

Fracture Mechanics of Concrete Structures  
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## MICROFRACTURING CAUSED BY ALKALI-SILICA REACTION OF WASTE GLASS IN CONCRETE

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### Abstract

The possibility of using waste glass as a substitute for part of the aggregate in concrete depends on resolving the problem of alkali-silica reaction. Recent experimental observations showed that when the glass particle size is reduced the adverse effects on the strength and expansion of concrete get worse, but only down to a certain critical (pessimum) size of particles. Surprisingly, for smaller particles, experiments at Columbia University revealed the opposite trend. The paper proposes an idea that can explain this experimental observation by fracture mechanics.

Key words: ASR, particle size, fracture, expansion, strength.

### 1 Introduction

The alkali-silica reaction (ASR) is the chemical reaction between the alkalis in Portland cement paste and certain kinds of silica found in a variety of natural concrete aggregates (e.g. Neville, A.M., 1981).

The reaction products form a gel which expands in time if the relative humidity in the pores exceeds about 85%. The expansion may cause extensive cracking of the cement matrix, thereby reducing the strength of concrete. An accelerated 14-day standard test, ASTM C-1260, was recently introduced to assess the potential reactivity of various aggregates.

The problem with recycling of waste glass could be greatly alleviated by using it as an aggregate in concrete, provided that the ASR problem could be solved. Recent experiments of C. Meyer et al. at Columbia University have demonstrated that the ASR-induced expansion, as measured by the ASTM C-1260 test, depends strongly on the size of the glass particles in concrete. A startling observation has been made: If the glass is ground to particles of diameter less than 0.25 mm, the measured expansions are not more than those of reference bars that contain no glass, and the strength of concrete is the same.

At first thought, one might expect that the expansion and strength loss would increase with particle refinement because the surface-to-volume ratio (at constant glass content) increases, causing increase of chemical reactivity. This is indeed observed in the 14-day test, but only down particle diameter 1.5mm ('pessimum' size), at which the maximum expansion of concrete and the greatest strength loss take place. For finer particles, the trend is reversed. The purpose of this brief article is to outline an idea that can explain this startling experimental observation by fracture mechanics. A detailed analysis is planned for a forthcoming journal article.

## 2 Analysis of Fracturing and Expansion

For a fixed duration of observation, which is 14 days in the accelerated ASTM test, the volume of  $V_a$  of glass that has undergone ASR (per unit volume of concrete) depends on diameter  $D$  of the glass particles, assumed to be spherical. Approximately, for  $D/2 > h_r$ :  $V_a = V_r$  and for  $D/2 \leq h_r$ :  $V_a = V_g$ , in which

$$V_r = (\pi/6) [D^3 - (D - 2h_r)^3] \bar{N}, \quad \bar{N} = V_g/(\pi D^3/6) \quad (1)$$

Here  $h_r \approx \text{constant}$  = thickness of the reaction layer of glass into which the ASR can penetrate during a given fixed time from the

surface of the particle,  $V_r$  = volume of the reaction layers in all the particles per unit volume of concrete, and  $\bar{N}$  = number of glass particles per unit volume of concrete. So we have

$$\begin{aligned} \text{for } D/2 > h_r: \quad V_a &= \left[ 1 - \left( 1 - \frac{2h_r}{D} \right)^3 \right] V_g \\ \text{for } D/2 \leq h_r: \quad V_a &= V_g \end{aligned} \quad (2)$$

We consider an idealized cubic array of spherical glass particles of equal diameters  $D$  in hardened cement mortar (Fig. 1). The number of glass particles per unit volume of concrete is  $\bar{N} = V_g/(\pi D^3/6)$ , and for a cubic array of particles  $(\bar{N})^{1/3}$  where  $s$  = particle spacing. So,  $D = (6V_g/\pi)^{1/3}s$ . The typical glass content  $V_g$  represents 10% of the volume of all aggregate in concrete.

