, it would apply only to the scatter observed in carefully controlled laboratory test series such as those conducted at the University of Toronto[3,4] and Northwestern University[8] (refer to Fig. 1(a) and Fig. 1 in Reference 2, for which the values of the CoV [standard error of regression normalized by data centroid] are only approximately 6.9 and 12%, respectively).

The errors of the current code formula $v_c = 2\sqrt{f_c'}$ are approximately characterized by the scatter seen in the ACI 445F database[20] (Fig. 1(b)), which originates from material randomness only to a minor extent. Because this formula must apply to a broad variety of beams used in practice, the database covers a wide range of secondary characteristics such as the steel ratio, shear-span ratio, and concrete type (which includes concrete strength, curing environment, water-cement ratio, aggregate-cement ratio, cement type, and other mixture proportions). While the scatter of these secondary characteristics is the result of human choices, it roughly reflects the range of characteristics occurring in practice (even though the distributions of these characteristics in design practice might not be exactly the same as in the database, there is no better information to use).

Even if one considered the recently proposed refinement[2] in which the effects of the secondary characteristics such as the steel ratio, shear-span ratio, and concrete type are incorporated into the formula for v_c, their representation would be only approximate, with a high degree of uncertainty. So, the scatter due exclusively to material randomness, exemplified roughly by the aforementioned laboratory tests at the University of Toronto and Northwestern University, would still be only a minor part of the overall scatter. This is revealed by the width of the scatter band seen in Fig. 5(b) through (d) of Reference 2 where the regression does take the secondary characteristics into account. The CoV of regression errors in that scatter band is of the order of $\omega_1 \approx$ 20%, while the CoV due to material randomness per se is of the order of $\omega_2 \approx 5$ to 10%.

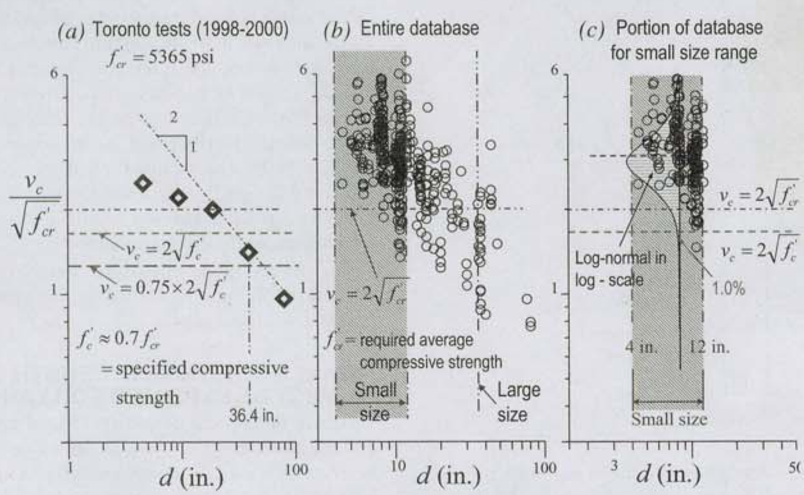

Fig. 1—(a) University of Toronto tests[3,4] of shear strength of beams of various sizes; (b) ACI 445F database of 398 data points; and (c) portion of the database for beams from 4 to 12 in. (100 to 300 mm) deep (v_c, f_{cr}', and f_c' are in psi). (Note: 1 in. = 25.4 mm; 1 MPa = 145 psi.)

To make this argument precise, note that if the points of a database whose CoV = ω_1 are perturbed by independent random scatter whose CoV = ω_2, then the resulting scatter of the perturbed database will have the CoV of $\omega_3 = (\omega_1^2 + \omega_2^2)^{1/2}$. In the present case, $\omega_3 \approx 20\%$ and $\omega_2 \approx 10\%$, which gives $\omega_1 \approx 17.3\%$. This is only 13% less than ω_3. Obviously, ω_2, ensuing from material randomness, has only a minor effect on the overall ω_3, and so its pdf type cannot matter much.

