CEMENT and CONCRETE RESEARCH. Vol. 14, pp. 793-806, 1984. Printed in the USA.
0008-8846/84 $3.00+00.

DOUBLE-POWER LOGARITHMIC LAW FOR CONCRETE CREEP

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(Communicated by F.H. Wittmann)
(Received Feb. 17, 1984)

ABSTRACT

An improvement of the double-power law for creep at constant temperature and moisture content is proposed. Comparisons with available test data indicate that the final slopes of long-term creep curves, as indicated by the double-power law, are predominantly on the high side. This is remedied by introducing a transition to a straight line in the logarithmic scale of load duration. The strain at the transition as well as the slope of the straight line are the same for all ages at loading. The strain and the slope at the transition point are continuous, while the curvature is discontinuous. The new law is also found to significantly limit the occurrence of divergence of the creep curves and of negative values at the ends of the relaxation curves calculated by the superposition principle. Extensive statistical comparisons with test data from the literature justify the proposed law.

Introduction

Although the double-power law [2,3] provides a relatively good description of the existing test data and the creep of concrete at constant temperature and water content, called the basic creep, one can detect some deviations which seem to be systematic rather than random. In particular, the final slope of the curves of strain versus the logarithm of load duration appears to be somewhat too steep when long-term tests are considered. This study examines whether this can be remedied by introducing a transition to a logarithmic law for long times.

Review of Double-Power Law

The basic creep of concrete may be relatively well described by the double-power law [1,2,3,7]:

$$J(t,t') = \frac{1}{E_0} + \frac{\phi_1}{E_0}(t'^{-m} + \alpha)(t - t')^n \tag{1}$$

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in which $J(t,t')$ is the compliance function (also called the creep function), which represents the strain at age t caused by a unit uniaxial constant stress acting since age t'; E_0 is the asymptotic modulus, which may be visualized as the left-hand side asymptotic value of the curve of $J(t,t')$ versus $\log(t-t')$, and n, m, α, and ϕ_1 are material parameters. Their typical values are $n \simeq 1/8$, $m \simeq 1/3$, and if t and t' are in days then $\alpha \simeq 0.05$ and $\phi_1 = 3$ to 6. Also, $E_0 \simeq 1.5E_{28}$ where E_{28} is the conventional elastic modulus at age 28 days. Since $(t-t')^n = \exp[n \ln(t-t')]$, the plots of $J(t,t')$ versus $\log(t-t')$ at constant t' have the shape of exponentials.

Eq. 1 has a remarkably broad range of applicability. It yields acceptable values for ages at loading from about 1 day to many years, and for load durations from 1 second to several decades. It also yields acceptable compliance values for rapidly (dynamically) applied loads, and the dynamic modulus is approximately obtained from Eq. 1 as the value of $1/J(t'+\Delta,t')$ for $\Delta \simeq 10^{-7}$ day, whereas the conventional (static) elastic modulus is obtained as the value of $1/J(t'+\Delta,t')$ for $\Delta \simeq 0.1$ day [24]. Since four parameters, namely E_0, ϕ_1, m and α are required to describe the age dependence of the elastic modulus we see that only one additional parameter, n, is needed to describe all creep.

Extensive statistical regression analyses of practically all test data available in the literature revealed that the double-power law exhibits, on the whole, smaller errors than other formulas for concrete creep proposed before [4,5,6,1,2,3,7]. The power function of load duration $t-t'$, involved in Eq. 1, was first introduced by Straub [8] and Shank [25]. Wittmann et al. [12] gave supporting arguments for the power function based on the activation energy theory, and Cinlar, Bažant and Osman [26] gave other supporting arguments based on a certain reasonable hypothesis for the stochastic nature of the physical mechanism of creep. Others, e.g. Branson [9,10], introduced a power function of age t'.

It has often been commented that a power function of $t-t'$ predicts too much creep for longer durations, exceeding 1 month. These critical comments were, however, incorrect since they resulted from using the power function to describe only that part of the creep strain that is in addition to the conventional short-time strain, approximately the strain for load duration 0.1 day [2]. With this approach, the horizontal asymptotic value in Fig. 1 is obtained too high, and in order to fit the test data for medium load durations (1 day to 30 days) one needs to introduce a relatively high curvature by using a high value for exponent n, about $n \simeq 1/3$. This then inevitably causes the power curve to shoot above the data points for longer load durations. Recently it has been discovered [2], however, that the applicability range becomes much broader if the power function is used to describe the entire creep strain including that which occurs for very short load durations (from 10^{-6} second to 0.1 day). This represents a fundamental difference from the earlier use of the power function, and invalidates the aforementioned critical comments. Since the left-hand asymptote (given by $1/E_0$) is much lower than considered in the classical studies (Fig. 1), exponent n required to fit the creep data between 1 and 30 day durations is much smaller, roughly 1/8. This then causes that the power curve has a much smaller positive curvature in the log-time plot, and thus does not overshoot the test data for long creep durations. (There remains, however, an overshoot for very long creep durations, and that is what we try to improve here.)