The ASR occurs at the surface of the particle. The expanding layer of the solid ASR gel applies radial pressure  $p$  on the surrounding cement mortar. Around each particle, there are likely to be pre-existing flaws. A pair diametrically opposite flaws shown in Fig. 1, which corresponds in three dimensions to a circular ring of flaws, or a ring crack around the sphere, is the most critical. The flaws or crack are likely to be penetrated and pressurized by the ASR gel.

Due to symmetry, we may restrict the analysis to a single cubical cell with one spherical glass particle in the center. As diameter  $D$  is changed, the crack depth  $c$  is assumed to scale in proportion. In other words, the relative crack depth  $\alpha = [c + (D/2)]/(D/2) = 1 + (2c/D)$  is constant. If a load is applied, the boundary of cubical cell is subject to average normal stress  $\sigma$ . If the applied stress is uniaxial,  $\sigma$  acts on two opposite sides of the cell only, and if it is equi-biaxial or triaxial hydrostatic,  $\sigma$  acts on four or all six of the sides. The energy release rate of the expanding ring crack may be expressed as

$$G = \frac{K_I^2}{E'} = \frac{(K_{Ip} + K_{I\sigma})^2}{E'} = \frac{D}{E'} [p k_p(\alpha) + \sigma k_\sigma(\alpha)]^2 \quad (3)$$

where  $K_I$  = total stress intensity factor;  $K_{Ip}, K_{I\sigma}$  = stress intensity factors caused by  $p$  and  $\sigma$ ; and  $k_p(\alpha), k_\sigma(\alpha)$  = dimensionless functions which could be determined by elastic finite element analysis (e.g. Bažant and Planas 1998). According to the general form of fracture mechanics formulas and the principle of superposition (e.g. Bažant

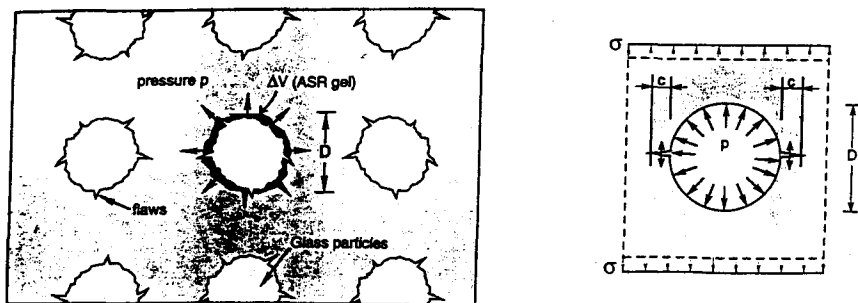


Fig. 1. Idealized cubic array of glass particles, and one cubical cell with a crack around the particle.

