

JOURNAL OF THE ENGINEERING MECHANICS DIVISION

THERMOVISCOELASTICITY OF AGING CONCRETE^a

By Zdeněk P. Bažant,¹ M. ASCE and Spencer T. Wu²

INTRODUCTION

Recently, creep of concrete at elevated temperatures has been receiving vivid attention, mainly because of its importance in the design of prestressed concrete reactor vessels. The effect of temperature is more intricate in concrete than it is in polymers and metals, and is basically two fold. First, temperature directly influences the deformation rate, and second, it affects the rate of aging, i.e., the change of material properties due to the progress of cement hydration.

A constitutive equation which accounts for both of these effects seems to have first been devised in Refs. 6, 39, and 40. Mukaddam and Bresler (39,40) formulated the acceleration of creep due to temperature according to the well-known concept of thermorheologically simple materials (17), which is based on the reduced time, t_{red} , whose increment dt_{red} is obtained from the actual time increment, dt , by multiplying it with a certain shift function, Ψ , of temperature T . This type of formulation describes very accurately the thermoviscoelastic response of time-invariable materials, e.g., polymers (17,24), and its extension to concrete has already been suggested by Sackman (54).

The basic idea of Mukaddam and Bresler was to introduce the age of concrete as a second variable into the shift function, putting $\Psi = \Psi_1(T) \Psi_2(t^*)$; in which Ψ_1 and $\Psi_2 =$ functions of one variable; and $t^* =$ the age at loading corrected for temperature T , such that $t^* = t_0 + (t' - t_0) e^{-A(T-T_0)}$; T_0 and $t_0 =$ the chosen reference temperature and reference age at loading; $t' =$ actual age at loading at temperature T ; and $A =$ constant. Mukaddam and Bresler's formulation was somewhat modified by Rashid (50), who omitted the temperature corrected age at loading, t^* , and introduced instead a different definition of reduced time, such that $dt_{red} = \Psi_1(T) \Psi_3(t - t') dt$ in which $\Psi_1, \Psi_3 =$

Note.—Discussion open until November 1, 1974. To extend the closing date one month, a written request must be filed with the Editor of Technical Publications, ASCE. This paper is part of the copyrighted Journal of the Engineering Mechanics Division, Proceedings of the American Society of Civil Engineers, Vol. 100, No. EM3, June, 1974. Manuscript was submitted for review for possible publication on December 22, 1972.

^aPresented at the October 29–November 1, 1973, ASCE Annual and National Environmental Engineering Meeting, held at New York, N.Y. (Preprint 2110).

¹Prof. of Civ. Engrg., Northwestern Univ., Evanston, Ill.

²Grad. Research Asst., Northwestern Univ.; presently Sr. Engrg. Analyst, Sargent and Lundy, Engrs., Chicago, Ill.

functions of one variable; and $\Psi_1(t-t') = d[\log(t-t' + 1/dt)]$.

The formulations of Mukaddam and Bresler (39,40) and Rashid (50) have the advantage that functions Ψ_1 and Ψ_2 or Ψ_3 may be obtained by simple graphical manipulations of the measured creep curves. However, generalizations to variable stress according to the principle of superposition leads to stress-strain relations in the form of an integral equation which cannot be converted to a system of first-order differential equations. This poses major difficulties in creep analysis of large structural systems arising, e.g., in finite element method. Namely, a step-by-step creep analysis for an integral-type creep law requires the values of stress from all previous time steps to be stored for each element of the structure and to be used in computation at each time step. Thus, the storage and machine time requirements become a multiple of those for the corresponding elastic problem.

The cumbersome history dependence can be avoided only by a transition to a rate-type creep law, i.e., a creep law consisting of a set of first-order differential equations involving hidden (internal) variables. [Certain forms of an integral-type creep law, e.g., those with a degenerate kernel, achieve the same; but such laws can always be converted to a rate-type form (5).] The structure of rate-type creep laws can always be visualized by spring-dashpot models, and a certain distaste has unfortunately developed toward these models during the 1960's, partly because of vulgarized microstructural interpretations, partly because of an avalanche of papers with only immaterial differences in the arrangement of model. Actually, there is justification for only two basic types of models, the Maxwell chain and the Kelvin chain, because either of these two models can describe a viscoelastic material with any desired accuracy, whether the material is age-independent (52), or age-dependent (10,11). These models define the well-known relaxation and retardation spectra and are equivalent to the expansion of memory kernel in a series of real exponentials (Dirichlet series), which is a counterpart of the Fourier series and plays a role as important in the decaying processes as the frequency response spectra do in the periodic processes. The rate-type laws also allow a much simpler formulation of thermodynamics which was first emphasized in a fundamental paper by Biot (14), and it is for this reason that the rate-type laws are now becoming the preferred approach in theoretical continuum mechanics (43). Finally, the rate-type formulation of the creep law is also necessary in deducing additional information on the creep law from a model of the microstructural processes (3,4,8,30) because their mathematical formulation always leads to differential rather than integral equations.

For constant temperature, the rate-type creep law of concrete has been developed in Refs. 10 and 11. An extension to variable temperature will be the primary objective of this study.

One important physical concept in temperature effects is that of the activation energy of thermally activated processes (26). This concept has been widely explored in creep of metals (19,36,62), and also creep of clays (38). Polivka and Best (49), analyzing the change of creep rate at a temperature rise, introduced it into the study of concrete creep. At the same time Hansen (28) proposed a temperature dependence of a certain viscosity parameter which is equivalent to the activation energy formulation. Later Ruetz (53), Klug and Wittman (32,64), Maréchal (35) and others (16,56), analyzed concrete creep using the activation

energy approach. However, in all of these studies the acceleration of aging due to temperature rise has been disregarded and creep curves have been fitted by expressions that are inapplicable to the case of variable stress.

REVIEW OF RATE-TYPE CREEP LAW AT CONSTANT TEMPERATURE

This study will be restricted to creep at constant or almost constant water

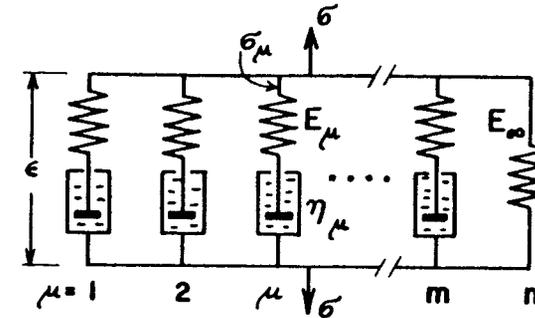


FIG. 1.—Maxwell Chain Model

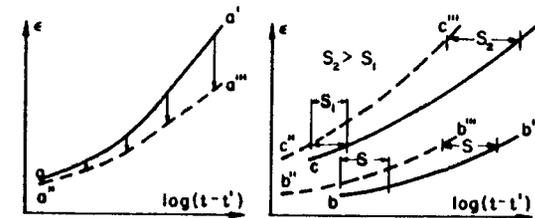


FIG. 2.—Sketch of Effect of Acceleration in Aging and of Change in Deformation Rate

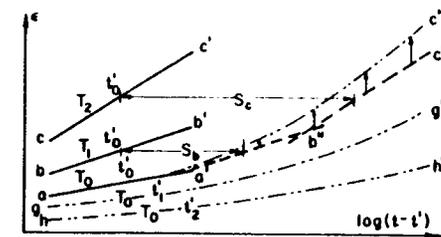


FIG. 3.—Graphical Procedure of Fitting Data for Different Temperatures

content and the case of variable water content will be examined in a subsequent paper. In a previous work (10,11) it has been shown that in the linear stress range, i.e., for stresses less than about 1/2 of the strength (45), the rate-type creep law for aging concrete at constant temperature may be based on either the Kelvin chain or the Maxwell chain. They both allow given creep data to

be fitted as closely as desired. However, identification of material parameters is easier to perform with the Maxwell chain (Fig. 1), and therefore this chain will be used throughout this study. The uniaxial stress-strain law then has the form:

$$\sigma = \sum_{\mu} \sigma_{\mu}; \quad \dot{\epsilon}_{\mu} = \frac{\dot{\sigma}_{\mu}}{E_{\mu}} + \frac{\sigma_{\mu}}{\eta_{\mu}}; \quad \mu = 1, 2, \dots, n \quad (1)$$

in which σ and ϵ = stress and strain; E_{μ} = spring moduli; and η_{μ} = dashpot viscosities; both E_{μ} and η_{μ} depending on age t of concrete; σ_{μ} values are called hidden stresses or internal state variables (14), whose current values fully characterize the previous history of the material. Superimposed dots stand for time derivatives, e.g., $\dot{\epsilon} = \partial \epsilon / \partial t$; and the stress-independent inelastic strains, such as shrinkage, are not included.

