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Conclusions for structural analysis and for formulation of standard design recommendations

Based on the state-of-the-art report of RILEM Committee TC69, several basic conclusions can be drawn with regard to the method of structural creep analysis as well as the formulation of improved standard design recommendations by engineering societies. These conclusions, which apply only to ordinary concretes under normal environmental conditions, may be succinctly stated as follows:

STRUCTURAL CREEP ANALYSIS

1. As the simplest approximation, structural creep and shrinkage analysis may be conducted under the assumption of linearity. The material creep properties are then fully characterized by the compliance function. The linear methods of structural creep analysis are at present well understood.

2. Approximate linear solutions by the age-adjusted effective modulus method may be recommended as most efficient and adequate for most cases. This method, which yields algebraic rather than differential or integral equations in time, is simpler than other existing linear methods. Its error is generally less, and often much less, than the errors of the compliance function used and of the linearity assumption itself. Because of these errors, the gain in accuracy of linear solutions for real structures achieved by more complicated linear methods (solving linear differential or integral equations in time) is normally only fictitious. The classical effective modulus method, which is not appreciably simpler than the age-adjusted one, is sufficient only for those cases where the stresses are almost constant in time (varying less than 5% to 10%).

3. The linearity assumption is applicable only in the service load range, and even then the linearity assumption suffers from large error when moisture effects and cracking dominate. The knowledge of nonlinear creep, particularly marked in the presence of moisture exchange and cracking, is still rather limited, and should be the principal objective of future research.

4. In theory, a realistic and accurate finite-element analysis of structures for creep and shrinkage must be nonlinear and coupled with the analysis of pore humidity and temperature distributions evolving in time as well as the nonuniform progress of hydration. The most important nonlinear effect to take into account is the cracking and strain-softening produced by nonuniformity of pore humidity, shrinkage and specific creep (and also temperature). Although modelling of these phenomena bears considerable promise, it is still in the process of development and the present formulations probably involve considerable errors, stemming from uncertainties in material data and the form of constitutive law, as well as from problems of convergence and fundamental questions with regard to localization of cracking or strain-softening (particularly their nonlocal and fracture mechanics aspects). An important goal of further research is to overcome these difficulties. Meanwhile, it is not inappropriate to use, in practice, simple material models (i.e. linear) and a simple method of structural analysis (such as the age-adjusted effective modulus).

COMPLIANCE FUNCTION AND CONSTITUTIVE RELATION

5. No special form needs to be imposed upon the compliance function in order to facilitate some simple method of structural analysis, such as the rate-of-flow method. This is because the age-adjusted effective modulus method works for any compliance function for concrete, and is at the same time simpler than the solutions by differential equations made possible by special forms of the compliance function.

6. In characterizing the creep and shrinkage properties, distinction must be made between the true material properties which are described by the constitutive equation (stress–strain relation) and refer to a

point in the structure, and the mean cross-section properties which characterize the overall creep and shrinkage in the cross-section, but do not represent a constitutive equation. Calculations on the basis of the mean compliance function and mean shrinkage function for the cross-section can yield only the internal force resultants in the cross-sections (i.e. axial forces and bending moments) but not the actual stress values, not even approximately. The stress values in such analysis are strictly nominal.

7. The desire for an overall characterization of creep and shrinkage in the cross-section is a source of large error as well as complexity. The mean cross-section compliance function for a drying environment in reality depends on the cross-section shape as well as the type of loading (bending, compression, tension, torsion, etc.). Without such distinctions, it apparently cannot be formulated with errors whose coefficient of variation would be below about 18% even if the most sophisticated formulation is used, and about 40% if simple formulations of the type of current codes are used. By contrast, the constitutive relation for a point in the structure, which must involve the pore humidity and temperature at the same point instead of the environmental ones, is far simpler. The true material creep and shrinkage properties (e.g. the compliance function) can in principle be formulated with a much smaller error.

8. When the cross-section is subdivided into layers or finite elements, one should not use for the individual layers or elements the mean compliance function of the cross-section as specified in codes. Rather, one should use the true material constitutive relation, which also must involve the humidity (and temperature) calculated for the same layer or element rather than the environmental humidity and temperature.

