THERMOVISCOELASTICITY OF AGING CONCRETE*

By Zdeněk P. Bažant,† M. ASCE and Spencer T. Wu‡

Introduction

Recently, creep of concrete at elevated temperatures has been receiving vivid attention, mainly because of its importance in the design of prestressed concrete reactor vessels. The effect of temperature is more intricate in concrete than it is in polymers and metals, and is basically two fold. First, temperature directly influences the deformation rate, and second, it affects the rate of aging, i.e., the change of material properties due to the progress of cement hydration.

A constitutive equation which accounts for both of these effects seems to have first been devised in Refs. 6, 39, and 40. Mukaddam and Bresler (39, 40) formulated the acceleration of creep due to temperature according to the well-known concept of thermorheologically simple materials (17), which is based on the reduced time, \( t_{\text{red}} \), whose increment \( dt_{\text{red}} \) is obtained from the actual time increment, \( dt \), by multiplying it with a certain shift function, \( \Psi \), of temperature \( T \). This type of formulation describes very accurately the thermoviscoelastic response of time-invariable materials, e.g., polymers (17, 24), and its extension to concrete has already been suggested by Sackman (54).

The basic idea of Mukaddam and Bresler was to introduce the age of concrete as a second variable into the shift function, putting \( \Psi = \Psi_1(T) \Psi_2(t^*) \); in which \( \Psi_1 \) and \( \Psi_2 \) = functions of one variable; and \( t^* = t^*_{0} + (t' - t^*_{0}) e^{-A(T - T^*)} \); \( T^* \) and \( t^*_{0} \) = the chosen reference temperature and reference age at loading; \( t' \) = actual age at loading at temperature \( T \); and \( A \) = constant. Mukaddam and Bresler's formulation was somewhat modified by Rashid (50), who omitted the temperature corrected age at loading, \( t^*_{0} \), and introduced instead a different definition of reduced time, such that \( dt_{\text{red}} = \Psi_1(T) \Psi_3(t - t') dt \) in which \( \Psi_1 \), \( \Psi_3 \), etc.

Note.—Discussion open until November 1, 1974. To extend the closing date one month, a written request must be filed with the Editor of Technical Publications, ASCE. This paper is part of the copyrighted Journal of the Engineering Mechanics Division, Proceedings of the American Society of Civil Engineers, Vol. 100, No EM3, June, 1974. Manuscript was submitted for review for possible publication on December 22, 1972.

†Prof. of Civ. Engrg., Northwestern Univ., Evanston, Ill.
functions of one variable; and \( \Psi_1(t - t') = d(\log(t - t' + 1/dt) \]

The formulations of Mukaddam and Bresler (39,40) and Rashid (50) have the advantage that functions \( \Psi_1 \) and \( \Psi_2 \) or \( \Psi_3 \) may be obtained by simple graphical manipulations of the measured creep curves. However, generalizations to variable stress according to the principle of superposition leads to stress-strain relations in the form of an integral equation which cannot be converted to a system of first-order differential equations. This poses major difficulties in creep analysis of large structural systems arising, e.g., in finite element method. Namely, a step-by-step creep analysis for an integral-type creep law requires the values of stress from all previous time steps to be stored for each element of the structure and to be used in computation at each time step. Thus, the storage and machine time requirements become a multiple of those for the corresponding elastic problem.

The cumbersome history dependence can be avoided only by a transition to a rate-type creep law, i.e., a creep law consisting of a set of first-order differential equations involving hidden (internal) variables. [Certain forms of an integral-type creep law, e.g., those with a degenerate kernel, achieve the same; but such laws can always be converted to a rate-type form (5).] The structure of rate-type creep laws can always be visualized by spring-dashpot models, and a certain distaste has unfortunately developed toward these models during the 1960's, partly because of vulgarized microstructural interpretations, partly because of an avalanche of papers with only immaterial differences in the arrangements of model. Actually, there is justification for only two basic types of models, the Maxwell chain and the Kelvin chain, because either of these two models can describe a viscoelastic material with any desired accuracy, whether the material is age-independent (52), or age-dependent (10,11). These models define the well-known relaxation and retardation spectra and are equivalent to the expansion of memory kernel in a series of real exponentials (Dirichlet series), which is a counterpart of the Fourier series and plays a role as important in the decaying processes as the frequency response spectra do in the periodic processes. The rate-type laws also allow a much simpler formulation of thermodynamics which was first emphasized in a fundamental paper by Biot (14), and it is for this reason that the rate-type laws are now becoming the preferred approach in theoretical continuum mechanics (43). Finally, the rate-type formulation of the creep law is also necessary in deducing additional information on the creep law from a model of the microstructural processes (3,4,8,30) because their mathematical formulation always leads to differential rather than integral equations.

For constant temperature, the rate-type creep law of concrete has been developed in Refs. 10 and 11. An extension to variable temperature will be the primary objective of this study.

