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Size Effect of Shear Failure in Prestressed Concrete Beams

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It is shown that the size effect predicted by dimensional analysis of fracture mechanics of failures due to progressive distributed microcracking agrees with the test data existing in the literature better than the presently used strength (ultimate load) criterion, which exhibits no size effect, and better than the linear elastic fracture mechanics criterion, which exhibits much too strong a size effect. This finding is similar to that made previously for unprestressed longitudinally reinforced beams. Based on the analysis of over 200 test results from the literature, a simple design formula involving the proper size effect is proposed. Comparison with other existing formulas, including the one presently used in the ACI Building Code, reveals an appreciable reduction in the coefficient of variation of the deviations from test data.

Keywords: beams (supports); cracking (fracturing); dimensional analysis; **failure; prestressed concrete; shear properties**; statistical analysis; structural analysis.

Shear failure of prestressed concrete beams has been studied by many authors and various valuable semi-empirical formulas have been proposed.[1-9] However, when confronted with the bulk of the existing test results, a large statistical scatter is observed. The scatter is, in fact, much larger than either that of the tensile strength of concrete or that of the fracture energy. This indicates that part of the scatter might be due to some systematic influence missing from the existing method of failure load prediction rather than solely to the inevitable inherent statistical nature of the material.

From the theoretical viewpoint, the existing formulas in essence utilize the concept of strength, either in the form of limit analysis or of the allowable stress elastic design. Strength criteria are justified, however, only in the case of ductile failures governed by the theory of plasticity. Rather than exhibiting a yield plateau, the material undergoes strain-softening, i.e., a decline of stress at increasing strain after the maximum stress has been reached. This phenomenon, due to progressive distributed tensile cracking, invalidates, in the strict theoretical sense, the use of limit analysis (plasticity theory) as well as allowable stress criteria. The reason is that, in the presence of strain-softening, the failure is not simultaneous, describable by some single-degree-of-freedom mechanism, but progressive. This causes that, while the stress approaches the strength value at some location, at another location the stress has already passed the maximum stress point and declined considerably below the strength value. But limit analysis is valid only if the maximum stress value exists simultaneously everywhere along the failure surface.

The difficulties due to strain-softening are clearly manifested in finite element analysis, in which it is found that the results based on the strength criterion for cracking can exhibit a spurious dependence on the choice of the mesh size and incorrectly predict failure at a vanishing load and vanishing energy dissipation as the mesh size tends to zero.[10] These difficulties are avoided by using an energy criterion of failure instead of a strength criterion. Such solutions are the purpose of fracture mechanics.

After many years of doubt, a new form of fracture mechanics that appears applicable to concrete has now emerged.[11] This new approach, exemplified by the work of Hillerborg, Bažant, Willam, Sture, Hilsdorf, Ingraffea, and others, does not treat fracture as a point phenomenon, but recognizes that in brittle heterogeneous materials such as concrete the front of a propagating crack is blunted by a relatively large fracture process zone in which progressive distributed microcracking reduces the tensile stress to zero gradually.[11] This property has an interesting consequence for the size effect, as seen in the variation of the apparent strength (the nominal stress at failure) with the size of the stucture when geometrically similar structures of different sizes are compared.

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Dimensional analysis of similitude for failure due to crack band propagation[12] has led to a simple approximate algebraic formula for the size effect. For a very small structure, this formula indicates the failure to be governed by the strength criterion (limit analysis or allowable stress), and for a very large structure (generally beyond the range of interest, except for dams) by the linear elastic fracture mechanics, which implies the fracture process zone to be reduced to a point. For most structures of the usual sizes, neither the linear elastic fracture mechanics nor the strength criterion is applicable, and the transitional range of the size effect, corresponding to nonlinear fracture mechanics, needs to be considered.

In a preceding study,[13] the size effect law based on nonlinear fracture mechanics[12] was applied to the diagonal shear failure of longitudinally reinforced unprestressed beams without shear reinforcement (stirrups). Analysis of over three hundred test results from the literature showed that a significant improvement of the failure load prediction is achieved if the size effect law is taken into account. This suggests that a similar improvement could be achieved for the shear failure of prestressed concrete beams, and a study of this question is the objective of the present paper.

SIZE EFFECT LAW FOR FAILURE DUE TO PROGRESSIVE DISTRIBUTED FRACTURING

First we need to review the size effect law, which follows by dimensional analysis from a certain reasonable simplifying hypothesis for distributed cracking.[12] We consider structures of different sizes but of geometrically similar shapes, e.g., beams of the same ratio of depth to shear span. In the case of prestressed structures we need to also require that the stresses due to prestressing be roughly the same because they affect the material cracking properties, which must be the same for all structures that are being compared. The criterion for strength (ultimate load or allowable load) can always be written in the form $\sigma_N = f_t'$ where f_t' = direct tensile strength of concrete and σ_N = nominal stress at failure. For reasons of dimensionality, we may always set $\sigma_N = c_N T/bd$ where T = failure load (or allowable load), d = characteristic size of the structure,

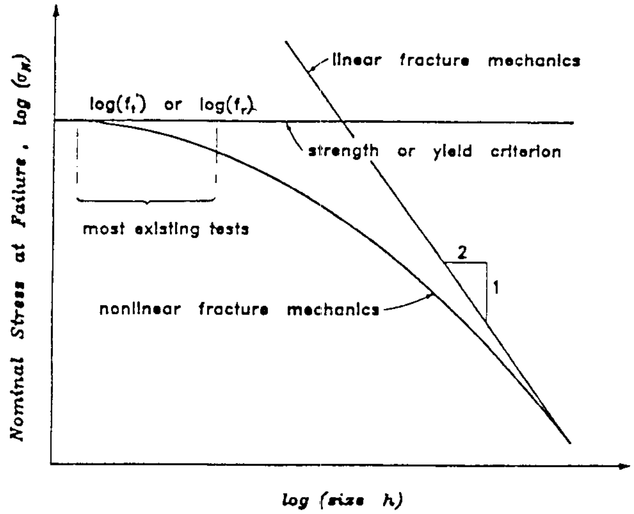

Fig. 1—Illustration of size effect according to various theories

e.g., the beam depth, and b = thickness of the structure = the beam width, in our case, and c_N = constant. The size effect law[12] may now be written as

$$\sigma_N = f_t' \ \phi(\lambda), \qquad \phi(\lambda) = \frac{1}{\sqrt{1 + \lambda/\lambda_0}} \tag{1}$$

in which $\lambda = d/d_a$ = relative structure size, d_a = maximum aggregate size, and λ_0 = empirical constant. Eq. (1) is plotted in Fig. 1.