9. Standard recommendations should specify the compliance function rather than the creep coefficient or specific creep. This approach prevents combining the creep coefficient value with an incorrect value of the elastic modulus, which has been a frequent source of error in practice. In particular, the elastic modulus as defined in current standards is rather different from the modulus that corresponds to the usual definition of the short-time initial strain in creep tests. These definitions should be brought into agreement or at least their applicability limitations specified.

10. The constitutive equation for creep must satisfy the principle of continuity. This principle dictates that the strain responses for any two stress histories that are infinitely close to each other must be also infinitely close to each other. For example, if one strain history is caused by a constant stress s_1 applied from time t_1 to time t_2, and another strain history by a constant stress s_0 applied from t_0 to t_1 and followed by constant stress s_1 applied from t_1 to t_2, then in the limit $s_0 \rightarrow 0$, the second strain history must approach the first stress history. Violations of continuity are not only unjustified physically but also destroy convergence of numerical solutions.

11. The mean cross-section compliance should be formulated as a sum of the basic creep compliance and the additional compliance due to drying. This separation is necessary to capture the different forms of both compliances, e.g. the fact that the drying creep compliance has a final asymptotic value and the basic creep compliance does not (see item 13), and the fact that the shape and magnitude of the drying compliance depend on the cross-section size and shape and the type of loading (bending, axial etc.) while those of the basic creep compliance do not.

12. Although a separation of creep into a reversible component (delayed elasticity) and an irreversible component (flow) can be introduced if desired, it is neither required nor justified by thermodynamics. The reason, briefly, is that in ageing materials the reversible creep component cannot be characterized as a unique function of the thermodynamic state of the material. The delayed elastic strain depends strongly on the age at unloading, the age at first loading, the duration of recovery, the hygrothermal conditions, etc.

13. The existing test results do not indicate the existence of any final asymptotic value of basic creep. Therefore, the basic creep formulae which smoothly extend in time the trend of available test data, are more reasonable than those which assume that beyond the available data range the creep curve changes its trend, approaching some fictitious final asymptotic value. This question would be moot if a slope decrease (in log-time) were assumed to start after the end of the lifetime of the structure, but this is not the case for the various formulae in use. No particular simplification of structural analysis is achieved if the compliance function is specified with a final asymptotic value.

14. Although thermodynamic restrictions do not prohibit the basic creep curves for two different ages at loading to diverge after a certain time, divergence-free expressions for the basic creep compliance function are preferable because otherwise application of the principle of superposition leads to a non-monotonic recovery curve. Non-monotonic recovery is found only in a minority of basic creep data, and is likely caused by other influences rather than by the shape of the compliance function.

15. Among the simplest formulae for the basic creep compliance function, a power function of load duration, with an exponent of about $\frac{1}{8}$, appears to be the best. The actual behaviour indicates a transition from a power law for short load durations to a logarithmic law for long load durations. The transition does not occur at any fixed time; rather, the transition time increases as the age at loading increases, while the strain value at the transition seems to be roughly the same for all ages at loading.

16. An accurate and physically justified formula for the compliance function is particularly important for extrapolation of short-time data. The classical Ross hyperbola as well as Dischinger's exponential are obsolete in view of the present experimental evidence and give incorrect long-time extrapolations.

17. The dependence of the basic creep strain on the age at loading is adequately described by an inverse power function.

18. Unlike thermorheologically simple materials, the compliance function of concrete cannot be expressed as a function of one time variable, the reduced time. Numerous attempts in this regard were unconvincing because they considered only very limited test data.

19. Strain-hardening and time-hardening, i.e. concepts in which the creep rate is expressed as a function of the current creep strain and the time elapsed from the first loading, are alone insufficient to describe the behaviour of concrete under variable stress.

