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Fracture Mechanics: Application to Concrete

Victor C. Li
Zdeněk P. Bažant
Editors

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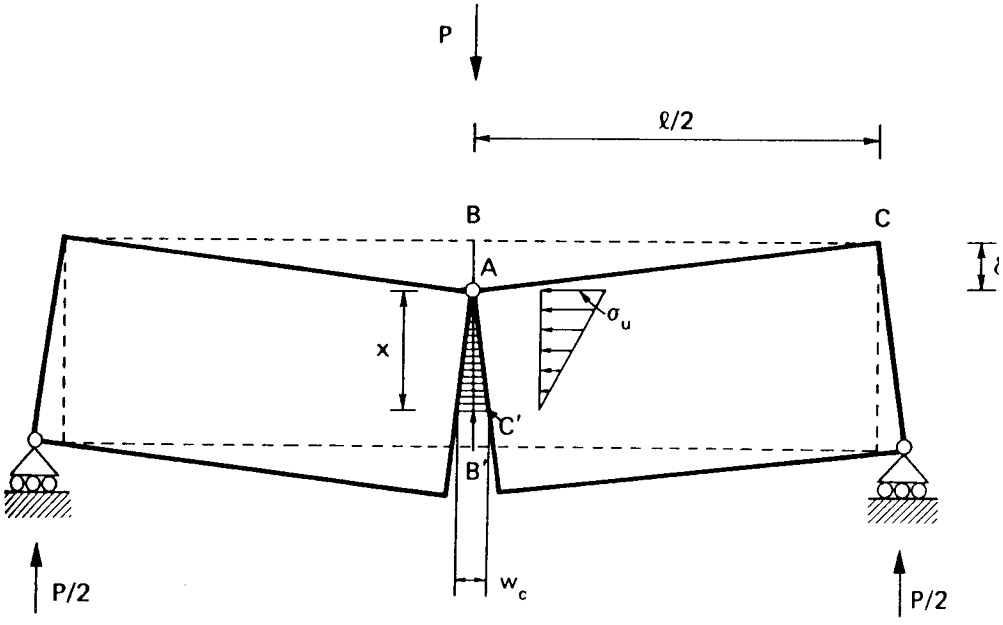

American Concrete Institute, Detroit

SP 118-8

Fracture Mechanics Size Effect and Ultimate Load of Beams Under Torsion

by Z.P. Bažant, S. Sener, and P.C. Prat

Synopsis: This symposium contribution gives a preliminary report on tests of the size effect in torsional failure of plain and longitudinally reinforced beams of reduced scale, made of micro-concrete. The results confirm that there is a significant size effect, such that the nominal stress at failure decreases as the beam size increases. This is found for both plain and longitudinally reinforced beams. The results are consistent with the recently proposed Bažant's size effect law. However, the scatter of the results and the scope and range limitations prevent it to conclude that the applicability of this law has been proven in general.

Keywords: beams (supports); dimensional analysis; fracture properties; loads (forces); reinforced concrete; tests; torsion

Center for Concrete and Geomaterials, Northwestern University,
Evanston, Illinois 60208
Zdeněk P. Bažant, Professor of Civil Engineering
Sıddık Şener, Visiting Scholar
Pere C. Prat, Graduate Research Assistant

INTRODUCTION

For many types of failure of concrete structures, the nominal stress at failure decreases as the size of the structure increases, given that the geometry is similar for all sizes. In theory the size effect can be basically of two types: (1) statistical, and (2) fracture. The statistical size effect, whose theory was originated by Weibull, has been the classical explanation. However, even though from the viewpoints of probability and influence of microscopic flaws the theory of the statistical size effect has been considerably refined, a consistent mechanical-statistical theory of such failures is not available [3].

The fracture-type size effect, long known in mechanics of fractures with isolated sharp cracks, has been extended by Bažant to failures which are due to propagation of zones of distributed cracking, which are typically observed in brittle failures of concrete structures. This fracture-type size effect is approximately described by Bažant's size effect law [2,3] which has been shown to agree quite well with the test results for many different types of brittle failures, including the diagonal shear failures of concrete beams, nonprestressed or prestressed, with or without stirrups, punching shear failure of slabs, pullout failure of bars, beam and ring failures of pipes, etc.