For small-size structures (relative to the aggregate size), i.e., for small λ, the value of λ/λ_0 in Eq. (1) may be neglected in comparison to unity, and then $\phi(\lambda) = 1$, in which case Eq. (1) reduces to $\sigma_N = f_t'$ = constant. This confirms that the strength criterion applies for small-size structures, which in fact covers the typical size of structures tested in laboratories.

The strength criterion, which shows no size effect, is represented in Fig. 1 by a horizontal line, regardless of whether the analysis is based on elasticity or plasticity (limit analysis). The only difference between elasticity and plasticity is the level at which the horizontal line is drawn. For structures that are very large compared to the aggregate size, 1 may be neglected compared to λ/λ_0 in Eq. (1), and then $\sigma_N = f_t' (\lambda/\lambda_0)^{-1/2}$. This is the well-known size effect of linear elastic fracture mechanics, which is represented in Fig. 1 by the inclined straight line of slope — ½. Evidently, Eq. (1) represents a gradual transition from the strength criterion for small structures to linear elastic fracture mechanics for very large structures. For $\lambda < \lambda_0$ the strength criterion prevails, and for $\lambda > \lambda_0$ the fracture mechanics aspect of failure prevails.

Note that Eq. (1) is only a first-order approximation. The most general size effect law of blunt fracture consists of the asymptotic series expansion $\phi(\lambda) = [1 + \lambda/\lambda_0 + \lambda_2/\lambda + (\lambda_3/\lambda)^2 + (\lambda_4/\lambda)^3 + \ldots]^{-1/2}$ where $\lambda_2, \lambda_3, \lambda_4, \ldots$ are additional empirical constants.[14] Obviously,

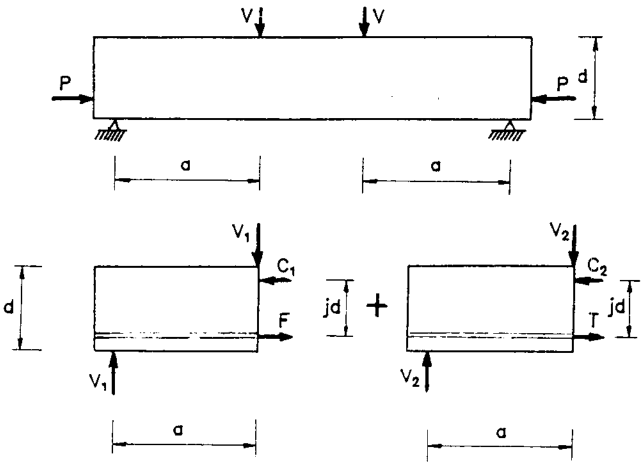

Fig. 2—Notation for the analysis of diagonal shear

the additional terms are negligible for large enough λ. Furthermore, when not only the structure size but also the aggregate size d_a is varied, f_t' in Eq. (1) should be replaced by $f_t^* = f_t' \, [1 + (c_0/d_a)^{1/2}]$ where c_0 = empirical constant (based on Bažant's paper in preparation). However, the data available for our purpose appear to be too scattered to permit meaningful evaluation of the additional constants $c_0, \lambda_2, \lambda_3, \ldots$ and so we content ourselves with the simple approximate form in Eq. (1).

SIMPLIFIED FORMULA FOR SHEAR STRENGTH

We consider a simply supported prestressed concrete beam with a constant shear force V within the end segment of length a (shear span) and bending moment $M = Va$ at distance a from the support. For the sake of simplicity, we assume that the ultimate shear force V_u may be subdivided into two parts. Part V_1 is balanced by the longitudinal prestress force F, and part V_2 is balanced by an additional force in the reinforcement due to bond $V_u = V_1 + V_2$. According to the moment equilibrium condition at the cross section at distance a from the support (Fig. 2), failure occurs when

$$M_u = V_u a = (F + T)jd \qquad (2)$$

in which F is the axial prestress force and T is the tensile force resultant from the reinforcement. We may substitute $F = f_{cc}bd$ where f_{cc} = mean prestress in concrete = stress due to F at neutral axis, b = beam width, and d = beam depth. Furthermore, because the additional force T is due to bond stresses which are approximately proportional to $\sqrt{f_c'}$, we introduce $T = \beta\sqrt{f_c'}\, bd$ where f_c' = standard 28 day cylindrical compression strength of concrete, and β is an empirical coefficient. Substituting these expressions into Eq. (2) we get

$$V_u = \frac{\beta\sqrt{f_c'} + f_{cc}}{a/d}\, jbd \qquad (3)$$

The ultimate shear stress $v_u = V_u/bd$, which may be considered to be the same as the nominal stress at failure for our case, is then expressed as

$$v_u = \frac{k_1\sqrt{f_c'} + k_2 f_{cc}}{a/d} \qquad (4)$$

in which $k_1 = \beta j$ and $k_2 = j$. This formula is similar to the one used in the ACI Building Code,[1] but it contains the effect of the shear span ratio in addition to the empirical parameters k_1 and k_2 to be found from test results.

The derivation of Eq. (4) did not take into account the size effect. According to Eq. (1), we should now multiply the nominal shear stress at failure, represented here by v_u, by function $\phi(\lambda)$

$$v_u = C_1\left(1 + \frac{d}{\lambda_0 d_a}\right)^{1/2},\ C_1 = \frac{k_1\sqrt{f_c'} + k_2 f_{cc}}{a/d} \qquad (5)$$

This formula (Fig. 3) is sufficiently simple to allow statistical linear regression analysis of test data.