The ratios $\tau_{\mu} = E_{\mu} / \eta_{\mu}$ are called relaxation times and the plot of E_{μ} versus $\log \tau_{\mu}$ is called the relaxation spectrum. It depends on age t and is discrete, while the actual spectrum must be continuous. Realizing that the E_{μ} values merely represent a discrete approximation of a continuous function, it is clear that any data set can be approximated equally well for any set of τ_{μ} values which are sufficiently densely distributed and cover the whole range of interest. Therefore, one should not try to determine τ_{μ} from the creep data (and indeed, if attempted, a problem with a nonunique solution would result); rather, the τ_{μ} values must be chosen. In particular, they may be chosen time-constant and a suitable choice is a geometrical series with quotient 10

$$\tau_{\mu} = 10^{\mu-1} \tau_1, \eta_{\mu} = 10^{\mu-1} \tau_1 E_{\mu}; \quad \mu = 1, 2, \dots, m \quad (2)$$

in which $m = n - 1$. (If smoothness of fit is not of concern, quotients up to approx 60, leading to fewer hidden stresses, are acceptable.) The last spring ($\mu = n$) in Fig. 1 is imagined not to be coupled with any dashpot, i.e., for $\mu = n$ there is $\eta_{\mu} \rightarrow \infty$ or $\tau_{\mu} \rightarrow \infty$. The τ_{μ} values must cover the entire range of response delays to be considered. The dependence of E_{μ} upon t may be satisfactorily fitted, e.g., by polynomials of the type (11):

$$E_{\mu}(t) = \sum_{p=0}^q E_{p\mu} x^p; \quad \text{with } x = t^{1/6}, q = 4 \quad (3)$$

in which $E_{0\mu} \dots E_{4\mu}$ = constants. Eq. 9 also works well for $x = \log t$, $q = 3$.

Generalization of Eq. 1 to multiaxial stress is defined by the creep Poisson's ratio, which may be assumed as constant and equal to $\nu = 0.18$, because no clear evidence to the contrary exists (45). Thus, equations analogous in form to Eqs. 1 and 3 may be obtained for the volumetric components and the deviatoric components of stresses and strains.

ACTIVATION ENERGY OF HYDRATION AND TEMPERATURE DEPENDENCE OF AGING

The change of properties of concrete with age, and the dependence of E_{μ} and η_{μ} upon t in particular, are explained by the progress of cement hydration. When temperature is increased, the hydration reaction is speeded up (as is any chemical reaction). Thus, the change of material parameters, e.g., E_{μ} and

η_{μ} , is accelerated. Accordingly, it is necessary to replace time t in the expressions for E_{μ} and η_{μ} by a certain variable t_e characterizing the degree of hydration or maturity of concrete (44,47). This variable, called the equivalent hydration period, has been defined (3,7,8) as follows:

$$dt_e = \beta_T dt \quad \text{or} \quad t_e = \int \beta_T dt \quad (4)$$

in which β_T = relative hydration rate coefficient = function of temperature T . Since hydration is a thermally activated process it is natural to assume that the hydration rate obeys the Arrhenius equation (19,26), i.e.:

$$\beta_T = \exp \left[\frac{U_h}{R} \left(\frac{1}{T_0} - \frac{1}{T} \right) \right] \quad (5)$$

in which U_h = activation energy of hydration; R = gas constant = 1.986 cal/°K; T = absolute temperature; and T_0 = chosen reference value of T for which $\beta_T = 1$.

It should be realized, however, that Eq. 5 is only approximate. Actually, hydration consists of a number of different chemical reactions, each with a different activation energy. This causes the apparent activation energy obtained by fitting the data with Eq. 5 to be temperature dependent. Nevertheless, on the basis of the rates at which hydration heat is evolved, Verbeck (18) has found U_h to be approximately constant between 0° C, and 100° C up to 90-day age. Evaluation of the data in Table 12 of Ref. 18 furnishes

$$\frac{U_h}{R} \approx 2,700^\circ \text{K} \quad (6)$$

This value has been assumed in all of the data fitting described in the sequel.

ACTIVATION ENERGIES FOR DEFORMATION RATES

The major source of creep in cement paste is probably the diffusion of solids and water along various types of thin layers between solid particles carrying load (9). These diffusion processes are interpreted in the model in Fig. 1 by the dashpots with viscosities η_{μ} , characterizing the associated diffusion rates. Because any diffusion is a thermally activated process, the diffusion rates, and thus also viscosities η_{μ} , may be assumed to obey, at least within a sufficiently narrow temperature range, the Arrhenius equation (19,26). Thus, with Eq. 2

$$\frac{1}{\eta_{\mu}} = \frac{1}{\eta_{\mu_0}} \exp \left[\frac{U_{\mu}}{R} \left(\frac{1}{T_0} - \frac{1}{T} \right) \right] = \frac{10^{1-\mu}}{E_{\mu} \tau_1} \exp \left[\frac{U_{\mu}}{R} \left(\frac{1}{T_0} - \frac{1}{T} \right) \right] \quad (7)$$

in which U_{μ} = activation energy of the diffusion process interpreted by the μ th dashpot in Fig. 1; η_{μ_0} = value of η_{μ} given by Eq. 2 for the same t_e and at chosen reference temperature T_0 .

The values of U_{μ} are likely to be different for various μ values. Those which correspond to higher τ_{μ} could have higher activation energy because higher τ_{μ} corresponds to diffusion in thinner and longer pores, in which diffusing molecules must be held more firmly. The simplest possible assumption about the distribution of U_{μ} over the Maxwell chain is:

$$U_\mu = U_1 + (\mu - 1) \Delta U \quad \dots \dots \dots (8)$$

in which U_1 and ΔU are constant. Substituting into Eq. 7 then gives

$$\eta_\mu = a^{\mu-1} \tau'_1 E_\mu; \quad \tau'_\mu = \tau'_1 a^{\mu-1} \quad \dots \dots \dots (9)$$

$$\text{in which } \tau'_1 = \tau_1 \exp \left[-\frac{U_1}{R} \left(\frac{1}{T_0} - \frac{1}{T} \right) \right] \quad \dots \dots \dots (10)$$

$$\text{and } a = 10 \exp \left[-\frac{\Delta U}{R} \left(\frac{1}{T_0} - \frac{1}{T} \right) \right] \quad \dots \dots \dots (11)$$

Note that the activation energy, U_μ , in Eq. 7 is not considered herein to be a function of time, because on the microstructural level U_μ is determined by the local distances and arrangements of atoms and molecules, which do not change in time unless significant changes occur in the material. This contrasts with the previous applications of the activation energy to concrete creep (32,35,64) in which it was necessary to assume that the activation energy decreases with the duration of loading because no distinction between various diffusion processes with different relaxation times was made and only a single thermally activated process was considered. For completeness, it should be mentioned that the activation energy concept requires, in general, that the creep rate be proportional to $\sinh(\sigma/\sigma_1)$ in which $\sigma_1 = \text{constant}$ (26,36,62). This dependence on stress is nonlinear but for small stress, $\sinh(\sigma/\sigma_1) \approx \sigma/\sigma_1$; i.e., the creep law becomes linear, as considered in the present study.

Moduli E_μ , representing the elastic properties of the material are, of course, also temperature dependent. However, as long as the local arrangements of atoms and molecules and the chemical composition of the material do not change appreciably, the change of E_μ is caused only by changes in atomic and molecular distances due to thermal dilatations, and because these are small, the changes in E_μ must also be small. It is known that the chemical composition of cement paste is rather stable up to about 80° C-100° C (34), and within this range moduli E_μ should thus be approximately constant. Measurements (16,20,29,46) reveal that from 25° C to 100° C the short-time modulus, E , drops no more than about 15%, which is not negligible but is still small. The drop in actual instantaneous modulus E is less, perhaps much less, because the effect of the increase of the short-time creep rate with temperature has not been separated in these measurements. For these reasons, moduli E_μ were all assumed to be independent of temperature up to about 80° C-100° C. But for a wider temperature range the change in E_μ would have to be considered.

Note that at varying temperature, the ratios η_μ/E_μ are not constant since η_μ includes activation energy effects while E_μ does not.