Note also that another advantage of including the entire creep strain in the power law is that $1/E_0$ can be considered age-independent, while the earlier approach in which only the creep strain after approximately 0.1 day was

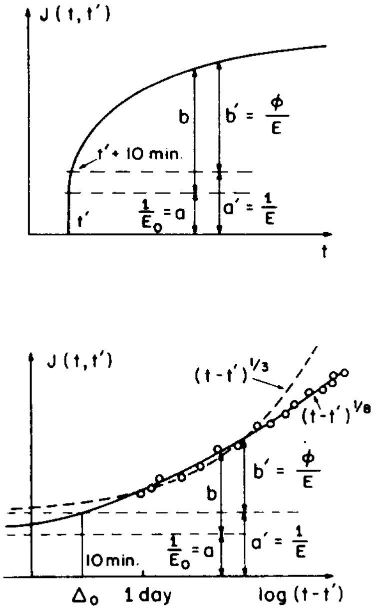

FIG. 1. Creep Curves in Actual and Log Time Scales (a = true elastic deformation, b = true creep, a' = conventional elastic deformation, b' = conventional creep).

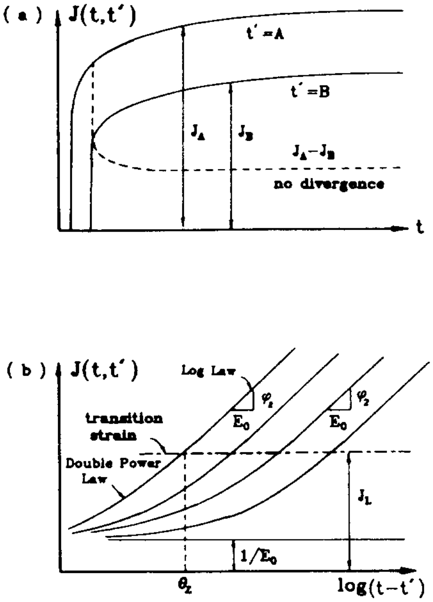

FIG. 2. (a) Theoretical Recovery Curve Obtained by Principle of Superposition, Showing No Divergence; (b) Typical Curves of Double-Power Logarithmic Law.

described by the power function, required considering an age-dependent elastic modulus $E(t')$.

Although the double-power law is intended to describe only the basic creep it may be used as an approximation for creep in drying environment provided the cross section is relatively massive, with a thickness over about 30 cm. For such cross sections, the average creep deformation in the cross section is closer to that of a sealed cylinder than to that of a standard six-inch cylinder exposed to a drying environment.

Proposed Formulation

When the compliance values $J(t,t')$ are plotted against $\log(t-t')$ for various constant values of h at constant t', it is found that for longer durations (over several years) the double-power law yields in most cases predictions that are somewhat on the high side. Especially, the final slope appears to be in most cases higher than the measured one (Fig. 1). A remedy can be achieved by introducing a transition to straight lines in the log-time scale at a certain creep duration θ_L. This may be accomplished by the following formulas

$$J(t,t') = \frac{1}{E_0} + \frac{\phi_1}{E_0}(t'^{-m} + \alpha)(t-t')^n \qquad \text{for } t-t' \le \theta_L \text{ (double-power law)}$$

$$= \frac{n\phi_L}{E_0}\ln\frac{t-t'}{\theta_L} + \frac{1+\phi_L}{E_0} \qquad \text{for } t-t' \ge \theta_L \text{ (log-law)} \qquad (2)$$

in which
$$\theta_L = \left(\frac{\phi_L}{\phi_1 (t'^{-m} + \alpha)}\right)^{1/n}. \tag{3}$$

According to these formulas, which may be called the Double-Power Logarithmic Law, the slope at the transition times θ_L (in log-time scale) is the same for all ages t' at loading, and the value of J(t,t') at which the transition occurs is also the same. So, the ranges of validity of the double-power law and of the logarithmic law are separated by a horizontal line (see Fig. 2b); for longer t' the transition occurs at longer creep duration. For very high ages at loading, the double-power law is valid throughout the entire lifetime.

Divergence of Creep Curves

If the principle of superposition is used, as a crude approximation, to calculate the creep recovery curves from the creep curves for various ages at loading, the recovery curves can be either monotonic, with a monotonic decay to an asymptotic value, or nonmonotonic, with a minimum followed by a monotonic rise up to a certain asymptotic value. Nonmonotonic recovery curves are thermodynamically impossible for a non-aging material, however, they are thermodynamically admissible in case of aging [11,30]. With some exceptions, which might be due to statistical scatter or the effect of drying, most test data show a monotonic decay [11,12,31]. Thus, although recovery cannot in fact be accurately predicted using the linear principle of superposition [12], it seems preferable to use compliance functions that cannot yield nonmonotonic recovery curves upon superposition. This condition is verified when, for the same age t, the creep curve for a higher age at loading t' has a higher slope, i.e. $\partial[\partial J(t,t')/\partial t]/\partial t' \geq 0$ or [11]

$$\frac{\partial^2 J(t,t')}{\partial t \partial t'} \geq 0. \tag{4}$$

It may be readily verified that the logarithmic law in Eq. 2 never violates the inequality in Eq. 4 (Fig. 2a). On the other hand, the double-power law (Eq. 2) violates this inequality beginning with a certain creep duration. It is possible to always satisfy with Eq. 2 the inequality in Eq. 4 if the following condition is verified

$$\phi_L \leq \phi_1 \left(\frac{1-n}{mf_{max}}\right)^n \tag{5}$$

in which
$$f_{max} = \mathrm{Max}\left[t'^{\frac{m-n}{n}} (1 + \alpha t'^m)^{-\frac{n+1}{n}}\right]. \tag{6}$$

