Part IV: Temperature effect on basic creep

Development of the model for basic creep in Part II is followed here by a prediction model for creep at various temperatures that are kept constant during creep. The model, which preserves the form of the double power law, reflects two opposing effects of temperature: the increase of creep rate due to heating, and the reduction of creep due to thermally accelerated hydration. Prediction of material parameters from mix composition is studied and extensive comparisons with test data indicate a good agreement.

INTRODUCTION

The double power law for basic creep, developed in Part II, will now be extended to model creep at various temperatures that are kept constant during creep. Unlike the preceding parts of this study, here we must not only model the composition influence but also decide on the proper form of the temperature effect because the model for variable temperature that we are going to investigate has not yet been proposed.

Realizing that the choice of reference temperature $T_0$ is subjective and largely arbitrary, we must conclude that the creep formula for any temperature (within a certain range) should have the same basic form. In particular, the form of double power law should be preserved for heated sealed concrete.

FORMULAS PROPOSED FOR TEMPERATURE EFFECT ON BASIC CREEP

Preserving its basic form, we may generalize the double power law as

\[ J(t, t') = \frac{1}{E_0} + C_0(t, t'), \]

\[ C_0(t, t') = \frac{\varphi_T}{E_0} (t' - t - t')^{\gamma}, \]

where

\[ t' = \int_0^{t'} \beta_T(t') \, dt', \]

\[ \varphi_T = \varphi_1 (1 + C_T), \quad \beta_T = \exp \left( \frac{4000}{T_0} - \frac{4000}{T} \right). \]

Here $C_T$, $n_T$ and $\beta_T$ are functions of temperature, and $t'_e$ represents the equivalent hydration period (or maturity) [5] (1), which is defined as the period at reference temperature $T_0$ needed to achieve the same degree of hydration as period $t'$ at temperature $T$. Equation (36) results from assuming that the temperature effect on hydration is governed by an activation energy, $Q$. In equation (36), $T$ and $T_0$ must be absolute temperatures. The constant 4000 K (representing $Q$ divided by gas constant) has been derived empirically from the data fitted in the sequel.

Following a theoretical analysis by Wittmann [58], function $C_T$ was previously [4] suggested to also obey the activation energy concept. However, an in-depth analysis of test data revealed that this is true only for a limited range of temperatures, from about 35 to about 75°C. Beyond this range significant deviations occur, which may be due to phase changes and chemical changes, as well as simultaneous operation of several processes controlled by different activation energies. Therefore, function $C_T$ has been identified empirically, although the basic product form of equation (36) for $\varphi_T$, as indicated by activation energy effects, has been retained. Function $c_T$, which is plotted in figure 26 a in comparison with the activation energy dependence, has the form

\[ C_T = c_T \tau_T, \]

\[ \tau_T = \frac{1}{1 + 60/t'_{e}^{0.69} + 0.78}, \]

\[ c_T = \frac{19.4}{1 + (100/(T - 253.2))^{3.5} - 1}, \]

where $c_0$ is a composition parameter, $t'_{e}$ is the age of concrete when temperature $T$ is applied and $T$ is absolute temperature. Note that $C_T$ is defined not only as a function of temperature but also as a function of $t'_{e}$.

According to the activation energy model for power-type creep functions [58], exponent $n_T$ would be a

(1) Reference numbers not listed at the end of this part are found in the preceding parts.
constant. Again, for a broader range of temperatures (−20 to 140°C) this is unacceptable. Nevertheless the form of equation (34), conforming to the activation energy model, may be retained and it suffices to take $n_T$ as temperature-dependent. By data fitting, the following empirical function has been found:

$$n_T = B_T n_T = \frac{0.25}{1 + (T/(T - 253.2))^2 + 1}. \quad (39)$$

Equation (37) is approximately valid from about −20°C to perhaps 120°C. Near the ends of the range the rise of $c_T$ with temperature is milder (fig. 26 a).

Function $B_T$ indicates that exponent $n_T$ increases with temperature, i.e., the ratio of long-time to short-time creep increases as temperature is raised. This may be explained by the larger effect of the acceleration of aging during the early creep period.