To decide which data to use for an empirical basis of the pdf choice, note that the scatter band in the ACI 445F database (Fig. 1(b) with 398 data points[20]) has a downward trend with respect to depth d (this is also confirmed by the earlier databases of 296 points assembled by Bažant and Kim[21] and 461 points assembled by Bažant and Sun[22]). The existence of a marked size effect trend becomes even clearer if the influences of shear span, steel ratio, and concrete strength are taken into account as subsidiary parameters in the regression (refer to Fig. 5 in Reference 2). Therefore, the entire ACI 445F database cannot be treated as a statistical population from which to identify the pdf of shear strength.

However, if one isolates from the database in Fig. 1 (b) the data in the small size range of depths d ranging from 100 to 300 mm (4 to 12 in.), centered at 200 mm (8 in.) as shown in Fig. 1(c), then the size effect trend is weak enough for treating the data as a population with no statistical trend (indeed, within this range, the size effect in the Toronto tests[3,4] causes a strength reduction of only approximately 10%). The mean and CoV of this population of data are found to be $\bar{y} = v_c/\sqrt{f_c'} = 3.2$ and $\omega = 27\%$. The relatively

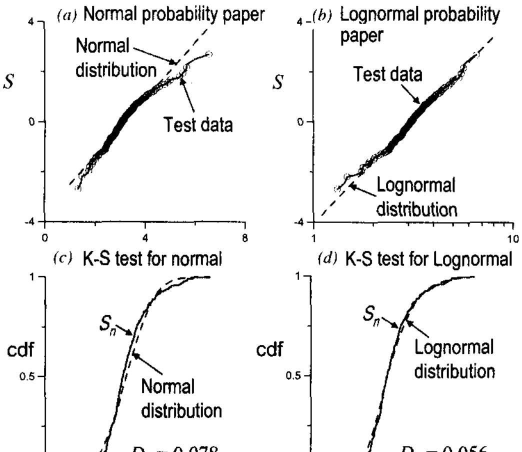

Fig. 2—(a) Cumulative histogram of data on normalized beam shear strength for small beams extracted from the ACI 445F database, plotted on normal probability paper, and their straight-line fit; (b) ditto on log-normal probability paper; (c) K-S test for normal distribution; (d) K-S test for log-normal distribution; and (e) Chi-square test for goodness of fit.

high value of ω is a consequence of variability of the secondary characteristics that have nonnegligible influence on the shear strength.

To determine the appropriate pdf of shear strength for the small size beams, one can plot the data points from the small size range as cumulative histograms on various types of probability paper. While several methods[23,24] to calculate the cumulative histogram are used in practice, Gumbel's method[25] is adopted herein due to clarity of its justification as well as simplicity; the plotting positions are $m/(n + 1)$, where m is the m-th point among the data arranged in the increasing order of normalized shear strength $v_c/\sqrt{f_c'}$, and n is the total number of points in the isolated database.

Figure 2(a) and (b) shows the cumulative histograms and their fits by cumulative distribution functions (cdf) in the normal and log-normal probability papers. Now note that the data points fit a straight line on the log-normal probability paper significantly better than they do on the normal probability paper (for the former, the mean and standard deviation are 3.22 and 0.895, and for the latter they are 3.22 and 0.885). Also note that if the Weibull probability paper were used, the fit of a straight line would be still worse. Hence, based on the information that exists, a log-normal pdf appears to be the best choice.

The type of pdf for small beams may alternatively be examined by the goodness-of-fit tests. The widely used Kolmogorov-Smirnov, or K-S test,[26] compares the observed cumulative probability S_n (solid curve) with the assumed normal distribution obtained by optimal fit (dashed curve), and generates a maximum discrepancy $D_n = D_{277} = 0.078$ (refer to Fig. 2(c)). This value satisfies the critical value for the 5% significance level ($D_{277}^{0.05} = 0.081$) but exceeds the critical value for the 10% significance level ($D_{277}^{0.10} = 0.073$). By contrast, the maximum discrepancy for log-normal distribution is $D_{277} = 0.056$, which is much less than that observed in K-S test for normal distribution and satisfies the critical values for both 5% and 10% significance level (refer to Fig. 2(d)).