Comparison With Test Data

Same as the double-power law, Eq. 2 should be applied only to creep at constant water content, called the basic creep. Only under such conditions the creep represents a constitutive property of the material. The creep observed on drying specimens is not a constitutive property but an average property of the specimen as a whole, since the drying causes in the specimen a highly nonuniform distribution of water content and of stress, produces microcracking, and leads to great differences in creep at various points. An empirical description of the mean creep of drying specimens requires, therefore, much more complicated formulas.

Eq. 2 has six material parameters, E_0, n, m, α, ϕ_0, ϕ_L, which have to be determined from test data. Similarly to previous works [2,3], this may be accomplished by minimizing the sum of squared deviations Δ of Eq. 2 from the given data. When all six material parameters are considered as unknown, the optimization problem is a nonlinear one. The optimum fits have been obtained by Marquardt-Levenberg algorithm, for which an efficient library subroutine exists. The test data used in optimization have been extracted from a computerized data bank set up at Northwestern University. Since most experimentalists did not take their readings at times uniformly spaced in the logarithmic time scale, the raw data as reported are biased in that some readings are crowded at certain times and others are too sparse. For this reason, the test data from the literature have been smoothed visually by hand. At the same time, the hand smoothing approximately achieves elimination of the measurement error, which needs to be done since structures do not feel this error. The hand-smoothed curves were characterized by data points placed at regular intervals in the log(t - t') scale, using two points per decade.

The deviations of Eq. 2 from test data have been characterized by the coefficient of variation defined [3,7, 31] as:

$$\bar{\omega} = \left(\frac{1}{N}\sum_{j=1}^{N}\omega_j^2\right)^{1/2}, \quad \omega_j = \frac{1}{\bar{J}_j}\left(\frac{1}{n-1}\sum_{i=1}^{n}\Delta_{ij}^2\right)^{1/2}, \quad \bar{J}_j = \frac{1}{n}\sum_{i=1}^{n}\tilde{J}_{ij} \tag{7}$$

in which $\tilde{J}_{ij}$ $(i=1,\ldots,n)$ are the characteristic points of the data set number j (placed at regular spacing in log-time scale) on the creep curves reported in the data set; n = number of all data points on all curves within the data set, Δ_{ij} = vertical deviations of Eq. 2 from these data points, $\bar{J}_j$ = mean ordinate of all data points from the data set number j, ω_j = coefficient of variation for data set number j, $\bar{\omega}$ = the overall coefficient of variation for all data sets combined $(j = 1,\ldots,N)$.

Fig. 3 shows the optimum fits of the data sets reported in Refs. 15, 16, 17, 18, 19, 20, 21 and 23. For comparison, Fig. 3 shows the optimal fits previously achieved with the double-power law [2,3]. The coefficients of variation are summarized in Table 1a. The overall coefficient of variation for the fits by the double-power law is $\bar{\omega}$ = 5.45% [2], and for the present Eq. 2 we achieve $\bar{\omega}$ = 3.9%. We see that some improvement is achieved by Eq. 2.

More importantly, we may note that the final slopes achieved with Eq. 2 are better. This is important for extrapolation to longer times. Therefore, we determine for all creep curves the final slope, which we draw graphically as illustrated in Fig. 3 by the dashed lines, and we compare these experimental slopes with the value of $\partial J(t,t')/\partial \log(t-t')$ according to Eq. 2 for the last sampling time of each curve. The combined coefficient of variation for the deviations from the measured final slopes for all data tests, $\bar{\omega}_f$, is found to be 34% for the optimum fits by the double-power law, and 29% for Eq. 2; see Fig. 3 and Table 1b. Thus, we see that the present formulation achieves an appreciable improvement in the representation of the final slopes of the creep curves, and therefore also in the extrapolation to longer times.

The foregoing fits of test data (Fig. 3) have been obtained without the nondivergence restriction (Eqs. 4 - 6). If this restriction is imposed in data fitting, the optimum fits must obviously get worse. Such fits are shown in Fig. 4, and the corresponding coefficient of variation for all data sets is found to be $\bar{\omega}$ = 5.54%, while for the double-power law it is $\bar{\omega}$ = 5.45%. Although here there is no improvement, one finds now a significant improvement for the final slopes of creep curves, as is shown in Fig. 4, in which case $\bar{\omega}_f$ = 22%, as compared to $\bar{\omega}_f$ = 34% for the double-power law.

