Finite Element Program for Creep Analysis in Concrete Structures

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Abstract

A new finite element program for the analysis of creep effects in concrete structures, developed at Northwestern University, is briefly described. The program represents an extension of the general purpose finite element code NONSAP developed at University of California at Berkeley. The program uses a rate type creep law based on expanding the relaxation function in Dirichlet series. The creep law is linear, the aging is accounted for, and the temperature effects on creep rate as well as the rate of aging are considered. Material properties can be input either as empirical creep test data or as the double power law for creep. The program can automatically set up the creep parameter from partial empirical information. The program uses a step-by-step integration in time. Applications are envisaged chiefly for nuclear concrete structures.

1. Introduction

With the increasing public concern about the safety of nuclear reactors, it is becoming imperative to evaluate the strengths and deformations of nuclear reactor vessels and containments very carefully and realistically, taking into account the actual inelastic behavior of the material. In concrete reactor containments as well as vessels acting as energy-absorbing barriers, the major source of inelastic behavior during the service life is the creep of concrete. During the past decade the knowledge of this phenomenon advanced tremendously and methods that allow an accurate and realistic analysis on the service conditions have been developed.

A general purpose finite element code for the creep analysis of nuclear concrete structures has recently been developed by Anderson and coworkers [1,7,10]. This program utilizes a rate type creep law proposed in 1971 by Bazant [3], which is based on expanding the compliance function for aging concrete (also called creep function) into a series of real exponentials, called Dirichlet series. Subsequently, it was found, however, [8] that this formulation has some theoretical disadvantages from the point of view of thermodynamic restrictions resulting from the aging of concrete; although the general form of the creep constitutive relation is correct from the thermodynamic viewpoint, when the experimental creep data for concrete are fitted some parameters of the model can be negative although according to thermodynamic limitations they should be positive. This is not necessarily incorrect, since these are internal variables while the thermodynamic restrictions must be satisfied only in total by the ensemble of the internal variables. However, such a feature is a matter of concern and it appears therefore safer to use a constitutive relation where the fulfillment of the thermodynamic restrictions is assured. This appears to be the case for an analogous rate type formulation which is based on a Dirichlet series expansion of the relaxation function rather than the compliance function and is modeled by an aging Maxwell chain model [2,4,5,6]. This formulation has been used in the extension of the finite element code NONSAP (University of California, Berkeley) described in the present paper.*

2. Constitutive Relation for Creep

The creep law is assumed to be linear, i.e., following the principle of superposition [2]. The linearity assumption is acceptable for concrete only for stresses less than about one-half of the strength, which is typical of nuclear vessels and containments under service conditions. Even within this range, significant deviations from linearity are observed upon unloading and in a regime with decreasing strains. In such cases, the program cannot yield very good results. Another phenomenon which spoils the linearity even within the service stress range is the simultaneous drying; however, this is not severe for nuclear concrete structures because they are massive and also protected by the steel liner on the interior face.

The aging of concrete, particularly the gradual change of creep properties as a function of age, is taken into account. This phenomenon causes that the parameters of Dirichlet series expansion of the relaxation function, representing the elastic moduli of the Maxwell chain model, are dependent on the age of concrete.

*Another large finite element program for creep in nuclear concrete structures has also been recently developed; we do not discuss it, however, since the form of the constitutive relation used in that program does not conform to the thermodynamic restrictions imposed by the phenomenon of aging.

The effect of temperature variations (limited to temperatures under 80°C) upon the creep behavior is taken into account. This includes both (a) the increase of the creep rate due to heating, and (b) the acceleration of aging, as modeled by the equivalent hydration period (maturity) [4,2]. Both of these effects are modeled using the activation energy concept, with different activation energies for the creep rate and for the process of aging (hydration).

The creep formulation used in the program is not capable of handling the general effects of time-variable pore humidity in concrete. These effects bring about a major complication of the constitutive relation [2] and require many additional variables to be used to characterize partial stresses in solid and water components, and they also require a good model for tensile cracking due to hygral stresses. Nevertheless, omission of moisture effects appears to be admissible for massive structures such as nuclear vessels and containments. This is because of their thickness, as well as the use of the steel liner of the interior face, which does not permit any significant drying to occur expect in a narrow layer near the exterior surface.