Furthermore, the type of pdf for small beams may be examined by the chi-square test.[27] In this test, one subdivides the range of coordinate $v_c/\sqrt{f_c'}$, which spans from 1.32 to 6.56, into several intervals and compares the frequencies n_i of the small beam data with the assumed frequencies e_i for all the intervals in the histogram. Herein, six intervals, labeled by 1, 2, 3, 4, 5, 6, and 7, are considered. They contain 18, 106, 107, 32, 13, and 1 data points, respectively (refer to the histogram in Fig. 2(e)). Compared with the frequencies corresponding to the normal distribution (dashed curve), one obtains $\Sigma(n_i - e_i)^2/e_i = 20.95$, which cannot satisfy the critical value $c_{0.95,3} = 7.81$ for 5% significance level. On the other hand, $\Sigma(n_i - e_i)^2/e_i = 3.45$ is obtained for log-normal distribution (solid curve), which satisfies the critical value for the 5% significance level.

The foregoing comparisons demonstrate that the log-normal pdf is the best choice for the small beam data from the ACI 445F database.

WHAT STATISTICAL STRENGTH DISTRIBUTION MUST BE EXPECTED FOR LARGE BEAMS?

Again, theoretical deductions based on the scatter in one and the same material[17] are inapplicable because this scatter is overwhelmed by the scatter due to random variability of steel ratio, shear span ratio, etc., in the ACI 445F database. As emphasized by Bažant and Yu,[1,2] the database is heteroscedastic in the plot of normalized shear strength $v_c/\sqrt{f_{cr}'}$ (resistance) versus size, but becomes nearly homoscedastic in the doubly logarithmic plot; in other words, the variance or

CoV of the data becomes almost independent of the structure size.[2] Furthermore, in view of the aforementioned origin of scatter, there is no reason for the type of pdf to change with the structure size. Therefore, it is logical to assume the pdf of the normalized shear strength in the ACI 445F database to be log-normal for all the sizes.

Figure 3(a) shows the same pdf (log-normal, with the same CoV) superposed on the series of individual tests of beams of various sizes made at the University of Toronto. Now, it should be noted that, for the particular type of concrete, steel ratio, shear span ratio, etc., used in the Toronto tests, the shear strength value in these tests lies (in the logarithmic scale) at certain distance a below the mean of the pdf (Fig. 3(a)). Because the width of the scatter band in Fig. 1(b) in logarithmic scale does not vary appreciably with the beam size, the same pdf and approximately the same distance a between the pdf mean and the University of Toronto data must be expected for every beam size d, including the size of d = 925 mm (36.4 in.), for which there is only one data point, and also the size of 1.89 m (74.4 in.). In other words, if the University of Toronto test for d = 925 mm (36.4 in.) were repeated for many different types of concrete, steel ratios, shear span ratios, humidity, and temperature conditions, etc., one would have to expect a pdf shifted downward in the logarithmic scale approximately by the same distance a, as shown in Fig. 3(a).

Instead of a deterministic shift a, it would be more realistic to consider a to be a random variable. To determine the mean and CoV of a, the University of Toronto tests would have to be repeated at least six times per size. They were not. Nevertheless, an approximate estimate of the CoV of a can be made, as shown in Appendix A. Such a more accurate statistical estimate, however, gives essentially the same result for failure probability P_f because the CoV of a is far smaller than the CoV of the database values.

Could the 22 test points in the size range 760 to 1000 mm (30 to 40 in.) be used directly to determine the distance a? No, because these 22 points cover only a portion of the entire range of the influencing parameters of interest and the distribution of these parameters is very different from that in the small size range. For example, the steel ratios in the small size range of the ACI 445F database vary from 0.25% to 6.64%, with the mean of 2.55%, whereas the aforementioned 22 points correspond on average to much lighter reinforcement, with the steel ratios varying from 0.14 to 2.1%, and the mean of 0.96%. A similar discrepancy exists for a/d. Therefore, using the few existing data points in this size range would be misleading (yielding for distance a the value of only 0.07 instead of 0.45).