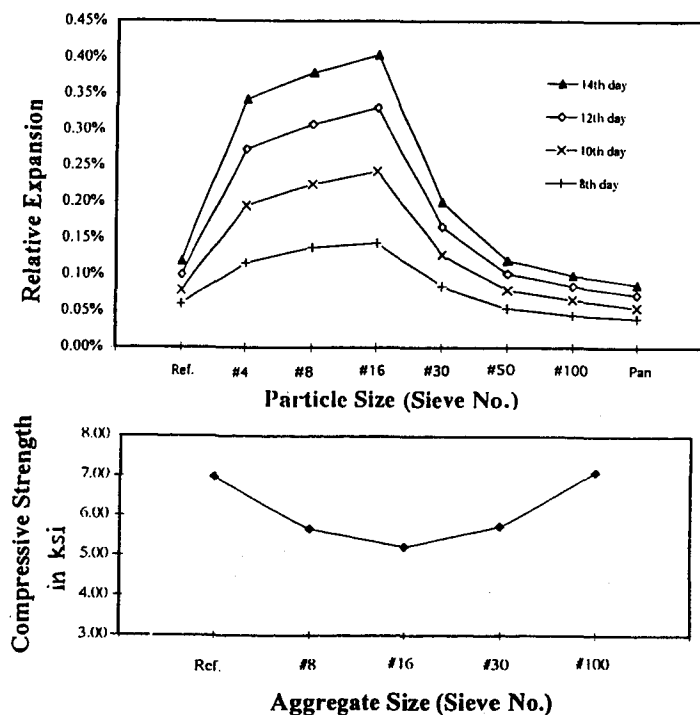


Fig. 2. Dependence of volume expansion  $\epsilon$  and compression strength  $f'_c$  due to ASR on the diameter  $D$  of glass particles, measured in the ASTM accelerated 14-day test by C. Meyer et al. at Columbia University.

and Planas 1998, p. 71), the increase of the combined volume of the crack and the sphere containing the reacting glass particle may be expressed as

$$\Delta V = \frac{p}{E'} D^3 \phi_{pp}(\alpha) + \frac{n\sigma}{E'} D^3 \phi_{p\sigma}(\alpha) \quad (4)$$

where  $n = 1$  or  $2$  or  $3$  for uni- bi- and tri-axial stress  $\sigma$ .  $\phi_{pp}(\alpha) = 2 \int_0^\alpha k_p^2(\alpha') d\alpha'$ ,  $\phi_{p\sigma}(\alpha) = 2 \int_0^\alpha k_p(\alpha') k_\sigma(\alpha') d\alpha'$  and  $\phi_{\sigma\sigma}(\alpha) = 2 \int_0^\alpha k_\sigma^2(\alpha') d\alpha'$ . Volume compatibility requires that

$$\Delta V = s^3 \left[ \left( e_g - \frac{p}{K_a} \right) - (V_g - V_a) \frac{p}{K_g} \right] \quad (5)$$

where  $e_g$  = ratio of the volume increase of the ASR gel at atmospheric pressure to the initial volume of glass from which the gel has formed,  $K_a, K_g$  = bulk moduli of the gel and of the remaining unreacted glass. The relative displacement between the opposite sides of the cell is  $\epsilon s$  where  $\epsilon$  = average strain of concrete; According to the principle of superposition,

$$s \epsilon = \frac{\sigma}{E_{ef}} s + \frac{p}{E'} \phi_{p\sigma}(\alpha) + \frac{\sigma}{E'} D \phi_{\sigma\sigma}(\alpha) \quad (6)$$

where  $E_{ef}$  = effective elastic modulus of the cell with spherical hole but crack, and  $\phi_{\sigma\sigma}(\alpha)$  = dimensionless function which could be determined by elastic finite element analysis.

Neglecting possible  $R$ -curve behavior of the matrix, we may write the conditions that the crack propagates and that the concrete is failing as  $\mathcal{G}(c) = G_f$  and  $\alpha = \alpha^*$  where  $G_f$  = fracture energy of the matrix and  $\alpha^*$  = critical value of  $\alpha$  which should be approximately constant. For the sake of simplification we set  $p/K_g \approx 0$  and  $p/K_a \approx 0$ , compared to ASR expansion  $e_g$ . One problem is to calculate the dependence of tensile strength  $\sigma = \sigma^*$  on  $D$ . In this case we need not solve  $\epsilon$ . Equations (4) with (5) and (3) lead to a system of two linear equations for  $p$  and  $\sigma$ . Setting  $s = (\pi/6V_g)^{1/3} D$ , we may bring the solution to the form:  $\sigma^* = C_1 D^{-1/2} - C_2 (V_a/V_g)$ , in which  $C_1, C_2$  are constants,  $C_1 = (2/\pi) \sqrt{E' G_f} \Delta$ ,  $C_2 = (2\pi) e_g E' \Delta$  with  $\Delta = [k_p^2(\alpha^*)]/[k_\sigma(\alpha^*) \phi_{pp}(\alpha^*) + k_p(\alpha^*) \phi_{\sigma\sigma}(\alpha^*)]$ .  $\sigma^*$  must represent the strength in addition to a certain residual strength  $f'_0$  governed by a plastic mechanism which is size-insensitive. So, the tensile strength

should be expressed as

$$f'_i = f'_0 + \frac{C_1}{\sqrt{D}} - C_2 \frac{V_a}{V_g} \quad (7)$$

Note that, for no glass, this formula has a similar form as the Petch formula for the effect of crystal size on the strength of polycrystalline metals (Cottrell 1964, Tetelman and McEvily 1967), as well as a formula for the effect of aggregate size on concrete strength derived by Bažant and Kim (1984).

