Creep and Shrinkage of Concrete

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50 FINITE ELEMENT MODELING OF RATE EFFECT IN CONCRETE FRACTURE WITH INFLUENCE OF CREEP

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Abstract
A time-dependent generalization of the smeared-crack model (crack band) or cohesive crack model is obtained by modeling rate-dependent fracture growth and creep in the material. The fracture growth is described by a rate-dependent relation between the crack bridging stress and the opening displacement, which is based on the activation energy concept. The creep of concrete is formulated according to the solidification theory. A numerical algorithm is developed and implemented in a finite element program. Numerical results illustrate the performance of the model and show that the model is capable of good representation of the behavior observed in recent experiments.
Keywords: Finite Elements, Concrete Fracture, Rate Effect, Creep, Crack Band Model, Cohesive Crack Model.

1 Introduction

The time-dependence of fracture is caused by three phenomena: (1) The effect of inertia weight propagation in the neighborhood of the crack tip, (2) the rate dependence of the process of bond breakages which produces the fracture surfaces, and (3) viscoelastic behavior or creep in the bulk of the material. The third phenomenon is negligible for very fast dynamic fracture, whilst the first phenomenon is negligible for very slow, static fracture. The time-dependence of concrete fracture has been studied extensively [1-6], but so far most studies focused on dynamic fracture. In regard to the time effect in static fracture, some investigators reported that it is effected by creep [e.g. 2, 4, 7-9]. Recently, an equivalent linear elastic fracture model based on the R-curve has been generalized to describe the rate effect and size effect in quasibrittle materials [7, 9] and extensive experimental data on the loading rate effect and the effect of a sudden change of loading rate, as well as the size effect at different rates, have been obtained [8, 10]. The stress-strain relations of the nonlocal strain softening microplane model for concrete have also been generalized to the rate effect [11], however, in a manner that produces the rate effect only within one order of magnitude of the loading rates.

A characteristic and difficult feature of concrete fracture is that the rate dependence is almost equally pronounced over many orders of magnitude of the loading rate, and this poses considerable modeling difficulties (such behavior has already been modeled in [9]). The purpose of this study is to present a rather general and fundamental model based on stress-strain or stress-displacement relations for the fracture process zone which give the rate effect over many orders of magnitude of the loading rate.

2 Mathematical Formulation

First one needs to formulate a model for the rate-dependence in fracture growth. In a fundamental approach, this model should be based on the rate process theory

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which describes the thermally activated nature of the breakages of bonds causing the formation of fracture surfaces. A mathematical formulation for this process is developed in a separate paper in this volume [12] and will now be briefly summarized. The rate of bond fractures is governed by the Maxwell-Boltzmann distribution of the frequency at which the energy of thermally vibrating atoms or molecules exceeds a certain specified energy level U. This distribution reads $f = k_b e^{U/RT}$ in which T = absolute temperature, R = gas constant, and k_b = constant. The potential of the bond forces, U (Fig. 1a) has a certain maximum called the activation energy Q which must be overcome for the bond to rupture. Now the activation energy barrier at no stress Q is modified by the presence of stress on the bond, σ_b (Fig. 1c), which causes that the rate of atom or molecule jumps over the activation energy barrier to one side exceeds the rate to the other side, thus leading to displacement with bond breakage. Based on this argument, which is similar to that used in material science models for creep or plastic flow, one obtains the following expression for the rate of opening of a crack due to thermally activated bond breakages [12],

$$\dot{v} = f(\sigma, v) = \dot{v}_r \sinh[\psi(\sigma, v)] \exp(Q/RT_0 - Q/RT) \tag{1}$$

in which $\dot{v}_r$ = constant (reference opening velocity), T_0 = reference absolute temperature, and $\psi(\sigma, v)$ = a function such that the equation $\psi(\sigma, v) = 0$ approximately describes the stress-displacement relation of the cohesive crack model for extremely slow opening, $\dot{v} \to 0$). By an extension and refinement of the arguments in [12] and in view of experimental data, this function has been introduced in the following form