The purpose of the present contribution to ACI Symposium in Seattle, based on report [5], is to give a preliminary brief report on the results of a small series of experiments with microconcrete beams which show that the fracture-type size effect, as described by the size effect law, is applicable to torsional failures as well. With some partial exceptions (e.g., the Swedish code), the size effect is ignored in the current design practica [1] although its existence has already been indicated by some previous experimental studies [6-9].

TEST SPECIMENS, MEASUREMENTS AND RESULTS

The specimens were made of microconcrete with maximum gravel size 4.8mm (0.19 in.) and maximum size 3.35mm (0.132 in.). The gravel consisted of crushed limestone, and the sand was a siliceous Illinois beach sand (Lake Michigan). The mix ratio cement:sand:gravel:water was 1:2:2:0.6. Portland cement C150 of ASTM Type I was used. The specimens were removed from their plywood molds one day after casting and were then cured for three weeks in water. After that they were kept in a moist room at 24°C (75°F) temperature until the time of test. The age of the specimens at the time of test was 28 days. The uniaxial compression strength of the concrete, measured on control cylinders of 76.2mm (3 in.) diameter, was on the average 43.6 MPa (6330 psi) with standard deviation 0.41 MPa (60 psi).

The test specimens, shown in Figs. 1-3, were square prisms of cross section side d and length L. Three specimen sizes characterized by d = 38.1, 76.2 and 152.4mm (1.5, 3 and 6 in.) were used, and the ratio L/d = 8/3 was the same for all the beams. The beams were loaded by opposite couples at their ends as illustrated in Figs. 1 and 3. The arms of the loading couples and beam ends were 19.1, 38.1 and 127mm (0.75, 1.5 and 5 in.). The forces of the couples were applied at distance a from the beam end, such that a/L = 3/32 for all beams. The loads were applied through spherical bearings on steel bearing plates glued by epoxy to the concrete surface. The bearing plates made of stainless steel were square, of sizes 9.5, 19.1 and 38.1mm (0.375, 0.75 and 1.5 in.), and the thickness of all the plates was 6.4mm (0.25 in.). To prevent kinematic indeterminacy and at the same time prevent axial forces, one bearing was prevented to roll in both directions and the other one in the transverse direction only.

Two types of specimens were tested: plain and reinforced. The reinforced specimens of the aforementioned three sizes contained four longitudinal bars placed in cross section corners with a cover of 8.1, 16.3 and 31.5mm (0.32, 0.64 and 1.25 in.), respectively, and the bar diameters were 3.18, 6.35 and 12.7mm (0.125, 0.25, and 0.5 in.). Deformed bars were used; they had yield strength f_y = 310 MPa (45,000 psi) for No. 1 bars, and 415 MPa (60,000 psi) for No. 2 and No. 4 bars. To ensure proper load transmission, both the reinforced and plain specimens were provided with additional steel stirrups at the ends, as shown in Fig. 1. From each batch of concrete mixed, one torsion specimen of each size as well as three control cylinders were cast. The torsion specimens were loaded under displacement control at a rate which reached the maximum load within about 5 minutes. The maximum torques measured for the individual specimens are given in Table 1. All specimens failed in the manner described in textbooks. The fracture surfaces had mean inclination of about 45° and were slightly warped on one beam side, terminating with a smaller inclination at the opposite side.

The nominal stress at failure (ultimate torsional shear stress) is usually defined as $v_u = T/\alpha d^3$ where T = maximum torque, d = the side of the square cross section and $\alpha = 1/3$. If the failure is purely brittle and is due to distributed cracking, the nominal shear stress at failure should depend on beam size d. According to Bažant's size effect law [2,3], one may expect, as an approximation, $v_u = C_1[1 + (d/\lambda_0 d_a)]^{-\frac{1}{2}}$, in which C_1 and λ_0 are empirical constants. This can be algebraically rearranged to the linear equation Y = AX + C in which

$$X = \frac{d}{d_a}, \quad Y = \frac{1}{v_u^2}, \quad C = \frac{1}{C_1^2}, \quad A = \frac{C}{\lambda_0}. \tag{1}$$

This means that the test results should ideally yield a straight-line plot in the graph of Y vs. X. In reality, there is scatter, and by linear regression one can determine the slope of the regression line A and the Y-intercept C, from which one can then easily evaluate the empirical constants C_1 and λ_0.