While the size effect law alone [Eq. (1)] is applicable only to geometrically similar structures, its combination with Eq. (4) allows us to apply Eq. (5) to geometrically dissimilar beams for which the values of a/d are different. This, of course, introduces an additional error due to Eq. (4). Ideally, the size effect should be tested comparing only beams that are geometrically similar (same a/d) and cover a broad range of sizes. However, such tests have not yet been carried out. A large funding would be required for that purpose since very large beams would have to be included in a meaningful test program.

STATISTICAL ANALYSIS OF AVAILABLE TEST DATA

Extensive test data that can be used for statistical evaluation of the formulas for the ultimate shear capacity of prestressed concrete beams have been accumulated over the years; see the test data of Arthur;[2] Hicks;[4] Sozen, Zwoyer, and Siess;[7] Kar;[5] Hanson and Hulsbos;[15] and Zwoyer.[16]

For the purpose of linear regression, Eq. (5) may be algebraically transformed to the following linearized form

$$\frac{1}{v_u^2} = \frac{1}{C_1^2} + \frac{1}{C_1^2\lambda_0}\frac{d}{d_a} \qquad (6)$$

This means that if we plot $1/v_u^2$ versus d/d_a, we should ideally obtain a straight line of slope $1/C_1^2\lambda_0$ and vertical axis intercept $1/C_1^2$. The vertical deviations from this straight line [Fig. 4(a)] then represent statistical errors.

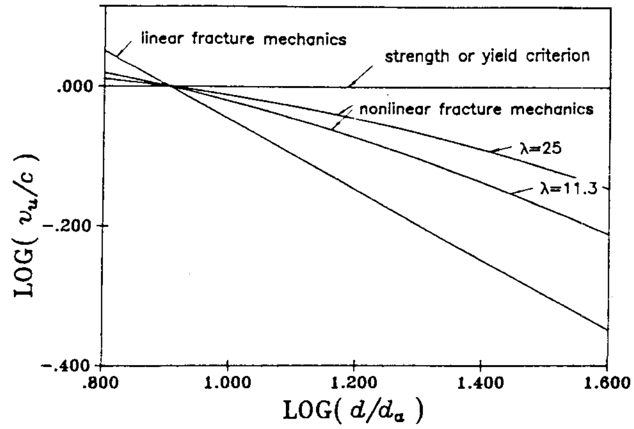

Fig. 3—Two curves according to Eq. (5) for nonlinear fracture mechanics

Unfortunately, none of the available test data for one particular concrete, taken alone, and for the same f_{cc}, f_c', and a/d, are sufficiently extensive to allow meaningful statistical analysis. Therefore, the existing test data that include some information on the size effect must be analyzed collectively, even though this inevitably entails statistical scatter due to differences between the concretes and test setups from various laboratories. A large statistical scatter must then be expected. Due to the need to analyze collectively the data for various concretes, coefficients k_1, k_2 (and also d_a, if not reported) must be identified simultaneously with the regression parameters from Eq. (6). In such a case linear regression cannot be used because coefficients k_1 and k_2 are involved nonlinearly. Therefore, a trial-and-error approach along with nonlinear computer optimization (Marquardt-Levenberg algorithm) has been used to determine the optimum values of these coefficients.

The optimum fit of the existing data achieved with Eq. (5) is shown in Fig. 4(a) as the linear size effect regression plot [Eq. (6)]. The same fit is shown in Fig. 4(b) as the plot of log v_u versus log(d/d_a). The very large scatter strikes the eye; however, this is probably inevitable when test data for different concretes, different beams, and from different laboratories are used in one plot. Even so, the size effect is clearly apparent. According to the strength criterion (no size effect), which is implied in the current design approach, the trend of the data points in Fig. 4(b) would have to correspond to a horizontal line, which is obviously not true. On the other hand, a straight line of slope − ½ in Fig. 4(b) would yield too strong a size effect, in disagreement with the data trend.

It should also be realized that the use of numerous test data from various laboratories in Fig. 4 has been inevitable, since each of these data, taken alone, does not reveal the size effect. This is because beams of sufficiently different sizes (depths) were not included in the test program. It is now, of course, desirable to carry out a sufficiently extensive test series involving both very small and very large beams; however, this will no doubt involve considerable costs, and in the meantime

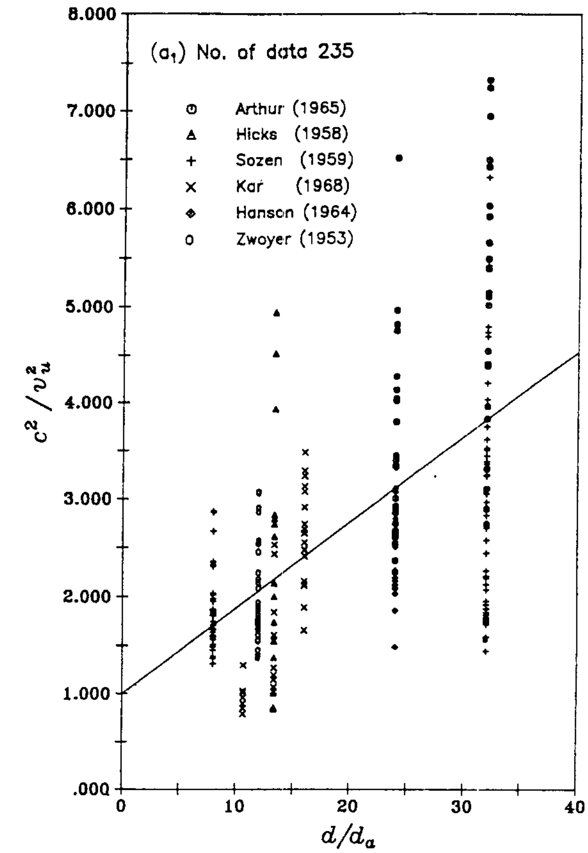

Fig. 4(a)—Comparison with existing test data for beams of different sizes (c = C_l)