One important physical concept in temperature effects is that of the activation energy of thermally activated processes 26. This concept has been widely explored in creep of metals (19,36,62), and also creep of clays (38). Polvaha and Best (49), analyzing the change of creep rate at a temperature rise, introduced it into the study of concrete creep. At the same time Hansen (28) proposed a temperature dependence of a certain viscosity parameter which is equivalent to the activation energy formulation. Later Ruetz (53), Klug and Wittman (32,64), Maréchal (35) and others (16,56), analyzed concrete creep using the activation energy approach. However, in all of these studies the acceleration of aging due to temperature rise has been disregarded and creep curves have been fitted by expressions that are inapplicable to the case of variable stress.

**Review of Rate-Type Creep Law at Constant Temperature**

This study will be restricted to creep at constant or almost constant water content and the case of variable water content will be examined in a subsequent paper. In a previous work (10,11) it has been shown that in the linear stress range, i.e., for stresses less than about 1/2 of the strength (45), the rate-type creep law for aging concrete at constant temperature may be based on either the Kelvin chain or the Maxwell chain. They both allow given creep data to
be fitted as closely as desired. However, identification of material parameters is easier to perform with the Maxwell chain (Fig. 1), and therefore this chain will be used throughout this study. The uniaxial stress-strain law then has the form:

$$\sigma = \sum \sigma_{\mu}; \quad \varepsilon = \frac{\sigma_{\mu}}{E_{\mu}} + \eta_{\mu} \varepsilon; \quad \mu = 1, 2, \ldots n \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (1)$$

in which $\sigma$ and $\varepsilon$ are stress and strain; $E_{\mu}$ are spring moduli; and $\eta_{\mu}$ are dashpot viscosities; both $E_{\mu}$ and $\eta_{\mu}$ depend on age $t$ of concrete; $\sigma_{\mu}$ values are called hidden stresses or internal state variables (14), whose current values fully characterize the previous history of the material. Superimposed dots stand for time derivatives, e.g., $\dot{\varepsilon} = d\varepsilon/dt$; and the stress-independent inelastic strains, such as shrinkage, are not included.

The ratios $\tau_{\mu} = E_{\mu}/\eta_{\mu}$ are called relaxation times and the plot of $E_{\mu}$ versus log $\tau_{\mu}$ is called the relaxation spectrum. It depends on age $t$ and is discrete, while the actual spectrum must be continuous. Realizing that the $E_{\mu}$ values merely represent a discrete approximation of a continuous function, it is clear that any data set can be approximated equally well for any set of $\tau_{\mu}$ values which are sufficiently densely distributed and cover the whole range of interest. Therefore, one should not try to determine $\tau_{\mu}$ from the creep data (and indeed, if attempted, a problem with a nonunique solution would result); rather, the $\tau_{\mu}$ values must be chosen. In particular, they may be chosen time-constant and a suitable choice is a geometrical series with quotient 10

$$\tau_{\mu} = 10^{\mu-1} \tau_1, \quad \eta_{\mu} = 10^{\mu-1} \tau_1, \quad E_{\mu}; \quad \mu = 1, 2, \ldots m \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (2)$$

in which $m = n - 1$. (If smoothness of fit is not of concern, quotients up to approx 60, leading to fewer hidden stresses, are acceptable.) The last spring ($\mu = n$) in Fig. 1 is imagined not to be coupled with any dashpot, i.e., for $\mu = n$ there is $\eta_n \to \infty$ or $\tau_n \to \infty$. The $\tau_{\mu}$ values must cover the entire range of response delays to be considered. The dependence of $E_{\mu}$ upon $t$ may be satisfactorily fitted, e.g., by polynomials of the type (11):

$$E_{\mu}(t) = \sum_{p=0}^{q} E_{\mu_p} t^p; \quad \text{with } x = t^{1/6}, \quad q = 4 \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (3)$$

in which $E_{\mu_p}$ ... $E_{\mu_q}$ are constants. Eq. 9 also works well for $x = \log t$, $q = 3$.

Generalization of Eq. 1 to multiaxial stress is defined by the creep Poisson's ratio, which may be assumed as constant and equal to $\nu = 0.18$, because no clear evidence to the contrary exists (45). Thus, equations analogous in form to Eqs. 1 and 3 may be obtained for the volumetric components and the deviatoric components of stresses and strains.