$$\psi(\sigma, v) = \frac{\sigma - \phi(v)}{k[\phi(v) + k_0 f_t']} \tag{2}$$

in which k, k_0, f_t' = constant (f_t' = tensile strength), and $\phi(v)$ = stress-displacement curve of the cohesive (fictitious) crack model for an infinitely slow loading (Fig. 1b) ($\dot{v} \to 0$). $k_0 f_t'$ is a constant added in denominator Eq. 2 in order to prevent the denominator from approaching 0. Function $\phi(v)$ is used in the denominator because it is assumed that the crack opening rate does not depend simply on the difference of the stress from the stress displacement curve $\phi(v)$ for infinitely slow loading, but on the ratio of this difference to the value on the curve, augmented by small constant. For the purpose of numerical calculations, it is convenient to express the crack bridging stress σ explicitly from Eq. 1 and 2, which yields

$$\sigma = F(v, \dot{v}) = \phi(v) + k[\phi(v) + k_0 f_t'] \sinh^{-1}(\dot{v}/\dot{v}_r) \qquad (\text{at } T = T_0) \tag{3}$$

(for the case of reference temperature). The constants involved in the model have been calibrated according to the experimental data, and the corresponding plots of the stress-displacement relations for various displacement rates are shown in Fig. 1c. The area under these curves represents the fracture energy of the material, G_f, for various opening rates, and its value for $\dot{v} \to 0$ is taken as a material property. H_0 represents the initial slope of the stress-displacement curve, shown in Fig. 1b. Coefficient k in the model can be approximately deduced by noting that, according to test results, the peak stress is increased approximately by 25% when the loading rate is increased by 4–8 orders of magnitude, that is,

$$\frac{\dot{v}(\sigma = 1.25 f_t,\ v = 0)}{\dot{v}_{cr}(\sigma = f_t,\ v = 0)} = 10^4 \sim 10^8 \tag{4}$$

in which $\dot{v}_{cr}$ is assumed to be a certain critical opening rate below which the rate effect vanishes (in practice, such a rate probably does not exist, but $\dot{v}_{cr}$ may be interpreted as a rate so slow that is beyond the range of interest). The range of the k values is thus estimated as 0.01 ~ 0.05.

Eq. 1 or 3 may be used directly as a rate-dependent generalization of the cohesive crack model. In this study, we choose to pursue a rate-dependent generalization of the

