Practical prediction of time-dependent deformations of concrete

Part V : Temperature effect on drying creep

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The model for drying creep from Part III is here extended for the temperature effect. The basic, additive form of the double power law and the shrinkage-like term are preserved, but certain coefficients become temperature-dependent to reflect the acceleration of drying and of aging. Satisfactory agreement with test data is achieved.

INTRODUCTION

The effects of temperature upon creep of sealed and unsealed specimens are known to be considerably different. Heating increases the rate of moisture loss from the unsealed specimens, and during this process creep is intensified, while after the moisture loss has been completed, creep gets substantially reduced. The model for the temperature effect on creep of sealed specimens, which has been presented in Part IV, will now be extended to cover the case of drying and dried concrete [3].

PROPOSED FORMULAS

From the physical point of view, for concrete there is nothing special about the room temperature, compared to moderate elevated temperatures, such as 40°C. Therefore, the basic formulas for drying creep, equation (25), must retain the same form at all constant moderate temperatures, although the coefficients in the formulas must be expected to vary as before. So, we may use again equation (25), i. e.,

$$J(t, t')=\frac{1}{E_0}+C_0(t, t')+C_d(t, t', t_0)-C_p(t, t', t_0), \tag{42}$$

where $C_0(t, t')$ now includes the effect of temperature on the basic creep part and is defined by equation (34). Temperature T is assumed to be constant from the loading time t' but may vary prior to t'. The remaining two terms are the same as in equation (25) but they now include the effect of temperature. They model the increase of creep during drying:

$$C_d(t, t', t_0)=\frac{\varphi'_d}{E_0}\, t_e'^{\,-m/2}\, k'_h\, \varepsilon_{sh_\infty}\, S_d(t, t') \tag{43}$$

and the decrease of creep after drying (predreid specimen):

$$C_p(t, t', t_0)=c_p\, k''_h\, S_p(t, t_0)\, C_0(t, t'). \tag{44}$$

The magnitude of the creep increase in equation (43) is determined by

$$\varphi'_d=\left(1+\frac{\Delta\tau'}{10}\right)^{-1/2}\varphi_d \tag{45}$$

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[3] Reference numbers less than 73 are found in the preceding parts, Vol. 11, No. 65 (Sept.-Oct. 1978), pp. 307-328, and Vol. 11, No. 66 (Nov.-Déc.), pp. 415-434.

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in which $\Delta\tau'$ is a reduced time lag of loading after heating ($t' \geqq t_0$):

$$\Delta\tau' = \int_{t_0}^{t'} \frac{k_T'' \, d\xi}{\tau_{sh}[T(\xi)]} = \frac{t_T' - t_0}{\tau_{sh}(T_0)} + \frac{t' - t_T'}{\tau_{sh}(T)} k_T''. \tag{46}$$

where $\tau_{sh}(T)$ is the value of τ_{sh} for temperature T and $k_T''=1$ for $T=T_0$. The shrinkage-like time function in equation (43) is defined as

$$S_d(t, t') = \left(1 + \frac{10\,\tau_{sh}(T)(k_T')^{1/4}}{t-t'}\right)^{-c_d n K_T^2} \tag{47}$$

The temperature-dependent parameters in these expressions are

$$\left.\begin{aligned} k_T' &= 0.42 + 17.6\left[1 + \left(\frac{100}{T-253.2}\right)^4\right]^{-1}, \\ k_T'' &= (k_T')^{5/4}, \end{aligned}\right\} \tag{48}$$

$$K_T = 1 + 0.4\left[1 + \left(\frac{93.5}{T-253.2}\right)^4\right]^{-1}. \tag{49}$$

where T = absolute temperature (K), while the remaining coefficients are given by the previous expressions: τ_{sh} by equations (4), (5), (9) and (48): φ_d by equations (31) and (32), C_T by equation (37), n by equations (17) and (18), t_e' by equation (35), m by equation (16), k_h' and k_h'' by equation (29), c_d and c_p by equation (30), and ε_{sh_∞} by equations (6) and (7). Equation (48) replaces equation (8) as a more accurate general expression for k_T'. Finally, function $S_p(t, t_0)$ giving the effect of drying time on the decrease of creep after drying is

$$S_p(t, t_0) = \left(1 + \frac{100}{\Delta\tau}\right)^{-n}, \tag{50}$$

where $\Delta\tau$ represents the reduced time lag which, however, differs from $\Delta\tau'$ in that it is taken up to the current time t rather than the loading time t': i.e.,

$$\Delta\tau = \int_{t_0}^{t} \frac{d\xi}{\tau_{sh}[T(\xi)]} = \frac{t_T' - t_0}{\tau_{sh}(T_0)} + \frac{t - t_T'}{\tau_{sh}(T)}. \tag{51}$$

where the last expression pertains to a step-wise temperature history, with a step from T_0 to T at time t_T'.