Equations (35), (37), (38) reflect the fact that the temperature effect on creep is twofold ([60], [5]): (a) an increase in temperature increases the creep rate, but (b) it also accelerates hydration, i.e., aging. These effects, modeled by coefficients $c_T$, $f_T$, and $t_T$, respectively, oppose each other. When a young concrete is heated well before it is loaded, the equivalent hydration period $t_T$ for the moment of load application may get sharply increased, causing a reduction of the creep increase due to heating. On the other hand, when an old concrete is heated, the change in $t_T$ has little effect on subsequent creep, and so a strong increase of creep with temperature takes place. Modeling of both these opposing tendencies is essential for successful fitting of test data.

The elastic modulus $E$ is known to decrease with temperature beyond 50°C, the drop reaching about 20% at 100°C ([61], [62]). Like the double power law which gives proper age-dependence of elastic modulus, equation (34) seems to give approximately correct temperature dependence of the elastic modulus:

$$\frac{1}{E(t')} = \frac{1}{E_{\text{stab}}(t')} = \frac{1}{E_{0}} + \frac{\varphi_T}{E_0} 10^{-\eta_T} (t' - t_0^{\text{m}} + \zeta). \quad (40)$$

**EFFECT OF COMPOSITION ON BASIC CREEP OF HEATED CONCRETE**

By fitting of test data ([59], [23], [61], [63], [64], [65], [66], [67], [68], [69], [22], [70], [71], [72]) it was verified that:

$$c_0 = \frac{1}{8} \left( \frac{w}{c} \right)^2 \left( \frac{a}{c} \right) a_1, \quad (41)$$

where $a_1$ accounts for the cement type and is the same as in equation (18) of Part II; $w/c$ = water-cement ratio; $a/c$ = aggregate-cement ratio. An increase of creep rate with the water-cement ratio, as given by equation (41), is logical to expect. The increase of $c_0$ with the aggregate-cement ratio means that the restraining effect of aggregate on creep is stronger at lower temperatures. Equation (41) does not involve strength, but since the strength depends on $w/c$ and $a/c$, the effect of strength is present indirectly.

**COMPARISONS WITH TEST DATA ON HEATED SEALED SPECIMENS**

Fits of numerous test data shown in figures 27–34 indicate a reasonable agreement of the present model with experiments. Basic information on the test data used is given in Appendix IV.

For some data sets important information was not reported and, therefore, had to be assumed. e.g., for England and Ross' data it has been assumed that the heat was applied at the age of 10 days, simultaneously with load application (i.e., no heat stabilization period before the test). Also, the initial "elastic" strains at elevated temperatures have been assumed using proportionality to the values of Marechal. For the tests of Silveira and Florentinio, it has been assumed that the heat was applied three days before loading.

For Nasser and Neville's data ([68], [69]), the sand-gravel ratio was not available, and so exponent $n$ has been assumed. The initial elastic strains have had to be assumed also (0.2 × 10$^{-5}$/psi). Papers [68] and [69] mentioned that $E$ was not a function of temperature; therefore, the value of 1/$E_0$ has been found by optimization. The $E$-modulus was reported to increase by 22% from $t' = 14$ days to $t' = 365$ days, and the value of $J(t' + 0.001, t')$ has been assumed to change in proportion. Moreover, these data indicate, independently of curing temperature, a 22% increase of elastic modulus upon heating, which conflicts with references [61] and [62]. The deviations from test data in figures 28 and 32 must be judged in the light of the preceding remarks.

When unspecified, the unit weight of concrete has been assumed as 2,400 kg/m$^3$.

It is interesting to compare $J$ (90 + 365, 90) for the data of Silveira and Florentinio [67], McDonald [22] and Kennedy [23]. At room temperature, the values are 0.425, 0.285, 0.285 (all in 10$^{-5}$/psi), and at elevated temperatures tested (45, 65.6 and 65.6°C respectively), the values are 0.748, 0.400 and 0.445. This is a considerable scatter in view of the fact that the mix parameters and test conditions were quite similar (see Appendix IV).