For small enough  $D$ ,  $V_a$  is independent of  $D$  (all glass reacted), and only the first two terms in (7) remain. For large enough  $D$ , the second term becomes negligible, and so

$$\text{for small } D: f'_i \approx f'_0 + \frac{C_1}{\sqrt{D}}; \quad \text{for large } D: f'_i \approx f'_0 - C_2 \frac{V_a}{V_g} \quad (8)$$

Both trends agree with the experimental findings of Meyer and co-workers at Columbia University (Fig. 2). The compression strength  $f'_c$  of concrete may be estimated on the basis of the well-known empirical formula  $f'_c = (f'_i/6)^2$ , valid in psi.

To calculate the expansion caused by ASR when no stress is applied, we substitute  $\sigma = 0$  with  $1/K_a \approx 0$ , solve  $p$  from (5) and (4) and introduce the linear approximation  $\phi_{pp}(\alpha) \approx \phi_{pp}^0 + \phi'_{pp}\Delta\alpha$ . This provides  $s^3 e_g V_a = (\phi_{pp}^0 + \phi'_{pp}\Delta\alpha)pD^3/E'$ . Here  $\Delta\alpha$  may be solved from the condition  $\mathcal{G} = G_f$ . Using the linear approximation  $k_p(\alpha) = k_p^0 + k'_p\Delta\alpha$  where  $k_p^0$  and  $k'_p$  are constants, one has  $\Delta\alpha = (1/k'_p)[(D/p)\sqrt{E'G_f} - k_p^0]$ , and substituting this into the foregoing equation, one may solve for  $p$  and substitute it into (6) in which  $\sigma = 0$ . After rearrangements,

$$\epsilon = A_2 V_a - \frac{A_1}{\sqrt{D}} \quad (9)$$

in which  $A_1, A_2 = \text{constants}$ ,  $A_1 = \Phi k'_p e_g s^2 / D^2$ ,  $A_2 = \Phi \phi'_{pp} (D/s) \sqrt{G_f/E'}$  with  $\Phi = \phi_{p\sigma}(\alpha) / [k'_p \phi_{pp}^0 - k_p^0 \phi'_{pp}]$ . For small enough  $D$ ,  $V_a = V_g = \text{constant}$ , which means that the expansion decreases with decreasing size  $D$ . For large enough  $D$ , the second term in (9) becomes negligible and the expansion is then proportional to the volume of gel  $V_a$ , which means that the expansion increases with decreasing size. Both trends agree with what has been found experimentally by Meyer and co-workers at Columbia University (Fig. 2).

## Explanation of Measured Trends and Conclusion

The fact that a decrease of particle size is observed to cause a decrease of tensile strength of concrete when the glass particles are sufficiently large may be explained by the increase of the surface-volume ratio of the glass particles. A larger volume of glass undergoes ASR, producing larger pressures. On the other hand, the fact that a decrease of the glass particle size is observed to cause the strength to increase when the particles are sufficiently small may be explained by the fact that all the glass has reacted while at the same time a decrease of particle size causes a reduction of the stress intensity factor at the flaws near the surface of the reacted particle. In the accelerated 14-day ASTM test, the worst (pessimum) particle size is obtained when both effects are balanced, which occurs for diameter  $D \approx 1.5$  mm. For  $D \leq 0.25$  mm, no adverse affect of glass in concrete is detectable.

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