ANALYSIS OF PROPOSED FORMULAS

The term $C_0(t, t')$ has the same form as before [equation (34)] because it describes the basic creep part. Coefficient φ_d', characterizing the creep increase due to drying, diminishes as the time lag $t'-t_0$ between the start of loading and the start of heating increases, i.e., as the specimen gets closer to hygral equilibrium at the start of loading. Because at higher temperatures the new hygral equilibrium gets established faster, the dependence of φ_d' upon $t'-t_0$ must evidently involve temperature. The simplest possible approach is to define an equivalent or reduced time lag $\Delta\tau'$ for various temperatures. (In fact, the use of reduced time seems to be the only proper choice because the acceleration of drying by heating is instantaneous, without memory effects.) The reduced time is introduced by the integral in equation (46) for generally variable temperature, whereas the last expression in equation (46) applies for a step-wise temperature history which equals T_0 from the time, t_0, of drying exposure to a certain intermediate time, t_T', and a given constant elevated temperature T from time t_T' to the time of loading, t'. Coefficient k_T'' is chosen to be 1.00 at reference temperature $T_0 = 23$°C and to grow with temperature. Since at high temperatures k_T'' is large, equation (46) will give a relatively large reduced time lag $\Delta\tau'$ even if the actual time lag $t'-t_0$ is small. For concrete loaded a long enough time after the start of drying, the drying creep term $C_d(t, t', t_0)$ must vanish, i.e., $\varphi_d'=0$, which is indeed properly indicated by equation (45). Maréchal's tests [75] (*see* also [76] and [77]) are an example of a relatively long time lag $t'-t_0$, which caused the response to be rather different from other data. Without equation (45) such differences could not be modeled.

Temperature affects not only the magnitude of the drying creep term (φ_d') but also the rate of development of this term and the dependence of this rate on size. This is modeled by coefficient k_T' [equation (48)], which enters the calculation of $\tau_{sh}(T)$ according to equations (4)-(5) (Part I). Without coefficient k_T'', $\Delta\tau'$ would be proportional to k_T' (because τ_{sh} is proportional to $1/k_T'$). This dependence is, however, insufficient and coefficient k_T'' serves to intensify the dependence of $\Delta\tau'$ upon T, making $\Delta\tau'$ in effect proportional to $(k_T')^{2.25}$. This models the fact that the effect of heating before loading upon the magnitude of the drying creep term appears to be stronger than the effect of heating upon the rate of development of the drying creep term, given by the shrinkage-square half-time in equation (47) for $S_d(t, t')$. The fact that k_T' enters through τ_{sh} in equation (47) automatically gives the temperature influence upon the size-dependence of the drying creep term; however, this dependence must further be adjusted by coefficient $(k_T')^{1/4}$, giving the overall dependence as $(k_T')^{5/4}$.

The optimum dependence of coefficients k_T' and k_T'' upon T has not come out to be of the activation energy type. Nevertheless, the reduced time approach, i.e., the use of $\Delta\tau'$ and $\Delta\tau$, is in agreement with the rate-process theory (activation energy model).

Temperature affects, however, not only the rate of the drying creep term, governed by τ_{sh} in equation (47), but also the time shape of this term. At a higher temperature, the rise of $S_d(t, t')$ in time is more gradual, and so the exponent in equation (47) ought to get smaller upon heating. This is achieved by correction of coefficient K_T in the exponent of equation (47).

In Part I, equation (8), the activation energy concept was assumed to describe the temperature dependence of coefficient k_T'. However, it appeared that according

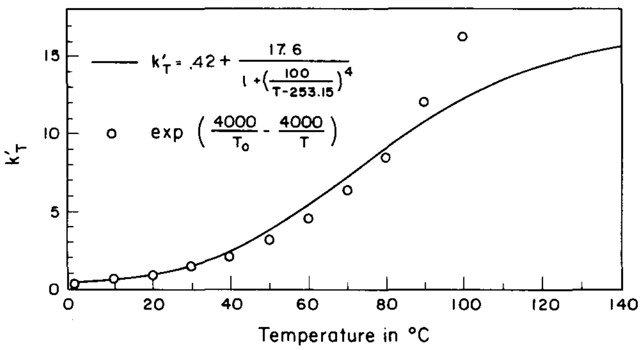

Fig. 35. — Coefficient k'_T as function of temperature.

to this concept the values of k'_T at higher temperatures (> 75°C) are much too high, which yields too small values of τ_{sh} and does not give proper size dependence. The temperature dependence introduced by equation (48) closely follows the activation energy dependence up to about 75°C but beyond this value k'_T rises slower (see *fig.* 35).