Now it is inescapable to recognize that the shifted pdf for $d \approx$ 925 mm (36.4 in.) reaches well below the line of required nominal strength $v_c = 2\sqrt{f_c'}$ of $y = v_c/\sqrt{f_c'} = 2$ (whereas the pdf for the small beam range lies almost entirely above this line). This means that if the type of concrete, steel ratio, shear span, humidity, and temperature conditions used in the single University of Toronto test were varied through the entire range occurring in practice (exemplified by the variation in the small size range), a large percentage of the beams would likely be found to be unsafe.

According to our assumption of log-normal pdf and equality of distances a for small and large sizes, the proportion of unsafe 925 mm (36.4 in.) deep beams would be approximately 40%, whereas for small beams 100 to 300 mm (4 to 12 in.) deep, it is only 1.0%. This is not acceptable. A design code known to have such an unsafe property cannot be adopted.

CAN FAILURE PROBABILITY FOR LARGE BEAMS BE ALLOWED TO BE GREATER THAN THAT FOR SMALL ONES?

To determine precisely the consequences for failure probability P_f of the beam, it is necessary to also consider the pdf of the extreme loads expected to be applied on the structure, which is denoted as $f(y)$. To calculate P_f, a certain value of the load factor needs to be considered. The present analysis considers only the load factor of 1.6, which is applicable to the cases where the live load dominates, as is the case for bridge beams up to 1 m (40 in.) deep (for load combinations with a significant dead load component, for which the blended load factor is less than 1.6, the failure probabilities for both small and large beams would be higher than obtained in what follows, but their ratio, which is of main interest, would be approximately the same).

The distribution of the applied extreme loads will be considered as log-normal (it is debatable whether the Gumbel distribution might be more realistic,[28,29] but it would make little difference for the ratio of probabilities and would make the calculation more tedious). The CoV of the applied extreme loads will be considered as 10%.

Under the foregoing assumptions, and based on the understrength factor ϕ = 0.75, the mean of the pdf of extreme applied loads and function $f(y)$ representing this pdf will be

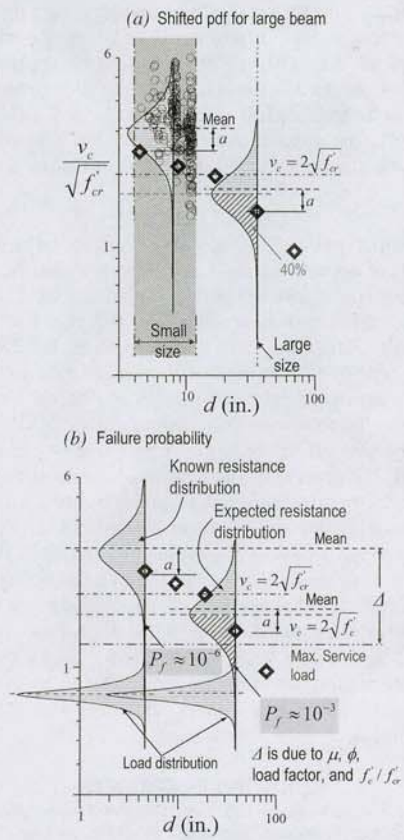

Fig. 3—(a) Probability distribution of shear strength of beams from 4 to 12 in. (100 to 300 mm) deep, based on the ACI 445F database; (b) distribution for beams 1 m (40 in.) deep inferred from the database (v_c, f_{cr}' and f_c' are in psi). (Note: 1 in. = 25.4 mm; 1 MPa = 145 psi.)

positioned as shown in Fig. 3(b). The failure probability may now be calculated from the well-known reliability integral[30-32]

$$P_f = \int f(y)R(y)dy \tag{1}$$

where $R(y)$ is the cdf of structural resistance, which is obtained by integrating the log-normal pdf in Fig. 3(b) and (c).