**Activation Energy of Hydration and Temperature Dependence of Aging**

The change of properties of concrete with age, and the dependence of $E_{\mu}$ and $\eta_{\mu}$ upon $t$ in particular, are explained by the progress of cement hydration. When temperature is increased, the hydration reaction is speeded up (as is any chemical reaction). Thus, the change of material parameters, e.g., $E_{\mu}$ and $\eta_{\mu}$, is accelerated. Accordingly, it is necessary to replace time $t$ in the expressions for $E_{\mu}$ and $\eta_{\mu}$ by a certain variable $t$, characterizing the degree of hydration or maturity of concrete (44,47). This variable, called the equivalent hydration period, has been defined (3,7,8) as follows:

$$dt = \beta_t dt \quad \text{or} \quad t = \int \beta_t dt \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (4)$$

in which $\beta_t = \text{relative hydration rate coefficient} = \text{function of temperature}$ $T$. Since hydration is a thermally activated process it is natural to assume that the hydration rate obeys the Arrhenius equation (19,26), i.e.:

$$\beta_t = \exp \left[ \frac{U_\mu}{R} \left( \frac{1}{T_0} - \frac{1}{T} \right) \right] \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (5)$$

in which $U_\mu = \text{activation energy of hydration}$; $R = \text{gas constant} = 1.986 \text{ cal/} \text{K}$; $T = \text{absolute temperature}$; and $T_0 = \text{chosen reference value of T}$ for which $\beta_t = 1$.

It should be realized, however, that Eq. 5 is only approximate. Actually, hydration consists of a number of different chemical reactions, each with a different activation energy. This causes the apparent activation energy obtained by fitting the data with Eq. 5 to be temperature dependent. Nevertheless, on the basis of the rates at which hydration heat is evolved, Verbeck (18) has found $U_\mu$ to be approximately constant between 0°C and 100°C up to 90-day age. Evaluation of the data in Table 12 of Ref. 18 furnishes

$$\frac{U_\mu}{R} = 2.70K \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (6)$$

This value has been assumed in all of the data fitting described in the sequel.

**Activation Energies for Deformation Rates**

The major source of creep in cement paste is probably the diffusion of solids and water along various types of thin layers between solid particles carrying load (9). These diffusion processes are interpreted in the model in Fig. 1 by the dashpots with viscosities $\eta_{\mu}$, characterizing the associated diffusion rates. Because any diffusion is a thermally activated process, the diffusion rates, and thus also viscosities $\eta_{\mu}$, may be assumed to obey, at least within a sufficiently narrow temperature range, the Arrhenius equation (19,26). Thus, with Eq. 2

$$\frac{1}{\eta_{\mu}} = \frac{1}{\eta_{\mu_0}} \exp \left[ \frac{U_\mu}{R} \left( \frac{1}{T_0} - \frac{1}{T} \right) \right] = \frac{10^{1/\mu}}{E_{\mu} \tau_1} \exp \left[ \frac{U_\mu}{R} \left( \frac{1}{T_0} - \frac{1}{T} \right) \right] \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (7)$$

in which $U_\mu = \text{activation energy of the diffusion process interpreted by the } \mu \text{th dashpot in Fig. 1}$; $\eta_{\mu_0} = \text{value of } \eta_{\mu} \text{ given by Eq. 2 for the same } t_\mu \text{ and at chosen reference temperature } T_0$.

The values of $U_\mu$ are likely to be different for various $\mu$ values. Those which correspond to higher $\tau_{\mu}$ could have higher activation energy because higher $\tau_{\mu}$ corresponds to diffusion in thinner and longer pores, in which diffusing molecules must be held more firmly. The simplest possible assumption about the distribution of $U_\mu$ over the Maxwell chain is:
In which \( U_i \) and \( \Delta U \) are constant. Substituting into Eq. 7 then gives

\[ \eta_n = a \tau_1 \tau_n^{n-1} \]

in which \( \tau_1 = \tau_1 \exp \left[ -\frac{\Delta U}{R \left( \frac{1}{T_0} - \frac{1}{T} \right)} \right] \)

and \( a = 10 \exp \left[ -\frac{\Delta U}{R \left( \frac{1}{T_0} - \frac{1}{T} \right)} \right] \)

Note that the activation energy, \( U_n \), in Eq. 7 is not considered herein to be a function of time, because on the microstructural level \( U_n \) is determined by the local distances and arrangements of atoms and molecules, which do not change in time unless significant changes occur in the material. This contrasts with the previous applications of the activation energy to concrete creep (32,35,64) in which it was necessary to assume that the activation energy decreases with the duration of loading because no distinction between various diffusion processes with different relaxation times was made and only a single thermally activated process was considered. For completeness, it should be mentioned that the activation energy concept requires, in general, that the creep rate be proportional to \( \sinh \left( \sigma_i / \sigma_1 \right) \) in which \( \sigma_i = \text{constant} \) (26,36,62). This dependence on stress is nonlinear but for small stress, \( \sinh \left( \sigma_i / \sigma_1 \right) \approx \sigma_i / \sigma_1 \); i.e., the creep law becomes linear, as considered in the present study.