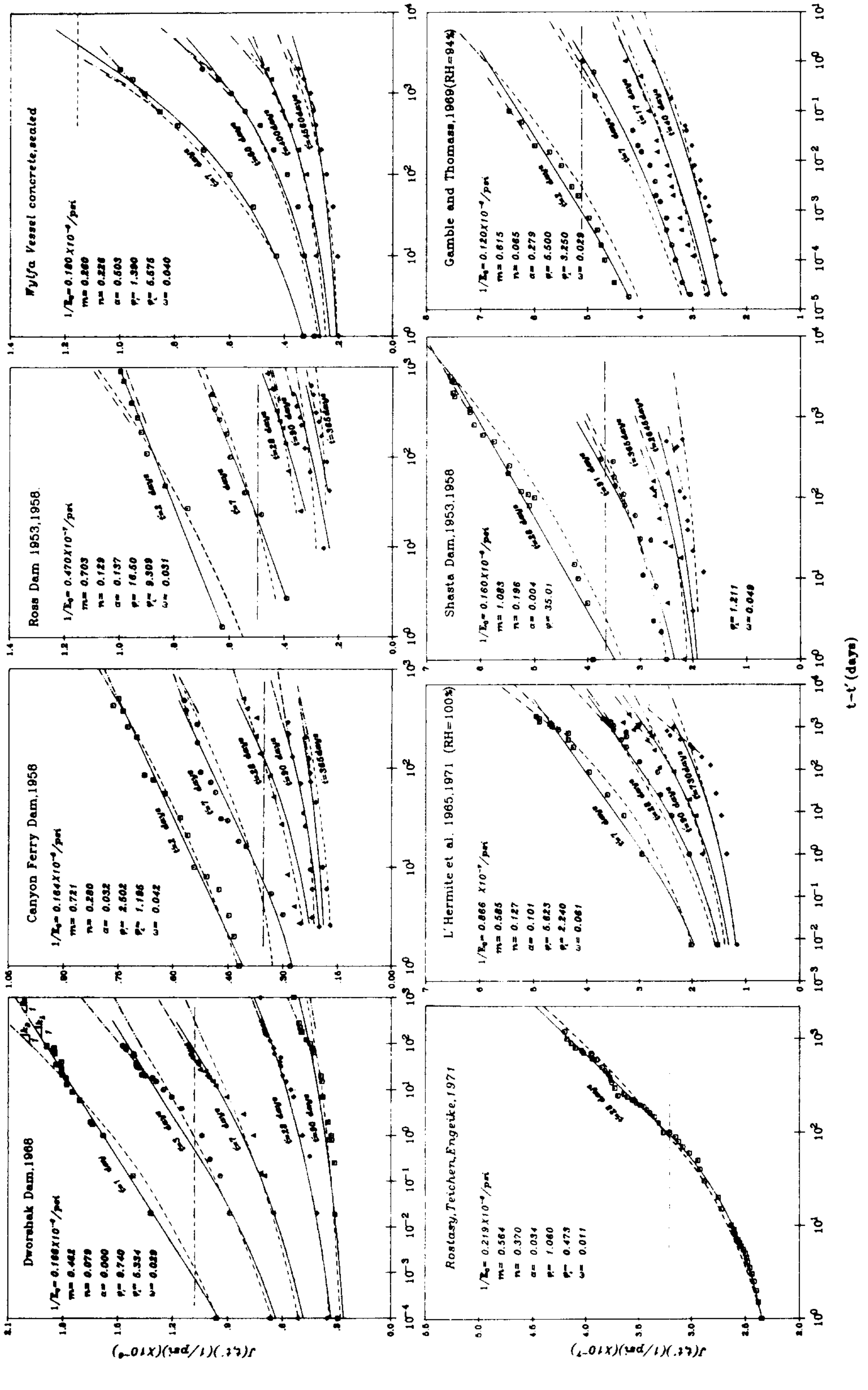

FIG. 3. Optimum Fits of Test Data by Double-Power Logarithmic Law without the Nondivergence Restriction.