The results of the present tests are plotted for the plain beams in Fig. 2 and for the reinforced beams in Fig. 3, which show the linear regression plots as well as the size effect plots in logarithmic scales. The inclined dashed line of slope -1/2 represents the size effect according to the linear elastic fracture mechanics [2,3]. Theoretically this slope cannot be exceeded; it represents the strongest possible size effect in brittle failures. The curves in Figs. 2 and 3 represent the optimum fits by the size effect law. The size effect law can be introduced into the code formula based on limit analysis in the manner already proposed by Bažant and Şener [4].

It may also be noted that some existing torsional test data for larger beams [4], even though rather scattered and contaminated by variation of factors other than the size, do also indicate a size effect consistent with the size effect law. In the present tests, it is noteworthy that the reinforced beams show a similar size effect as the unreinforced ones. This observation appears to confirm the code approach which assumes that the longitudinal bars have no effect on the failure load. The scatter of the results for the reinforced beams is larger, which might be due to the fact that bond failure, a phenomenon of considerable uncertainty, is involved.

CONCLUSION

In agreement with the analysis of some previous test data [4], the test results confirm the existence of a significant size effect. This suggests that the existing design code formulas, which exhibit no size effect, could be improved. As far as the scatter of the results permits it to be seen, the test results are consistent with Bažant's size effect law (Eq. 1).

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REFERENCES

1. ACI Standard, "Commentary on Building Code Requirements for Reinforced Concrete," ACI 318M-83J, ACI, Detroit, June 1984.
2. Bažant, Z. P., "Size Effect in Blunt Fracture: Concrete, Rock, Metal," Journal of Engineering Mechanics, Proc. ASCE, 110, April 1984, pp. 518-535.
3. Bažant, Z. P., "Fracture Energy of Heterogeneous Material and Similitude," Proc. SEM/RILEM Intern. Conf. on Fracture of Concrete and Rock, Houston, June 1987, ed. by S. P. Shah and S. Swartz, publ. by Soc. for Exper. Mechanics, pp. 390-402.
4. Bažant, Z. P., and Şener, S., "Size Effect in Torsional Failure of Concrete Beams," J. of Structural Engineering, Proc. ASCE 113, (10), Oct. 1987, pp. 2125-2136.
5. Bažant, Z. P., Şener, S., and Prat, P., "Size Effect Tests of Torsional Failure of Plain and Reinforced Concrete Beams," Report 86-12/498s, Center for Concrete and Geomaterials, Northwestern University, Dec. 1986; also Matériaux et Constructions (RILEM, Paris), in press.
6. Hsu, T.T. C., "Torsion of Structural Concrete-Plain Concrete Rectangular Sections," Torsion of Structural Concrete, SP-18, American Concrete Institute, Detroit, 1968, pp. 203-238.
7. Hsu, T. T. C., "Torsion of Reinforced Concrete," Van Nostrand Reinhold Company, New York, 1984, 516 pp.
8. Humphreys, R., "Torsional Properties of Prestressed Concrete," The Structural Engineer, London, 35, No. 6, June 1957, pp. 213-224.
9. McMullen, A. E., and Daniel, H. R., "Torsional Strength of Longitudinally Reinforced Beams Containing an Opening," ACI Journal, 72, No. 8, August 1975, pp. 415-420.

TABLE 1--TEST RESULTS

Plain Beams				Reinforced Beams			
Beam No.	d in.	f'_c psi	T kip × in.	Beam No.	d in.	f'_c psi	T kip × in.
A1	6.0	6325	24.01	B1	6.0	6250	21.00
A2	6.0	6325	24.73	B2	6.0	6250	22.80
A3	6.0	6400	24.00	B3	6.0	6400	27.05
A4	3.0	6325	4.20	B4	3.0	6250	3.44
A5	3.0	6325	3.96	B5	3.0	6250	3.87
A6	3.0	6400	4.16	B6	3.0	6400	4.16
A7	1.5	6325	0.61	B7	1.5	6250	0.50
A8	1.5	6325	0.60	B8	1.5	6250	0.72
A9	1.5	6400	0.61	B9	1.5	6400	0.67

