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WHY DIRECT TENSION SPECIMENS FLEX AND BREAK AT MIDLENGTH

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ABSTRACT

In a direct tension test, unnotched specimens of quasibrittle materials that exhibit post-peak strain softening do not deform symmetrically. The equilibrium path bifurcates and the thermodynamic criterion of stable path requires the specimen to flex to one side, even if the geometry is perfect and if the straightening effect of the moment of the axial force about the centroid of the deflected cross section is taken into account. In absence of rotational restraints at the ends, the specimen fails right after the peak load. But if the attachments to the loading machine exert a sufficient restraint against rotation, the flexing to the side is retarded and failure occurs at midlength. The phenomenon (which is similar to the recently discovered behavior of notched tensile fracture specimens) is illustrated using a simple model in which the specimen consists of two bars of unequal length, joined by a strain-softening link.

INTRODUCTION

For brittle or quasibrittle materials which exhibit strain softening, for example concrete, rock or ceramics, the interpretation of the direct tensile test is not easy. As Rots and de Borst [8, 9] and Hordijk, Reinhardt and Cornelissen [6] have recently shown, symmetrically notched prismatic specimens subjected to centric tension do not deform symmetrically but flex to one side as cracks propagate from the notches. A similar loss of symmetry and lateral flexing was analytically demonstrated by Bažant [1, 2] and Bažant and Tabbara [11]. Subsequently Bažant [3] and Bažant and Cedolin [4] analyzed bifurcation and stable response path of a simple model of a centrically tensioned unnotched specimen, and they again found that this idealized specimen, too, flexes to one side if certain, not abnormal, conditions are met. A similar phenomenon was demonstrated by Pijaudier-Cabot and Akrib [7] by layered finite element analysis. The objective of this paper is to present a summary of the results obtained in Ref. [5] through a generalization of the previously presented model.

RIGID-BAR MODEL

Let us consider the tensile specimen in Fig. 1a, which consists of two rigid bars of unequal lengths L_1 and L_2, and denote by N the axial (tensile) load and by u the corresponding axial displacement. The bars are linked by two deformable short flanges of length h ($h \ll L$, $L \simeq L_1 + L_2$) and cross section areas A_f, each of negligible width and located at distances b from the cross section center. These flanges can exhibit strain-softening, characterized by tangent softening modulus E_t ($E_t < 0$, Fig. 1d). The specimen is supported at the ends by hinges with springs of rotational stiffnesses C. Let w be the deflection of the link, and θ_1 and θ_2 the inclination angles of bars L_1 and L_2, respectively. The link is assumed to transmit a shear force without deforming in shear.

Let us assume that initially $\theta_1 = \theta_2 = 0$ (perfect structure). In general the equilibrium path in the (P, u) plane bifurcates, with two possible branches: (1) the primary (symmetry preserving) branch $\delta\theta_1 = \delta\theta_2 = 0$, for which both flanges are loading, obeying modulus E_t; and (2) the secondary (symmetry breaking) branch $\delta\theta_1 \neq \delta\theta_2 \neq 0$, for which one flange is loading, obeying modulus E_t ($E_t < 0$), and the other is unloading, obeying unloading modulus E_u ($E_u > 0$, $E_u \le E$ = initial elastic modulus). For the latter path, writing the equilibrium conditions of each bar for vertical forces and for moments about the center of the link, expressing the strains in the left and right flanges of the link in terms of displacements, and assuming that the right flange is loading and the left flange is unloading, one can show (see Ref. [5]) that the condition that the left flange during this deformation unloads requires that

$$-E_t \ge \bar{E}_t = \frac{h}{4b^2 A_f}\left[N\frac{L}{2} + C - 2\Delta^2\left(\frac{N}{L} - 2\frac{C}{L^2}\right)\right] \tag{1}$$

where $\Delta = (L_1 - L_2)/2$ = distance of the location of failure from the specimen midlength, and $\bar{E}_t$ represents the minimum magnitude of E_t for which flexing to the side can occur. From this equation one can see that $\bar{E}_t$ (1) increases with the value of C, i.e. with the stiffness of the rotational restraints at the bar ends ; and (2) increases