For temperatures beyond 95°C, the present model gives only very crude estimates. Even though all
Fig. 27. – Fits of Tests of Temperature Effect on Creep by Hannant (1967) [61], Arthanari and Yu (1967) [63], Nishizawa and Okamura (1970) [64], England and Ross (1962) [65], McDonald (1975) [22] and Johansen and Best (1962) [66]. $C_T$ optimized – solid line; subscript number refers to corresponding temperature. $C_T$ with formula – dashed line. $1/E_0$ calculated from experimental $E_{230}$ or optimized from basic creep data.
specimens considered here were sealed, moisture may have moved out of concrete and collected under a bulged jacket. Also, rapid redistribution of moisture within the heated specimen may have had considerable effect on creep. In particular, the present model does not describe the decrease in creep rate (i.e., in $C_T$) that is sometimes observed upon passing 100°C; see the curves near 100°C in figure 28 for Nasser and Neville's data, and the reversed order of temperatures for the curves near 100°C in figure 27 for England and Ross's data.

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**Fig. 28.** Fits of Tests of Temperature Effect on Creep by York, Kennedy and Perry (1970) [23], Da Silveira and Florestino (1968) [67], Nasser and Neville (1965) [68], Nasser and Neville (1967) [69], Seki and Kawasumi (1970) [70] and Siciinski and Sadowski (1973) [71]. $C_T$ optimized—solid line, $C_T$ with formula—dashed line. $1/E_0$ optimized from basic creep data.

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APPENDIX IV

Basic Information on Test Data Used

Hannant’s Tests of Temperature Effect on Creep (1967) [61]. — Cylinders 4:1 1/2 inch (105 x 305 mm), cured for 24 hours in molds under wet rags, then 5 months under water at 20°C, and then one month sealed in copper. Heating rates about 10°C/hr. For temperature stabilization all specimens were heated for 24 hours before loading. Stress 2,000 psi (13.8 N/mm²). Water-cement-sand-gravel ratio 0.47 : 1 : 1.845 : 2.655. Sulphate resisting Portland cement with Plastocrete plasticizer. Coarse aggregate limestone max. size 3/8 inch (10 mm). 28-day cube strength 9,350 psi (64.5 N/mm²).

Nishizawa and Okamura’s Tests of Temperature Effect on Creep (1970) [64]. — Specimens 15 x 15 x 55 cm, sealed in copper, prestressed to compressive stress 120 kg/cm² (11.8 N/mm²) at the age of 28 days. After 7 days of loading at 20°C, specimens exposed to temperature of 70 or 90°C. Water-cement-ratio 0.40, cement content 377 kg/m³, sand percentage 36.5% (In calculations water-cement-sand-gravel ratio 0.40 : 1 : 1.85 : 3 : 22 was used.) Max. size of coarse aggregate = 25 mm, normal cement. Cylinder strength 459 kg/cm² (45 N/mm²).

McDonald’s Tests of Temperature Effect on Creep (1975) [22]. — Cylinders 6 x 16 inch (152 x 406 mm) demolded after 24 hours, coated with epoxy and returned to fog room. After 24 hours another coat of epoxy, and sealed in copper. At age of 83 days, specimens recoated with epoxy, sealed in neoprene, and placed to environment of test temperature, loaded at age of 90 days. Water-cement-sand-gravel ratio 0.425 : 1 : 2.03 : 2.62. Type II Portland cement (404 kg/m³). Limestone aggregate, max. size 3/4 inch (19 mm). 28-day average cyl. strength 6,300 psi (43.4 N/mm²).

ArthMANi and Yi’s Tests of Temperature Effect on Creep (1967) [63]. — Slabs 12 x 12 x 4 inch (305 x 305 x 102 mm), cured under wet hessian for 7 days. For test under mass-concrete conditions sealed by epoxy resin and two coats of plastic emulsion paint. Loaded at age of 15 days, stress 1,000 psi (6.9 N/mm²). Heating began 1 day before loading. Water-cement-sand-gravel ratio 0.564 : 1 : 1.125 : 2.625. Thames river gravel of size 3/16-3/8 inch (4.76-9.5 mm), ordinary portland cement. 28-day average cube strength 6,000 psi (41.4 N/mm²).

England and Ross’ Tests of Temperature Effect on Creep (1962) [65]. — Cylinders 4.5 x 12 inch (114 x 305 mm), demolded at age of 1 day, placed under water for additional 3 days, after which stored at 17°C and 90% R.H. until tested at age of 10 days in a sealed state. The seal was a polyester resin, with fibre glass reinforcement. Water-cement-sand-gravel ratio 0.45 : 1 : 2 : 4. Compressive strength of 4 inch (102 mm) cubes at age of 14 days = 5,500 psi (37.9 N/mm²). Elastic modulus 5 x 10⁶ lb/in² (34,480 N/mm²).