20. To save computer time and storage, the integral-type creep law based on the compliance function should be converted to a rate-type form corresponding to an ageing spring–dashpot rheologic model, such as the Maxwell or Kelvin chain. The conversion can be accomplished in the computer program by an efficient algorithm, applicable to any compliance function provided as input. The rate-type stress–strain relation may then be most effectively solved by the exponential algorithm. The spring moduli and dashpot viscosities obtained from this algorithm unfortunately do not always satisfy certain thermodynamic restrictions for the stress–strain relation; however, this problem is likely to be caused by trying to make a linear theory fit compliance data which involve nonlinear phenomena.

PHYSICAL BASIS

21. The stress–strain relation should be based to the maximum possible extent on the mathematical theory of creep and shrinkage mechanism, because creep tests inevitably yield information that is of limited range and imprecise, owing to random scatter and experimental error. The physical considerations and mechanistic theories which have so far yielded information on the compliance function and the material model include: (i) thermodynamic restrictions; (ii) ageing interpreted as time variability of nonageing components; (iii) activation energy theory, which governs the creep rate, ageing (hydration), rate diffusion rate, etc; (iv) diffusion theory for moisture and temperature effects; (v) theories of capillarity, adsorption, and diffusion between small and large pores; (vi) models of cracking and fracture mechanics; (vii) basic restrictions of continuum mechanics, including the principles of objectivity, tensorial invariance, form invariance, etc; (viii) stochastic process modelling.

22. Proposals for physical mechanisms whose consequences for the stress–strain relation (or compliance function) cannot be formulated mathematically are of little use for modelling creep and shrinkage because they can be neither proved nor disproved. It is therefore essential to develop mechanistic models that can be described in mathematical terms.

23. Interpretation of macroscopic strain and other measurements in terms of a microscopic physical mechanism must take into account the so-called apparent mechanisms which are not inherent in the local material properties *per se*, but are the results of spatial interactions which depend on structure shape and size. These include the effect of nonuniformity of the distributions of stress, strain, moisture content and temperature throughout the specimen, the nonuniform creep and shrinkage caused by them, the residual stresses, and cracking or tensile strain-softening.

24. The diffusion equation for water movement through concrete is strongly nonlinear, principally because of a strong decrease of diffusivity as the specific water content increases.

25. The relevance of diffusion theory of moisture transport is confirmed by experimental evidence. The consequences of diffusion theory should be observed in the formulation of the mean shrinkage function and the mean compliance function for drying creep in the cross-section. In particular, the diffusion theory indicates that: (1) the half-time of shrinkage and drying creep is proportional (a) to the size-square, (b) to the drying diffusivity, and (c) the shape factor (which are the same as obtained from weight loss measurements); (2) the shrinkage function and the drying creep term initially evolve as the square root of time. These laws are true for both the simplified solutions according to linear diffusion theory and the more realistic solutions according to nonlinear diffusion theory. Because of the effects of ageing, cracking or strain-softening (with its irreversibility), and creep due to residual stresses, the foregoing laws are only approximate. Nevertheless, instead of abandoning diffusion theory, deviations should be introduced as additional corrective terms because they vary from one situation to another, being in one case larger, in another case smaller.

26. One should distinguish a two-fold effect of temperature on creep: the effect on the creep rate (or viscosity coefficients in a rate-type model), and the effect on the ageing rate (hydration rate). These two effects counteract each other and must be described by separate activation energy expressions, because sometimes one effect dominates, sometimes another. Activation energy also controls the effect of temperature on moisture diffusion, which in turn affects shrinkage and drying creep.

27. To model the drying creep effect (called also the Pickett effect, stress-induced shrinkage, or the mechanosorptive effect), it appears necessary (according to studies based on finite-element fitting of measured strains) to introduce into the constitutive relation a cross effect between the stress and the time rates of local pore humidity and local temperature. Particularly, this cross effect may be described as an additional strain rate component that depends on the product of stress with the rate of pore humidity or temperature. This cross effect may be explained by the hypothesis that the rate of bond rupture that controls the creep rate depends on the flux of moisture between macropores and micropores in cement gel. A related effect, which explains the so-called transitional thermal creep and probably has the same physical mechanism, is an additional strain rate which depends on the product of stress and temperature rate and may be called

the stress-induced thermal expansion. In three dimensions, the stress-induced shrinkage and the stress-induced thermal expansion are anisotropic, because of dependence of the creep rate on the stress tensor.

