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Size Effect in Pullout Tests


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The results of tests of the pullout strength of reinforcing bars embedded in concrete are reported. The test specimens are 1.5, 3, and 6-in. cubes with geometrically similar bars. The results are found to be consistent with Bažant's size effect law for the nominal stress at softening failures due to distributed cracking. Based on the size effect law, an approximate formula predicting pullout strength is developed.

Keywords: bonding; ***cracking (fracturing); dimensional analysis***; embedment; failure; ***pullout tests***; reinforced concrete; ***reinforcing steels***; slippage; tests.

Although a number of investigators, e.g., Balarini, Shah, and Keer,[1] Stang and Shah,[2] Krenchel and Shah,[3] and Gerstle, Ingraffea, and Gergely,[4] have demonstrated the fracture mechanics aspects of pullout failure of reinforcing bars embedded in concrete, pullout strength is currently predicted on the basis of strength. The strength concept, however, is theoretically justified only for ductile failures. For brittle failures, in which the load decreases after reaching the maximum (as is the case for pullout failures), the strength concept (or limit analysis) is not justified theoretically.[5-7] The failure is ductile only if the load remains constant at increasing deformation after the ultimate state is reached; whereas, in brittle failures, the load decreases after the maximum.

The phenomenon of load decrease after the peak profoundly influences the behavior and safety margins of the structure. While for plastic behavior, geometrically similar structures of different sizes fail at the same nominal stress level, for brittle behavior the nominal stress at failure decreases as the size increases. This is caused by the fact that in the presence of softening (due to distributed cracking) the failure cannot be simultaneous but must occur through propagation of a failure zone across the structure, with one part of the cross section having already failed as another part is approaching its peak capacity. In a larger structure, this propagating, nonsimultaneous nature of failure is more pronounced, since a larger amount of energy is available to flow into the currently failing zone and thus help to drive the failure.

In previous studies,[6-15] the size effect has been analyzed for the diagonal shear failure of concrete beams with longitudinal reinforcement (both without and with stirrups), the diagonal shear failure of prestressed concrete beams, the torsional failure of beams, the punching shear failure of slabs, and the beam and ring failures of unreinforced pipes. In all these brittle failures, the size effect was clearly apparent and, as far as the experimental scatter permits it to be seen, was consistent with Bažant's[5-7,16] approximate size effect law for failures due to distributed cracking. The agreement was better for tests in which all the specimens were made of the same concrete and were geometrically similar (the punching shear[10] and torsion[14] tests at Northwestern University).

In pullout failure of reinforcing bars embedded in concrete, the existence of the size effect must clearly be expected, due to the brittle nature of these failures as well as the previously observed formation of cracks. The purpose of this study is to report the results of reduced-scale tests of microconcrete specimens designed to examine the applicability of the size effect law and develop an approximate prediction formula. A preliminary report on these tests was given at a recent symposium.[17]

REVIEW OF SIZE EFFECT LAW

In concrete structures, the size effect is intermediate between the linear elastic fracture mechanics, for which it is much too strong, and the plastic limit analysis, for which it is absent. As deduced by dimensional analysis and similitude arguments,[6,16,18] the following approximate size effect law appears to be applicable to brittle failure of heterogeneous materials including concrete (Fig. 1)

$$\sigma_N = B f_t' \phi(t), \qquad \phi(t) = (1 + \lambda/\lambda_0)^{-1/2} \quad (1)$$

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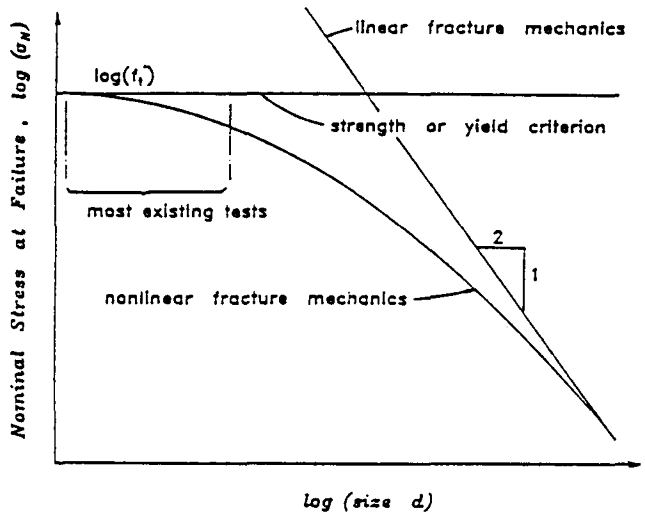

Fig. 1 — Bažant's size effect law[16]

in which f_t' = direct tensile strength of concrete, B and λ_0 = empirical constants depending on the shape of the structure, $\lambda = d/d_a$ relative structure size, $\sigma_N = P/d^2$ = nominal stress at failure for specimens similar in three dimensions[5] (which is the present case), P = maximum load of the structure, d = characteristic dimension of the structure, and d_a = maximum aggregate size. In the case of constant B and λ_0, Eq. (1) is applicable only to structures that are geometrically similar. The generalization to different geometrical shapes is possible if the dependence of B and λ_0 on the structure geometry is determined.