IDENTIFICATION OF MATERIAL PARAMETERS FROM GIVEN DATA

Identification of the material parameters requires the coefficients of a differential equation to be determined from a given solution. This is a typical problem of control theory (12). A simple general method for its solution is outlined in Appendix I. It succeeds, however, only if a sufficiently good initial estimate of the material parameters can be found. Therefore, a special procedure has

been developed in which the identification problem is broken in a sequence of simpler subproblems. This approach allows one to gain better insight into the problem, utilize experience, and substitute at intermediate steps some intuitive analogical judgments, which is desirable in view of the fact that the fit of the incomplete data presently available is not unique.

First, it is necessary to determine $E_\mu(t)$ at a chosen reference temperature T_0 . To this end, the creep data at temperature T_0 must first be converted on the basis of the superposition principle to the stress relaxation function, $E_R(t, t')$, representing stress relaxation curves for various ages t' at imposition of constant strain. This may be accomplished by a method presented in Ref. 5. Then, each relaxation curve must be expanded into the Dirichlet series, $\sum_{\mu=1}^m E_\mu(t') \exp[-(t-t')/\tau_\mu] + E_\infty(t')$, which represents solution σ of Eqs. 1 for ϵ , as a step function. This can be done by minimizing a sum of the squares of deviations from given $E_R(t, t')$ plus an additional term giving a penalty for unsmooth dependence of E_μ upon μ . A subroutine which implements this task and yields $E_{0\mu}, \dots, E_{4\mu}$ from Eq. 3 is described in Ref. 11 (and a FORTRAN listing of an improved version which suppresses the random components of E_μ is given in a paper submitted to Cement and Concrete Research).

Subsequently, the data for other temperatures T are analyzed. In this regard, it is instructive to make the following three observations on the creep curves plotted in $\log(t, t')$ in which $t = \text{observation time}$; and $t' = \text{instant of loading}$.

1. Acceleration of aging by temperature rise, when considered alone, does not shift the creep curve parallel to the time axis because relaxation times $\tau_\mu = \eta_\mu/E_\mu$ are not affected by aging. It rather scales down the ordinates because the increase of moduli E_μ with age is faster at a higher temperature (Fig. 2, curves aa' , $a''b''$).

2. Let aging be now left out of consideration, i.e., regard E_μ as constants. The creep rate at time $t - t' = \tau_\mu$ depends mainly upon the stress in the μ th Maxwell unit in Fig. 1. A change of τ_μ to τ'_μ means that the response of the μ th Maxwell unit will come at an earlier time that appears in the $\log(t - t')$ scale to be shifted by distance $\log(\tau'_\mu/\tau_\mu)$ to the left from the time $t - t' = \tau_\mu$. If all U_μ are the same ($\Delta U = 0$), the response of all Maxwell units will be shifted to the left by equal distance (S in Fig. 2), i.e., the whole creep curve will be shifted to the left without any change in shape and slopes (Fig. 2, curves bb' , $b''b'''$). (This is the case of the thermorheologically simple material, closely approximated by most polymers.) However, if U_μ are distributed linearly, the shift to the left will be, according to Eqs. 9-11, by the distance $\log(\tau'_\mu/\tau_\mu) = \log \tau'_1 + (\mu - 1) \log a$, i.e., a distance linearly increasing with μ (distances S_1 and S_2 in Fig. 2). This means that the slope of the creep curve will be increased in the ratio $1/\log a$. This is also apparent from the fact that at reference temperature T_0 the relaxation times, τ_μ , are distributed at distances $\log = 1$, while at temperature T they are distributed according to Eqs. 2 and 9, at distances $\log a$.

3. Because temperature rise causes a shift of the response to shorter times, the creep curves at a lower temperature in each data set ought to be known up to much longer elapsed times $t - t'$ than those at reference temperatures and, conversely, the creep curves at a higher temperature ought to be known

down to much shorter elapsed times than those at reference temperature. Unfortunately, this is not the case for any data presented so far. Therefore, certain extensions of the creep curves must be estimated and, as a consequence, the problem of identification of the material parameters from given data does not have a unique solution for all data sets known at present.

Considering first the special case, $U_\mu = \text{constant} = U$ (i.e., $\Delta U = 0$), the

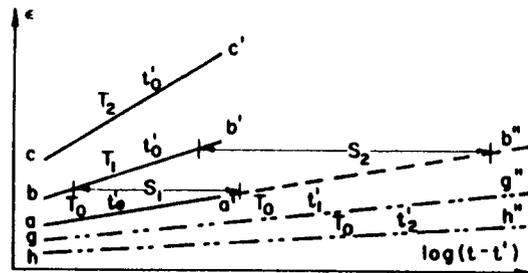


FIG. 4.—Modified Graphical Procedure when Activation Energy is Distributed Linearly

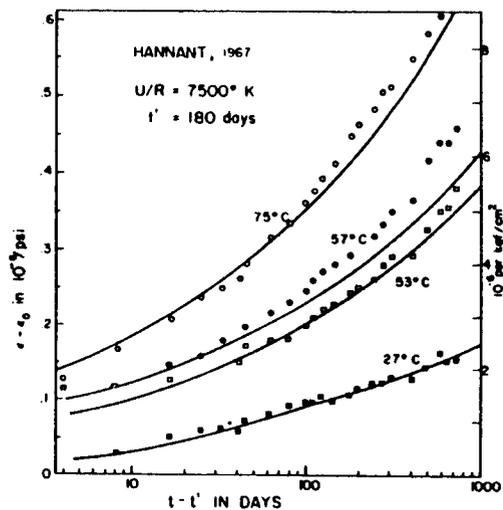


FIG. 5.—Fit of Creep Data of Hannant (27)

procedure of fitting the data at temperatures other than T_0 may be arranged as follows:

1. The given creep curves for various temperatures and the chosen reference age at loading, t'_0 (aa' , bb' , and cc' in Fig. 3) are shifted parallel to the logarithmic time axis to make them coincide with one continuous creep curve at reference temperature T_0 (curve $aa' b' c'$ in Fig. 3). This succeeds well enough for the test data known at present [as was observed by Sackman (54) and Mukaddam and Bresler (40), although for the purpose of a different theory].

2. The foregoing shift to a lower reference temperature, T_0 , does not account for the fact that the aging is slower at T_0 than it is at T . Therefore, as stated previously, the actual creep curve at T_0 , sketched as curve ac''' in Fig. 3, must deviate upward from the curve $aa' b' c'$ (or downward, when $T_0 > T$). This deviation must be estimated, and the correct estimate is easily found after several trials (which may be regarded as the iterates in the regula falsi method). Curve ac''' represents an extended creep curve at reference temperature

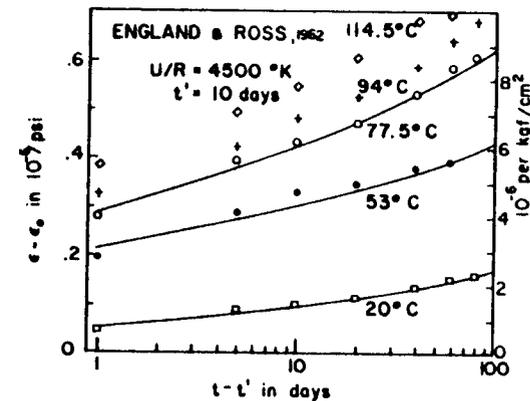


FIG. 6.—Fit of Creep Data of England and Ross (22)

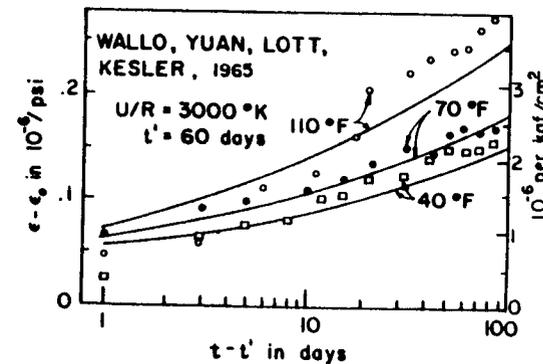


FIG. 7.—Fit of Creep Data of Wallo, et al. (60)

and age t'_0 at loading. Similar extensions (curves gg''' and hh''' in Fig. 3) must then be made for other ages t'_1 , t'_2 at loading, and in absence of elevated temperature data for these ages, these extensions can only be based on the trend of curve ac''' .

3. Analyzing the creep curves at reference temperature and various ages at loading (ac''' , gg''' , hh''' in Fig. 3) with the program from Ref. 10, parameters $E_{0\mu}, \dots, E_{4\mu}$ of Eq. 3 for E_μ are ascertained.