When this integral is evaluated for small beams within the range of depths d from 100 to 300 mm (4 to 12 in.) centered at d = 200 mm (8 in.), and also for the large beams of 1 m (40 in.) depth, one obtains the following failure probabilities

$$\text{For beams of 200 mm (8 in.) mean depth: } P_f \approx 10^{-6} \tag{2}$$

$$\text{For beams of 1 m (40 in.) depth: } P_f \approx 10^{-3} \tag{3}$$

For alternative or more accurate calculations, refer to Appendixes B and C. The failure probability of 10^{-6}, that is, one in a million, which is obtained for small beams, corresponds to what the risk analysis experts generally consider as the maximum acceptable for engineering structures in general[33-35] because it does not significantly increase the inevitable risks that people face anyway. But the probability of 10^{-3} is unacceptable.

Therefore, if the size effect in beam shear were ignored for beams without stirrups up to 1 m (40 in.) deep, the probability of failure for 1 m (40 in.) depth would be approximately 1000 times greater than it is for 200 mm (8 in.) depth. This should not be tolerated. If there should be any difference, it should be in the opposite sense because, for large beams, the failure consequences are usually more serious than for small ones.

CONCLUSIONS

The main hypothesis of analysis is that, for large beams of the depth of approximately 1 m (40 in.), which featured in only a few size effect test series, the variations of concrete type, steel ratio, and shear span ratio that occur in practice would lead to about the same scatter (with the same CoV and the same type of probability distribution) as they do for small beams of approximately 200 mm (8 in.) depth. Under this hypothesis, the following conclusions can be made:

1. If the size effect for beams up to 1 m (40 in.) deep is neglected, the percentage of beams whose load capacity is less than the nominal strength required by the Code is only approximately 1.0% for beams of 200 mm (8 in.) depth, but increases roughly to 40% for beams of 1 m (40 in.) depth;

2. Failures of beams 1 m (40 in.) deep must be expected to be approximately three orders of magnitude more frequent than failures of beams 200 mm (8 in.) deep. From the viewpoint of safety, this is unacceptable; and

3. Design safety requires the size effect to be introduced into the Code for all beams sizes, including beams less than 1 m (40 in.) deep.

ACKNOWLEDGMENTS

Financial support from the U.S. Department of Transportation through the Infrastructure Technology Institute of Northwestern University under Grant No. 0740-357-A210 is gratefully acknowledged.

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APPENDIX A—EFFECT OF UNCERTAINTY IN SHIFT *a*

The values of resistance $Y = v_c/\sqrt{f_c'}$ measured in the University of Toronto tests (Fig. 1) represent realizations of a random variable Y characterized by a certain probability distribution $p_Y(Y)$. The CoV (ω_Y) is due to the scatter of material properties of one and the same concrete and the same test conditions. The scatter of the database points is represented by random variable $X = v_c/\sqrt{f_c'}$, characterized by distribution $p_X(X)$ with a CoV (ω_X), which mainly reflects the effects of random variation of the type of concrete, steel ratio, and shear span, and dwarfs ω_Y.

Although the five data points from the University of Toronto tests at different sizes (Fig. 1) represent only a single test per size, the CoV (ω_Y) may be estimated from the standard error of regression of these five data points, obtained by optimally fitting them with Bažant's[36] size effect law. The result is $\omega \approx 7\%$. This estimate gives an overall measure of scatter for all the sizes. But it may be taken as a rough estimate of ω_Y for $d \approx 1$ m (40 in.) because ω_Y does not appear to vary significantly with the size.

Another estimate can be based on the reduced-scale size effect tests at Northwestern University,[8] in which three geometrically similar beams, made with aggregate of maximum size 4.8 mm (0.19 in.), were tested for each of the five sizes, spanning the size range of 1:16. The CoV for the subsequent sizes were 6, 7, 8, 6, and 8%. This again gives the mean value of 7%.