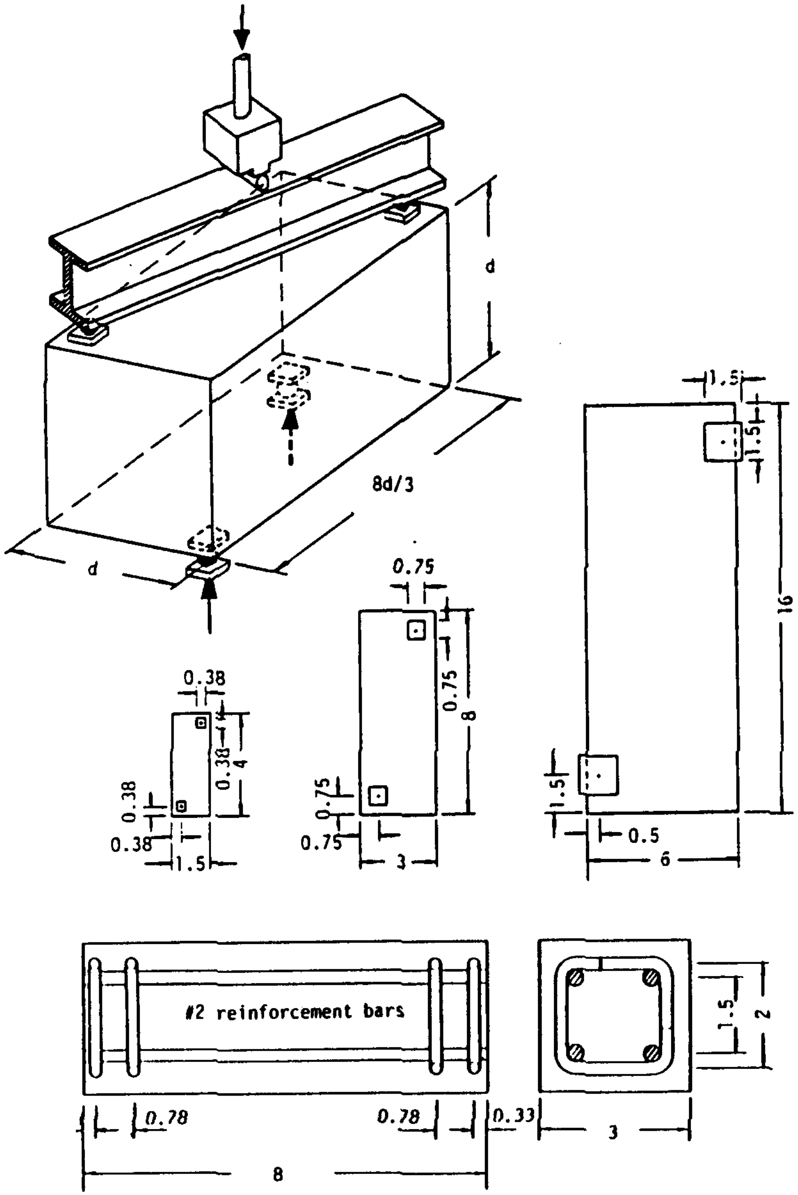

Fig. 1--Test specimens

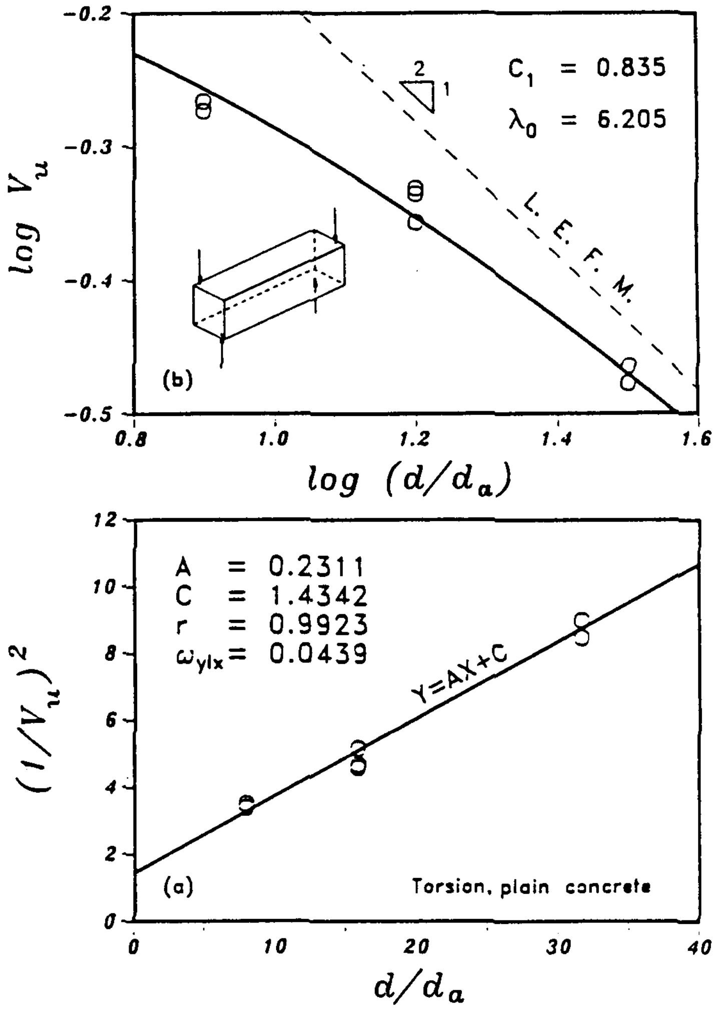

Fig. 2--Observed size