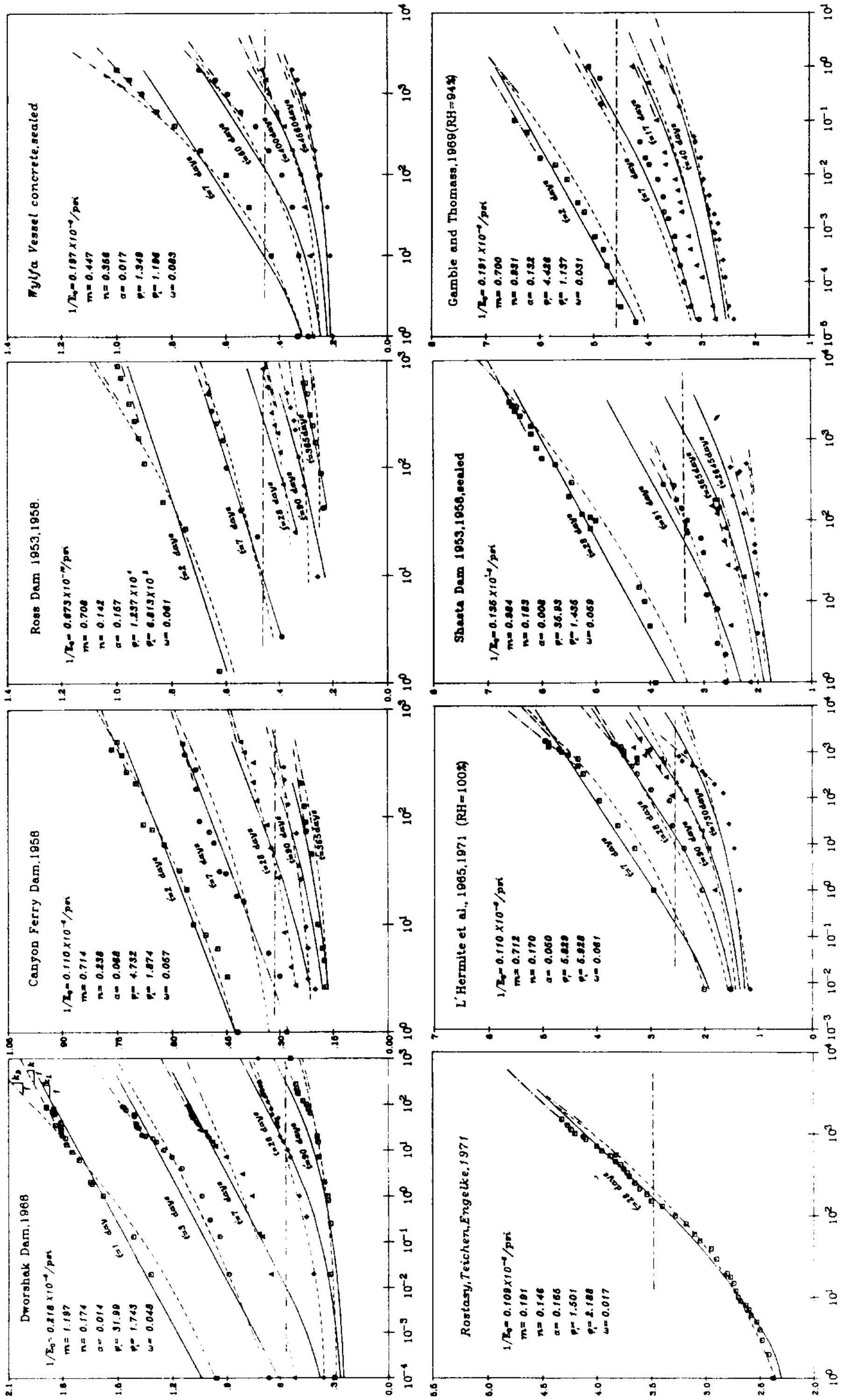

FIG. 4. Optimum Fits of Test Data by Double-Power Logarithmic Law with the Nondivergence Restriction.

Due to the nonlinearity of the optimization problem, it is hard to carry out a more refined statistical analysis which would indicate the increase of error as a function of the distance from the centroid of measured data. To obtain information on this aspect, it is necessary to linearize Eq. 2 so that linear regression statistics can be applied. For this purpose we may introduce the notations

$$y = E_0 J(t,t') - 1, \quad x = \phi_1 (t'^{-m} + \alpha)(t - t')^n \quad \text{for } (t - t') \le \theta_L$$
$$x = (n+1)\phi_L \ln \frac{t - t'}{\theta_L} \quad \text{for } (t - t') \ge \theta_L \tag{8}$$

This allows writing Eq. 2 as $y = c_0 + c_1 x$, in which c_0 and c_1 are coefficients to be found by linear regression analysis. Variable y may be regarded as the creep coefficient relative to instantaneous value $1/E_0$. If Eq. 2 represented the test data perfectly, then c_1 would be 1 and c_0 would be 0. The deviations of c_1 from 1 and of c_0 from 0 characterize the errors. Statistical regression analysis yields not only the mean values of c_1 and c_0, but also their coefficients of variation, as well as the confidence intervals for the values of y and for the mean of y.

Using the material parameters found before, the test data from the literature, used in Fig. 3, can be all plotted in one regression diagram; see Fig. 5. The regression line is shown as a solid line. Furthermore, the hyperbolas representing the 95% confidence intervals, are shown as the dashed lines. We may note the widening of the confidence interval with the distance from the centroid of the test data. The values of c_1 and c_0 found from the linear regression analysis may be interpreted as corrections to the values of ϕ_1 and of $1/E_0$. Comparing the statistical regressions in Figs. 5a and 5b for the present formulation and the double-power law, we again see very little difference. As we already showed, an appreciable improvement is achieved only in the final slopes to which the linear regression does not apply.