Johansen and Best’s Tests of Temperature Effect on Creep (1962) [66]. — Cylinders 10 x 30 cm and 15 x 30 cm cast in steel molds and remolded at age of 1 day, then stored at 100% rel. humidity and 20°C. At age of 42 days, specimens were sealed and moved to test environment. At the end of 3 days stabilization period specimens were loaded at their respective temperatures to 30% of their ultimate strength in compression as measured at 20°C. Water-cement-sand-gravel ratio 0.7 : 1 : 3 : 5 : 3.5. Normal portland cement. Max. size of aggregate 3/8 inch (9.5 mm). Average compressive strength 179 kp/cm² (17.6 N/mm²) at the age of 42 days on cylinders 15 x 30 cm.

York, Kennedy and Perry’s Tests for Temperature Effect on Creep (1970) [23]. — Cylinders 6 x 16 inch (152 x 406 mm), removed from molds 24 hours after casting. Then epoxy coat applied and specimens stored in fog room. Next, 48 hours after casting, specimens sealed in copper and placed in test environment at 73.4°F (23°C). At age of 83 days specimens sealed in neoprene jacket, and exposed to test temperature. Loaded at age of 90 days. Water-cement-sand-gravel ratio 0.425 : 1 : 2.03 : 2.62. Cement type II (404 kg/m³). Limestone aggregate, max. size 3/4 inch (19 mm); 28-day cyl. strength 6,560 psi (45.2 N/mm²).
Fig. 30. — Fits of Tests of Temperature Effect on Creep by Komendant, Polivka and Pirtz (1976) [72]. $C_T$ optimized—solid line, $C_T$ with formula—dashed line. $1/E_0$ optimized from basic creep data.
Fig. 31. — Fits of Tests of Temperature Effect on Creep by Hannant (1967) [61], Arthanari and Yu (1967) [63], Nishizawa and Okamura (1970) [64], England and Ross (1962) [65], McDonald (1975) [22] and Johansen and Best (1962) [66]. $C_T$ and $1/E_0$ both calculated with formula.
Fig. 32. — Fits of Tests of Temperature Effect on Creep by York, Kennedy and Perry (1970) [23], Da Silveira and Florentino (1968) [67], Nasser and Neville (1965) [68], Nasser and Neville (1967) [69], Seki and Kawasumi (1970) [70], Zielinski and Sadowski (1973) [71]. $C_T$ and $1/E_0$ both calculated with formula.
Da Silveira and Florentino’s Tests of Temperature Effect on Creep (1968) [67]. — Prisms 20 × 20 × 60 cm, in copper jackets. Heat is assumed to be applied 3 days before loading. Water-cement-sand-gravel ratio 0.5 : 1 : 2.35 : 3.84. Granite aggregate, modified portland cement, similar to ASTM type II. Cement content 314.6 kg/m³, 8-day cube strength 297 kp/cm² (29.1 N/mm²).

Nasser and Neville’s Tests of Temperature Effect on Creep (1965) [68]. — Cylinders 3 × 9 1/2 inch (76 × 235 mm), sealed in polypropylene jackets, stored from 24 hours onwards in a water bath at the desired temperature and loaded at age of 14 days. Water-cement ratio 0.6 and aggregate-cement ratio 7.15. Max. size of aggregate 3/4 inch (19 mm). Aggregate was a mixture of dolomite and hornblende. Cement type II (320 kg/m³). Strength 5,660 psi (39 N/mm²) at 14 days measured on cylinders 3 × 9 1/2 inch (76 × 235 mm). Stress/strength ratio 0.35.

Nasser and Neville’s Tests of Temperature Effect on Creep (1967) [69]. — Cylinders 3 × 9 1/2 inch (76 × 235 mm), stored in water at 70°F (21°C) up to 1 week prior to application of load. Concrete 1 : 7.15 mix; water-cement ratio of 0.6. Max. size of dolomite and hornblende aggregate was 3/4 inch (19 mm), cement type III (320 kg/m³). Specimens loaded at age of 1 year and remained under water while loaded. Mean strength at the time of load application (determined on specimens of same size) = 7,250 psi (50 N/mm²).

