Bažant, Z.P., Pijaudier-Cabot, G., and Berthaud, Y. (1990). "Modeling of cracking induced damage in particu- late and fiber-reinforced composites" (ASME *Winter Annual Meeting*, Dallas, TX, Nov. 1990), AMD-Vol. 111, ed. by G.J. Dvorak and D.C. Lagoudas, 87–94.

AMD-Vol. 111
MD-Vol. 22

MICROCRACKING-INDUCED DAMAGE IN COMPOSITES

PRESENTED AT
THE WINTER ANNUAL MEETING OF
THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS
DALLAS, TEXAS
NOVEMBER 25–30, 1990

SPONSORED BY
THE APPLIED MECHANICS DIVISION AND
THE MATERIALS DIVISION, ASME

EDITED BY
GEORGE J. DVORAK
RENSSELAER POLYTECHNIC INSTITUTE

DIMITRIS C. LAGOUDAS
RENSSELAER POLYTECHNIC INSTITUTE

THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS
345 East 47th Street ■ United Engineering Center ■ New York, N.Y. 10017

MODELING OF CRACKING INDUCED DAMAGE IN PARTICULATE AND FIBER-REINFORCED COMPOSITES

Zdenek P. Bazant
Center for Advanced Cement-Based Materials
Northwestern University
Evanston, Illinois

Gilles Pijaudier-Cabot and Yves Berthaud
Laboratoire de Mecanique et Technologie
ENS Cachan/CNRS
University of Paris
Cachan, France

Abstract. -Micromechanics analysis of damage in heterogeneous media and composites cannot ignore the interactions among cracks as well as the interactions between cracks and inclusions or voids. Several previous investigations came to this conclusion upon finding that states of (diffuse) distributed cracking (damage) cannot be obtained mathematically merely be analyzing crack systems in a homogeneous medium although stable states with distributed damage have been experimentally observed in heterogeneous materials such as concrete. This paper presents a method for analyzing interactions between a crack and many inclusions which may be arbitrarily distributed. The problem is solved by superposition; it is decomposed into several standard problems of elasticity for which well known solutions are available, and is finally reduced to a system of algebraic linear equations. The calculated estimates of the stress intensity factors are checked by comparison with exact solutions. The errors appear to be less than ten percent provided the crack or the inclusions are not very close to each other. As a simplified approach, crack propagation in a composite can be treated as propagation of a crack in an equivalent homogeneous material for which the fracture toughness increases or decreases as a function of the crack length. These variations of apparent fracture toughness are analogous to R-curves in nonlinear fracture mechanics. They depend on the volume fraction of the inclusions, their spatial distribution, and the difference in elastic properties of the inclusions and the matrix. Large variations of the apparent fracture toughness are found depending on the location of the crack and its direction of propagation with respect to the inclusions.

INTRODUCTION

Particulate and fiber-reinforced composites usually do not fail by propagation of a single microcrack. In concrete, for example, microscopic observations and acoustic emission analyses of tensile specimens (see e.g. Mazars and Bazant, 1989) have proven that there exists a number of distributed propagating microcracks in the material prior to damage localization.

The fact that microcracking remains fairly well distributed lead many researchers to propose constitutive equations that are based on homogenization theories such as the self-consistent method (Budiansky and O'Connell, 1976; Krajcinovic and Sumarac, 1989; Ju, 1990), or homogenization theories for periodic composites. Nevertheless, these theories rarely address the problem of evolution of the crack density under increasing load or displacement conditions. In most homogenization techniques, the state of degradation, i.e. the crack density or the volume fraction of voids, is a fixed parameter referring to a fixed equilibrium state. Crack propagation is usually controlled at the microscopic level by Griffith criterion applied to each crack separately (Krajcinovic and Sumarac, 1989), and the influence of the interactions between the constituents of the composite (particles, fibers, matrix, cracks, and voids) is neglected as far as the evolution of damage is concerned.

In particle or fiber-reinforced composites, there exist various types of interactions: the interaction among the cracks, the interaction among the inclusions (particles or fibers), and the interaction between the cracks and the inclusions.

The effect of crack interaction has recently been considered in studies of the micromechanics of damage in concrete or ceramics (Horii, 1989; Ortiz, 1988; and Bažant and Tabbara, 1989), and several approximation schemes for estimating crack interaction effects have been proposed (see e.g Kachanov, 1987; Horii and Nemat-Nasser, 1985). The influence of the crack interaction on the rate of growth of damage was shown to cause failure at a much lower crack density than that predicted by the self-consistent method (Berthaud and Pijaudier-Cabot, 1990). Furthermore, the change in the regime of interaction during the transition between diffuse and localized damage was also proven to be a fundamental aspect which justifies the use of partially nonlocal constitutive relations for homogenized composites such as the nonlocal damage model (Pijaudier-Cabot and Berthaud, 1990).

To determine the effect of the interaction on the damage evolution, stability analyses of interacting crack systems must be performed. Surprisingly, such investigations (Bažant, 1987a; Bažant and Tabbara, 1989) revealed that most regular crack systems such as parallel cracks, periodic arrays of cracks, and some periodic colinear crack systems cannot be reached by a stable path under usual load or displacement control conditions. Such models predict that a single crack ought to propagate. Thus, stable states of diffuse damage consisting of a system of microcracks cannot exist according to the mathematical model in the first place, although they have been observed experimentally. Furthermore, the predicted shape of the softening post-peak load-displacement curve does not agree with experience and snapback instability is predicted to occur earlier than seen in tests (Bažant, 1987b). These discrepancies prove that the mechanical effect of heterogeneities cannot be ignored in modeling the evolution of damage and its progressive localization in concrete-like materials.

Solutions for the interaction between a crack and an inclusion in an elastic matrix exist (see e.g Kunin and Gommerstadt 1985, Erdogan et al. 1974). They are based on systems of singular integral equations, which however appear to be intractable in cases where several inclusions interact with the crack. Mura's equivalent inclusion method (Furuhashi et al., 1981) poses similar problems as it requires computation of integrals which may not converge absolutely when the inclusions are periodically distributed in an infinite medium.