However, an approximate consideration of the humidity effect which does not cause any major complication of the mathematical formulation, is possible with the present program. The user is allowed to specify for each finite element the mean ambient humidity (not the pore humidity in concrete, and not a time variable quantity). The coefficients of the creep law are then adjusted to give a creep function that corresponds to concrete cylinders exposed to this environmental humidity, kept constant for the duration of the creep test. The approximate practical creep prediction model given in parts I and III of Ref. 7 is used for this purpose. As for the definition of shrinkage, the program can use the formulas from Ref. 7 or a set of experimentally observed shrinkage values can be specified. It must be remembered, that the foregoing consideration of humidity effects is very crude and yields realistic values only for the overall internal force resultants caused by humidity effects within the cross section of the structure. The method does not permit evaluating the distribution of the stresses produced by humidity effects throughout the cross section.

3. Numerical Algorithm

As has been clearly demonstrated before e.g.,[2], a rate type creep law allows a much more efficient finite element analysis than the integral-type creep law based directly on the principle of superposition. As already mentioned, the present program uses a rate-type creep law based on expanding the relaxation function of concrete into a series of real exponentials, called Dirichlet series. This creep formulation may be visualized by the Maxwell chain model, the elastic moduli of which depend on the age of concrete, more precisely the equivalent hydration period. This creep formulation is completely specified by a set of relaxation times, which are automatically selected in the program according to the rules given in Ref. 2 and 5, and by the age dependent elastic moduli of the Maxwell chain. These moduli are automatically generated from the relaxation function, which itself is automatically determined from the compliance function (creep function). The program also makes a check by reproducing the creep curve on the basis of the Maxwell chain model (as presented in Ref. 6).

The rate type creep law [6,2] consists of a set of first order differential equations for the partial stresses of the Maxwell chain and for the total strain of the Maxwell chain. It has been shown before that the usual step-by-step methods for ordinary differential equations cannot be applied to this system of equations. This is because either a numerical instability or a gross loss of accuracy results when the time step of these methods greatly exceeds the shortest relaxation time, which is a necessity when the long time values, corresponding for example to a 40-year service lifetime, are to be reached. The problem is completely avoided by the so-called exponential algorithm [4,5,2], which is based on an exact solution of the differential equations for the Maxwell chain under the assumption that all parameters are constant within the time step. This exponential algorithm was proven and demonstrated to be unconditionally stable, and highly accurate even for very long time steps. This allows increasing the time step roughly in a geometric progression, keeping it approximately constant in the log time scale.

4. Flow Chart of the Program

Instead of exhibiting the flow chart graphically, it is sufficient to describe it as follows.

The program implements a step-by-step stable integration procedure combined with iterations in each time step. A typical time step consists of a two-cycle computation. In the first cycle, estimates of the incremental quantities for the step are computed using the material properties at the beginning of the time step. In the second cycle, the material properties are evaluated for the midstep based on the results of the previous cycle. The flow of the program is outlined below (the subroutines involved in each step of the program are mentioned in the parentheses).

1. Read and check the input data (INPUT, MATPAR).
2. Compute the material parameters for the Maxwell chain model (MATPAR, PRINT1).
3. Initialize and set the step number to zero (MAINC).
4. Increment the step number and check whether the final step is completed. If it is, the final output is printed and the program ends (MAINC, PRINT2)
5. For the current time step, the age of the concrete, applied external loads and temperature are determined and the iteration parameter is set to zero (MAINC, CONSTI).
6. The iteration parameter is incremented and the material properties are established based on the current age of concrete (CONSTI, TEMHUM, DIFFUS, EMUF).
7. The instantaneous stiffness matrix is established for each element (ASREEK).
8. The incremental inelastic strains and the corresponding load vector are evaluated for each element (ASREEO).
9. The structural linear equation system for the nodal displacement increments and the load vector are assembled (MERGE).
10. The prescribed boundary constraints are imposed and the equation system is solved for the incremental displacements (BOCOND, BANSOL).
11. If the iteration parameter equals one, the midstep stresses and strains are computed and the program returns to step 6 (RETRCK, STRESS, ASREEP). If the iteration parameter equals two, program proceeds to step 12.
12. The incremental strains and stresses are computed, the total stresses, strains and displacements are computed and selective output is printed. The program then returns to step 4 (RETRCK, STRESS, PRINT2).

The foregoing flow chart is, of course, routine, used in many inelastic computer programs.

5. Determination of Creep Properties

The creep properties are determined by the following subroutine (MATPAR), calling for the subroutines as specified.