4. Measuring the horizontal distances, e.g., S_c and S_b in Fig. 3, one estimates the value of activation energy U , and using the unconditionally stable step-by-step

algorithm of the type presented in Ref. 10, one computes from $E_{0,\mu}, \dots, E_{4,\mu}$ the creep curves for the temperatures and the ages, t' , available in the given data. The plots of the curves (e.g., obtained by Calcomp plotter) are then compared visually with given data (curves aa' , bb' , cc' in Fig. 3). The estimate of U

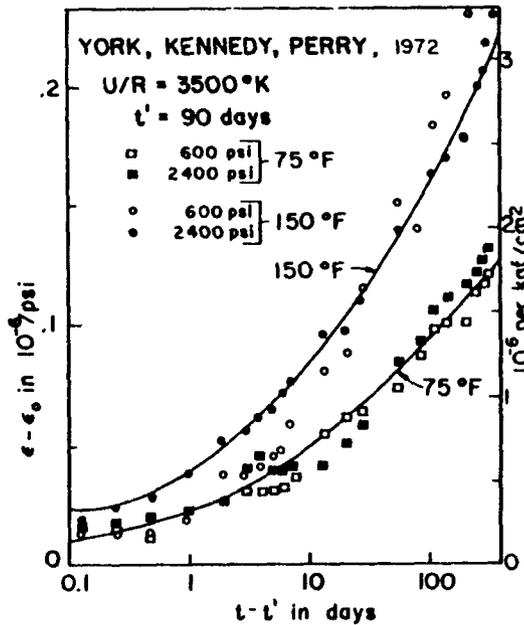


FIG. 8.—Fit of Creep Data of York, Kennedy, and Perry (65)

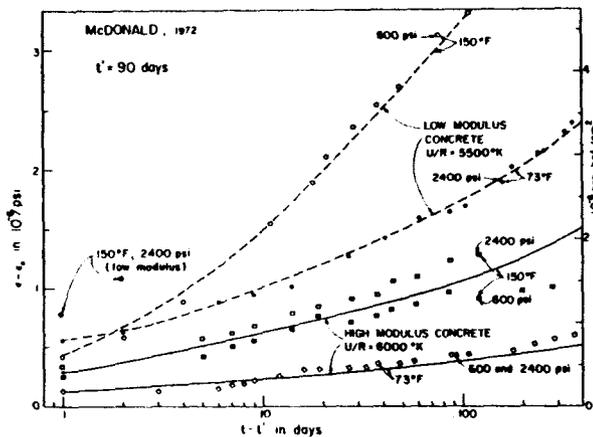


FIG. 9.—Fit of Creep Data of McDonald (37)

is then corrected repeating step 4 (in the spirit of the regula falsi method).

5. Finally, the whole procedure from step 2 is repeated, in the sense of regula falsi method, for other possible extensions of the creep curves for reference

temperature (ac''' , gg''' , hh'''), until a satisfactory fit is achieved. (After some experience no more than one repetition was needed.)

When U_μ is considered to be distributed linearly with μ (Eq. 8), a slightly different procedure is more convenient. First one estimates the extension of the creep curve at reference temperature (curve aa' in Fig. 4). Lacking detailed information, the most natural assumption is perhaps a straight-line extension, as shown in curve $a' b'$ in Fig. 4. Then, measuring the horizontal distances (e.g., S_1 and S_2) from the creep curves at elevated temperatures (e.g., bb'), the values of U_1 and ΔU in Eq. 8 can be estimated, though only roughly because aging distorts the picture. Step 3 from the previous procedure is then implemented and after computing the creep curves, their comparison with data is made, as in step 4. Using the regula falsi method, the procedure is then repeated for improved estimates of U_1 and ΔU_1 and improved extensions of

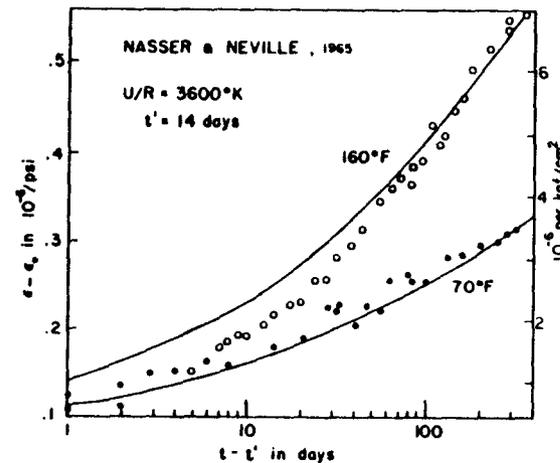


FIG. 10.—Fit of Creep Data of Nasser and Neville (41)

creep curves at reference temperature, until the best fit is found.

NUMERICAL RESULTS AND ANALYSIS

Using the foregoing procedure, all of the more extensive published data on temperature dependence of creep (15,22,27,37,41,42,55,61,65) were analyzed. The data are shown (by points or dotted lines) in Figs. 5-14 and further information on the data is compiled in Table 1.

First the data were fitted under the assumption that $U_\mu = U = \text{constant}$ (or $\Delta U = 0$). The fits are drawn in Figs. 5-14 by continuous solid or dashed lines, using as ordinates the total stress-dependent strain, ϵ (excluding thermal expansion and shrinkage), minus the instantaneous (or short-time) strain ϵ_0 at load application. The solid data points in Fig. 14 are from tests under biaxial compression, in which case $\epsilon = (1 - \nu) \epsilon_{x_1}$, in which ϵ_{x_1} is the strain under uniaxial compression; and $\nu = \text{creep Poisson's ratio} = 0.2$ (1). The activation

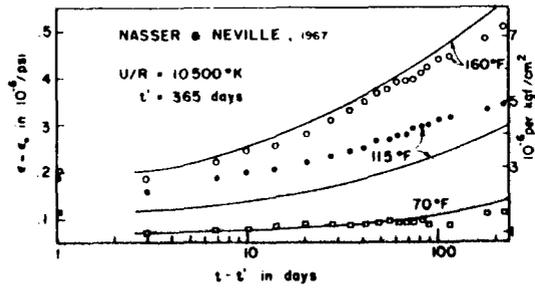


FIG. 11.—Fit of Creep Data of Nasser and Neville (42)

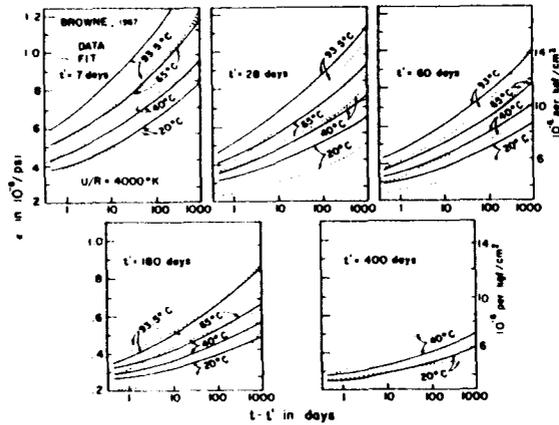


FIG. 12.—Fit of Creep Data of Browne (15)

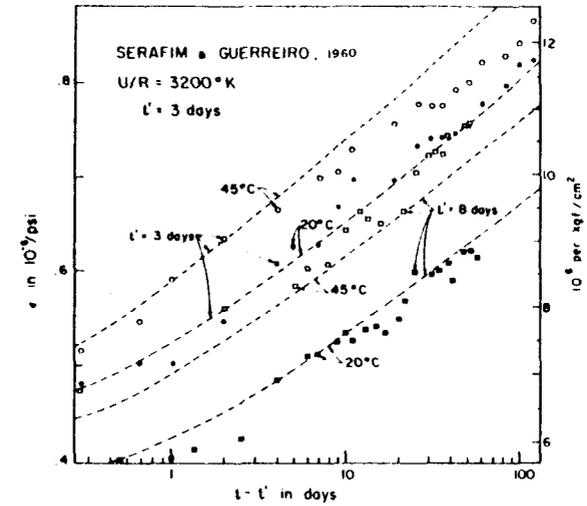
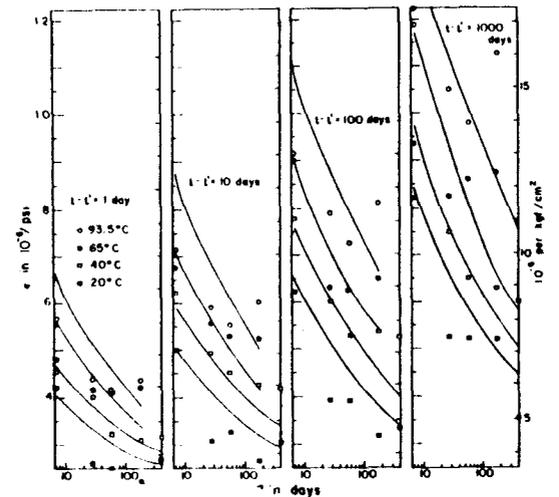