The present paper has two objectives: First, an approximation scheme for solving the problem of interaction between cracks and inclusions proposed by Pijaudier-Cabot and Bažant (1990) is presented. The method can be viewed as an extension of Kachanov's superposition scheme (1987) for an interacting crack system without inclusions. The formulation is restricted to cases in which the bond between the matrix and the inclusion is perfect. Partial debonding and interfacial cracking is not considered. Second, it is proposed that the effect of the inclusions on crack propagation be interpreted in terms of the apparent variation of the fracture toughness of the homogenized composite with the crack length, similar to an R-curve in non linear fracture mechanics. Applications show that the apparent fracture toughness depends on the volume fraction of inclusions, on the spatial distribution of inclusions, and on the ratio of the elastic stiffnesses of the inclusions and the matrix.

INTERACTION BETWEEN A CRACK AND SEVERAL INCLUSIONS

Consider an infinite two-dimensional solid subjected to remote uniform boundary tractions producing a uniform stress field $\underset{\sim}{\sigma}_\infty$ at infinity. The solid is made of a linear elastic material of stiffness matrix $\underset{\sim}{D}_m$. It contains a crack of length 2c and N elastic circular inclusions (inhomogeneities) of radius R and stiffness matrix $\underset{\sim}{D}_a$; see Fig. 1a. The inclusions are arbitrarily distributed in the matrix and their contours are denoted as Γ_i $(i = 1,\ldots,N)$.

We seek for such a solid an estimate of the stress intensity factors at each crack tips. This problem is solved by superposing the solution of two simpler problems (Fig. 1a).

Subproblem I : the solution for the infinite solid without crack containing the inclusions and loaded by the remote tractions corresponding to $\underset{\sim}{\sigma}_\infty$.

Subproblem II : the solution for the given infinite cracked solid loaded by distributed normal and tangential forces $\underset{\sim}{p}(\underset{\sim}{x})$ on the crack faces Γ_c.

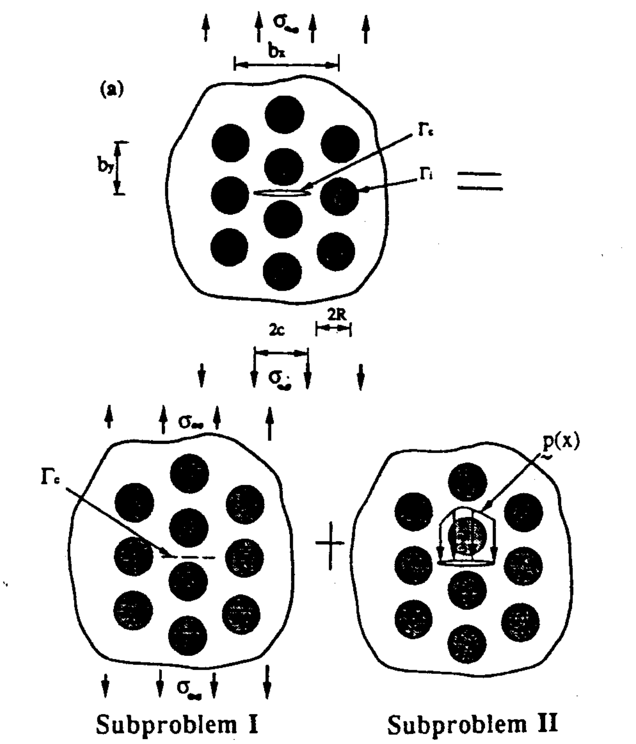
By superposition, the equilibrium condition for the crack surfaces may be written as

$$\underset{\sim}{\sigma} \cdot \underset{\sim}{n}(\underset{\sim}{x}) + \underset{\sim}{p}(\underset{\sim}{x}) = 0 \text{ on } \Gamma_c \tag{1}$$

in which $\underset{\sim}{\sigma}$ denotes the stress field solution of subproblem I calculated at the imaginary crack surfaces Γ_c, and $\underset{\sim}{n}(\underset{\sim}{x})$ is the vector of a unit outward normal to Γ_c at a point with cartesian coordinates $\underset{\sim}{x}$. Ideally, Eq. 1 should be satisfied at every point of Γ_c and superposition would then yield an exact result. For the sake of simplification, we will assume that Eq. 1 is satisfied only approximately, in the average sense, that is

$$< \underset{\sim}{\sigma} \cdot \underset{\sim}{n}(\underset{\sim}{x}) + \underset{\sim}{p}(\underset{\sim}{x}) > = 0 \tag{2}$$

where the brackets < > denote the spatial averaging over domain Γ_c. This simplification is inspired by Kachanov's scheme (1987) for interacting crack systems in homogeneous solids without inclusions, which has been showed to be satisfactory in most situations. In Kachanov's scheme as well as here, the averaging is justified by St. Venant principle: The errors represent a self-equilibrated stress field



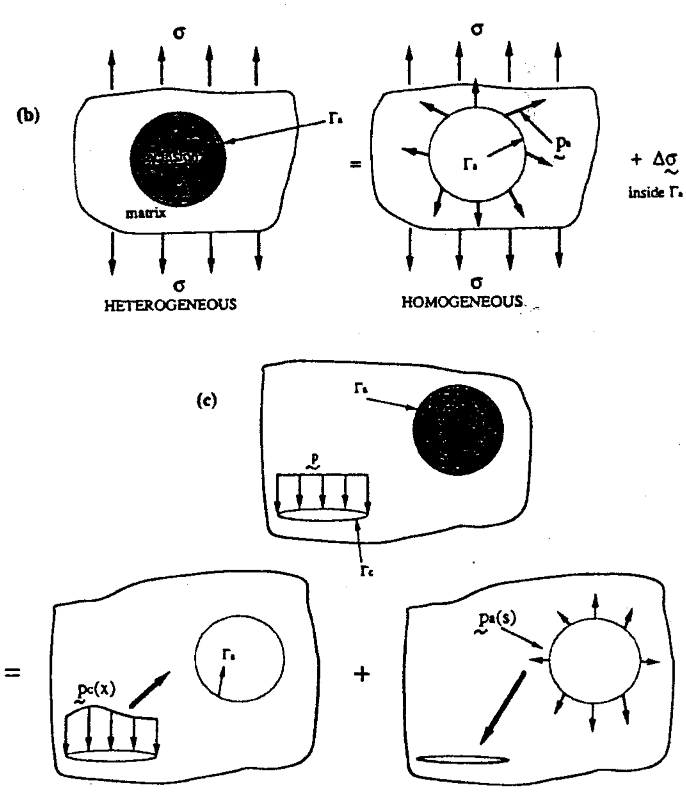


Figure 1 : Crack Interacting with Inclusion: (a) Superposition Scheme ; (b) Duhamel - Neuman Analogy ; (c) Superposition in Subproblem II.