Moduli \( E_n \), representing the elastic properties of the material are, of course, also temperature dependent. However, as long as the local arrangements of atoms and molecules and the chemical composition of the material do not change appreciably, the change of \( E_n \) is caused only by changes in atomic and molecular distances due to thermal dilatations, and because these are small, the changes in \( E_n \) must also be small. It is known that the chemical composition of cement paste is rather stable up to about 80\(^\circ\) C–100\(^\circ\) C (34), and within this range moduli \( E_n \) should thus be approximately constant. Measurements (16,20,29,46) reveal that from 25\(^\circ\) C to 100\(^\circ\) C the short-time modulus, \( E_n \), drops no more than about 15\%, which is not negligible but is still small. The drop in actual instantaneous modulus \( E \) is less, perhaps much less, because the effect of the increase of the short-time creep rate with temperature has not been separated in these measurements. For these reasons, moduli \( E_n \) were all assumed to be independent of temperature up to about 80\(^\circ\) C–100\(^\circ\) C. But for a wider temperature range the change in \( E_n \) would have to be considered.

Note that at varying temperature, the ratios \( E_n / \eta_n \) are not constant since \( \eta_n \) includes activation energy effects while \( E_n \) does not.

**Identification of Material Parameters From Given Data**

Identification of the material parameters requires the coefficients of a differential equation to be determined from a given solution. This is a typical problem of control theory (12). A simple general method for its solution is outlined in Appendix 1. It succeeds, however, only if a sufficiently good initial estimate of the material parameters can be found. Therefore, a special procedure has been developed in which the identification problem is broken in a sequence of simpler subproblems. This approach allows one to gain better insight into the problem, utilize experience, and substitute at intermediate steps some intuitive analogical judgments, which is desirable in view of the fact that the fit of the incomplete data presently available is not unique.

First, it is necessary to determine \( E_n(t) \) at a chosen reference temperature \( T_0 \). To this end, the creep data at temperature \( T_0 \) must first be converted on the basis of the superposition principle to the stress relaxation function, \( E_n(t, t') \), representing stress relaxation curves for various ages \( t' \) at imposition of constant strain. This may be accomplished by a method presented in Ref. 5. Then, each relaxation curve must be expanded into the Dirichlet series, \( E_n(t') \exp \left[ -\left( t - t' \right) / \tau_n \right] + E_{\infty} \), which represents solution \( \varepsilon \) of Eqs. 1 for \( \varepsilon \) as a step function. This can be done by minimizing a sum of the squares of deviations from given \( E_n(t, t') \) plus an additional term giving a penalty for unsmooth dependence of \( E_n \) on \( t' \). A subroutine which implements this task and yields \( E_n, E_{\infty} \) from Eq. 3 is described in Ref. 11 (and a FORTRAN listing of an improved version which suppresses the random components of \( E_n \) is given in a paper submitted to Cement and Concrete Research).

Subsequently, the data for other temperatures \( T \) are analyzed. In this regard, it is instructive to make the following three observations on the creep curves plotted in \( \log (t, t') \) in which \( t = \) observation time; and \( t' = \) instant of loading.

1. Acceleration of aging by temperature rise, when considered alone, does not shift the creep curve parallel to the time axis because relaxation times \( \tau_n = \eta_n / E_n \) are not affected by aging. It rather scales down the ordinates because the increase of moduli \( E_n \) with age is faster at a higher temperature (Fig. 2, curves \( a \alpha', a \beta' \)).

2. Let aging be now left out of consideration, i.e., regard \( E_n \) as constants. The creep rate at time \( t - t' = \tau_n \) depends mainly upon the stress in the \( \mu \)th Maxwell unit in Fig. 1. A change of \( \tau_n \) to \( \tau_n' \) means that the response of the \( \mu \)th Maxwell unit will come at an earlier time that appears in the log \( (t - t') \) scale to be shifted by distance log \( \left( \tau_n'/\tau_n \right) \) to the left from the time \( t - t' = \tau_n \). If all \( U_n \) are the same \( (\Delta U = 0) \), the response of all Maxwell units will be shifted to the left by equal distance \( (S \text{ in Fig. 2}) \), i.e., the whole creep curve will be shifted to the left without any change in shape and slopes (Fig. 2, curves \( b \alpha', b \beta', b \beta' \)). (This is the case of the thermomechanically simple material, closely approximated by most polymers.) However, if \( U_n \) are distributed linearly, the shift to the left will be, according to Eqs. 9–11, by the distance \( \log \left( \tau_n'/\tau_n \right) = \log \left( \tau_n' / \tau_n \right) + (\mu - 1) \log a \), i.e., a distance linearly increasing with \( \mu \) (distances \( S_1 \) and \( S_2 \) in Fig. 2). This means that the slope of the creep curve will be increased in the ratio \( 1 / \log a \). This is also apparent from the fact that at reference temperature \( T_0 \) the relaxation times, \( \tau_n \), are distributed at distances \( \log 1 \), while at temperature \( T \) they are distributed according to Eqs. 2 and 9, at distances \( \log a \).