FIG. 13.—Fit of Creep Data of Serafim and Guerreiro (55)

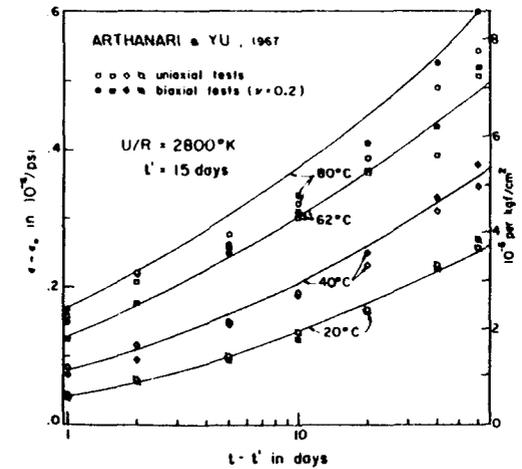


FIG. 14.—Fit of Uniaxial and Biaxial Creep Data of Arthanari and Yu (1)

energy values corresponding to the fits are indicated in the figures in terms of U/R . Unfortunately, it is found that either different concretes exhibit different activation energies or the scatter of the data presently known is large and the control of test conditions is inadequate. The mean value, which could reasonably

fit most data, is roughly

$$\frac{U_{\mu}}{R} = \frac{U}{R} \approx 5,000^{\circ} \text{K} \text{ or } U \approx 10,000 \text{ cal} \dots \dots \dots (12)$$

Reference temperature T_0 was always taken as the lowest temperature shown,

TABLE 1.—Test Specimen

Figure (1)	Reference (2)	Strength, in pounds per square inch (3)	Stress-strength ratio (4)	Type of cement (5)
5	27	7,800 (cylinder)	< 0.25	—
6	22 ^a	5,500 4-in. cubes at 614 days	0.18	—
7	61 ^a	6,000 (cylinder)	< 0.25	I
8	65 ^b	6,000 (cylinder)	< 0.4	II
9	37 ^c	6,000 (cylinder)	< 0.4	II
10	41	5,500 9-1/2-in. (cylinder) at 14 days	< 0.35 (0.6, 0.7 not fitted)	III
11	42	7,250 (cylinder)	< 0.45	III
12	15	6,600 (cylinder)	< 0.36	II
13	55	—	—	—
14	1	6,000 (cube)	0.22	“ordinary”

^a Instantaneous deformation not reported.

^b Used are tests F-33, E-39, B-4, B-7.

^c Reported partly, in which case the initial points were arbitrarily adjusted.

except for the values of 70° F in Fig. 7 and 40° C in Fig. 12. None of the data completely defines the creep at T_0 . Different ages at loading are available only for the data in Fig. 12. (Their fit at first seems quite poor but a closer examination reveals that a better smooth fit could hardly be desired because of the very high scatter of these data.) Thus, the creep curves for ages other than the one given, as well as extensions in length of the curve at the one basic age given, had to be estimated except in Fig. 12. The estimates were based on analogy with the data on the aging effect given in Ref. 10. The estimates were then slightly (but not unreasonably) modified, to give the best fit at other than reference temperatures. However, the information on the aging effect is not as limited as the foregoing comments may suggest, since the most important effect, e.g., the acceleration of aging due to temperature change, is based on

Properties for Published Data

Aggregate (6)	Maximum aggregate size, in inches (7)	Water-cement ratio (8)	Sealing, curing, temperature history (9)
Oldbury limestone	—	0.47	5 months in water, 20° C, 1 month sealed by copper foil
—	—	0.45	6 days in water, 6 days at 90% humidity, 17° C, then sealed
—	—	0.53	in water
limestone	3/4	0.425	83 days sealed at 73° F, then to test temperature; sealed by epoxy and copper jacket same as above
graywacke, chert	3/4	—	
dolomite, hornblende	3/4	0.6	raised to test temperature at 1-day age, in water
dolomite, hornblende	3/4	0.6	raised to test temperature 1 week before test, in water
foraminiferous limestone	1-1/2	0.42	cured at room temperature till 1 day before test, then sealed
river gravel	1	0.5	in copper jacket, raised to test temperature 1 day ($t' = 3$) or 3 days ($t' = 8$) before test
	3/8	0.564	raised to test temperature 1 day before test, sealed by epoxy with two plastic coats

the hydration heat data in Ref. 18, as previously mentioned, and not on the data shown. The complete creep curves at T_0 for various ages, corresponding to the fits in Figs. 8, 12, 5, and 15, are shown in Fig. 16 as cases, *a*, *b*, and *c*, respectively. They are drawn only up to the elapsed times, $t - t'$, to which they are needed for fitting the data. Note that the creep data at reference temperature must completely cover the whole age interval (approximate):

$$t'_{c_{min}} + (t - t')_{min} \exp \left[\frac{U}{R} \left(\frac{1}{T_0} - \frac{1}{T_{min}} \right) \right] \leq t \leq t'_{c_{max}} + (t - t')_{max} \exp \left[\frac{U}{R} \left(\frac{1}{T_0} - \frac{1}{T_{max}} \right) \right] \dots \dots \dots (13)$$

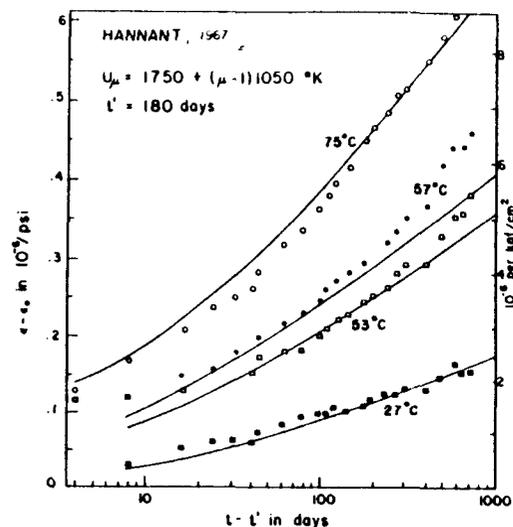


FIG. 15.—Fit of Hannant's Data (27), Assuming Linearly Distributed Activation Energy

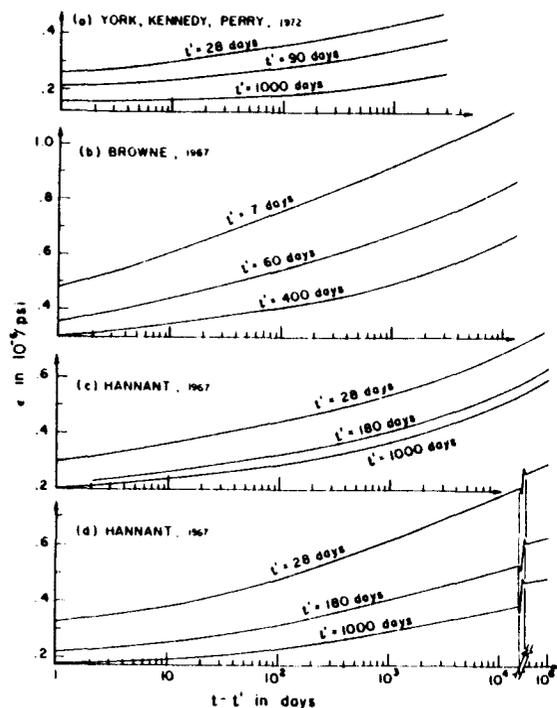


FIG. 16.—Extrapolated Creep Curves Obtained in Fitting Data in Figs. 8, 12, 5, and 15

in which T_{max} , T_{min} = the maximum and minimum temperatures considered; t'_{max} , t'_{min} = the maximum and minimum age at loading t' expressed in terms of t_0 ; and $(t - t')_{max}$, $(t - t')_{min}$ = the maximum and minimum time lag after loading. In judging the quality of fits, it should be kept in mind that the creep curves shown also fit exactly (with the same stress-strain relations) the values of instantaneous deformation ϵ_0 , assumed to correspond to $t - t' = 0.001$ day. These points would lie to the far left, out of the figures, and are not shown.