which must be decaying exponentially with the distance from the crack and is, therefore, negligible for a sufficient separation of crack and inclusion.

Subproblem I

For the sake of simplicity, attention is restricted to plane elasticity. The local stress and strain fields in the matrix could be calculated via Mura's equivalent inclusion method (Furuhashi et al., 1981). Since we intend to deal with many interacting inclusions as well as interacting cracks, we prefer to use a simpler iterative solution which is based on Duhamel-Neuman analogy (Fig. 1b). The main advantage of this analogy is that it transforms a problem of elasticity of a heterogeneous solid into an equivalent problem of a homogeneous solid which can be decomposed into a superposition of standard problems for which analytical solutions exist.

Given the stress field $\underset{\sim}{\sigma}$ that is a solution of this subproblem, the so-called unbalanced stress field denoted as $\Delta\underset{\sim}{\sigma}_i$ inside the inclusion of contour Γ_i is calculated as follows:

$$\left.\begin{aligned} \Delta\underset{\sim}{\sigma}_i &= \left(\underset{\sim}{D}_a - \underset{\sim}{D}_m\right) : \underset{\sim}{\varepsilon} \\ \text{with } \underset{\sim}{\varepsilon} &= \underset{\sim}{D}_m^{-1} : \underset{\sim}{\sigma} \end{aligned}\right\} \tag{3}$$

while in the matrix outside Γ_i the stress fields $\Delta\underset{\sim}{\sigma}_i$ vanish. $\underset{\sim}{n}_i$ is the unit outward normal of the boundary curve Γ_i of inclusion i.

The unbalanced stresses $\Delta\underset{\sim}{\sigma}_i$ can be equilibrated by applying tractions $\Delta\underset{\sim}{\sigma}_i \cdot \underset{\sim}{n}_i$ on interface Γ_i. Since these tractions do not exist in reality, the opposite unbalanced interface tractions must act on the interface Γ_i in the composite

$$\underset{\sim}{p}_i = -\Delta\underset{\sim}{\sigma}_i \cdot \underset{\sim}{n}_i \qquad \text{on } \Gamma_i \tag{4}$$

Thus, the stress field in subproblem I may be written as

$$\begin{aligned} \underset{\sim}{\sigma} &= \underset{\sim}{\sigma}^* && \text{outside } \Gamma_i \ (i = 1,\ldots,N) \\ \underset{\sim}{\sigma} &= \underset{\sim}{\sigma}^* + \Delta\underset{\sim}{\sigma}_i && \text{inside } \Gamma_i \ (i = 1,\ldots,N) \end{aligned} \tag{5}$$

where $\underset{\sim}{\sigma}^*$ is an equilibrium stress field solution of the new problem of elasticity of the homogeneous material of stiffness $\underset{\sim}{D}_m$ subjected to the uniform stress field $\underset{\sim}{\sigma}_\infty$ and to the pressures $\underset{\sim}{p}_i$ on each contour Γ_i. Obviously, the unbalanced stress fields $\Delta\underset{\sim}{\sigma}_i$ are unknown. Their computation calls for an iterative procedure which may proceed as follows:

1. The initial stress field $\underset{\sim}{\sigma}^*$ is

$$\underset{\sim}{\sigma}^* = \underset{\sim}{\sigma}_\infty + \sum_{i=1}^{N} \underset{\sim}{\sigma}_i \tag{6}$$

where $\underset{\sim}{\sigma}_i$ is the stress due to the presence of inclusion i alone in the matrix (Eshelby's solution, see e.g. Mura 1987). The unbalanced pressures $\underset{\sim}{p}_i$ on the contour Γ_i of each inclusion i are calculated from $\underset{\sim}{\sigma}$ according to Eqs. 3-4. The stress $\underset{\sim}{\sigma}_i$ due to $\underset{\sim}{p}_i$ is then calculated as if each inclusion i were alone in the infinite solid i.e.

$$\underset{\sim}{\sigma}_i = \int_{\Gamma_i} \underset{\sim}{f}\left(\underset{\sim}{p}_i(\underset{\sim}{s})\right) ds \tag{7}$$

in which $\underset{\sim}{f}$ denotes the well known two dimensional solution for the concentrated force $\underset{\sim}{p}_i(\underset{\sim}{s})ds$ applied at point $\underset{\sim}{s}$ of an infinite homogeneous elastic space which belongs to the contour Γ_i (see e.g. Timoshenko and Goodier 1976). A new total stress field is computed from Eq. 6.

2. From $\underset{\sim}{\sigma}^*$, the unbalanced pressures $\underset{\sim}{p}_i$ on each contour Γ_i are recalculated using Eq. 4. Then again the stress $\underset{\sim}{\sigma}_i$ due to $\underset{\sim}{p}_i$ is calculated from Eq. 7 as if the inclusions were alone, and by superposing $\underset{\sim}{\sigma}_i$, the new total stress field is obtained from Eq. 6.

3. Step 2 is iterated until the unbalanced tractions $\underset{\sim}{p}_i^\ell$ resulting from $\underset{\sim}{\sigma}$ in iteration ℓ differ negligibly from those at iteration ℓ-1. Typically, an admissible error is 1% and convergence is reached in less than 10 iterations. For a small enough error the exact solution can be approximated as closely as desired (Pijaudier-Cabot and Bazant, 1990).