As in Part III the decrease of creep after drying, given by function $C_p(t, t', t_0)$, is proportional to the basic creep term $C_0(t, t')$, because the shape of creep curves after drying is similar, and is vertically scaled according to the degree of drying reached up to the current time t, as given by the shrinkage-like function $S_p(t, t_0)$ in equation (50).

COMPARISON WITH TEST DATA

Only relatively limited test data on drying creep of heated concrete exist, and they mostly deal with concretes which are typical of reactor vessels. Five different comprehensive test data sets available in the literature ([73], [63], [74], [70], [75]) have been fitted by the preceding formulas (see *fig.* 36). In these fits, $1/E_0$ has been determined by optimizing the fit of the drying creep data at room temperature. Thus, the fits shown indicate the ability of the formulas to describe the effect of temperature upon drying creep when there is no error in $1/E_0$. Different fits, in which $1/E_0$ is also predicted with a formula [equation (19)], are exhibited in figure 37.

The most comprehensive and consistent data set is that of Hickey [73]. A rather unusual curing history (specimens demolded after 24 hours and stored thereafter at 50% relative humidity) was used in Seki and Kawasumi's tests [70], and the total predictions in figure 37 must be judged in this light. They are poor because the value of $1/E_0$ determined from basic creep data (Part IV), as well as that from equation (19), was 19% smaller (0.105×10^{-6}/psi) than the value indicated by the optimum fits from drying creep data at T_0 (*fig.* 36). Because the early drying exposure in these tests unusually impaired the strength development, the value of $1/E_0$ was higher compared to that from the prediction formulas. Moreover, this caused that the basic creep curves $J(t, t')$ for $t' = 29$ days and 105 days were unusually close to each other. The fits are also rendered more difficult by the fact that the relative humidity was not controlled and only an approximate mean value was given.

The drying creep of heated concrete very strongly depends on the movement of water, which itself is a rather scattered phenomenon. So, highly accurate formulas are impossible without a good model for the diffusion of water.

The tests of Gross [74] were of relatively short creep duration. In these tests, the curves for 40, 60 and 80°C are very close to each other, compared to other data. For the tests of Arthanari and Yu [63] the relative humidities for 20, 58, 84 and 102°C were not reported and they have been, therefore, assumed to be 70, 25, 3 and 1% respectively. Nevertheless, these estimates must be good enough for higher temperatures (84 and 102°C) because the humidities obtained by heating room air at constant moisture content of the air always fall within a few percent from zero. The initial elastic deformation for these tests had to be estimated, too, which was done by comparing the creep values for very short times to the corresponding creep values reported for sealed specimens of the same composition. Due to scatter of these data, the curves for 58 and 84°C cross each other, which is illogical and makes it, anyway, impossible to achieve accurate fits. So, the results must be regarded more in the qualitative sense.

Maréchal's tests [75] differed from the other tests mainly by the use of a long preheating period (15 days) and also by a rather small specimen size. This caused the specimens at high temperature to lose much of their water before loading while at lower temperatures the 15 day period was not sufficient to produce severe drying of the specimens. This explains why in Maréchal's tests the creep at 70°C was less than the creep at 50°C. Figure 38 shows that the present formulation is capable of modeling this reverse dependence on temperature; this is achieved by decreasing the magnitude of $C_d(t, t', t_0)$ by means of φ'_d, and by increasing the magnitude of $C_p(t, t', t_0)$, which is still small at 50°C but becomes very large beyond 100°C. Maréchal's data show a peak value of creep at 50°C while the present formulas gave a peak at 70°C, which could not be further adjusted without causing the temperature increase of creep to be too small in fitting other test data. For this reason the fit of Maréchal's data is not too good, although the behavior predicted is qualitatively correct.

The initial elastic deformation for Maréchal's data was not reported as part of the creep data in [75], but a temperature dependence of E that is probably pertinent to these data was reported in reference [62], which was used in fitting. The relative humidities and mix proportions had to be also assumed, which was done so as to get typical average values of m, n and α.