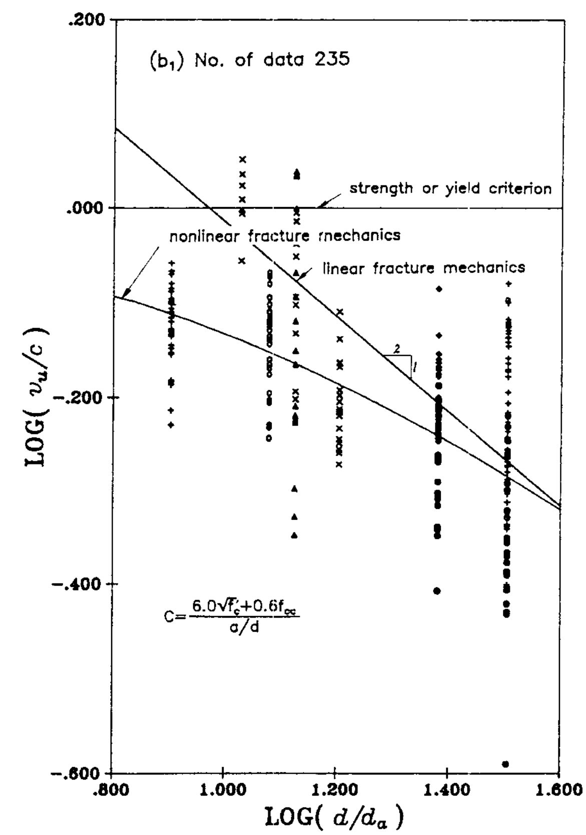

Fig. 4(b)—Comparison with existing test data for beams of different sizes (c = C_l)

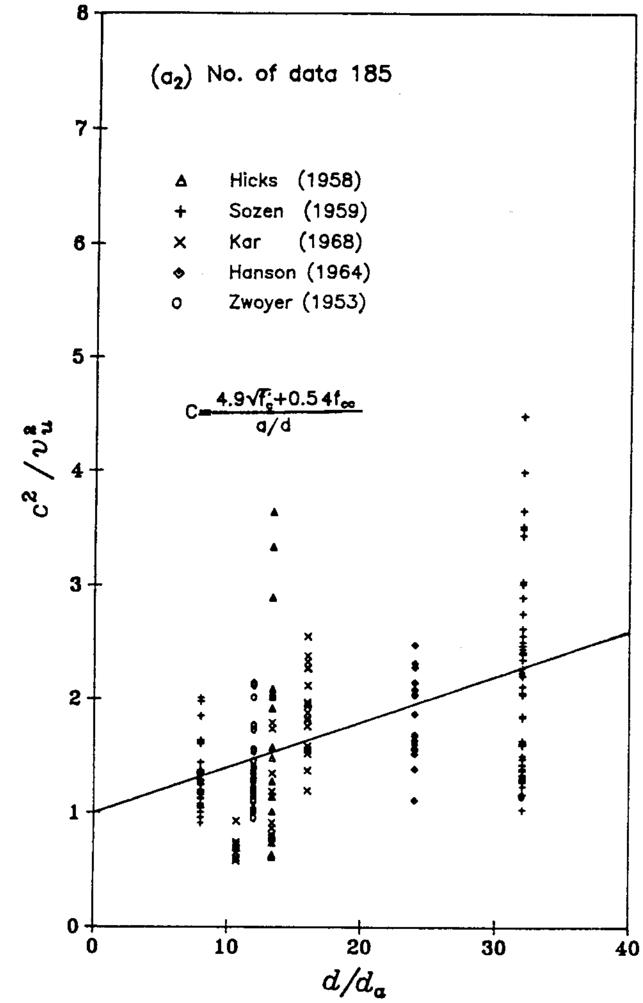

Fig. 4(c)—Comparison with existing test data for beams of different sizes (c = C_l)

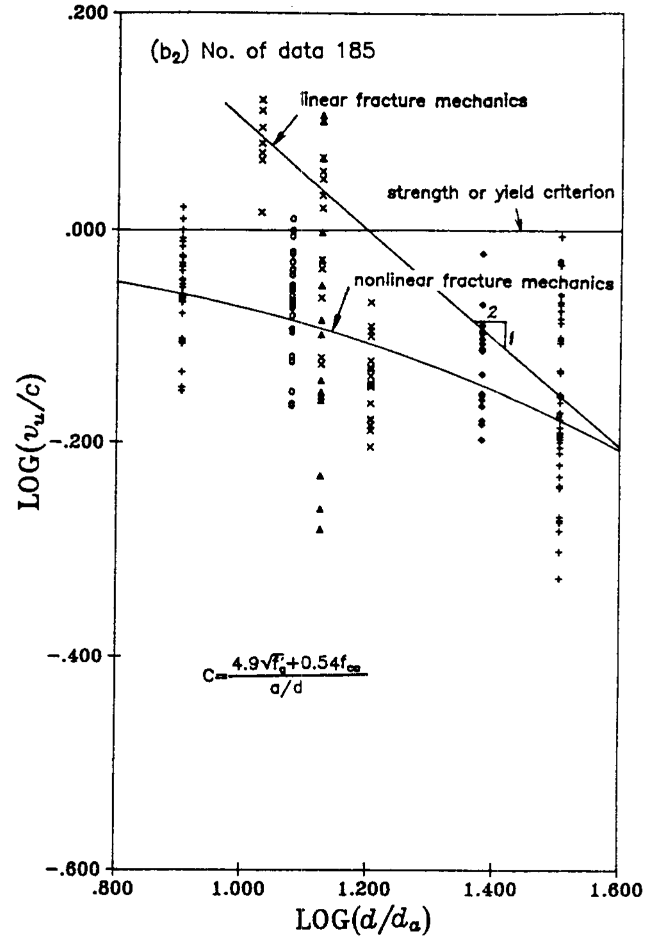

Fig. 4(d)—Comparison with existing test data for beams of different sizes (c = C_l)

the best approach is to make use of what is available. Note also that there are numerous test data in the literature that are not included in Fig. 4; these data correspond to the smallest beams that could be cast with a given aggregate, and thus they reveal nothing about the size effect.