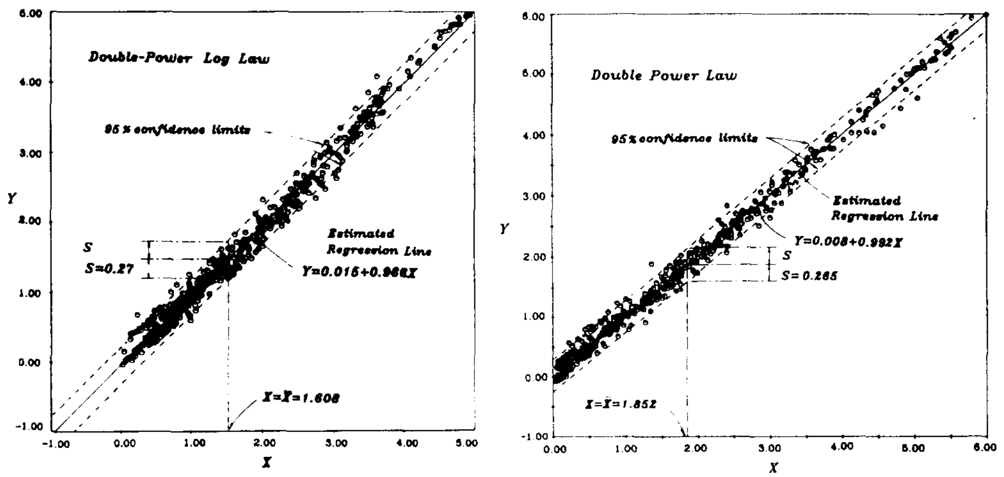

FIG. 5
Regression Plots for Double-Power Logarithmic Law and Double-Power Law.

Prediction of Creep

The capability of close fitting the available test data is an indication of the correctness of the mathematical formula for creep. It is, however, another matter to predict creep of a given concrete if no measurements were taken. The prediction is notoriously far more uncertain. Due to the very large uncertainty in predicting the parameters $E_0, m, n, \alpha, \phi_1$ and ϕ_L from concrete strength and composition, hardly any improvement can be achieved with the present formulation (Eq. 2) compared to the double-power law, and therefore the prediction formulas in Eqs. 16, 17, 18, 15 and 19 from Ref. 3 (or Eqs. 9a-9e from Ref. 7) are recommended to be also used with the present formulation. An additional formula is needed for parameter ϕ_L, for which the following expression may be recommended

$$\phi_L = \frac{\phi_1}{1.53 - 0.06\, f'_c} \qquad (9)$$

with f'_c in ksi (1 ksi = 6.895 MPa); see Fig. 6. This equation does not guarantee nondivergence, in general. Nevertheless, the increase in the separation of any two adjacent creep curves for adjacent ages at loading is quite small, much smaller than that obtained with the double-power law.

Table 1 shows the coefficient of variation for all data sets and Fig. 7 gives the predictions of creep curves. When all parameters are predicted (for the concretes considered) on the basis of the aforementioned formulas, $\bar{\omega}$ is 21.3%, as compared to 24.1% for the double-power law. We see that the prediction of final slope is greatly improved by double-power logarithmic law. For final slopes of creep curves, as shown in Table 1b, $\bar{\omega}$ = 34% as compared with 61% for double-power law!

As has been shown before, the predictions of creep are drastically improved if the initial elastic value of deformation is measured. From such an initial value, one can determine one material parameter, E_0. The uncertainty of predicting creep when the initial elastic deformation is known may be illustrated by comparing the formula predictions obtained for these data under the condition that the value of E_0 is optimized. Such statistical comparisons are shown in Table 1.

Stress Relaxation Predictions

Unlike creep recovery, the stress relaxation at constant strain may be rather closely predicted on the basis of the principle of superposition. This is confirmed by relaxation measurements, although measurements of very long

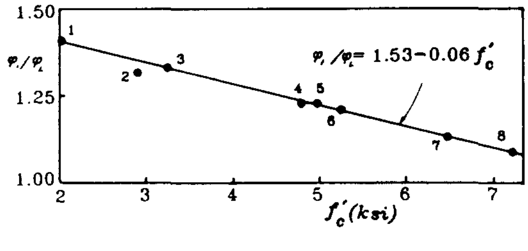

FIG. 6
ϕ_1/ϕ_L Versus the 28 Day Cylinder Strength f'_c. (1. Dworshak Dam, 2. Canyon Ferry Dam, 3. Shasta Dam, 4. Gamble and Thomass, 5. Ross Dam, 6. L'Hermite et al., 7. Rostasy et al., 8. Wylfa Vessel).