Subproblem II

The crack is loaded by a uniform internal pressure $\underset{\sim}{p}(\underset{\sim}{x})$ on its contour Γ_c. From superposition (Fig. 1c)we obtain:

$$\underset{\sim}{p}(\underset{\sim}{x}) = \underset{\sim}{p}_c(\underset{\sim}{x}) + \sum_{k=1}^{N} \underset{\sim}{p}_{int}^{k}(\underset{\sim}{x}) \text{ on } \Gamma_c \tag{8}$$

In this equation, $\underset{\sim}{p}_c(\underset{\sim}{x})$ is the distribution of internal pressure applied on Γ_c at a point of cartesian coordinates $\underset{\sim}{x}$, and $\underset{\sim}{p}_{int}^{k}$ $(k = 1, \ldots, N)$ are the interaction terms due to the presence of the inclusions. $\underset{\sim}{p}_{int}^{k}$ is computed at the imagined location of the crack as if the composite were uncracked.

There are two type of terms contributing to $\underset{\sim}{p}_{int}^{k}$:

- The first type of contribution arises from the loading $\underset{\sim}{p}_c$ on the inclusion k that is assumed to be alone in the matrix. This term is:

$$\underset{\sim}{p}_k^k = \left[\int_{\Gamma_k} \underset{\sim}{f}\left(-\Delta\underset{\sim}{\sigma}_c \cdot \underset{\sim}{n}_k(\underset{\sim}{s})\right) ds\right] \cdot \underset{\sim}{n} \tag{9}$$

Here $\underset{\sim}{\sigma}_c$ is the stress field due to the crack loaded by $p_c(\underset{\sim}{x})$, calculated for the infinite solid without inclusions. $\Delta\underset{\sim}{\sigma}_c$ is the resulting unbalanced stress computed on the imagined contour Γ_k of the inclusion k and $\underset{\sim}{n}_k$ is the outward normal to Γ_k.

The second type of contribution is the interaction between inclusions j and k ($j \neq k$), and its influence on the crack faces. Each inclusion in the composite is subjected to the stress σ_c. The value of

p_j^k can be computed in the same manner as in subproblem I but the stress fields σ_c is substituted to the remote field σ_∞. From Eqs. 4 and 7 we obtain :

$$p_j^k(x) = \left[\int_{\Gamma_k} f\left(-\Delta\sigma_j \cdot n_k(s) \right) ds \right] \cdot n \tag{10}$$

in which $\Delta\sigma_j$ is the stress field due to the unbalanced pressure p_j acting on contour Γ_j :

$$\sigma_j = \int_{\Gamma_j} f\left(-\Delta\sigma_c \, n_j(s) \right) ds \tag{11}$$

Superposition yields :

$$p_{int}^k(x) = \sum_{j=1}^{N} p_j^k(x) \tag{12}$$

and after substitution into Eq. 8 :

$$p(x) = p_c(x) + \sum_{k=1}^{N} \sum_{j=1}^{N} p_j^k(x) \tag{13}$$

We now assume that Eq. 13 needs to be satisfied only in the average sense:

$$<p(x)> = \left[1 + \sum_{k=1}^{N} \sum_{j=1}^{N} \Lambda_j^k \right] \cdot <p_c(x)> \tag{14}$$

where Λ_k^k is the transmission factor due to inclusion k considered to be alone with the crack, Λ_j^k is the transmission factor due to interaction between inclusions k and j, and 1 is the 2x2 identity matrix. In plane elasticity with in-plane displacements, the transmission factors are 2x2 matrices which couple mode I and mode II crack opening.

It should be noticed that if σ_c is assumed to be the stress field due to the crack alone subjected to the uniform internal pressure $<p_c(x)>$, Eq. 14 is linear in $<p_c(x)>$ and has a single vector unknown. According to this assumption the transmission factors do not depend on the shape of the distribution of $p_c(x)$. This simplifying assumption is acceptable if the distances between any two inclusions, and between the crack and the inclusions, are not too small.

Superposition

The superposition equation 2 is used to compute the average internal pressure $<p_c(x)>$ on the crack surface. Substitution of Eq. 14 in Eq. 2 yields

$$<p_c(x)> = -\left[1 + \sum_{k=1}^{N} \sum_{j=1}^{N} \Lambda_j^k \right]^{-1} \cdot <\sigma \cdot n> \tag{15}$$

The stress intensity factors at each crack tip are computed using the right-hand side of Eq. 14. For instance, the stress intensity factors for mode I crack opening are :

$$K_I(\pm c) = \frac{1}{\sqrt{\pi c}} \int_{-c}^{+c} \sqrt{\frac{c \pm x}{c \mp x}} \; p_c(x) . n \; dx \tag{16}$$

As an example, figure 2 shows the result for the mode I stress intensity factors for a crack in an epoxy matrix which is tangential to a void or an elastic inclusion. The remote loading is uniaxial tension perpendicular to the crack faces and plane strain is assumed. Since the average tangential pressure is zero, Eq. 14 has a single scalar unknown. The radius of the inclusion is such that R / c = 2, and the material properties are $E_a / E_m = 23$, $\nu_a = 0.3$, and $\nu_m = 0.35$ in the case of a metallic inclusion, and $E_m = 3.5$ GPa, ($\nu_m = 0.35$) in the case of a void; E_a, E_m, ν_a and ν_m are the Young's elastic moduli and Poisson's ratio of the inclusion and of the matrix respectively. In this figure, K_I is normalized with respect to the stress intensity factor K_{I0} for a crack in an infinite homogeneous solid, which is $K_{I0} = \sigma_\infty \sqrt{\pi c}$. The approximation is compared to Erdogan et al.'s (1974) analytical solution. The quality of the approximation is quite acceptable unless crack and inclusion (or void) become very close. The reason is that the stress fields in subproblems I and II have a large variation over the imagined crack length which is not compatible with the averaging of the internal distribution of pressure over the crack faces.