3. Because temperature rise causes a shift of the response to shorter times, the creep curves at a lower temperature in each data set ought to be known up to much longer elapsed times \( t - t' \) than those at reference temperatures and, conversely, the creep curves at a higher temperature ought to be known
down to much shorter elapsed times than those at reference temperature. Unfortunately, this is not the case for any data presented so far. Therefore, certain extensions of the creep curves must be estimated and, as a consequence, the problem of identification of the material parameters from given data does not have a unique solution for all data sets known at present.

Considering first the special case, $U_\mu = \text{constant} = U$ (i.e., $\Delta U = 0$), the procedure of fitting the data at temperatures other than $T_0$ may be arranged as follows:

1. The given creep curves for various temperatures and the chosen reference age at loading, $t_0'$ ($aa'$, $bb'$, and $cc'$ in Fig. 3) are shifted parallel to the logarithmic time axis to make them coincide with one continuous creep curve at reference temperature $T_0$ (curve $aa'$ $bb'$ $cc'$ in Fig. 3). This succeeds well enough for the test data known at present [as was observed by Sackman (54) and Mukaddam and Bresler (40), although for the purpose of a different theory].

2. The foregoing shift to a lower reference temperature, $T_0'$, does not account for the fact that the aging is slower at $T_0$ than at $T$. Therefore, as stated previously, the actual creep curve at $T_0$, sketched as curve $ac'$ in Fig. 3, must deviate upward from the curve $aa'$ $bb'$ $cc'$ (or downward, when $T_0 > T$). This deviation must be estimated, and the correct estimate is easily found after several trials (which may be regarded as the iterates in the regula falsi method). Curve $ac'$ represents an extended creep curve at reference temperature $T_0$.

3. Analyzing the creep curves at reference temperature and various ages at loading ($ac'$, $gg'$, $hh'$ in Fig. 3) with the program from Ref. 10, parameters $E_{a0}$, $E_{b0}$ of Eq. 3 for $E_\mu$ are ascertained.

4. Measuring the horizontal distances, e.g., $S_a$ and $S_b$ in Fig. 3, one estimates the value of activation energy $U$, and using the unconditionally stable step-by-step
algorithm of the type presented in Ref. 10, one computes from \( E_0, \ldots, E_\infty \) the creep curves for the temperatures and the ages, \( t' \), available in the given data. The plots of the curves (e.g., obtained by Calcomp plotter) are then compared visually with given data (curves aa’, bb’, cc’ in Fig. 3). The estimate of \( U \)

temperature \((ac^m, gg^m, hh^m)\), until a satisfactory fit is achieved. (After some experience no more than one repetition was needed.)

When \( U_0 \) is considered to be distributed linearly with \( \mu \) (Eq. 8), a slightly different procedure is more convenient. First one estimates the extension of the creep curve at reference temperature (curve aa’ in Fig. 4). Lacking detailed information, the most natural assumption is perhaps a straight-line extension, as shown in curve a’ b’ in Fig. 4. Then, measuring the horizontal distances (e.g., \( S_1 \) and \( S_2 \)) from the creep curves at elevated temperatures (e.g., bb’), the values of \( U_i \) and \( \Delta U \) in Eq. 8 can be estimated, though only roughly because aging distorts the picture. Step 3 from the previous procedure is then implemented and after computing the creep curves, their comparison with data is made, as in step 4. Using the regula falsi method, the procedure is then repeated for improved estimates of \( U_i \) and \( \Delta U_i \) and improved extensions of

creep curves at reference temperature, until the best fit is found.

**Numerical Results and Analysis**

Using the foregoing procedure, all of the more extensive published data on temperature dependence of creep \((15,22,27,37,41,42,55,61,65)\) were analyzed. The data are shown (by points or dotted lines) in Figs. 5-14 and further information on the data is compiled in Table 1.

First the data were fitted under the assumption that \( U_0 = U = \text{constant} \) (or \( \Delta U = 0 \)). The fits are drawn in Figs. 5-14 by continuous solid or dashed lines, using as ordinates the total stress-dependent strain, \( \epsilon \) (excluding thermal expansion and shrinkage), minus the instantaneous (or short-time) strain \( \epsilon_a \) at load application. The solid data points in Fig. 14 are from tests under biaxial compression, in which case \( \epsilon = (1 - \nu) \epsilon_x \), in which \( \epsilon_x \) is the strain under uniaxial compression; and \( \nu = \text{creep Poisson's ratio} = 0.2 \) \((1)\). The activation
energy values corresponding to the fits are indicated in the figures in terms of $U/R$. Unfortunately, it is found that either different concretes exhibit different activation energies or the scatter of the data presently known is large and the control of test conditions is inadequate. The mean value, which could reasonably fit most data, is roughly

$$\frac{U_0}{R} = \frac{U}{R} = 5000^\circ K \text{ or } U = 10,000 \text{ cal}$$

Reference temperature $T_0$ was always taken as the lowest temperature shown.
### TABLE 1.—Test Specimen