After determining the values of the coefficients in Eq. (5) by computer optimization, it is possible to calculate from Eq. (5) for each beam the theoretical nominal shear strength at failure v_u and compare it to the measured value of v_u. If the formula were perfect, the plot would have to be a straight line of slope 1, passing through the origin. Thus, the deviations from the straight-line plot represent statistical errors. The plot of this type is shown, for the proposed Eq. (5), in Fig. 5(c). The standard deviation s and the coefficient of variation ω of the vertical deviations from the regression line is also given in the figure, along with the correlation coefficient r. The values plotted are all in psi (1 psi = 6895 Pa) and so is the strength of concrete. When the cubic strength f_c (in psi) was reported, it was converted to the cylindrical strength f'_c according to the formula[17] $f'_c = [0.76 + 0.20 \log (f_c/2840)] f_c$.

Based on the foregoing statistical comparisons, the following formula is proposed for the mean ultimate nominal shear strength

$$v_u = \frac{6\sqrt{f'_c} + 0.6 f_{cc}}{a/d}\left(1 + \frac{d}{10 d_a}\right)^{-1/2} \tag{7}$$

For comparison, the available test data have also been fitted with the formula used in the present ACI 318-77[1] as well as another previous semi-empirical formula.[7] Note that the coefficients indicated for these formulas have to be adjusted for the present purpose since they are not intended to give the mean values of the ultimate nominal shear strength but the values for the initiation of cracking.

Thus, the formula for ACI 318-77[1] may be written as

$$v = k_1\sqrt{f'_c} + k_2 f_{cc} \tag{8}$$

in which the values of coefficients k_1 and k_2 must now be found by nonlinear optimization. The values of the optimum coefficients found are listed in Table 1 and the resulting optimal fits are shown in Fig. 5(a). It is apparent that the scatter, characterized by the standard

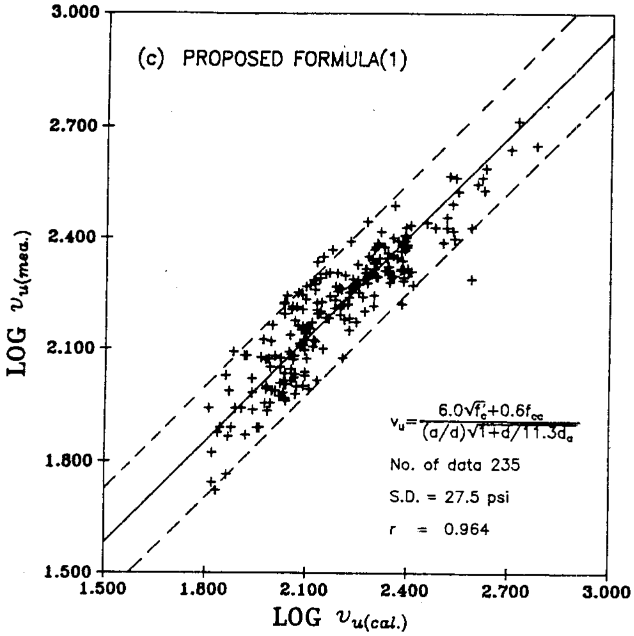

Fig. 5(a)—Comparison of various formulas with the bulk of the existing test data on the ultimate shear strength (S.D. = standard deviation, r = correlation coefficient)

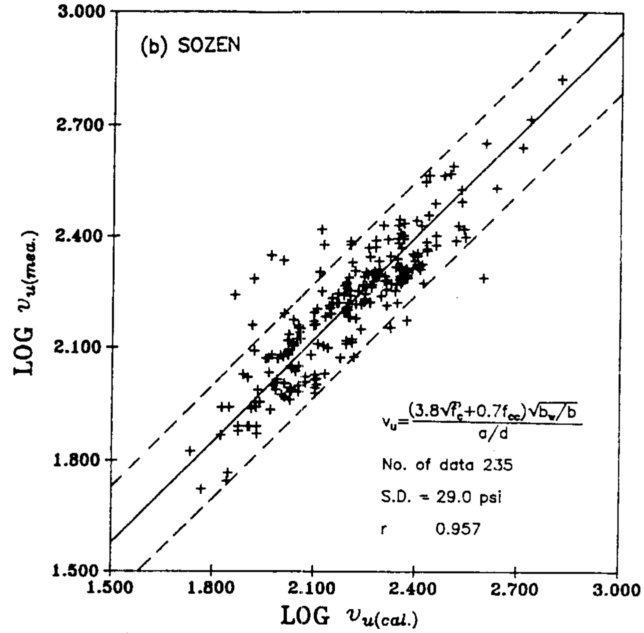

Fig. 5(b)—Comparison of various formulas with the bulk of the existing test data on the ultimate shear strength (S.D. = standard deviation, r = correlation coefficient)

deviation and the correlation coefficient indicated in the figure, is much larger than that of the proposed formula.

Fig. 5(b) shows a comparison with the formula proposed by Sozen, Zwoyer, and Siess,[7] which reads

$$V = (k_1\sqrt{f'_c} + k_2 f_{cc})\frac{(b_w/b)^p}{(a/d)^q} \qquad (9)$$

in which b_w is the web thickness, b is the flange width, and k_1, k_2, p, and q are four empirical coefficients. The values of these constants have been optimized by fitting the available data shown in Fig. 5 and listed in Table 1. The comparison shows that this formula agrees with the available data nearly as well as the proposed formula.

It is also useful to consider the population of the values of $Y = (v_{mea}/v_{cal})$-1, in which v_{cal} is the calculated value and v_{mea} is the measured value of the nominal shear strength. Fig. 6 shows the plot of these values as a function of the logarithm of the relative size. In this comparison, the proposed formula [Fig. 6(c)] appears to be the best. Nevertheless, the formula of Sozen, Zwoyer, and Seiss is nearly as good [Fig. 5(b) and 6(b)]. The scatter of the ACI formula [Fig. 6(a)] is much larger.