effect for unreinforced beams

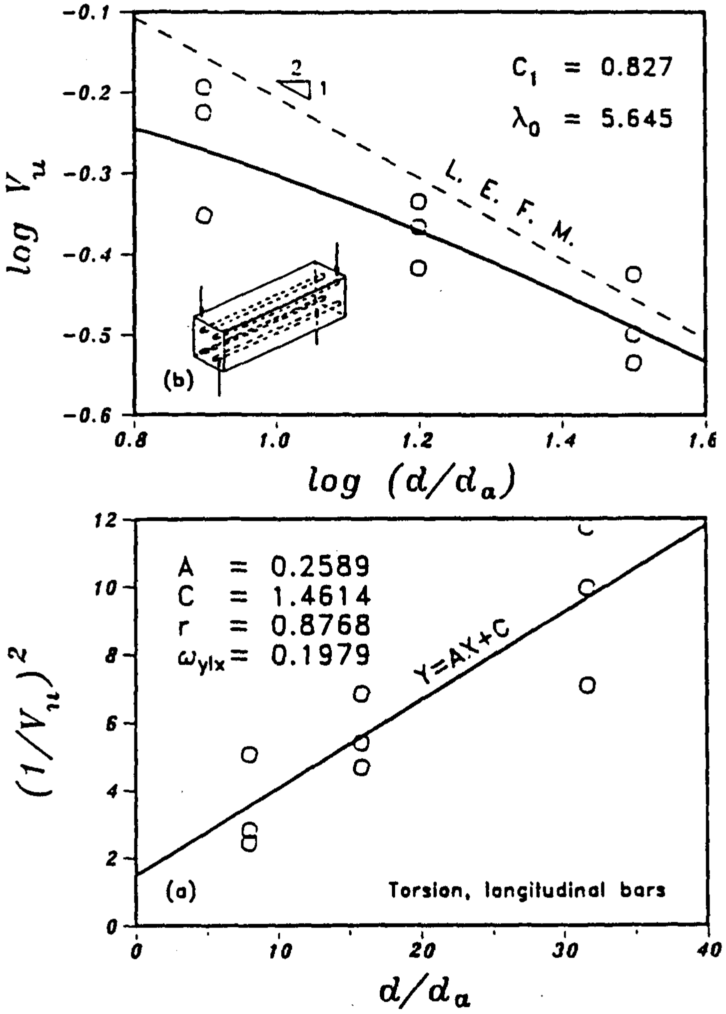

Fig. 3--Observed size effect for longitudinally reinforced beams