In view of the high scatter and incompleteness, it would be unjustified to try determining a greater number of material parameters from the present data, even if some of the fits could be considerably improved (in Fig. 10, for instance, by using $\Delta U \neq 0$). Nevertheless, the fit with a linearly distributed U_μ (Eq. 8), was tried at least for one case, the data from Fig. 5, which belong to the most complete and consistent ones. The fit is shown by solid lines in Fig. 15 along with the values of U_i and ΔU found. The corresponding reference creep curves are shown as case d in Fig. 16 and they differ considerably from case c pertaining to the same data fitted with constant U_μ (Fig. 5). This underscores the lack of uniqueness of fit of the data available.

Assumptions also had to be made with regard to other circumstances not reported by the experimenters. For instance, for the data in Figs. 6 and 7, the value of instantaneous deformation had to be estimated, using analogy with other concretes. But for the whole set of data for one concrete the same value of the instantaneous deformation was assumed throughout, of course. The time at which heating was started was not reported for the data in Figs. 6 and 7; it was assumed to occur shortly before loading, as in most tests. In some of the tests, however, heating started at the instant of loading, which distorted the data because temperature could not be constant throughout the specimen at the beginning of the test.

Preservation of constant moisture content has been inadequate in some of the tests, especially the early ones (59). At a higher temperature concrete can lose water much faster, and after this happens, the creep rate drops, which overrides the activation energy effect. This has probably also affected to some extent the data in Fig. 6. Using the present theory it would be possible to fit successfully other data of more limited scope as well (2,21,28,33,51,58,60).

Test data show that on approach to 100°C the creep first ceases accelerating and beyond a certain temperature it begins slowing down, while according to Eq. 7 creep rate must always rise when temperature rises. This can be explained by changes in the chemical composition and microstructure of cement paste. For this reason, the data points close to 100°C have not been fitted by the present theory even if given in the original data set (temperatures 94°C and 114.5°C in Fig. 6, 205°F in Fig. 11). However, if the activation energy concept were dropped and η_μ were regarded as an arbitrary function of temperature, the rate-type creep law (1) could probably fit even the data beyond 100°C . The material is also changed by the freezing of water and the present formulation thus does not apply below 0°C without further refinements (as is confirmed by the tests in Refs. 31 and 48).

According to the present formulation a drop in temperature would always result in a reduction of the creep rate. However, it has been observed on small specimens that during and immediately after a very abrupt temperature drop

the creep is not slowed down, but accelerated at first (1,23,31,56,61,64). One way of explaining this effect is by considering that the dependence of the rates of the diffusion of solids and water causing creep upon chemical potential gradient is progressively nonlinear when the gradient is very high (9). Then the rate increase due to this nonlinearity offsets the effect of activation energy when a large gradient due to sudden temperature drop is superimposed on the gradient due to the load. (This is analyzed in more detail by the senior writer in *Mechanics Today*, Vol. 2, Pergamon Press, 1974, in press.) The present stress-strain relations can thus be applied only at sufficiently slowly varying temperatures, which are usually the case for all massive concrete bodies.

CONCLUSIONS

1. Creep of aging concrete at variable temperature can be modeled by a rate-type creep law with hidden stresses as internal state variables (Eq. 1). The formulation presented herein is restricted to the linear stress (i.e., working stress) range, to constant (or almost constant) water content, to constant or slowly varying temperatures, conditions which are typical for massive concrete structures, and to temperatures between about 2° C and 90° C.

2. The main purpose in formulating the rate-type creep law is to avoid the storage of stress history and the evaluation of hereditary integrals, required by previous formulations. This makes the numerical creep analysis of large structural systems feasible.

3. Introduction of the activation energy concept (applicable from 2° C to approx 80° C-100° C) is advantageous because it specifies the form of the temperature-dependence and thus enables determination of the creep law from more limited data than would be needed otherwise. Yet the problem of identification of the material parameters from the limited data presently available does not have a unique solution (although the activation energy concept reduces the degree of arbitrariness considerably). For example, the data presently known can be fitted, assuming the activation energies for various relaxation times to be either all equal, in which case the creep curve at reference temperature extended into very long times curves upward in the logarithmic time scale (Figs. 3, 16), or to be distributed linearly over the Maxwell chain (Eq. 8), in which case the extension just mentioned may be straight (Figs. 4, 16). To decide which alternative is true would necessitate data for a much longer test duration than is presently available.

4. In contrast with some previous studies, the artificial and unjustified assumption that the activation energy depends on the duration of loading appears to be unnecessary.

5. Introducing the equivalent hydration period (Eq. 4), the acceleration of aging due to temperature rise is formulated with the help of activation energy of hydration (Eqs. 5, 6). This concept (30) is corroborated by data fits.

6. All of the more extensive data on the temperature dependence of creep have been fitted satisfactorily, by the method described (see Figs. 5-14).

7. The activation energy of creep appears to differ widely for various concretes, the mean value being approx $U = 10,000$ cal. Many of these differences may be due, however, to experimental scatter and differences in test procedures.

ACKNOWLEDGMENT

The present research results have been obtained under United States National Science Foundation Grant GK-26030.

APPENDIX I.—ALTERNATIVE GENERAL METHOD OF MATERIAL IDENTIFICATION

To discretize the problem, the measured creep data may be characterized by certain suitably chosen characteristic data points, numbered as $\alpha = 1, 2, \dots, N$. [Suitable choice requires that these points be uniformly distributed in $\log(t - t')$ - scale.] The measured values at these points, characterized by $T_\alpha, t'_\alpha, (t - t')_\alpha$, may be denoted as \bar{F}_α . The theoretical solution at these same points, F_α , may be regarded as a function of the unknown material parameters, p_m ($m = 1, 2, \dots, m_0$), which represent $U_1, \Delta U$, and those of the characteristic values of the creep function $J(t, t - t')$ at reference temperatures that have not been measured and are unknown, usually the values for high $t - t'$ and high t' . Assuming that a suitable estimate p_m^0 of the unknown material parameters, p_m , has been made the solution, F_α (in all points α), corresponding to p_m^0 is computed. Further solutions, m_0 in number, are determined for m_0 sets of material parameters $p_m^0 + \delta_{mk} \Delta p_k$ ($k = 1, 2, \dots, m_0$) in which Δp_k are chosen small changes of p_k^0 ; and $\delta_{mk} = 0$ for $m \neq k$; and $\delta_{mk} = 1$ for $m = k$. Then, assuming F_α as a function of p_m to be continuous and sufficiently smooth, the solution for any material parameters p_m close to p_m^0 is

$$F_\alpha = F_\alpha^0 + \sum_k F_{\alpha,k}^0 (p_k - p_k^0) \dots \dots \dots (14)$$

$$\text{in which } F_\alpha^0 = F_\alpha(p_m^0), F_{\alpha,k}^0 \approx \frac{F_\alpha(p_m^0 + \delta_{mk} \Delta p_k) - F_\alpha(p_m^0)}{\Delta p_k} \dots \dots \dots (15)$$

Eq. 14 is a Taylor series expansion by which the problem is linearized, and $F_{\alpha,k}^0$ are approximations to partial derivatives with respect to p_k . The objective of optimum fit of data points \bar{F}_α may be achieved by imposing a least square criterion, that is by minimizing the quadratic form

$$\Phi = \sum_\alpha w_\alpha \left[F_\alpha^0 + \sum_k F_{\alpha,k}^0 (p_k - p_k^0) - \bar{F}_\alpha \right]^2 + \Phi_1 \dots \dots \dots (16)$$

in which w_α = chosen weights; and Φ_1 = a certain chosen quadratic penalty term (25,63) which may be added, e.g., to prevent unreasonably large changes in p_m (or to guarantee a smooth distribution of p_m values when required, as in the case when p_m are the unknown ordinates of some reference creep curve). The minimization problem is, strictly speaking, a constrained one (63) because usually the parameters, p_m , must be positive and some of them may have to satisfy further inequalities (e.g., expressing the fact that the reference creep curve must always rise). Constrained minimization can be, however, usually avoided by transformations (63). The minimizing condition then simply is $\partial \Phi / \partial p_j = 0$ for all $j = 1, 2, \dots, m_0$. This yields a system of linear algebraic equations for p_m , from which improved p_m values may be obtained. These values may then be labeled as new p_m^0 and, repeating the procedure just described, a further

improvement in p_m values can be made, etc.

The method outlined is in essence equivalent to the method of Gauss and Legendre (63). It belongs to the techniques of quasilinearization and can be brought into connection with the well-known Newton-Raphson method (12).