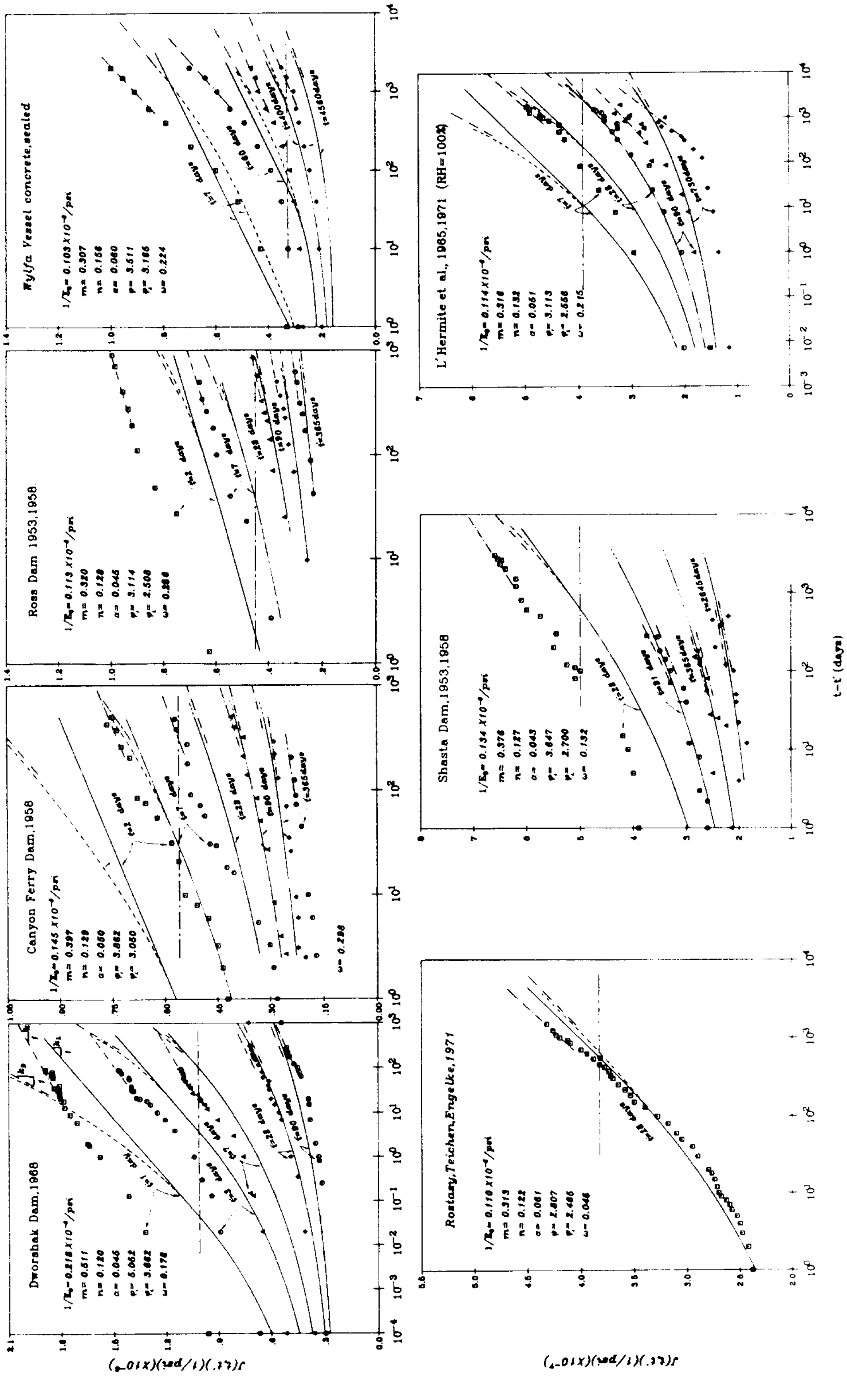

FIG. 7. Fits of Test Data by Prediction Formulas.

TABLE 1. Coefficient of Variation for Test Data

TABLE 1a. For sampling points (in percent)

j	creep law / data set	Optimum Fits DPL [3]	DPL [2]	DPLL	DPLL*	Prediction Formula DPL [3]	DPLL	DPLL**
1	Canyon Ferry Dam [15,16]	4.60	5.58	5.70	4.20	39.6	29.8	6.20
2	Ross Dam [15,16]	3.50	7.00	6.10	3.09	27.7	28.6	16.5
3	Dworshak Dam [17]	5.46	5.63	4.80	2.90	21.2	17.8	11.1
4	Rostasy et al. [22]	1.00	1.20	1.70	1.12	5.10	4.50	3.50
5	L'Hermite et al. [18]	4.90	6.28	6.11	6.10	25.2	21.5	7.30
6	Shasta Dam [15,16]	4.10	5.37	5.90	4.90	13.6	13.2	8.80
7	Wylfa Vessel [19,20,21]	4.14	4.15	8.30	4.00	21.0	22.4	9.50
8	Gamble-Thomass [23]	2.82	6.20	3.10	2.90	-	-	-
	$\bar{\omega}$	4.03	5.45	5.54	3.91	24.12	21.30	9.72

TABLE 1b. Final Slopes

Data Sets	t'	Optimum Fits DPL[2] Δ_D	DPLL Δ_L	DPLL* Δ_L	Prediction Formulas DPL[3] Δ_D	DPLL Δ_L
1	2	0.38	0.0	0.0	1.20	0.08
	7	0.17	0.30	0.20	0.85	0.0
	28	0.0	0.30	0.30	0.0	0.0
	90	0.40	0.45	0.40	0.10	0.10
	365	0.32	0.50	0.10	0.20	0.20
2	2	0.70	0.0	0.0	1.00	0.0
	7	0.0	0.0	0.0	0.50	0.24
	28	0.26	0.65	0.30	0.0	0.0
	90	0.50	0.30	0.05	0.50	0.50
	365	0.60	0.40	0.0	0.40	0.40
3	1	0.60	0.0	0.0	1.64	0.62
	3	0.05	0.31	0.0	1.00	0.30
	7	0.20	0.0	0.0	0.80	0.60
	28	0.0	0.22	0.0	0.0	0.0
	90	0.50	0.10	0.50	0.30	0.30
4	28	0.25	0.0	0.0	0.11	0.08