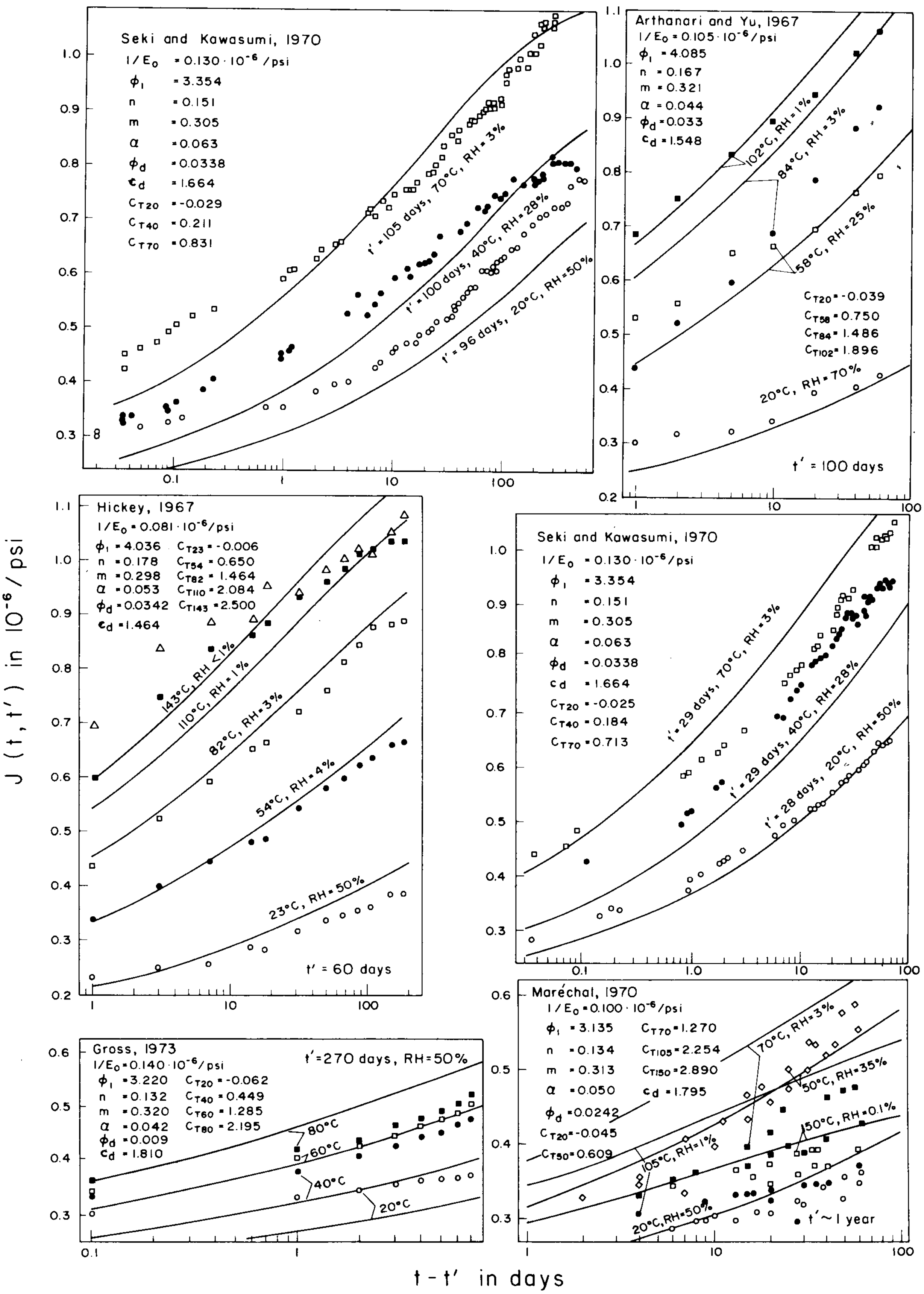

Fig. 36. — Fits of Tests by Seki and Kawasumi, 1970 [70]; Arthanari and Yu, 1967 [63]; Hickey, 1967 [73]; Maréchal, 1970 [75]; and Gross, 1973 [74] ($1/E_0$ optimized).