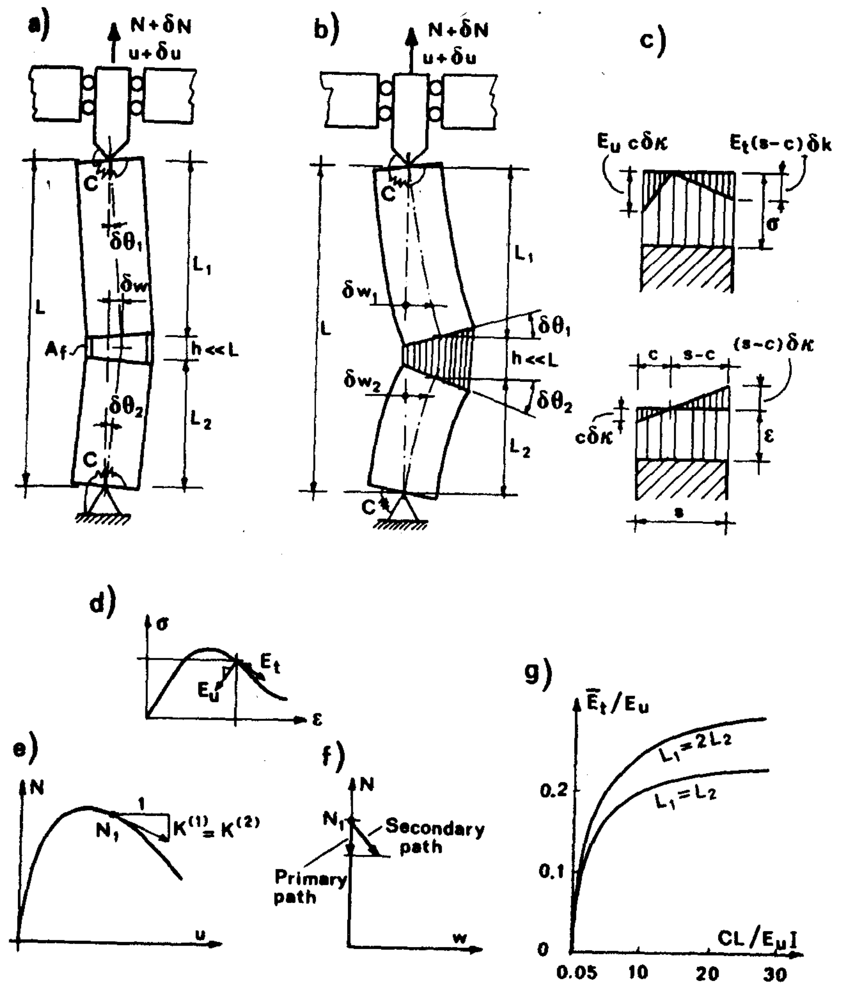

Figure 1. (a) Rigid-Bar Model; (b) Deformable Bar Model; (c) Variations of Stresses and Strains in Strain-Softening Slice; (d) Loading and Unloading Tangential Moduli; (e, f) Bifurcation of Equilibrium Path; (g) Minimum Magnitude of the Loading Tangential Modulus at Bifurcation.

with distance Δ provided that $C > NL/2$, i.e. that the rotational restraint at the bar ends exceeds a certain limit. If this condition is met, the smallest $\bar{E}_t$ occurs for $L_1 = L_2$.

In the same Ref. [5] it is also shown that the value of $K^{(2)}$ corresponding to $E_t = -\bar{E}_t$ coincides with the tangent stiffness $K^{(1)} = 2A_f E_t/h$ for the primary branch, for which $\delta\theta_1 = \delta\theta_2 = 0$ and symmetry is preserved (Fig. 1e, f). As shown by thermodynamic arguments in Ref. [4] (Sec. 10.2 - 10.3), the equilibrium path branch that actually occurs is that for which the tangent slope given by $K^{(1)}$ or $K^{(2)}$ is steeper. Thus, the symmetry-breaking secondary branch must occur when $K^{(2)} < K^{(1)}$, and it is found that this condition is always satisfied for $E_t < -\bar{E}_t$. This means that, assuming a softening stress-strain diagram with a gradually steepening donward slope, the specimen will flex to the side as soon as the limit condition $E_t = -\bar{E}_t$ is attained. Furthermore, comparing the $\bar{E}_t$ values for various Δ values for a specimen with rotational end restraints, we see that the minimum slope occurs for $\Delta = 0$. This proves that the direct tensile specimen must break at midlength.