Bentz and Buckley[37] conducted partly similar tests, with normal aggregate size, but with the size range of only 1:4 (refer also to discussions[38,39]). They tested several specimens for each size. The CoV was 10.7% for $d = 82$ mm (3.3 in.), 2.76% for $d = 168$ mm (6.6 in.) and 2.65% for $d = 333$ mm (13.1 in.). According to these tests, the value $\omega_Y \approx 7\%$ for $d \approx 1$ m (40 in.) is a conservative high estimate.

For the ACI 445F database, the CoV (ω_X) for large sizes may be estimated from the 22 test points falling in the size range of 760 to 1000 mm (30 to 40 in.). This gives $\omega_X = 27.9\%$ and is almost the same as ω_X for the small size range. This confirms that the scatter band width in the logarithmic plot does not change significantly with the size.

The predicted resistance distribution $p(Z)$ for $d \approx 1$ m, sketched in Fig. 3, is obtained as the distribution of $Z = X - a$ + constant, in which the CoV of shift a is the same as ω_Y. Thus the CoV of Z may be estimated as

$$\omega_Z = \sqrt{\omega_X^2 + \omega_Y^2} = \sqrt{27.9^2 + 7^2} = 28.8\% \qquad \text{(A-1)}$$

(this value is exact only for normal distributions[32]). Compared with 27.9%, 28.8% represents an insignificant correction that will not appreciably affect the estimate $P_f \approx 10^{-3}$ made under the assumption of a deterministic shift a.

APPENDIX B—ALTERNATIVE EQUIVALENT CALCULATION OF FAILURE PROBABILITIES

Because both the load L and structural resistance R are assumed to be log-normal (and statistically independent), the failure probabilities in Eq. (2) and (3) would be obtained upon assuming the stochastic variable $Z = \ln(R/L)$ to be Gaussian. Then the mean and the standard deviation of Z are[32]

$$\bar{Z} = \lambda_R - \lambda_L,\ \sigma = \sqrt{\zeta_R^2 + \zeta_L^2} \qquad \text{(B-1)}$$

Herein, λ and ζ are the parameters of the log-normal distribution for resistance (that is, of v_c) and the load, respectively. They both have the form

$$f(x) = \frac{1}{\sqrt{2\pi}x\zeta}e^{-(\ln x - \lambda)^2/2\zeta^2} \quad (0 \le x < \infty) \qquad \text{(B-2)}$$

where x stands for either the load or the resistance. Then the failure probability is $P_f = \text{Prob}(\bar{Z} < 0)$. Using, for example, a table of the error function, one readily finds again that P_f for 200 mm (8 in.) deep beams is 10^{-6} and P_f for 1 m (40 in.) deep beams is 10^{-3}.

APPENDIX C—MORE ACCURATE CALCULATION OF RISK OF DESIGN FORMULA

A more precise calculation of failure probability P_f would require distinguishing among several probability density functions[10,15]: 1) distribution of the applied load, $p_L(v_c)$, or the distribution of shear strength v_c considered as a load parameter; 2) distribution $p_\alpha(\alpha)$ due to variations of shear span ratio $\alpha = a/d$ used in practice; 3) distribution $p_s(\rho)$ due to variations of longitudinal steel ratios used in practice; 4) distribution $p_v(v_c')$ of v_c due to random variation of strength f_c' of all the concretes used in practice; and 5) distribution $p_e(e)$ of the errors of the design formula. If these distributions are assumed to be independent, then

$$P_f = \iiiint\!\!\int_{L>R} p_L(v_c)p_\alpha(\alpha)p_s(\rho)p_v(v_c')p_e(e)\,de\,d\rho\,dv_c'\,d\alpha\,dv_c \qquad \text{(C-1)}$$

At present, however, such calculations cannot be meaningfully carried out because the required probability distribution functions are unavailable.