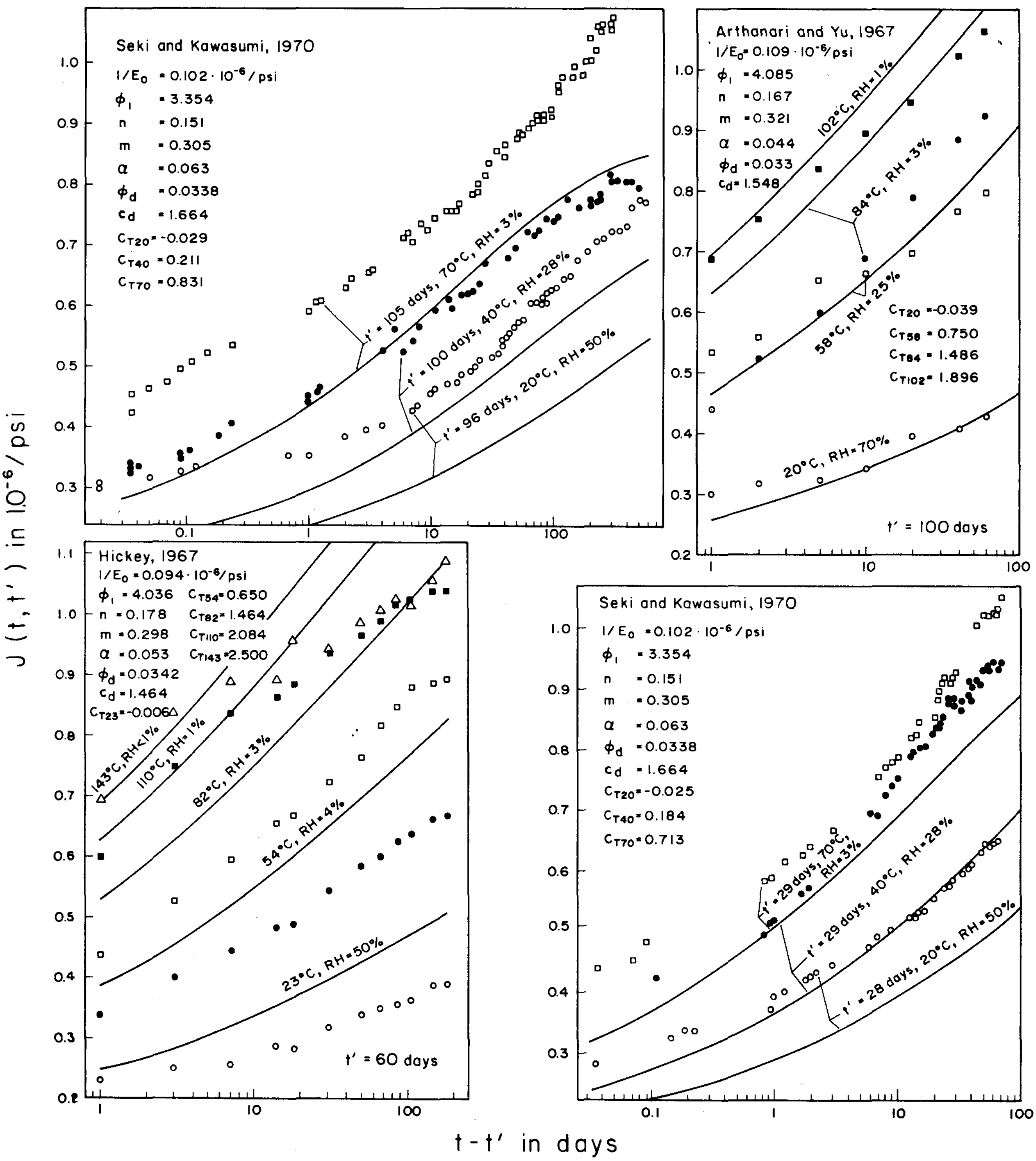

Fig. 37. — Fits of Tests for Temperature Effect on Drying Creep by Seki and Kawasumi, 1970 [70]; Arthanari and Yu, 1967 [63]; and Hickey, 1967 [73] ($1/E_0$ predicted by formula).