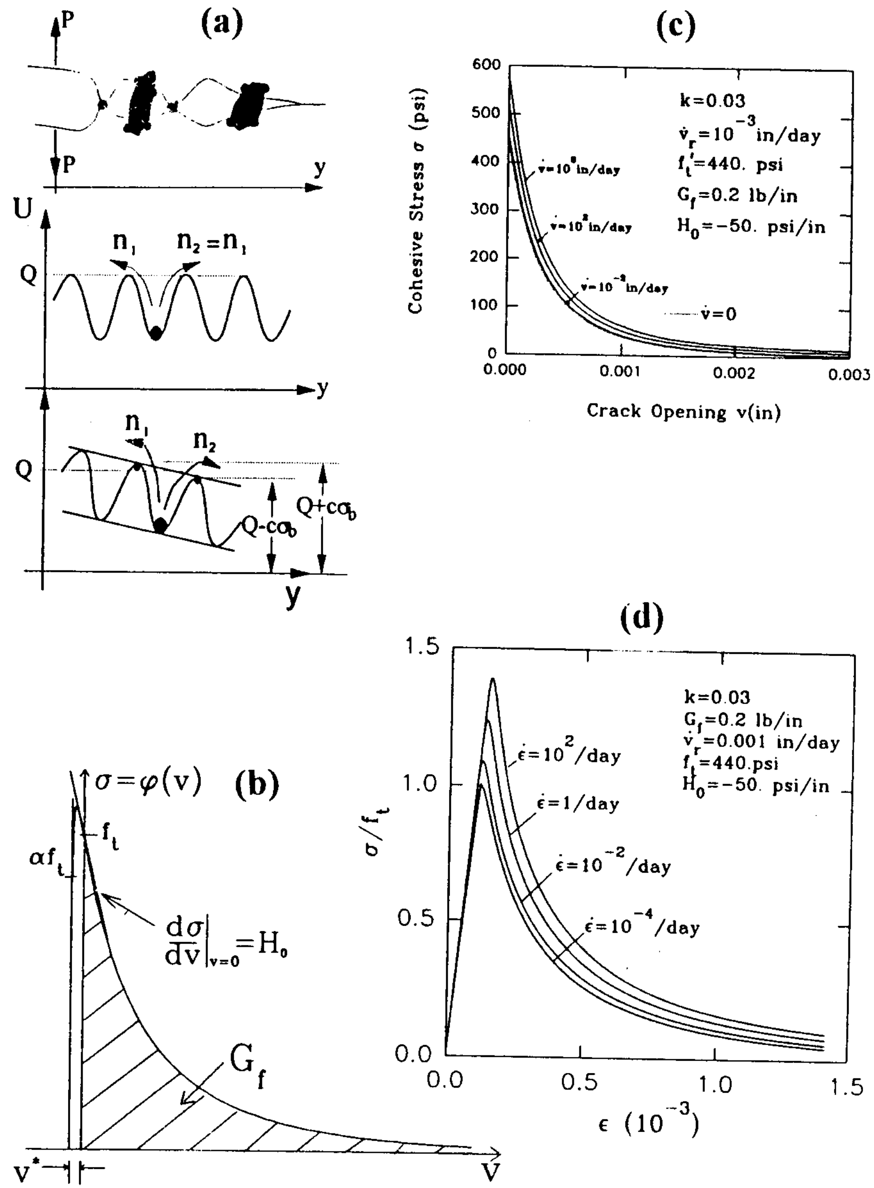

Fig. 1 Activation energy concept (a), stress-displacement relation used as input (b), and the obtained responses (c, d).

crack band model (which is similar to that of a nonlocal continuum model). In that case, the fracturing strain corresponding to the opening displacement v may approximately be taken as $\epsilon_f = v/h$ where h is approximately the width of the crack band front or the characteristic length of the material in a nonlocal model. Similarly, $\dot{\epsilon}_f = \dot{v}/h$. The total strain in the continuum model with cracking is assumed to be a sum of the elastic strain, creep strain, fracturing strain, and shrinkage and thermal strains.

The creep of concrete, along with the effect of aging, has been described according to the solidification theory [13]. The aging is in the theory modeled by a growth of the volume fraction of the solidified load-bearing matter. The creep of the solidifying constituent is considered to be nonaging, characterized by a nonaging compliance function or, for the purpose of numerical computation, a nonaging Kelvin chain model. The retardation spectrum of this material is characterized as a continuous spectrum [14], based on approximating a log-power creep law for the solidifying matter.

The foregoing mathematical description of the fracturing strain and creep has been combined with the smeared-crack concept (e.g. [15]). For the purpose of numerical calculations with the smeared-crack concept, Eq. 3 is approximated in an incremental form and is transformed into the relation $\Delta\sigma = H' d\epsilon_f + m d\dot{\epsilon}_f$, in which $H' = h\partial\sigma(v,\dot{v})/\partial\dot{v}$, $m = h\partial\sigma(v,\dot{v})/\partial v$. This equation is then generalized to a matrix form and is combined with the elastic and creep strain according to the smeared-crack model. The numerical algorithm for time steps Δt is of a forward gradient type, with a linear interpolation for the fracture strain rate used in each time increment. The algorithm is similar to that used by Needleman [15]. To handle post-peak softening, the modified Newton-Raphson method is combined with the arc-length method of Crisfield [16] (an excellent discussion of numerical algorithms for this type of problems has recently been given by Sluys [17]).

3 Numerical studies and comparisons with test data

Several examples of finite element analysis with the proposed constitutive model will now be presented. The calculations have been performed using four-node isoparametric finite elements with a 2 × 2 Gauss integration scheme. First, the model performance is checked for a uniaxial concrete bar in tension at various imposed strain rates. The calculated results are shown in Fig. 1d, in which $\dot{v}_r = 0.001$ in./day and $k = 0.03$. We see we achieve rate sensitivity many several orders of magnitude of the displacement rate, however, due to the choice of constants, the rate sensitivity decreases for $\dot{\epsilon} < 10^{-4}$/day. The peak stress in the strain range shown varies by 25%, which roughly agrees with experiments on concrete.

The fracture specimens considered and the corresponding mesh are shown in Fig. 2a. This type of specimens was tested in [8] at various crack mouth opening displacement (CMOD) rates and for three specimen sizes in the ratio 1 : 2 : 4, with geometrically similar shapes. Fig. 2b shows the load-CMOD curve as affected by the shear reduction factor β of the smeared-crack model. The effect of aging in creep on the fracture behavior is shown for two different Mode durations in Fig. 2c. In Fig. 2d,e the performance of the model is compared with experimental results [8] ($k = 0.03, \dot{v}_r = 10^{-3}$ in/day, $G_f = 0.2$ lb./in.), and the tensile strength f_t is related to the Young's modulus according to the ACI formula. In Fig. 2d, the CMOD rate is constant and t_p is the time to reach the peak stress. Fig. 2e shows that the model is also capable of approximating the experimentally observed size effect.