1. Read the number of decades in log-time scale to be considered, the number of steps per decade, the number of elements in Maxwell chain, the time for the start of the first time step and the first relaxation time.
2. Read the input option number (Sec. 6) and read the corresponding material data from which the characterization of the compliance function is developed. In case the drying is specified, read also the characteristics for shrinkage and drying creep.
3. Compute the discrete values of the relaxation function for various strain deviations and various ages at the start of relaxation. (This consists in direct numerical solution of a linear Volterra integral equation.) (RELAX).
4. Compute the discrete values of the Maxwell chain moduli at all discrete ages (MAXWL1).
5. As a check, compute discrete values of the compliance function from the discrete values of Maxwell chain moduli for various ages (CRCURV). Print the resulting values of the compliance function and of their deviations from the initially specified values of the compliance function. Also calculate and print the coefficient of variation of these deviations.

During the determination of the relaxation function, a subroutine for the compliance function is repeatedly called. This subroutine has three options to be specified by the user:

1. The compliance function is evaluated by interpolation from a given array of discrete values. The interpolation is linear in the logarithm of creep duration and in the logarithm of age at loading. For durations and ages falling outside the range, extrapolation is automatically used.
2. The compliance function is evaluated from a formula corresponding to the double power law [7].
3. If drying is specified, the compliance function is evaluated from a formula consisting of the double power law enhanced by a drying term [7].

The ultimate result obtained in the subroutine MATPAR is the array of the discrete variables of the moduli of the Maxwell chain. For any specified age, the value of the Maxwell chain moduli is computed from the discrete values (in subroutine function EMUF) by a linear interpolation in the logarithm of the age of concrete. For arguments falling outside the time range, a linear extrapolation from the extreme values is used.

6. Input Options for Material Characterization

For convenience of the user, a number of input options are provided in the subroutine MATPAR:

1. The compliance function is specified as an array of discrete values for various load durations and various ages at loading. No drying is considered.
2. Drying is considered, and the mean compliance function for the cross section is specified as in option 1. Also, the values of the mean shrinkage strain of the cross section are specified for various durations of drying and the given age at the start of drying.
3. The compliance function is given by the double power law, for which all of its five parameters are read. No drying is considered.
4. Same as option 3 but all double power law parameters except the 28-day elastic modulus are generated from the given strength and composition parameters of concrete (to mix ratio of water, cement, sand, gravel, cement-type, and the unit weight).

5. Same as option 4 except that the 28-day elastic modulus is also predicted from the strength and composition parameters.
6. Drying is specified and the compliance function is defined by the double power law enhanced by the term for the creep increase due to drying [7]. A formula for the shrinkage strain development (according to Ref. 7) is also specified. And all parameters in these formulas are read.
7. Same as option 6 but all material parameters except the 28-day elastic modulus and the final shrinkage strain are predicted from strengths and composition of concrete (and also the given ambient humidity).
8. Same as 6 but all parameters except the 28-day elastic modulus are predicted from the strengths and composition of concrete.
9. Same as option 6 but all parameters are predicted from the strength and composition of concrete.
10. Two of the five double power law parameters (namely, E_0 and ϕ_1, see Ref. 7) are determined so as to obtain the best fit (in the least square sense) of the given array of discrete values of the compliance function at various load durations and various ages. The remaining three double power law parameters are given. This option is used when the given array of discrete values for the compliance function is of limited range in time and/or age. No drying is considered in this option. As a check, the coefficient of variation for the deviations of the formula from the given array of values for the compliance function is automatically computed and printed.
11. Same as option 10 but the remaining three double power law parameters are not specified; they are predicted from given strengths and composition of concrete.
12. Same as option 10 but drying is considered.
13. Same as option 11 but drying is considered.

The user selects his input option depending on the amount of information available to him before the analysis. If sufficient test data have been obtained, as is frequently done for nuclear concrete structures, then the measured discrete values of the compliance function should be used (if the range of these variables is too limited, it is preferable to approximate these values by the double power law. If no experimental information is available, the double power law parameters are predicted from composition. However, if there is some information on the double law parameters for a similar concrete, an adjustment of some of its parameters is appropriate [11]. Moreover, if the short-time deformation (elastic modulus) is measured, it should be also used to improve the parameters of the double power law. The choice of the proper input option has a great effect on the accuracy in representing the concrete properties.

7. Conclusion

It may be expected that the new computer program* will allow a rather efficient, versatile, and realistic analysis of the creep effects in nuclear concrete structures under long-time loading, and prescribes temperature variations (however, without significant drying effects).

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*At the time of writing, the program described herein is still under development, although rather near completion.

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