<table>
<thead>
<tr>
<th>Figure (1)</th>
<th>Reference (2)</th>
<th>Strength, in pounds per square inch (3)</th>
<th>Stress-strength ratio (4)</th>
<th>Type of cement (5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>27</td>
<td>7,800 (cylinder)</td>
<td>&lt; 0.25</td>
<td>—</td>
</tr>
<tr>
<td>6</td>
<td>22*</td>
<td>5,500</td>
<td>0.18</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4-in. cubes at 614 days</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>61*</td>
<td>6,000 (cylinder)</td>
<td>&lt; 0.25</td>
<td>I</td>
</tr>
<tr>
<td>8</td>
<td>65*</td>
<td>6,000 (cylinder)</td>
<td>&lt; 0.4</td>
<td>II</td>
</tr>
<tr>
<td>9</td>
<td>37*</td>
<td>6,000 (cylinder)</td>
<td>&lt; 0.4</td>
<td>II</td>
</tr>
<tr>
<td>10</td>
<td>41</td>
<td>5,500</td>
<td>&lt; 0.35</td>
<td>III</td>
</tr>
<tr>
<td></td>
<td></td>
<td>9-1/2-in. (cylinder) at 14 days</td>
<td>(0.6, 0.7 not fitted)</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>42</td>
<td>7,250</td>
<td>&lt; 0.45</td>
<td>III</td>
</tr>
<tr>
<td>12</td>
<td>15</td>
<td>6,600 (cylinder)</td>
<td>&lt; 0.36</td>
<td>II</td>
</tr>
<tr>
<td>13</td>
<td>55</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>14</td>
<td>1</td>
<td>6,000 (cube)</td>
<td>0.22</td>
<td>“ordinary”</td>
</tr>
</tbody>
</table>

*Instantaneous deformation not reported.
*Used are tests F-33, B-39, B-4, B-7.
*Reported partly, in which case the initial points were arbitrarily adjusted.

**Properties for Published Data**

<table>
<thead>
<tr>
<th>Aggregate (6)</th>
<th>Maximum aggregate size, in inches (7)</th>
<th>Water-cement ratio (8)</th>
<th>Sealing, curing, temperature history (9)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oldbury limestone</td>
<td>—</td>
<td>0.47</td>
<td>5 months in water, 20° C, 1 month sealed by copper foil</td>
</tr>
<tr>
<td>limestone</td>
<td>—</td>
<td>0.45</td>
<td>6 days in water, 6 days at 90% humidity, 17° C, then sealed in water</td>
</tr>
<tr>
<td>graywacke, chert</td>
<td>3/4</td>
<td>0.425</td>
<td>83 days sealed at 73° F, then to test temperature; sealed by epoxy and copper jacket same as above</td>
</tr>
<tr>
<td>dolomite, hornblende</td>
<td>3/4</td>
<td>0.6</td>
<td>raised to test temperature at 1-day age, in water</td>
</tr>
<tr>
<td>dolomite, hornblende</td>
<td>3/4</td>
<td>0.6</td>
<td>raised to test temperature 1 week before test, in water cured at room temperature till 1 day before test, then sealed in copper jacket, raised to test temperature 1 day (t = 3) or 3 days (t = 8) before test</td>
</tr>
<tr>
<td>foraminifer limestone</td>
<td>1-1/2</td>
<td>0.42</td>
<td>raised to test temperature 1 day before test, sealed by epoxy with two plastic coats</td>
</tr>
<tr>
<td>river gravel</td>
<td>3/8</td>
<td>0.564</td>
<td></td>
</tr>
</tbody>
</table>

Except for the values of 70°F in Fig. 7 and 40°C in Fig. 12. None of the data completely defines the creep at T0. Different ages at loading are available only for the data in Fig. 12. (Their fit at first seems quite poor but a closer examination reveals that a better smooth fit could hardly be desired because of the very high scatter of these data.) Thus, the creep curves for ages other than the one given, as well as extensions in length of the curve at the one basic age given, had to be estimated except in Fig. 12. The estimates were based on analogy with the data on the aging effect given in Ref. 10. The estimates were then slightly (but not unreasonably) modified, to give the best fit at other than reference temperatures. However, the information on the aging effect is not as limited as the foregoing comments may suggest, since the most important effect, e.g., the acceleration of aging due to temperature change, is based on the hydration heat data in Ref. 18, as previously mentioned, and not on the data shown. The complete creep curves at T0 for various ages, corresponding to the fits in Figs. 8, 12, 5, and 15, are shown in Fig. 16 as cases a, b, and c, respectively. They are drawn only up to the elapsed times, t - t', to which they are needed for fitting the data. Note that the creep data at reference temperature must completely cover the whole age interval (approximate):

\[
t'_{\text{min}} + (t - t') \exp \left[ \frac{U}{R} \left( \frac{1}{T_0} - \frac{1}{T_{\text{min}}} \right) \right] \leq t \leq t'_{\text{max}} + (t - t') \exp \left[ \frac{U}{R} \left( \frac{1}{T_0} - \frac{1}{T_{\text{max}}} \right) \right]
\]  