4 Conclusions

The rate dependent generalization of the cohesive crack model based on the activation energy concept and combined with a model for concrete creep in the bulk of the specimen approximately agrees with the experimental evidence on the responses of direct tension specimens and fracture specimens at different loading rates as well as at

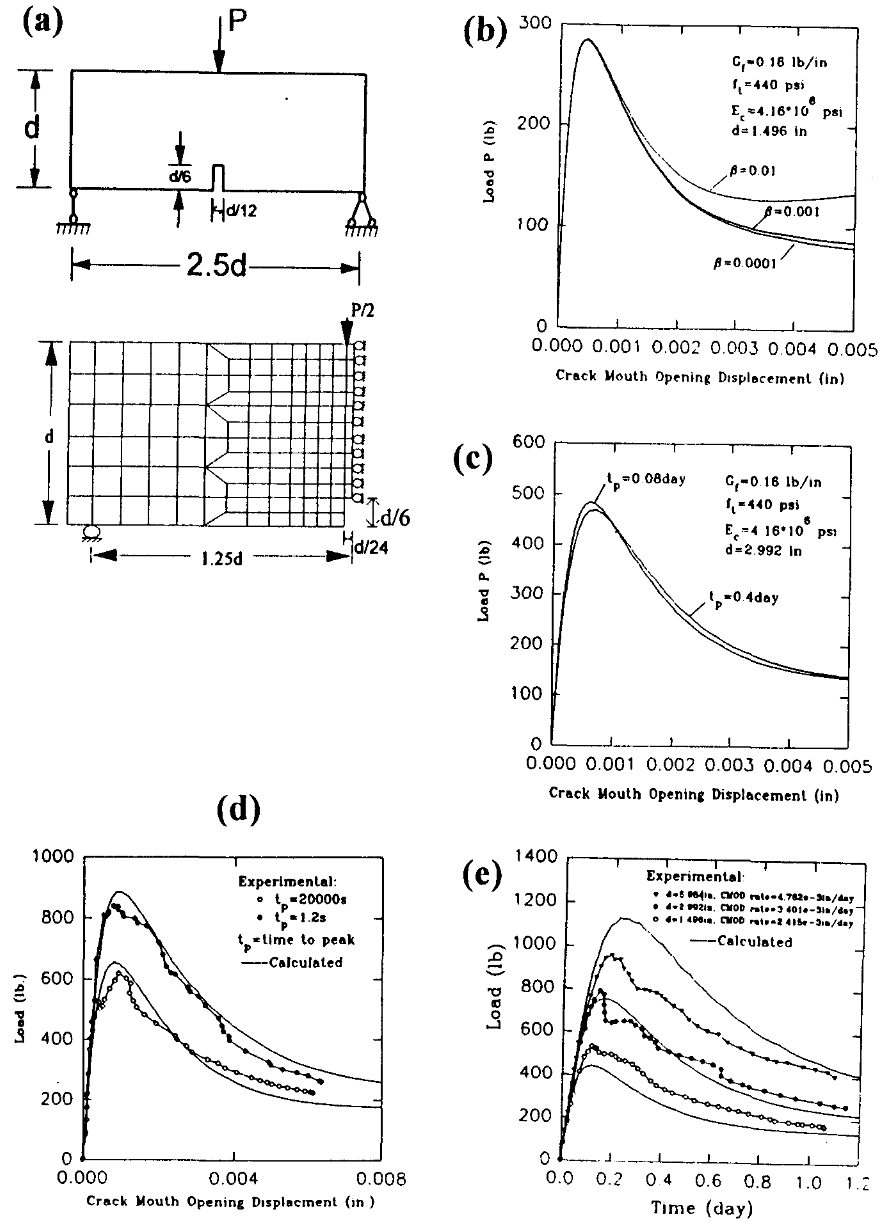

Fig. 2 Fracture specimens analyzed and the corresponding mesh (a), load-CMOD curves obtained (b, c), and comparison with test results from [8] (d, e).

different specimen sizes. However, close representation of the experimental results will require further refinements and calibration, which is in progress.

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