Fig. 3 shows an example of the calculated variation of the mode I stress intensity factor K_I at the tip of a crack located between two circular voids as a function of the crack length and of the spacing between the voids. The remote loading is uniaxial tension perpendicular to the crack. The distances between the center of the crack and the centers of the voids are equal. The results are compared with the known analytical solution given in Tada et al. (1985)

If the distance between the voids is large compared to their radius, the approximation is seen to be quite accurate (error less than 5 %). However, when the crack length increases, the effect of the voids becomes more localized on a small segment of the crack contour Γ_c and the agreement with the analytical solution is less than satisfactory. This discrepancy is again due to the two successive averagings of the distributions of internal pressures on the crack faces (averaging of $p(x)$ first, and of $p_c(x)$ second). For the same reason the approximation looses its accuracy when the voids get too close to each other.

APPARENT FRACTURE TOUGHNESS OF A COMPOSITE

The present approximate method could no-doubt be combined with Kachanov's method (1987) and thus generalized for a system of cracks in a composite. However, programing the computation of the various transmission coefficients seems to be too tedious. As we have observed from Figs. 2,3, the effect of the inclusions is to cause the ratio K_I / K_{I0} to increase or to decrease when the crack propagates in the matrix. It means that the stress intensity factor at the crack tips in the composite changes during crack propagation or alternatively, that the equivalent homogeneous material behaves as if the apparent fracture toughness was following an R-curve.

The knowledge of such an apparent R-curve would permit a much simpler calculation of crack propagation conditions in composites. In this approach, the interaction among cracks is uncoupled from the interaction between the crack and the inclusions. Similar simplifications have been proposed by Mori et al. (1988) and Gao and Rice (1988), who used a perturbation method to analyze fiber-reinforced composites in which the values of the elastic moduli

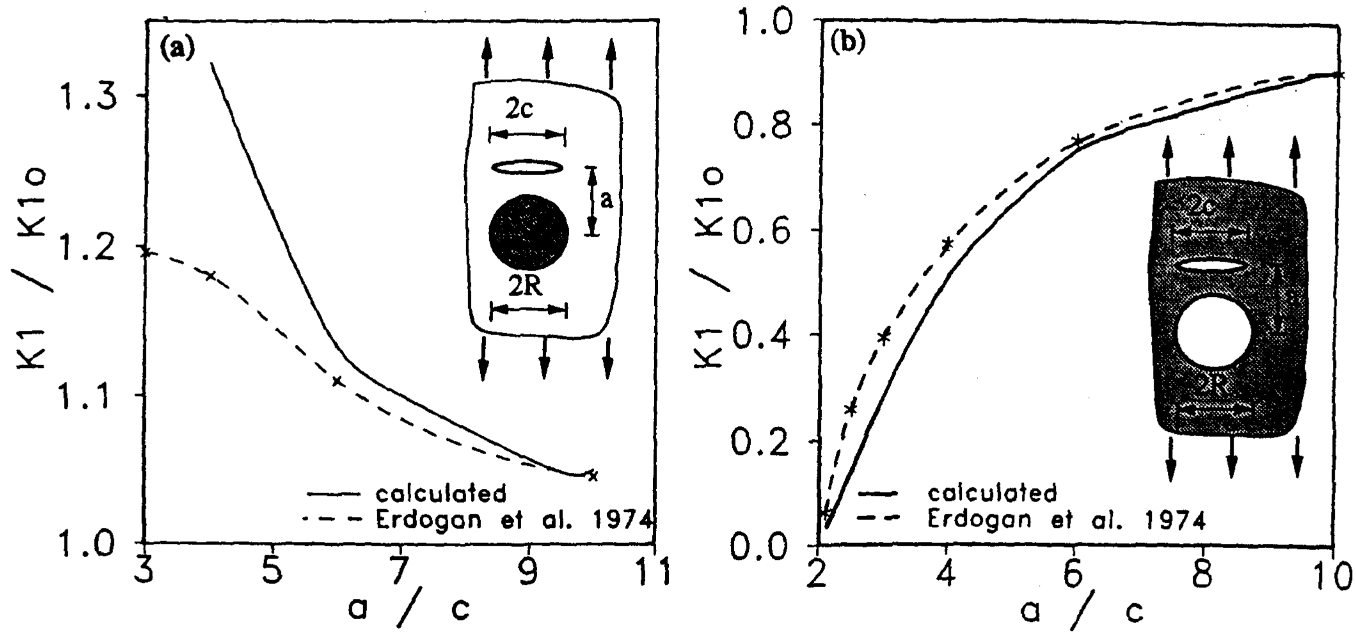


Figure 2 : S.I.F for a Crack Tangential to an Inclusion (a), and a Void (b)