\[\text{(13)}\]
in which \( T_{\text{max}}, T_{\text{min}} \) = the maximum and minimum temperatures considered; \( t'_{\text{max}}, t'_{\text{min}} \) = the maximum and minimum age at loading \( t' \) expressed in terms of \( t'_{\text{u}} \) and \( t' - t' \) = the maximum and minimum time lag after loading. In judging the quality of fits, it should be kept in mind that the creep curves shown also fit exactly (with the same stress-strain relations) the values of instantaneous deformation \( \varepsilon_{\text{u}} \) assumed to correspond to \( t' = 0.001 \) day. These points would lie to the far left, out of the figures, and are not shown.

In view of the high scatter and incompleteness, it would be unjustified to try determining a greater number of material parameters from the present data, even if some of the fits could be considerably improved (in Fig. 10, for instance, by using \( \Delta U \neq 0 \)). Nevertheless, the fit with a linearly distributed \( U_{\text{u}} \) (Eq. 8), was tried at least for one case, the data from Fig. 5, which belong to the most complete and consistent ones. The fit is shown by solid lines in Fig. 15 along with the values of \( U_{\text{u}} \) and \( \Delta U \) found. The corresponding reference creep curves are shown as case d in Fig. 16 and they differ considerably from case c pertaining to the same data fitted with constant \( U_{\text{u}} \) (Fig. 5). This underscores the lack of uniqueness of fit of the data available.

Assumptions also had to be made with regard to other circumstances not reported by the experimenters. For instance, for the data in Figs. 6 and 7, the value of instantaneous deformation had to be estimated, using analogy with other concretors. But for the whole set of data for one concrete the same value of the instantaneous deformation was assumed throughout, of course. The time at which heating was started was not reported for the data in Figs. 6 and 7; it was assumed to occur shortly before loading, as in most tests. In some of the tests, however, heating started at the instant of loading, which distorted the data because temperature could not be constant throughout the specimen at the beginning of the test.

Preservation of constant moisture content has been inadequate in some of the tests, especially the early ones (59). At a higher temperature concrete can lose water much faster, and after this happens, the creep rate drops, which overrides the activation energy effect. This has probably also affected to some extent the data in Fig. 6. Using the present theory it would be possible to fit successfully other data of more limited scope as well (2, 21, 28, 33, 51, 58, 60).

Test data show that on approach to 100°C the creep first ceases accelerating and beyond a certain temperature it begins slowing down, while according to Eq. 7 creep rate must always rise when temperature rises. This can be explained by changes in the chemical composition and microstructure of cement paste. For this reason, the data points close to 100°C have not been fitted by the present theory even if given in the original data set (temperatures 94°C and 114.5°C in Fig. 6, 205°F in Fig. 11). However, if the activation energy concept were dropped and \( \eta_{\text{u}} \) were regarded as an arbitrary function of temperature, the rate-type creep law (1) could probably fit even the data beyond 100°C. The material is also changed by the freezing of water and the present formulation thus does not apply below 0°C without further refinements (as is confirmed by the tests in Refs. 31 and 48).

According to the present formulation a drop in temperature would always result in a reduction of the creep rate. However, it has been observed on small specimens that during and immediately after a very abrupt temperature drop...
the creep is not slowed down, but accelerated at first, (123, 31, 56, 61, 64). One way of explaining this effect is by considering that the dependence of the rates of the diffusion of solids and water causing creep upon chemical potential gradient is progressively nonlinear when the gradient is very high (9). Then the rate increase due to this nonlinearity offsets the effect of activation energy when a large gradient due to sudden temperature drop is superimposed on the gradient due to the load. (This is analyzed in more detail by the senior writer in Mechanics Today, Vol. 2, Pergamon Press, 1974, in press.) The present stress-strain relations can thus be applied only at sufficiently slowly varying temperatures, which are usually the case for all massive concrete bodies.

Conclusions

1. Creep of aging concrete at variable temperature can be modeled by a rate-type creep law with hidden stresses as internal state variables (Eq. 1). The formulation presented herein is restricted to the linear stress (i.e., working stress) range, to constant (or almost constant) water content, to constant or slowly varying temperatures, conditions which are typical for massive concrete structures, and to temperatures between about 2° C and 90° C.

2. The main purpose in formulating the rate-type creep law is to avoid the storage of stress history and the evaluation of hereditary integrals, required by previous formulations. This makes the numerical creep analysis of large structural systems feasible.