28. The second major source of drying creep (Pickett effect) and transitional thermal creep is an apparent mechanism: the cracking and microcracking produced by nonuniform stress distributions that arise in drying or heating. Description of this effect requires consideration of tensile strain-softening. Its irreversibility further contributes to the Pickett effect, and so does ageing.

29. Calculations of pore humidity distributions must take into account the strong dependence of moisture diffusivity on the pore relative humidity, which causes the diffusion problem to be highly nonlinear.

30. Since thermodynamics deals only with substances invariant in time, ageing should be described mathematically as a consequence of varying composition of a mixture of reacting constituents which are themselves nonageing, i.e. time invariant. These constituents may approximately be considered as the hydrated and unhydrated volume portions of cement paste (although in view of silicate polymerization such distinctions are more difficult – for the C–S–H component of cement gel, perhaps the volume fraction or surface area fraction of polysilicate). In such a formulation, thermodynamics yields restrictions on the form of the compliance function as well as on the elastic moduli and viscosities of the rate-type model. In particular, it follows that the age-dependent elastic modulus in a spring of rheological model must not be used to relate the stress and strain in the spring, but their rates.

UNCERTAINTY

31. Prediction of creep and shrinkage from concrete composition and design parameters such as strength is highly uncertain. For the current standard recommendations of ACI (1971, 1982) and CEB–FIP (1978), the 95% confidence limits on strain predictions are about ± 77% and ± 92%, respectively. For the best available predictive model for mean cross-section properties, i.e. the BP model, which is considerably more involved but is to a larger extent based on physical considerations and has a broader range of applicability, these confidence limits are about ± 37%. To reduce these errors further would no doubt require a much better understanding of the effect of concrete composition on creep, which is, however, a difficult goal to achieve. Also, predictions would have to be made for material behaviour at a point rather than for its mean over the cross-section.

32. At present the only available means of substantially improving the aforementioned confidence limits is to measure short-time values and then extrapolate in time. The simplest way to extrapolate is statistical regression. A better way is the Bayesian approach, in which further information is acquired from the prior knowledge of all concretes. With Bayesian extrapolation, the 95% confidence limits for long-time predictions can be reduced to about ± 12%. Short-time measurements of 3 to 30 days appear to be sufficient.

33. With the use of present-day standard recommendations, the errors in prediction are largely model errors rather than actual randomness of creep and shrinkage. Therefore, the standard design recommendations of engineering societies should not only specify the recommended creep and shrinkage prediction model, but they should also indicate its coefficient of variation or confidence limits (and bias). Inevitably, adoption of a more sophisticated model will be rewarded with a smaller error. The choice of model should be left up to the engineer. He would be able to choose a model of proper simplicity or sophistication depending on the error which he deems to be acceptable for his design. The acceptable value of error will depend on whether the structure to be built is creep-sensitive or creep-insensitive.

34. The errors of the creep and shrinkage prediction models in the present standard design recommendations are so large that finite-element analysis (and even statically indeterminate frame analysis) of a creep-sensitive structure hardly makes any sense unless statistics are calculated. The error due to the use of these models is much larger than the error involved in the traditional simplified solutions of frames such as the portal method.

35. The probabilistic method of structural creep and shrinkage analysis is now available and ready for use in practice. Its simplest version treats creep and shrinkage as a function of several random material parameters and calculates the statistics by a sampling method (latin hypercube sampling).

36. More realistic and more sophisticated probabilistic models separate internal uncertainty and external uncertainty. The external uncertainty, owing to randomness of the environment, affects the surface and interior regions of a structure differently and its correct modelling must take into account the size effect that results from diffusion theory describing water and heat transport through concrete. Some form of spectral approach seems appropriate for such analysis.

37. In most structures, creep and shrinkage affect long-time deflections and stress redistributions which may produce cracking. In these situations, creep and shrinkage influence primarily long-term serviceability while the effect on structural safety against collapse is minor and indirect. However, in certain structures such as slender columns and especially shells, creep may have a major effect on structural safety because it can cause long-time buckling collapse.