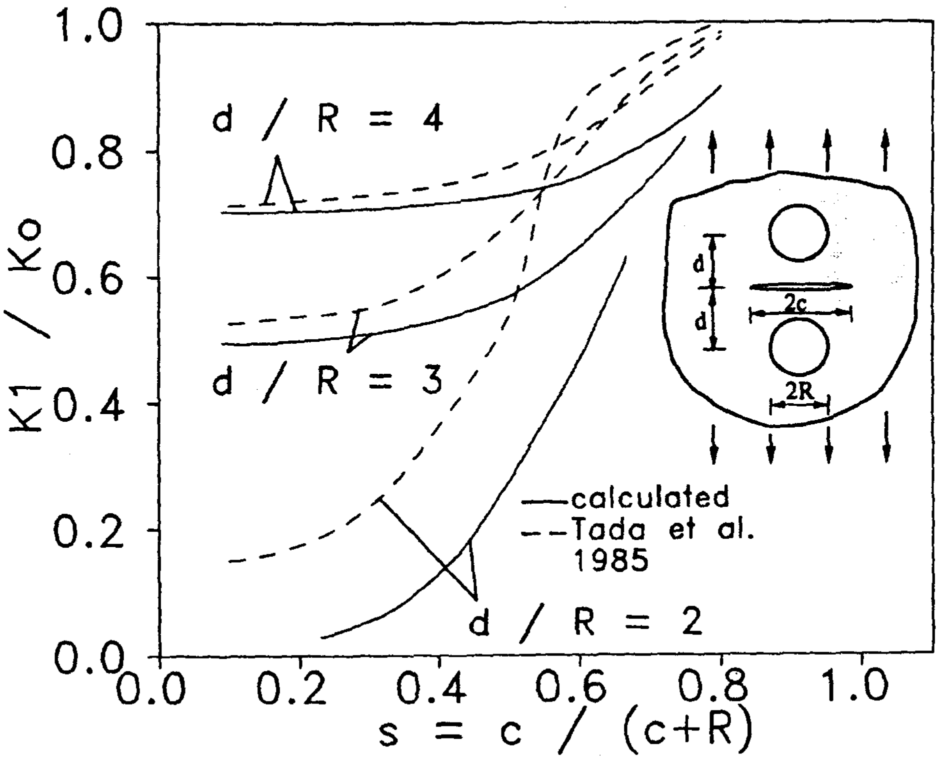


Figure 3 : S.I.F for a Crack Interacting with Two Circular Voids.

of the matrix and the inclusions are sufficiently close. More precisely, let K_{co} be the fracture toughness of the matrix. According to Griffith's criterion, crack propagation occurs when $K_I = K_{co}$. For a crack length c in a macro-homogeneous composite loaded with tensile stress σ_∞, we may write $K_I = K_{Io}\ F(c)$ with $K_{Io} = \sigma_\infty \sqrt{\pi c}$ where F(c) is a certain amplification function that is computed from the crack-inclusion interaction. The estimation of K_I yields the apparent fracture toughness K_c of the composite

$$K_c = \frac{K_{co}}{F(c)} \qquad (17)$$

In most studies (see e.g. Zaitsev, 1985), F(c) was assumed to remain constant or to change only when the crack touches an inclusion (Huang and Li, 1989). Fig. 4 presents the approximate variation of fracture toughness for a crack propagating symmetrically in a composite made of regular staggered circular inclusions embedded in an elastic matrix. The radii of the inclusions are equal and denoted as R. The volume fraction of inclusions is $V_f = 0.7$ corresponding to equal spacings between the centers of the inclusions in the x and y directions ($b_x = b_y = 3R$). Plane stress is assumed with $E_a / E_m = 3$ and $\nu_a = \nu_m = 0.2$. The remote boundary traction is uniaxial tension perpendicular to the crack faces.

Three configurations have been analyzed (Fig. 4a) : In configuration 1 the crack propagates toward two inclusions. In configuration 3, the center of the crack is at equal distances from two rows of inclusions. Configuration 2 is intermediate between configurations 1 and 3.

Fig. 4b shows the variation of apparent fracture toughness with the crack length according to Eq. 17. We see that these variations may be radically different depending on the configurations analyzed. Configurations 1 and 3 are the upper and lower bounds on the apparent mode I fracture toughness, respectively. The more drastic variation is obtained, when the crack propagates toward an inclusion and corresponds to the maximum possible toughening. These variations of fracture toughness are of primary importance for stability analyses of crack systems. As we see, the mechanical effect of the inclusions cannot be neglected in crack propagation analyses as the fracture toughness of the equivalent medium may vary by as much as 100 %. It should be stressed that these curves are valid only if the crack does not touch the inclusions. Otherwise, the singular stress field at the tips of the crack would need to be modified.

To exemplify the influence of the spatial distribution of the inclusions at a constant volume fraction, Fig. 5 shows the variation of apparent toughness for a regular ($b_x = b_y = 3R$) staggered distribution of inclusions and a non regular staggered distribution of inclusion ($b_x = 4R$, $b_y = 2.25R$). The inclusion spacings are such that the volume fraction is the same, $V_f = 0.7$. Configuration 1 is chosen with the same material properties as in Fig.4. Again, there is a large difference between the two cases. The non-regular staggered distribution (dashed curve) provides the lowest apparent fracture toughness. This suggests that inclusions that are radial to the crack have the largest influence since b_x has been increased. The toughening effect, which is important for the regular distribution, is delayed as the tips of the crack are more distant from the inclusions.

Finally, the effect of the variation of the ratio E_a / E_m of the elastic modulus of the inclusion to that of the matrix is shown in Fig. 6. The apparent fracture toughness of the composite has been computed for the crack length 2c = R with $\nu_a = \nu_m = 0.2$. The composite contains a regular staggered distribution of inclusions with $V_f = 0.7$. The solid line corresponds to configuration 1 and the dashed line corresponds to configuration 3 (see Fig.4a). We obtain the upper and lower bounds of the variation of toughness for a crack opened under mode I, as a function of the ratio E_a / E_m. For configuration 1 this curve is certainly not linear. It should be pointed out that for large values of E_a / E_m, convergence in subproblem I could not be reached ($E_a / E_m > 7$). The range of variation of E_a /