PROPOSED DESIGN FORMULA

For the purpose of design, two formulas of the type of Eq. (7) have been considered, with different parameter values as shown in Table 1. Formula No. 1 is based on the complete available set of 235 data, while formula No. 2 is based on a reduced set of 185 data. The reason for reducing the number of data is to mitigate

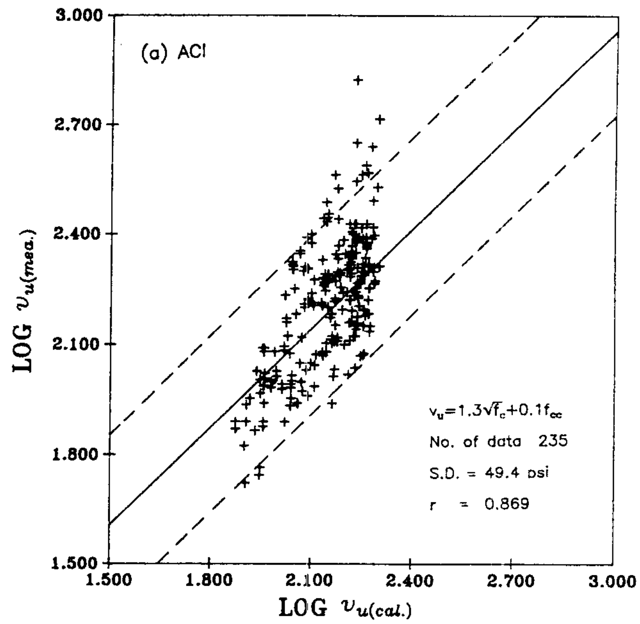

Fig. 5(c)—Comparison of various formulas with the bulk of the existing test data on the ultimate shear strength (S.D. = standard deviation, r = correlation coefficient)

Table 1 — Coefficients obtained by nonlinear regression for v_u

Model	k_1	k_2	λ_0	p	q	Number of test data
ACI 318-77	2.56	0.22				235
Sozen	3.84	0.66		0.49	0.96	235
with size effect, No. 1	6.01	0.59	11.3			235
with size effect, No. 2	4.85	0.54	26.7			185

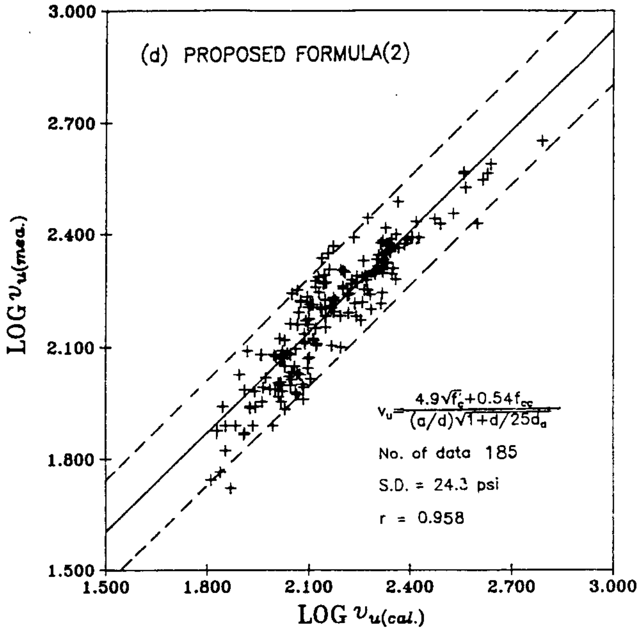

Fig. 5(d)—Comparison of various formulas with the bulk of the existing test data on the ultimate shear strength (S.D. = standard deviation, r = correlation coefficient)

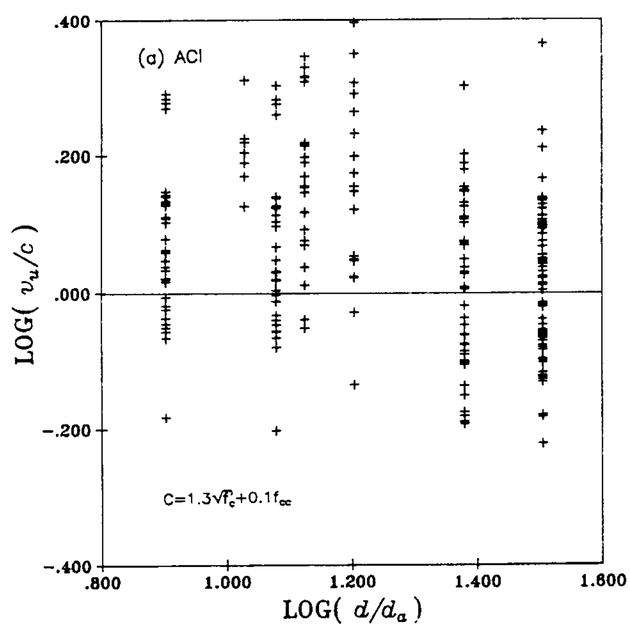

Fig. 6(a)—Same data as in Fig. 4 compared with formulas in different manner (c = C_l)

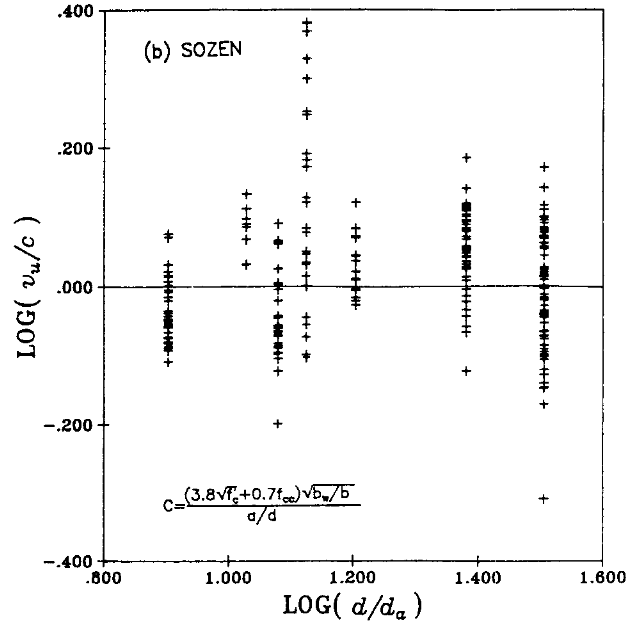

Fig. 6(b)—Same data as in Fig. 4 compared with formulas in different manner (c = C_l)