APPENDIX II.—REFERENCES

1. Arthanari, S., and Yu, C. W., "Creep of Concrete under Uniaxial and Biaxial Stresses at Elevated Temperatures," *Magazine of Concrete Research*, Vol. 19, 1967, pp. 149-156.
2. Barrett, N. T., and Murray, J. T., "Creep in Pressure Vessels at Elevated Temperatures," Conference on Prestressed Concrete Pressure Vessels, Institution of Civil Engineers, London, England, 1967, Paper C14, pp. 33-46.
3. Bažant, Z. P., "Constitutive Equation for Concrete Creep and Shrinkage Based on Thermodynamics of Multiphase Systems," *Materials and Structures*, Reunion Internationale des Laboratoires d'Essais et de Recherches sur les Matériaux et les Constructions, Vol. 3, 1970, pp. 3-36.
4. Bažant, Z. P., "Delayed Thermal Dilatations of Cement Paste and Concrete due to Mass Transport," *Nuclear Engineering and Design*, Vol. 14, 1970, pp. 308-318.
5. Bažant, Z. P., "Numerical Solution of Non-Linear Creep Problems with Application to Plates," *International Journal of Solids and Structures*, Vol. 7, 1971, pp. 83-97.
6. Bažant, Z. P., "Thermodynamic Theory of Concrete Deformation of Variable Temperature and Humidity," *Report No., 69-11*, Division of Structural Engineering and Structural Mechanics, University of California, Berkeley, Calif., Aug., 1969; abstract, *American Concrete Institute Special Publication 27*, 1971, pp. 411-421.
7. Bažant, Z. P., "Numerical Determination of Long-Range Stress History from Strain History in Concrete," *Materials and Structures*, Reunion Internationale des Laboratoires d'Essais et de Recherches sur les Matériaux et les Constructions, Vol. 5, 1972, pp. 135-141.
8. Bažant, Z. P., "Thermodynamics of Interacting Continua with Surfaces and Creep Analysis of Concrete Structures," *Nuclear Engineering and Design*, Vol. 20, 1972; pp. 477-505; *Cement and Concrete Research*, Vol. 2, 1972, pp. 1-16.
9. Bažant, Z. P., and Moschovidis, Z., "Surface Diffusion Theory for the Drying Creep Effect in Portland Cement Paste and Concrete," *American Ceramic Society Journal*, Vol. 56, 1973, pp. 235-241.
10. Bažant, Z. P., and Wu, S. T., "Dirichlet Series Creep Function for Aging Concrete," *Journal of the Engineering Mechanics Division*, ASCE, Vol. 99, No. EM2, Proc. Paper 9645, Apr., 1973, pp. 367-387.
11. Bažant, Z. P., and Wu, S. T., "Rate-Type Creep Law of Concrete Based on Maxwell Chain," *Materials and Structures*, Reunion Internationale des Laboratoires d'Essais et de Recherches sur les Matériaux et les Constructions, Vol. 7, Jan.-Feb., 1974.
12. Bellman, R. E., and Kalaba, R. E., *Quasilinearization and Nonlinear Boundary Value Problems*, American Elsevier, New York, N.Y., 1965.
13. Berwanger, C., "The Modulus of Concrete and the Coefficient of Thermal Expansion below Normal Temperatures," *Temperature and Concrete*, American Concrete Institute, SP-25, 1971, pp. 191-234.
14. Biot, M. A., "Variational Principles of Irreversible Thermodynamics with Application to Viscoelasticity," *Physical Review*, Vol. 97, 1955, pp. 1463-1469.
15. Browne, R. D., "Properties of Concrete in Reactor Vessels," *Proceedings*, Conference on Prestressed Concrete Pressure Vessels, Institution of Civil Engineers, London, England Group C, Paper 13, 1967, pp. 11-31.
16. Browne, R. D., and Blundell, R., "The Influence of Loading Age and Temperature on the Long Term Creep Behavior of Concrete in a Sealed, Moisture Stable State," *Materials and Structures*, Reunion Internationale des Laboratoires d'Essais et de Recherches Sur les Matériaux et les Constructions, Vol. 2, 1969, pp. 133-144.
17. Christensen, R. M., *Theory of Viscoelasticity*, Academic Press, New York, N.Y., 1971.
18. Copeland, L. E., Kantro, D. L., and Verbeck, G., "Chemistry of Hydration of Portland Cement," *Proceedings*, Fourth International Symposium on the Chemistry of Cement, Washington, D.C., 1960, National Bureau of Standards Monograph 43, Paper 3, pp.

- 429-465; Portland Cement Association Bulletin 153.
19. Cottrell, A. H., *The Mechanical Properties of Matter*, John Wiley and Sons, Inc., New York, N.Y., 1965.
20. Cruz, C. R., "Elastic Properties of Concrete at High Temperatures," *Journal of the Portland Cement Association Research and Development Laboratories*, Jan., 1967, pp. 37-45; Portland Cement Association Bulletin, 191.
21. Da Silveira, A. F., and Florentino, C. A., "Influence of Temperature on the Creep of Mass Concrete," *Temperature and Concrete*, American Concrete Institute Special Publication SP-25, Detroit, Mich., 1971, pp. 173-190.
22. England, G. L., and Ross, A. D., "Reinforced Concrete under Thermal Gradients," *Magazine of Concrete Research*, Vol. 14, No. 40, Mar., 1962, pp. 5-12.
23. Fahmi, H. M., Polivka, M., Bresler, B., "Effect of Sustained and Cyclic Elevated Temperature on Creep of Concrete," *Cement and Concrete Research*, Vol. 2, 1972, pp. 591-606.
24. Ferry, J. D., *Viscoelastic Properties of Polymers*, 2nd ed., John Wiley and Sons, Inc., New York, N.Y., 1970.
25. Fox, R. L., *Optimization Methods for Engineering Design*, Addison-Wesley Publishing Co., Inc., Reading, Mass., 1971.
26. Glasstone, S., Laidler, K. J., and Eyring, H., *The Theory of Rate Processes*, McGraw-Hill Book Co. Inc., New York, N.Y., 1941.
27. Hannant, D. J., "Strain Behavior of Concrete up to 95° C under Compressive Stresses," *Proceedings*, Conference on Prestressed Concrete Pressure Vessels, Institution of Civil Engineers, London, England, Group C, Paper 17, 1967, pp. 57-71.
28. Hansen, T. C., "Creep and Stress Relaxation of Concrete," *Proceedings NR 31*, Swedish Cement and Concrete Research Institute of Royal Institute of Technology, Stockholm, Sweden, 1960.
29. Hansen, T. C., and Eriksson, L., "Temperature Change Effect on Behavior of Cement Paste, Mortar and Concrete," *American Concrete Institute Journal*, Vol. 37, 1966, pp. 489-503.
30. Hickey, K. B., "Creep, Strength and Elasticity of Concrete at Elevated Temperatures," *Report No. C-1257*, Bureau of Reclamation, Denver, United States Department of the Interior, Washington, D.C., Dec., 1967.
31. Johansen, R., and Best, C. H., "Creep of Concrete with and without Ice in the System," *Bulletin Reunion Internationale des Laboratoires d'Essais et de Recherches sur les Matériaux et les Constructions No. 16*, 1962, pp. 47-57.
32. Klug, P., and Wittmann, F., "Activation Energy of Creep of Hardened Cement Paste," *Materials and Structures*, Reunion Internationale des Laboratoires d'Essais et de Recherches sur les Matériaux et les Constructions, Vol. 2, 1969, pp. 11-16.
33. Lankard, D. R., et al., "Effects of Moisture Content on Concrete Exposed to Temperatures up to 500° F," *Temperature and Concrete*, American Concrete Institute SP-25, 1971, pp. 59-102.
34. Lea, F. M., *The Chemistry of Cement and Concrete*, 3rd ed., Chemical Publishing Co., New York, N.Y., 1971.
35. Maréchal, J. C., "Fluage du Béton en Fonction de la Température," *Materials and Structures*, Reunion Internationale des Laboratoires d'Essais et de Recherches sur les Matériaux et les Constructions, Vol. 2, 1969, pp. 111-115; Vol. 3, 1970, pp. 395-406.
36. McClintock, F. A., and Argon, A. S., eds., *Mechanical Behavior of Materials*, Addison-Wesley, Reading, Mass., 1966.
37. McDonald, J. E., "An Experimental Study of Multiaxial Creep in Concrete," *Concrete for Nuclear Reactors*, American Concrete Institute Special Publication No. 34, 1972, pp. 732-768.
38. Mitchell, J. K., Campanella, R. G., and Singh, A., "Soil Creep as a Rate Process," *Journal of the Soil Mechanics and Foundation Division*, ASCE, Vol. 94, No. SM1, Proc. Paper 5751, Jan., 1968, pp. 231-153.
39. Mukaddam, M. A., "Behavior of Concrete under Variable Temperature and Loading," thesis presented to the University of California, at Berkeley, Calif., in 1969, in partial fulfillment of the requirements for the degree of Doctor of Philosophy.
40. Mukaddam, M. A., and Bresler, B., "Behavior of Concrete under Variable Temperature and Loading," presented at the American Concrete Institute International Seminar on Concrete for Nuclear Reactors, held at West Berlin, West Germany, in 1970;