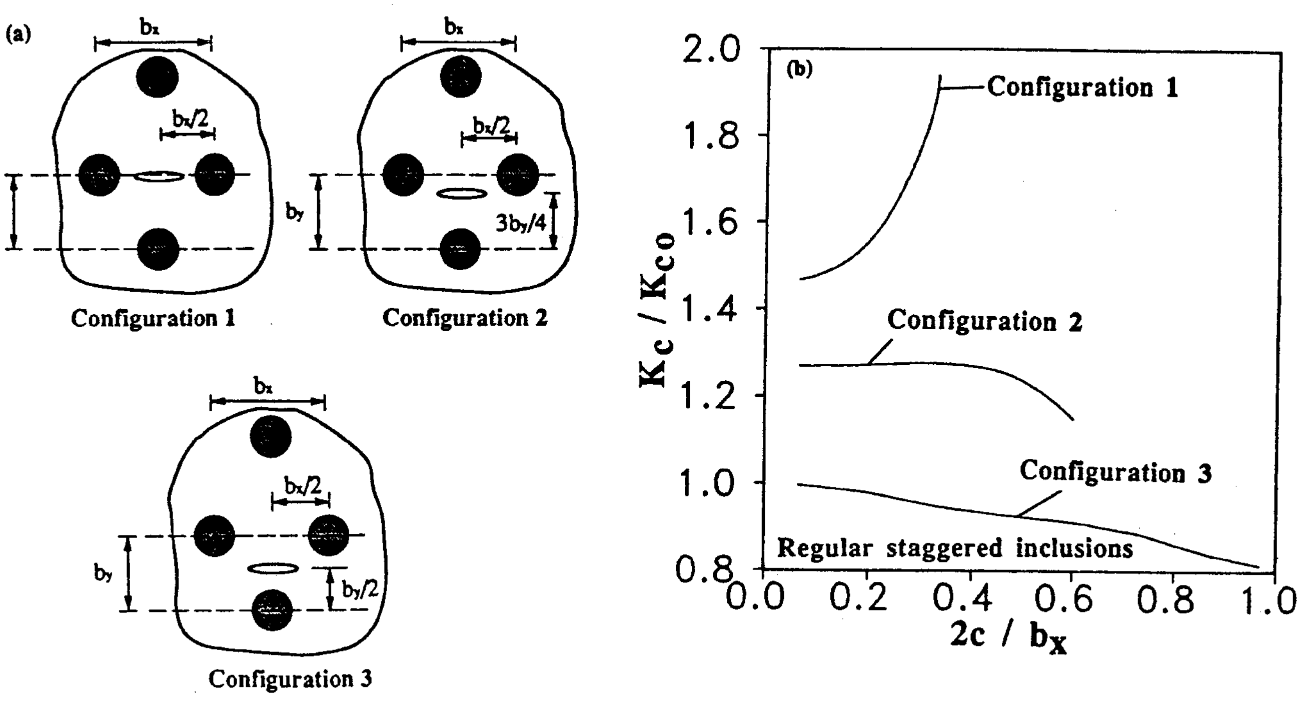


Figure 4 : Apparent Fracture Toughness of a Composite with Staggered Inclusions : (a) Configurations Analyzed ; (b) Fracture Toughness vs. Crack Length.

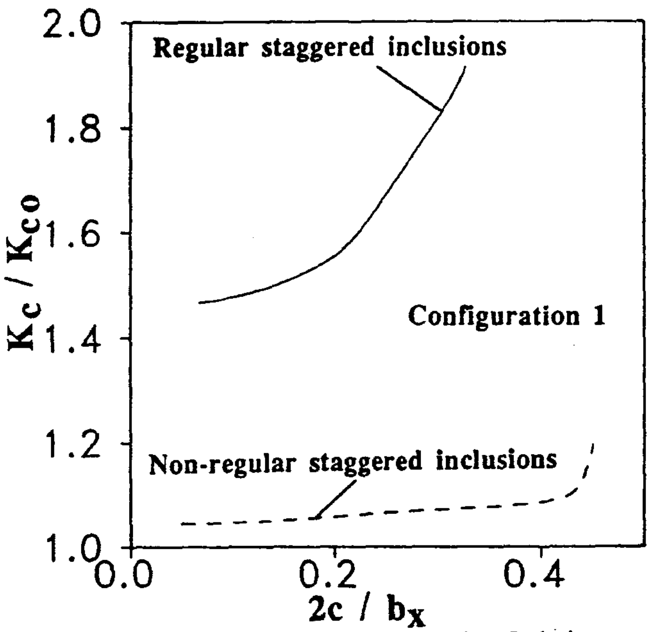


Figure 5 : Influence of the Spatial Distribution of the Inclusions on the Apparent Toughness of a Composite.

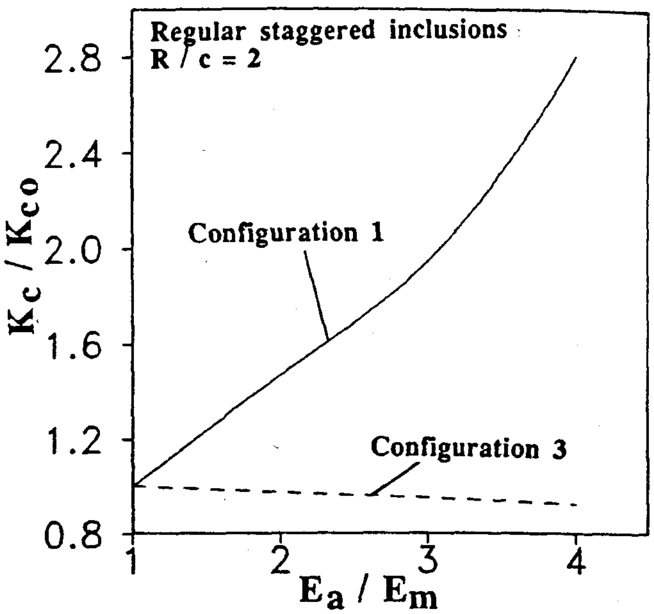


Figure 6 : Influence of the Relative Elastic Properties of the Inclusions on the Apparent Toughness of a Composite.

E_m showed in Fig. 6 corresponds to the usual values for normal concrete.

CONCLUSION

The interaction between a crack and several inclusions can be analyzed by superposing known solutions of standard problems of elasticity. The method uses first Duhamel-Neuman analogy in order to transform the problem into a problem of elasticity of a homogeneous body in which the inclusions are replaced by the matrix and the boundary conditions are modified. The proposed superposition scheme is similar to Kachanov's method for interacting crack systems.

The variation of apparent fracture toughness of the equivalent homogeneous medium is the inverse of the calculated variation of the mode I stress intensity factor at the tip of a crack propagating in the composite. These variations of toughness are analogous to R-curves. Calculations show that the apparent toughness depends on the volume fraction of the inclusions, on their spatial distribution, and on the elastic properties of the constituents of the composite. The largest toughness is obtained when the crack propagates toward an inclusion (mode I crack propagation) and the lowest toughness corresponds to a crack propagating between two inclusions.