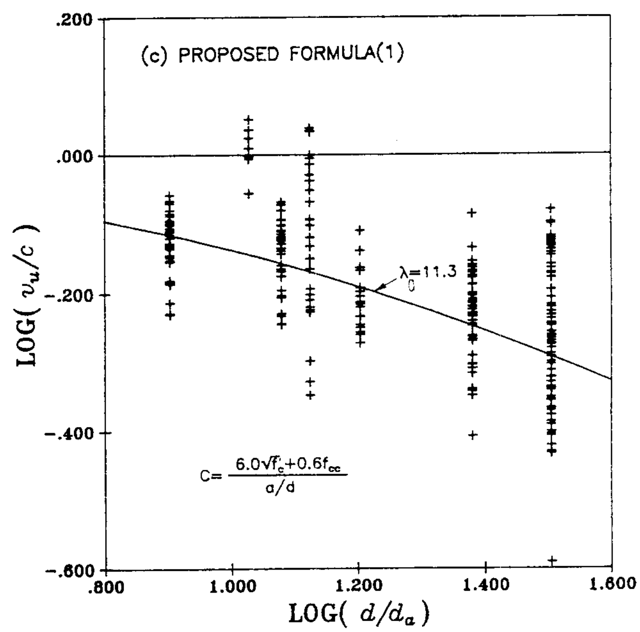

Fig. 6(c)—Same data as in Fig. 4 compared with formulas in different manner (c = C_l)

the bias caused by nonuniformity of data distribution. The problem is that various values of the relative size d/d_a are not evenly represented within the data set. The number of the data with d/d_a = 24 and 32 is much larger than that with d/d_a = 8, 10.67, 12, and 16. Consequently, in the complete data set the weight of the data with d/d_a = 24 and 32 is much larger than that of other data, which causes considerable bias in the optimized coefficient values. Therefore, certain data, notably those of Arthur, have been deleted from the complete data set, which caused coefficient λ_0 to change from 11.3 to about 25 (the exact optimum is 26.7 rather than 25, but the difference is unimportant). The latter value, which gives a weaker size effect, seems to be more reasonable for prestressed concrete practice. Moreover, the latter λ_0-value is much closer to that found for unprestressed longitudinally reinforced concrete beams.[13] Consequently, the following formula is proposed for mean v_u

$$v_u = \frac{4.9\sqrt{f'_c} + 0.54 f_{cc}}{a/d}\left(1 + \frac{d}{25\, d_a}\right)^{-1/2} \qquad (10)$$

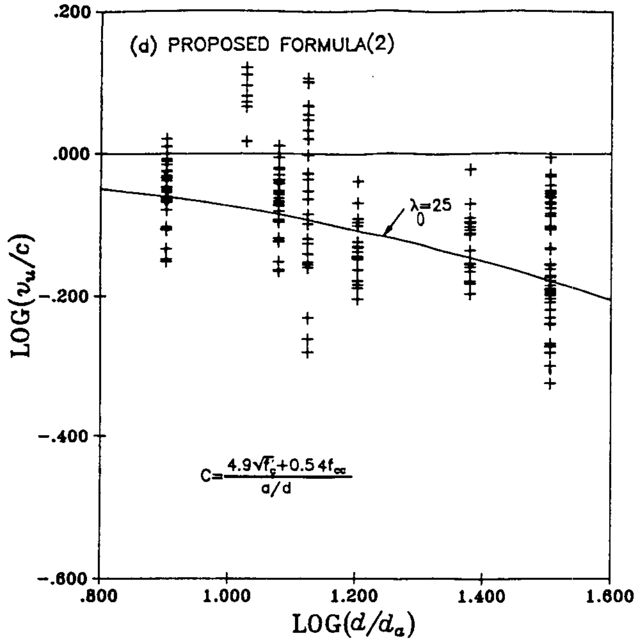

Fig. 6(d)—Same data as in Fig. 4 compared with formulas in different manner (c = C_l)