Browne’s Tests of Temperature Effect on Creep (1967) [59]. — Cylinders 6 × 12 inch (152 × 305 mm), sealed at casting in 1/16 inch (1.6 mm) polypropylene jackets, cured at room temperature. Heat applied 1 day before loading. Water-cement-sand-gravel ratio 0.42 : 1 : 1.45 : 2.95. Ordinary portland cement, crushed foraminiferal limestone, max. size 1.5 inch (38 mm). Average 6 inch (15.2 cm) cube strength = 7,250 psi (50 N/mm²).

Zielinski and Sadowski’s Tests of Temperature Effect on Creep (1973) [71]. — Cylinders 160 × 480 mm within the first 70 days stored in atmosphere 100% relative humidity and temperature 20-23°C, then sealed with rubber coat. Specimens were heated at the age of 120 days and loaded three days later. Water-cement-aggregate ratio 0.456 : 1 : 4.154. Sand/cement-gravel/cement ratio assumed to be 1.9 : 2.254. Cement ordinary Portland cement type I, 450 kg/m³, aggregate crushed basalt and river sand, max. size 20 mm. 120 day compressive cylinder (160 × 160 mm) strength 430 kg/cm².

Seki and Kawasumi’s Tests of Temperature Effect on Creep (1970) [70]. — Cylinders 150 × 600 mm were cast into 0.2 mm copper jackets. Specimens loaded at room temperature (20°C) at the age of 28 and 96 days. The temperature 40°C was applied at the age of 28 and 97 days and loaded at the age of 29 and 100 days. The temperature 70°C was applied at the age of 27 and 104 days and specimens loaded when they were 29 and 105 days old. Water-cement-sand-aggregate ratio 0.4 : 1 : 1.761 : 3.834. Normal Portland cement 343 kg/m³, fine aggregate Fuji-Gawa river sand, coarse aggregate from the river Ara-Kawa. 28-day cylinder strength 445 kp/cm².

Komendant, Polivka and Pritz’s Tests of Temperature Effect on Creep (1976) [72]. — Cylinders 6 × 16 inch (152 × 406 mm) sealed with butyl rubber against moisture loss and cured at 73°F (23°C) until five days prior to the age of loading. The specimens were then heated to test temperatures 110 and 160°F (43 and 71°C) at a rate of 24°F/day (13.3°C/day) and remained for the duration of the creep test. Specimens were loaded at the age of 28, 90 and 270 days. Cement, Medusa type II. Mix A: water-cement-sand-gravel ratio 0.381 : 1 : 1.734 : 2.685; 28-day cylinder strength 6,590 psi (45.4 N/mm²). Cement 706 lbs/ey (419 kg/m³). Max size of aggregate 1.5 inch.
Fig. 34. — Fits of Tests of Temperature Effect on Creep by Komendant, Polivka and Pirtz (1976) [72]. $C_T$ and $1/E_0$ both calculated with formula.
RéSUMé

Un modèle de prévision pratique des déformations du béton en fonction du temps. III. Fluxage en séchage. — Le modèle pratique de détermination du fluxage et du retrait exposé dans les parties I et II de ce mémoire est à présent appliqué au fluxage en ambiance sèche et à température constante. L'augmentation du fluxage due au séchage est reliée au retrait. On donne les formules pour déterminer les paramètres des matériaux à partir de la résistance du béton et de la composition du mélange, et on les vérifie par des comparaisons nombreuses avec les résultats d'essai publiés.

IV. Influence de la température sur le fluxage de base. — Le développement d'un modèle pour le fluxage de base qui est l'objet de la deuxième partie de ce mémoire est suivi ici par un modèle de détermination du fluxage à différentes températures maintenues constantes durant le phénomène. Ce modèle qui préserve la loi de double puissance traduit deux effets contraires de la température : l'augmentation de la vitesse du fluxage due à la chaleur et la diminution du fluxage due à l'accélération de l'hydratation par la chaleur. L'étude comprend la détermination des paramètres des matériaux à partir de la composition du mélange et de nombreuses comparaisons avec les résultats d'essai indiquent une bonne concordance.

To be continued by Parts V and VI.