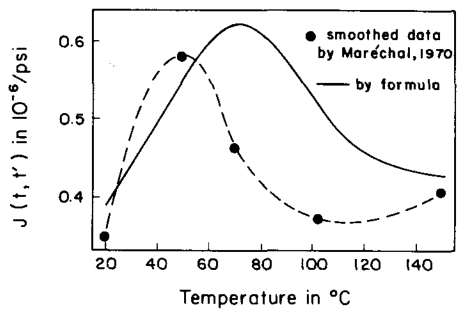

Fig. 38. — **$J(t, t')$ predicted with formulas (solid line) at $t-t'=60$ days as function of temperature compared with experimental test result (mean values) by Maréchal.**

APPENDIX V

Basic Information on Test Data Used

Hickey's Tests of Temperature Effect on Drying Creep (1967) [73]. — Cylinders 6 × 16 in. (152 × 406 mm), in molds for 24 hours at 73°F (23°C); then stripped and cured in a fog room at 73°F (23°C) for 1 month; then exposed to 50 % R.H. at 73°F (23°C) for another month. One day before elevated temperature tests the specimens were wrapped with fiberglass insulation (to minimize temperature changes). Specimens were loaded at the age of 2 months. Immediately after loading, the chamber temperature was slowly increased to the test temperature over a period of about 24 hours. Water-cement-sand-gravel ratio 0.468 : 1 : 1.775 : 3.864. Cement type V, 332 kg/m^3, and pozzolan 83 kg/m^3. Coarse aggregate: good quality amphibole schist river aggregate, max. size 1 in. (25 mm). 39-day strength of specimens 7,700 psi. In calculations, 28-day standard cylinder strength was assumed as 7,500 psi (51.7 N/mm^2).

Arthanari and Yu's Tests of Temperature Effect on Drying Creep (1967) [63]. — Slabs 12 × 12 × 4 in. (305 × 305 × 102 mm) cured under water until 3 days before loading. Heating began one day before loading. Loaded at age of 100 days, stress 1,000 psi (6.895 N/mm^2). For transforming biaxial test data to equivalent uniaxial ones, Poisson's ratio was assumed to be 0.2. The initial (elastic) values of $J(t, t')$ at $t-t'=0.001$ day and 20, 58, 84 and 102°C were assumed to be 0.250, 0.273, 0.288, 0.298 10^{-6}/psi, and the corresponding R.H. were assumed as 70, 25, 3 and 1 %. Water-cement-sand-gravel ratio 0.564 : 1 : 1.125 : 2.625; Thames river gravel of size (3/16) – (3/8) in. (4.73-9.45 mm), ordinary portland cement; its content assumed as 380 kg/m^3; 28-day average cube strength 6,000 psi (41.4 N/mm^2), cold.

Gross' Tests of Temperature Effect on Drying Creep (1973) [74]. — Cylinders 60 × 180 mm stored in water up to the 7th day, then kept under damp hessian for 4 weeks, thereafter in the air at 50 % R.H. and 20°C. Specimens were preheated for one day at the relevant temperature. Load applied at the age of 9 months; stress-strength ratio 0.2 (cold strength). Water-cement-sand-gravel ratio 0.6 : 1 : 2.2 : 2.39. Ordinary portland cement; its content assumed as 350 kg/m^3. Aggregate: Thames River deposits, max. size 9.5 mm. 28-day average cube strength 42 N/mm^2, cold.

Seki and Kawasumi's Tests of Temperature Effect on Drying Creep (1970) [70]. — Cylinders 150 × 600 mm demolded after 24 hours and cured at 20°C and 50 % R.H.; sealed at the top and bottom. The 40°C specimens were loaded at 29 days and 100 days; placed into a room of 40°C at 28 and 97 days. R.H. was not controlled, but was about 28 % at 40°C. The 70°C specimens were loaded at 29 and 105 days; placed into the test chamber at the age of 27 and 104 days. R.H. not controlled but was about 3 %. Water-cement-sand-gravel ratio 0.4 : 1 : 1.761 : 3.834. Ordinary portland cement, 343 kg/m^3. Fine aggregate natural sand from the River Fuji-Gawa, coarse aggregate from the River Ara-Kawa, max. size 40 mm. Pozzolan was added to the mixture. 28-day average cylinder strength 445 kp/cm^2 (43.6 N/mm^2), cold.

Maréchal's Tests of Temperature Effect on Drying Creep (1970) [75]. — Prisms 70 × 70 × 280 mm cured in water for 1 year then heated 0.25°C/h up to the test temperature. Temperature stabilization time 15 days before loading. Specimens at temperatures 20, 50, 70, 105, 150°C were assumed to dry at 50, 35, 3, 1 and 0.1 % R.H. Cement content 400 kg/m^3. For fitting, the following was assumed: water-cement-sand-gravel ratio 0.5 : 1 : 1.8 : 2.8; and 28-day average cylinder strength 5,500 psi (37.9 N/mm^2). The initial (elastic) values of $J(t, t')$ at elevated temperatures were evaluated using reference [62].

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