- Reports of the Working Commissions International Association of Building and Structural Engineers, Vol. 6; Final Report, International Association of Building and Structural Engineers Symposium on "Design of Concrete Structures for Creep, Shrinkage and Temperature," held in Madrid, Spain, in 1970, pp. 301-310.
41. Nasser, K. W., and Neville, A. M., "Creep of Concrete at Elevated Temperatures," *Journal of the American Concrete Institute*, Vol. 62, 1965, pp. 1567-1579.
 42. Nasser, K. W., and Neville, A. M., "Creep of Old Concrete at Normal and Elevated Temperatures," *Journal of the American Concrete Institute*, Vol. 64, 1967, pp. 97-103.
 43. Nemat-Nasser, S., "On Nonlinear Thermoviscoelasticity and Nonequilibrium Thermodynamics," *Nonlinear Elasticity*, Academic Press, New York, N.Y., 1973.
 44. Neville, A. M., *Properties of Concrete*, I. Pitman and Sons, London, England, 1963, pp. 249.
 45. Neville, A. M., and Dilger, W., *Creep of Concrete: Plain, Reinforced, Prestressed*, American Elsevier, New York, N.Y., 1970.
 46. Philleo, R., "Some Physical Properties of Concrete at High Temperature," *American Concrete Institute Journal*, Vol. 29, 1958, pp. 857-864.
 47. Plowman, J. M., "Maturity and Strength of Concrete," *Magazine for Concrete Research*, Vol. 8, 1965, pp. 13-22.
 48. Podvalnyi, A. M., Creep of Freezing Concrete, *Doklady Akademii Nauk, U.S.S.R.*, Vol. 148, 1963, pp. 1148-1151.
 49. Polivka, M., and Best, C. H., "Investigation of the Problem of Creep in Concrete by Dorn's Method," Department of Civil Engineering Report, University of California, Berkeley, Calif., Jan., 1960; 39th Annual Meeting, Highway Research Board, Washington, D.C.
 50. Reshid, Y. R., "Nonlinear Analysis of Two-Dimensional Problems in Concrete Creep, Transactions, American Society of Mechanical Engineers, Series E, *Journal of Applied Mechanics*, Vol. 39, 1972, pp. 475-482.
 51. Rickenstorff, G., and Liscovius, R., "Kriechen und Relaxation des Betons bei hohen Temperaturen," *Wissenschaftliche Zeitschrift, der Technischen Universität Dresden*, Dresden, Germany, Vol. 17, 1968, pp. 1563-1565.
 52. Roscoe, R., "Mechanical Models for the Representation of Viscoelastic Properties," *British Journal of Applied Physics*, Vol. 1, 1950, p. 171-173.
 53. Ruetz, W., "A Hypothesis for the Creep of Hardened Cement Paste and the Influence of Simultaneous Shrinkage," *Proceedings, International Conference on the Structure of Concrete*, London, England, 1965, Cement and Concrete Association, 1969, pp. 111-115; Deutscher Ausschuss Für Stahlbeton, Heft 183, Berlin, Germany, 1966.
 54. Sackman, J. L., "Creep in Concrete and Concrete Structures," *Proceedings, The Princeton University Conference on Solid Mechanics*, Princeton, N.J., 1963, pp. 15-48.
 55. Serafim, L. J., and Guerreiro, M. Q., "Influence of Temperature on Creep of Mass Concrete," *Reunion Internationale des Laboratoires d'Essais et de Recherches sur les Matériaux et les Constructions Bulletin No. 6*, Mar., 1960, pp. 23-32.
 56. Sullivan, P. J. E., "Time-Temperature Extrapolation Techniques as Applied to Concrete Creep Strains," *Proceedings, First International Conference on Structural Mechanics, in Reactor Technology*, Berlin, Germany, 1971, T. A. Jaeger, ed., Paper H2/2.
 57. Sullivan, P. J., and Poucher, M. R., "The Influence of Temperature on Concrete and Mortar in the Range 20° C to 400° C," *American Concrete Institute. SP-25*, 1971, pp. 103-135.
 58. Sullivan, P. J. E., and Zaman, A. A. A., "Explosive Spalling of Concrete at High Temperature," *Proceedings, First International Conference on Structural Mechanics in Reactor Technology*, Berlin, Germany, 1971, Paper No. H1/5.
 59. Theuer, A. V., "Effect of Temperature on the Stress Deformation of Concrete," *Journal of Research of the National Bureau of Standards*, Vol. 18, 1937, pp. 195-204.
 60. Torroja, E., and de la Peña, C., discussion of "Volume Changes of Concrete," by R. L'Hermite, 4th International Symposium on Chemistry of Cement, Washington, D.C., 1960, *United States National Bureau of Standards Monograph 43*, Vol. II, pp. 695-699; also de la Peña, *Bulletin Reunion Internationale des Laboratoires d'Essais et de Recherches sur les Matériaux et les Constructions No. 3*, 1959, pp. 60-70.
 61. Wallo, E. M., et al., Sixth Progress Report, *Theoretical and Applied Mechanics Report No. 658*, University of Illinois, Urbana, Ill. "Prediction of Creep in Structural Concrete from Short Time Tests," 1965.

62. Weertman, J., and Weertman, J. R., "Mechanical Properties, Strongly Temperature Dependent," *Physical Metallurgy*, R. W. Cahn, ed., North Holland, Amsterdam, The Netherlands, 1965, p. 793.
63. Wilde, D. J., and Beightler, C. S., *Foundations of Optimization*, Prentice-Hall, Inc., Englewood Cliffs, N.J., 1967, Chap. 7-06.
64. Wittman, F., "Kriechen bei gleichzeitigem Schwinden des Zementsteins," *Rheologica Acta*, Vol. 5, 1966, pp. 198-204; cf. also *Rheologica Acta*, Vol. 6, 1967, pp. 303-306, Vol. 10, 1971, pp. 422-428.
65. York, G. P., Kennedy, T. W., and Perry, E. S., "Experimental Investigation of Creep in Concrete Subjected to Multiaxial Compressive Stresses and Elevated Temperatures," *Research Report 2864-2*, University of Texas, Austin, Tex., June, 1970; "Concrete for Nuclear Reactors," *American Concrete Institute Spec. Publ. No. 34*, 1972, pp. 647-700.

APPENDIX III.—NOTATION

The following symbols are used in this paper:

- $E = \sum_{\mu} E_{\mu}$ = instantaneous elastic modulus;
 E_{μ} = elastic modulus of μ th Maxwell unit (Fig. 1, Eq. 1);
 $J(t, t')$ = creep function = strain in time t due to constant unit stress applied in time t' ;
 $R = 1.986 \text{ cal/}^{\circ}\text{K}$ = gas constant;
 T, T_0 = absolute temperature and its chosen reference value;
 t = time from casting of concrete;
 t', t'_0 = time at load application and its reference value;
 U = U_{μ} when all U_{μ} are equal (Fig. 1, Eq. 7);
 U_h = activation energy of hydration (Eq. 5);
 U_{μ} = activation energy of creep for μ th Maxwell unit;
 $U_1, \Delta U$ = activation energy parameters in Eq. 8;
 β_T = relative hydration rate (Eq. 4);
 ϵ_0 = initial instantaneous (or short-time) strain at load application (in figures);
 ϵ, σ = strain and stress (Eq. 1);
 η_{μ} = viscosity of μ th dashpot (Fig. 1, Eq. 1);
 σ_{μ} = hidden stress = stress in μ th Maxwell unit (Fig. 1, Eq. 1);
 τ_{μ} = η_{μ} / E_{μ} = μ th relaxation time (Eq. 2) and;
 τ'_{μ} = μ th relaxation time (Eq. 9).