The foregoing analysis shows that, if there is a rotational restraint of nonzero stiffness at the specimen ends, the minimum magnitude of the tangential modulus for which flexing to the side occurs is attained when the specimen breaks in the middle and increases with C . It appears, then, that in order to increase the stability of the primary path, one should increase the stiffness of the rotational restraints at the bar ends.

The limit condition $E_t = \bar{E}_t$, expressed through Eq. 1, gives an implicit equation [5] exactly analogous to that which defines Shanley's load in compression [10]; for a review see for example Ref. [4] (Sec. 8.1 and 10.2-10.4). Overall we may conclude that the strain-softening feature of the constitutive law engenders bifurcation of the equilibrium path in tension (Fig. 2f), a fact that has not been suspected.

DEFORMABLE-BAR MODEL

A more realistic analysis requires consideration of a deformable specimen, as shown in Fig. 1b, which has a rectangular cross section of width r and height s . We assume that strain softening, if it occurs, must be localized in a beam slice of a short length h ($h \ll L$), located at distances L_1 and L_2 from the specimen top and bottom (Fig. 1b); $L_1 + L_2 \simeq L$ = specimen length. Outside the slice, there is no loading anywhere. The variations of the rotations of the ends of the slice at the moment of bifurcation are denoted as $\delta\theta_1$ and $\delta\theta_2$. The variation of curvature in the slice is $\delta\kappa = (\delta\theta_1 + \delta\theta_2)/h$. The variations of strains at the left and right faces of the slice are $c\delta\kappa$ and $(h-c)\delta\kappa$ where c is the distance from the left face to the neutral axis, which separates loading from unloading (Fig. 1c). Finally, the variation of the axial deformation of the link is expressed by $\delta\epsilon = \delta u/h = (s/2 - c)\,\delta\kappa$.

Writing the appropriate equilibrium and compatibility equations, and taking into account the nonlinear geometric effects, one can show [5] that the limit condition $c = 0$ gives for $K^{(2)}$ a value which coincides with $K^{(1)}$, similarly to what we found for the rigid bar specimen. The corresponding value of E_t is given by

$$-E_t = \bar{E}_t = \frac{12G_2 h}{kG_1 r s^3} \tag{2}$$

where G_1 and G_2 are known expressions. Again one finds that, for $E_t < -\bar{E}_t$, the condition $K^{(2)} < K^{(1)}$ is always satisfied.

Numerical calculations show that the effects of the stiffness of the end restraints and of the position of the strain-softening zone along the bar are analogous those found for the rigid-bar specimen. This is illustrated by Fig. 1g , which shows the value of $\bar{E}_t$ as a function of the stiffness C for different values of the ratio L_1/L_2 , for the case that $L = 100$ cm , $h = s = 5$ cm, $r = 10$ cm , $I = 500$ cm^3, $N = 15000$ N and $E_u = 3\times10^6$ N/cm^2. One can see that the effect of increasing the stiffness of the end restraints is to enhance the stability of the primary path (provided that $C > C_{lim} = 0.05\,E_u I$) .

CONCLUSIONS

1. The equilibrium path in a direct tension test of a strain softening material exhibits a bifurcation in which the secondary, symmetry-breaking path corresponds to flexing of the specimen to the side.

2. The lateral flexing can be retarded by providing rotational restraints at the specimen ends. The stiffer they are, the steeper is the post-peak slope of the strain-softening diagram at which bifurcation occurs.

3. The analysis shows that lateral flexing favors a break at midlength of the specimen over a break away from the middle.

4. The bifurcation behavior is analogous to Shanley's bifurcation in elastoplastic columns.

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