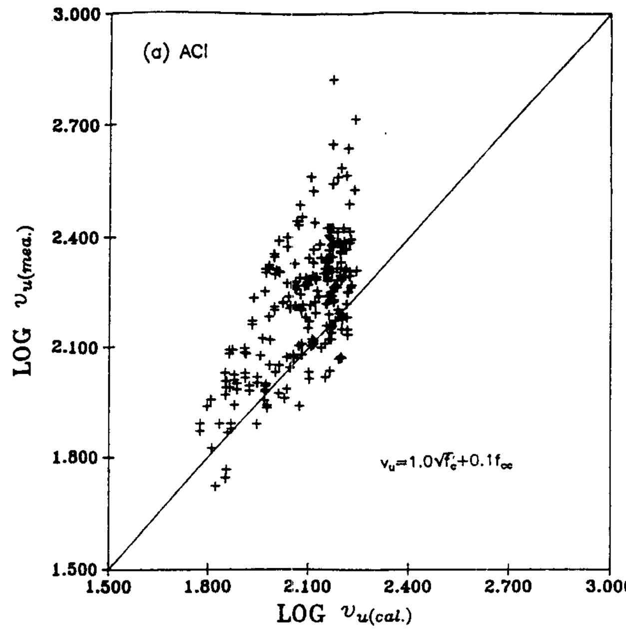

Fig. 7(a)—Comparison of the design formulas with the bulk of existing data

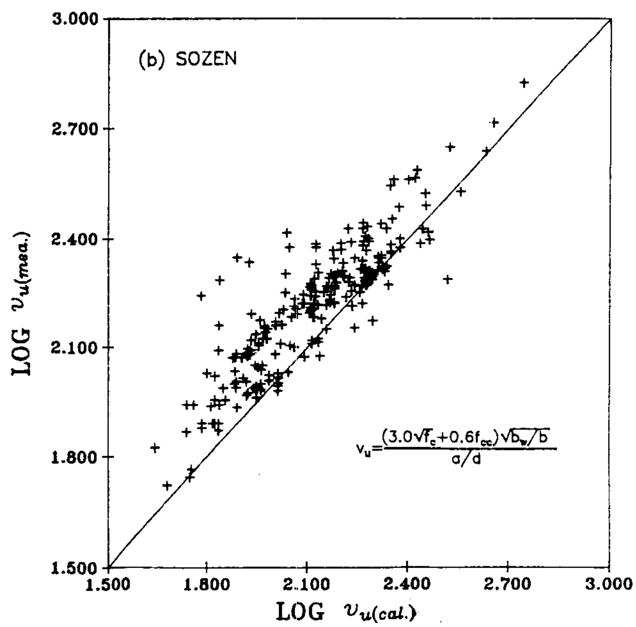

Fig. 7(b)—Comparison of the design formulas with the bulk of existing data

In the plot of measured versus theoretical values (Fig. 7), a perfect formula would yield a straight line through the origin of slope 1. In reality there is scatter, and for a design formula it is required that most of the data points lie above the straight line for the formula and only a few of them lie slightly below it. A formula that has this property, and is shown as the straight line in Fig. 7(d), is

$$v = \frac{4\sqrt{f'_c} + 0.4 f_{cc}}{a/d}\left(1 + \frac{d}{25\, d_a}\right)^{-1/2} \qquad (11)$$

Note that, unlike Fig. 7(a), no data points fall significantly below the proposed formula. Also note that the band of data points for this formula is somewhat narrower than that for the formula of Sozen, Zwoyer, and Siess shown in Fig. 7(b). Also note that coefficient 25 is the same as in the analogous formula for the size effect in nonprestressed longitudinally reinforced beams.[13] Eq. (11) is proposed for use in design practice.

CONCLUSIONS

1. Since the shear failure of prestressed concrete beams is a brittle failure caused primarily by tensile cracking of concrete rather than plastic yielding of steel, a size effect typical of fracture mechanics should, in theory, be observed. The size effect must be based on nonlinear fracture mechanics which takes into account the disperse nature of cracking.

2. The size effect law, previously derived by dimensional analysis of the progressive failure due to distributed microcracking,[12] appears to agree with the bulk of the test data available in the literature, and a new formula based on this size effect is therefore proposed for design. The size effect is the same as previously found for unprestressed longitudinally reinforced beams. The available test data agree neither with the strength (ultimate load) criterion, which exhibits no size effect and is implied in the present design formulations, nor with the linear elastic fracture mechanics, which exhibits too strong a size effect. The present size effect represents a gradual transition between the strength (ultimate load) criterion and the linear elastic fracture mechanics.

3. Since no test data reported in the literature involved a sufficient range of beam depths, the existing data must be analyzed collectively. This introduces large scatter due to comparing different concretes and

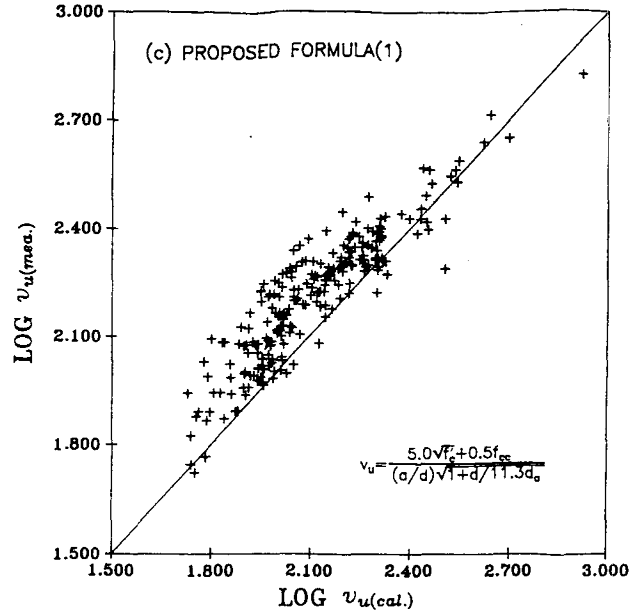

Fig. 7(c)—Comparison of the design formulas with the bulk of existing data

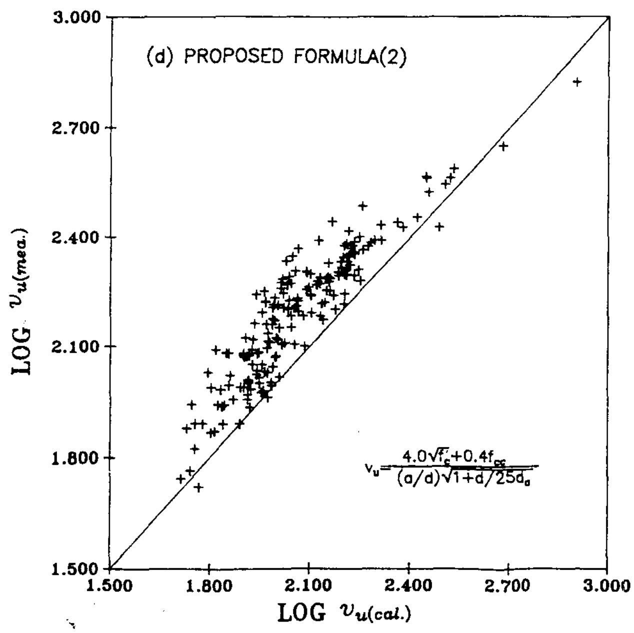

Fig. 7(d)—Comparison of the design formulas with the bulk of existing data

different beams produced in different laboratories. Although the size effect is apparent despite this large scatter, the evidence would be stronger if a sufficiently large data series with a broad range of beam sizes were obtained for one concrete in one and the same laboratory.

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