3. Introduction of the activation energy concept (applicable from 2° C to approx 80° C -100° C) is advantageous because it specifies the form of the temperature-dependence and thus enables determination of the creep law from more limited data than would be needed otherwise. Yet the problem of identification of the material parameters from the limited data presently available does not have a unique solution (although the activation energy concept reduces the degree of arbitrariness considerably). For example, the data presently known can be fitted, assuming the activation energies for various relaxation times to be either all equal, in which case the creep curve at reference temperature extended into very long times curves upward in the logarithmic time scale (Figs. 3, 16), or to be distributed linearly over the Maxwell chain (Eq. 8), in which case the extension just mentioned may be straight (Figs. 4, 16). To decide which alternative is true would necessitate data for a much longer test duration than is presently available.

4. In contrast with some previous studies, the artificial and unjustified assumption that the activation energy depends on the duration of loading appears to be unnecessary.

5. Introducing the equivalent hydration period (Eq. 4), the acceleration of aging due to temperature rise is formulated with the help of activation energy of hydration (Eqs. 5, 6). This concept (30) is corroborated by data fits.

6. All of the more extensive data on the temperature dependence of creep have been fitted satisfactorily, by the method described (see Figs. 5-14).

7. The activation energy of creep appears to differ widely for various concretes, the mean value being approx $U = 10,000$ cal. Many of these differences may be due, however, to experimental scatter and differences in test procedures.

EM3

Acknowledgment

The present research results have been obtained under United States National Science Foundation Grant GK-26030.

Appendix I.—Alternative General Method of Material Identification

To discretize the problem, the measured creep data may be characterized by certain suitably chosen characteristic data points, numbered as $a = 1, 2, ..., N$. [Suitable choice requires that these points be uniformly distributed in log $(t − t')$ - scale.] The measured values at these points, characterized by $U_a, t'_a, (t − t')_a$, may be denoted as $F'_a$. The theoretical solution at these same points, $F_a$, may be regarded as a function of the unknown material parameters, $p_m$ (m = 1, 2, ..., m), which represent $U_a, ΔU$, and those of the characteristic values of the creep function $I(t, t' − t')$ at reference temperatures that have not been measured and are unknown, usually the values for high $t' − t'$ and high $t'$. Assuming that a suitable estimate $p_m^0$ of the unknown material parameters, $p_m$, has been made the solution, $F_a$ (in all points $a$), corresponding to $p_m^0$ is computed. Further solutions, $m$ in number, are determined for $m$ sets of material parameters $p_m^0 + δ_m Δp_k (k = 1, 2, ..., m)$ in which $Δp_k$ are chosen small changes of $p_m^0$; and $δ_m = 0$ for $m ≠ k$; and $δ_m = 1$ for $m = k$. Then, assuming $F_a$ as a function of $p_m$ to be continuous and sufficiently smooth, the solution for any material parameters $p_m$ close to $p_m^0$ is

$$F_a = F'_a + \sum_k F_{a,k}(p_k - p_k^0)$$

in which

$$F_{a,k} = \frac{F_a(p_m^0 + δ_m Δp_k) - F_a(p_m^0)}{Δp_k}$$

Eq. 14 is a Taylor series expansion by which the problem is linearized, and $F_{a,k}$ are approximations to partial derivatives with respect to $p_k$. The objective of optimum fit of data points $F_a$ may be achieved by imposing a least square criterion, that is by minimizing the quadratic form

$$Φ = \sum_a w_a \left[ F'_a + \sum_k F_{a,k}(p_k - p_k^0) - \bar{F}_a \right]^2 + Φ_1$$

in which $w_a = $ chosen weights; and $Φ_1 = $ a certain chosen quadratic penalty term (25,63) which may be added, e.g., to prevent unreasonable large changes in $p_m$ (or to guarantee a smooth distribution of $p_m$ values when required, as in the case when $p_m$ are the unknown ordinates of some reference creep curve). The minimization problem is, strictly speaking, a constrained one (63) because usually the parameters, $p_m$, must be positive and some of them may have to satisfy further inequalities (e.g., expressing the fact that the reference creep curve must always rise). Constrained minimization can be, however, usually avoided by transformations (63). The minimizing condition then simply is $δΦ/δp_j = 0$ for all $j = 1, 2, ..., m$. This yields a system of linear algebraic equations for $p_m$, from which improved $p_m$ values may be obtained. These values may then be labeled as new $p_m^0$ and, repeating the procedure just described, a further
improvement in $p_e$ values can be made, etc.

The method outlined in essence equivalent to the method of Gauss and Legendre (63). It belongs to the techiniques of quasilinearization and can be brought into connection with the well-known Newton-Raphson method (12).

**APPENDIX II.—REFERENCES**


37. McDonnell, J. E., “An Experimental Study of Multiaxial Creep in Concrete,” *Concrete for Nuclear Reactors, American Concrete Institute Special Publication No. 34, 1972*, pp. 732-768.


40. Mukaddam, M. A., and Bresler, B., “Behavior of Concrete under Variable Temperature and Loading,” presented at the American Concrete Institute International Seminar on Concrete for Nuclear Reactors, held at West Berlin, West Germany, in 1970.