The numerical applications show that, for a given composite and for a fixed crack configuration, the mechanical effect of the interaction between the crack and the inclusion is not negligible. This effect should be important for explaining the stable simultaneous propagation of many interacting cracks in a heterogeneous medium, as well as for determining the conditions under which stable states of diffuse damage can exist.

Acknowledgements : Partial financial support from AFOSR contract F49620-87-0030DEF with Northwestern University and from the Center for Advanced Cement-Based Materials at Northwestern University is gratefully acknowledged.

REFERENCES

BAŽANT, Z.P. (1987a), "Why Continuum Damage is Nonlocal : Justification by Quasiperiodic Microcrack Array", Mechanics Research Communications, Vol. 14, No 5/6, pp. 407-419.

BAŽANT, Z.P. (1987b), "Snapback Instability at Crack Ligament Tearing and Its Implication for Fracture Micromechanics", Cement and Concrete Research, Vol. 17, pp. 951-967.

BAŽANT, Z.P. and TABBARA, M.R. (1989), "Stable Propagation of Interacting Cracks Systems and Modeling of Damage", Proc. of 10th Conf. on Structural Mechanics in Reactor Technology SMiRT 10, A.H. Hadjian Ed., Vol. Q, pp. 1-8.

BERTHAUD, Y. and PIJAUDIER-CABOT, G. (1990), "Nonlocal Continuum and Micromechanics of Damage in Quasi-Brittle Materials", in "Micomechanics of Failure in Quasi-Brittle Materials", Ed. by S.P. Shah et al., Elsevier Pub., pp. 125-134.

BUDIANSKY, B. and O'CONNELL, R.J. (1976), "Elastic Moduli of Cracked Solids", Int. J. of Solids and Structures, Vol. 12, pp. 81-97.

ERDOGAN, F., GUPTA, G.D. and RATWANI, M. (1974) "Interaction Between a Circular Inclusion and an Arbitrary Oriented Crack", J. of Appl. Mech. ASME, Trans. of ASME, pp. 1007-1013,

FURUHASHI, R., KINOSHITA, N. and MURA, T. (1981), "Periodic Distribution of Inclusions", (1981) "Periodic Distribution of Inclusions", Int. J. of Engng. Science, Vol. 19, pp. 231-236.

GAO, H., and RICE, J.R. (1988) "A First Order Perturbation Analysis of Crack Trapping by Arrays of Obstacles", Report Division of App. Sciences, Harvard Univ. Cambridge MA.

HORII, H., ZIHAI, S. and GONG, S.X (1989), "Models of Fracture Process Zone in Concrete, Rock, and Ceramics", in Cracking and damage", J. MAZARS and Z.P BAŽANT Eds, Elsevier App. Sciences Pub., pp. 104-115

HORII, H. and NEMAT-NASSER, S. (1985) "Elastic Fields of Interacting Inhomogeneities", Int. J. of Solids and Struct, Vol. 21, pp. 731-745.

HUANG, J. and LI, V.C (1989), "A Meso-Mechanical Model of the Tensile Behavior of Concrete", Parts I and II", Composites, Vol. 20, No.4, pp. 361-378.

JU, J. W. (1990), "On Two-Dimensional Self Consistent Micromechanical Damage Models for Brittle Solids", Int. J. of Solids and Structures, in press.

KACHANOV, M (1987) "Elastic Solids with Many Cracks-Simple Method of Analysis", Int. J. of Solids and Struct., Vol. 23, pp. 23-43.

KRAJCINOVIC, D. and SUMARAC, D. (1989), "A Mesomechanical Model for Brittle Deformation Processes", J. of Applied Mech. ASME, Vol. 56, pp. 51-62.

KUNIN, I. and GOMMERSTADT, B. (1985), "On Elastic Crack-Inclusion Interaction", Int J. of Solids and Structures, Vol. 21, No 7, pp. 757-766.

MAZARS, J. and BAŽANT, Z.P. eds. (1989), "Cracking and Damage - Strain Localization and Size Effect", Elsevier Pub.

MORI, T., SAITO, K. and MURA, T. (1988) ,"An Inclusion Model for Crack Arrest in a Composite Reinforced by Sliding Fibers", Mech. of Materials, Vol. 7, pp. 49-58.

MURA, T. (1987), "Micromechanics of Defects in Solids", 2nd Edition, Martinus Nijhoff Pub.

PIJAUDIER-CABOT, G., BAŽANT, Z.P. (1990) "Cracks Interacting with Particles of Fibers in Composites Materials", Report No. 105 Laboratoire de Mécanique et Technologie, jointly with ACBM at Northwestern University; see also PIJAUDIER-CABOT, G., BAŽANT, Z.P., and BERTHAUD Y. (1990), "Interacting Crack Systems in Particulate or Fiber Reinforced Composites", Proceedings of the fifth Conf. on Numerical Methods in Fracture Mechanics. D.R.J. Owen et al. eds., Pineridge Pub., pp. 403-414.

PIJAUDIER-CABOT, G. and BERTHAUD, Y. (1990), "Effets des Interactions dans l'endommagement d'un milieu fragile - formulation nonlocale", Comptes Rendus acad. Sc., t. 310, Serie II, pp. 1577-1582.

ORTIZ, M. (1988), "Microcrack Coalescence and Macroscopic Crack Growth Initiation in Brittle Solids", Int. J. of Solids and Structures, Vol. 24, pp. 231-250.

TADA, H., PARIS, P.C., and IRWIN, J.K (1985), "The Stress Analysis of Cracks Handbook", 2nd Ed., Paris Prod. Inc., 226 Woodbourne Dr. St Louis Mo. USA.

TIMOSHENKO, S.P., and GOODIER, J.N. (1970), "Theory of Elasticity", 3rd ed., Mc Graw Hill, New York, p. 127

ZAITSEV, Y.V (1985), "Inelastic Properties of Solids with Random Cracks", in Mechanics of Geomaterials, Z.P BAŽANT Ed., J. Wiley Pub. pp. 89-128.