Data Sets	t'	Optimum Fits DPL[2] Δ_D	DPLL Δ_L	DPLL* Δ_L	Prediction Formulas DPL[3] Δ_D	DPLL Δ_L
5	7	0.04	0.41	0.35	0.54	0.20
	28	0.17	0.32	0.17	0.50	0.05
	90	0.17	0.0	0.0	0.25	0.20
	730	0.60	0.53	0.50	0.60	0.60
6	28	0.39	0.0	0.0	0.35	0.05
	91	0.0	0.0	0.12	0.08	0.10
	365	0.0	0.24	0.0	0.10	0.10
	2645	0.50	0.15	0.25	0.10	0.10
7	7	0.39	0.30	0.35	0.20	0.56
	60	0.17	0.25	0.17	0.40	0.43
	400	0.05	0.41	0.05	0.30	0.20
	4560	0.29	0.15	0.35	0.15	0.15
8	2	0.20	0.14	0.05	1.0	0.70
	7	0.0	0.0	0.0	0.80	0.80
	17	0.0	0.10	0.05	0.60	0.60
	40	0.28	0.24	0.20	0.15	0.15
$\bar{\omega}_f^{\dagger}$		0.34	0.29	0.22	0.61	0.34

Note: Δ_D, Δ_L are normalized errors of final slopes for curves of double-power law (k_D) and Double-Power Logarithmic Law (k_L) in comparisons with the final slopes of test data (k) and defined as $|k_D/k - 1|$, $|k_L/k - 1|$, respectively.

* Double-Power Logarithmic Law without the nondivergence restriction.

**Double-Power Logarithmic Law with optimum E_0.

† $\bar{\omega}_f = \left[\frac{1}{n-1}\sum_{i=1}^{n} \Delta_{D_i}^2\right]^{\frac{1}{2}}$ or $\left[\frac{1}{n-1}\sum_{i=1}^{n} \Delta_{L_i}^2\right]^{\frac{1}{2}}$.

durations (over 10 years) are lacking. It has been noted before that, for small ages at the instant of initial straining, the stress relaxation curves cross into negative stress values at very long times (over 10 years) when they are calculated by the principle of superposition from the double-power law. The same thing happens when the calculation is done by the principle of superposition directly from certain measured creep curves. From the thermodynamic viewpoint [30], there exists no fundamental prohibition against such negative values when one deals with an aging material. Nevertheless, it is not clear whether the negative values at the end of early-age relaxation curves are not caused merely by the choice of the creep formula or by an experimental error in creep measurements.

To examine this aspect, the creep data for Dworshak Dam [17] and for Canyon Ferry Dam [15,16] were optimally fitted by the double-power law and by the present formulation (Eq. 2), and for both cases the stress relaxation curves were calculated on the basis of the superposition principle using a highly accurate step-by-step algorithm [13,14]. The results of the calculation are shown in Fig. 8 by the solid curves for the present formulation and by the dashed curves for the double-power law. We see that the present formulation limits the occurrence of negative stress values at the end of early-age relaxation curves, which seems to be a desirable feature.

Conclusions

1. The double-power law yields for the creep curves plotted in log-time final slopes that are predominantly on the high side when compared with long-time measurements. This may be remedied by a creep law which exhibits a transition from the double-power law to a straight line in the logarithmic scale of creep duration at a certain transition time. The straight line has the same slope for all ages at loading and the transition occurs later for an older concrete.

2. The aforementioned improvement in the representation of the final slope of the creep curves should allow a better extrapolation of creep data into very long times.

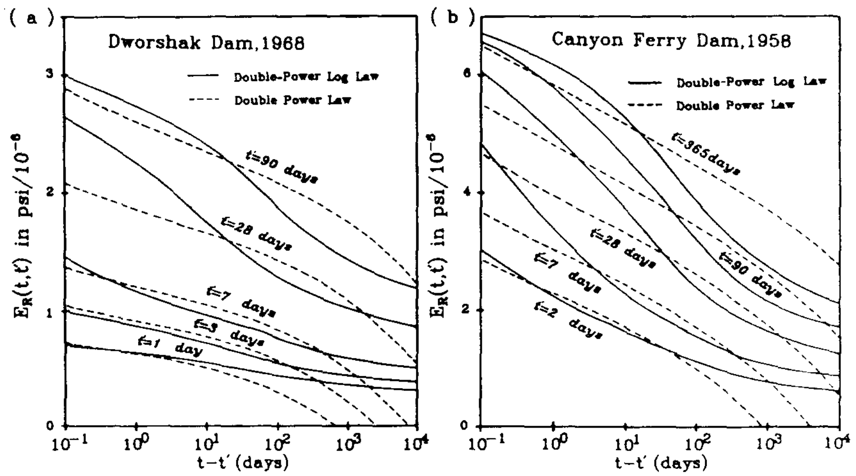

FIG. 8
Comparisons of Stress Relaxation Predictions for Dworshak Dam [17] and Canyon Ferry Dam [15,16].

3. The new formulation reduces the occurrence of divergence of creep curves. Although the divergence could be eliminated, for certain values of material parameters, completely, the representation of existing test data would be impaired.

4. The new law also greatly reduces the possibility that the relaxation curves calculated on the basis of the superposition principle would cross at very long times into negative values.

Acknowledgment - Support by the U.S. National Science Foundation under Grant No. CEE-8303148 to Northwestern University is gratefully acknowledged.

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