There are two simple limiting cases of Eq. (1). If the structure is very small, then λ/λ_0 is negligible compared to 1. Then σ_N is proportional to the strength f_t', which is the case of plastic limit analysis (or elastic allowable stress design). If the structure is very large, then 1 is negligible compared to λ/λ_0, in which case σ_N is inversely proportional to $\lambda^{1/2}$. This is typical of linear elastic fracture mechanics. While most laboratory tests carried out in the past are close to the plastic limit analysis case (negligible λ/λ_0), real structures are often in the transitional range between the limit of plastic limit analysis and elastic fracture mechanics.

The approximate size effect law for failures due to distributed cracking [Eq. (1)] was previously compared

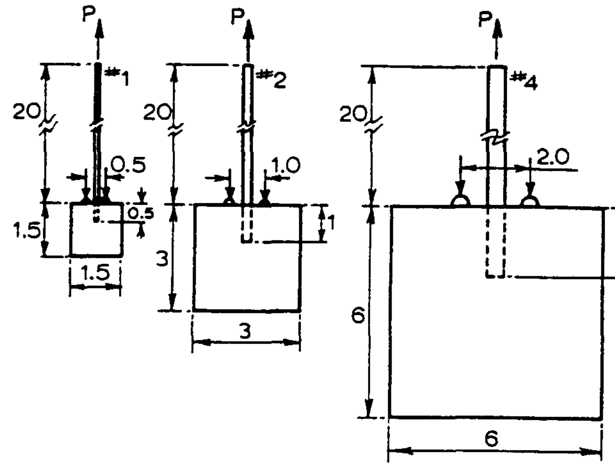

Fig. 2 — Specimen geometry (all dimensions are in inches)

to the available test data on the diagonal shear failure of longitudinally reinforced nonprestressed and prestressed beams[8,11,15] available in the literature, as well as the data on beam and ring failures of unreinforced pipes.[9] Furthermore, this law was verified by tests on punching shear failure of slabs[10] and torsional failure of beams of widely different sizes[14] made at Northwestern University.

Eq. (1) is strictly applicable only to structures made of the same concrete, which includes the same maximum aggregate size d_a. As an approximation, this law may apparently be used even when there are small differences in d_a. When there are large differences in d_a, a correction term must be introduced into Eq. (1).[7]

PULLOUT TESTS

The test specimen was a cube with a steel bar parallel to one edge of the cube and sticking out at the center of one face (Fig. 2 and 3). To determine the size effect, geometrically similar specimens with cube sides d = 1.5, 3, and 6 in. (38.1, 76.2, and 152.4 mm) were tested. Deformed reinforcing bars of yield strength 60,000 psi (414 MPa) and diameters 0.113, 0.25, and 0.5 in. (2.9, 6.4, and 12.7 mm) scaled in proportion to the cube size were used. The embedment lengths of the steel bars were 0.5, 1.0, and 2.0 in. (12.7, 25.4, and 50.8 mm). These lengths prevented the yielding of the bar before its pullout, as predicted by ACI 318-83[19] and verified by the tests.

All specimens (of all sizes) were cast from the same batch of concrete. The concrete mix ratio of water:cement:sand:gravel was 0.6:1:2:2 (all by weight). The maximum gravel size was d_a = 0.25 in. (6.4 mm), and the maximum sand grain size was 0.132 in. (3.35 mm). Mineralogically, the aggregate consisted of crushed limestone and siliceous river sand. The aggregate and sand were air dried prior to mixing. Portland cement 150 (ASTM Type I), with no admixtures or air entrainment, was used. Three companion cylinders 3 in. (76.2 mm) in diameter and 6 in. (152.4 mm) in length were cast from the concrete mix to determine the compression strength, whose mean was measured to be

Fig. 3 — Test specimens of three sizes after test (top left); failure mode of the largest specimen (top right); and test in progress (bottom)

f'_c = 6650 psi (45.8 MPa) after standard 28 day moist curing. The specimens were removed from their plywood forms 1 day after casting and were subsequently cured until the moment of the test, for 28 days, in a moist room of 95 percent relative humidity and a temperature of about 75 F (25 C).

All the specimens were tested in a 60 ton Baldwin frame modified as a servocontrolled closed-loop machine with an MTS electronic controller. The tests were displacement controlled. For the pullout tests, the free end of the embedded reinforcing bars was pulled from above by the edge grips of the machine. The cube was held down by a square sleeve made of split reinforcing bars, as shown in Fig. 2 and 3. The sides of these squares were 0.5, 1, and 2 in. (12.7, 25.4, and 50.8 mm) (measured to the axis of the split bar) for the 1.5, 3, and 6-in. (38.1, 76.2, and 152.4-mm) cubes, respectively. These square supports were glued to concrete by epoxy shortly before the test.

FORMULA FOR PULLOUT STRENGTH

Applying the regression analysis to the results from a large number of previously performed tests, Orangun, Jirsa, and Breen[20] developed, in 1977, an empirical bond strength formula that takes into account the effect of cover thickness, bar diameter, and embedment length. Their formula is based on the strength (or limit analysis) concept, thus ignoring the size effect. The formula reads

$$v_u = k_1\,(1.22 + 3.23\,C/d_b + 53 d_b/\ell_d)\sqrt{f'_c} \qquad (2)$$

in which v_u = theoretical 28 day bond strength in psi (1 psi = 6895 N), f'_c = 28 day concrete cylinder strength in psi, C = minimum clear cover of concrete in inches (1 in. = 2.54 cm), ℓ_d = embedment length in inches, d_b = nominal diameter of the bar in inches, and k_1 = an empirical nondimensional coefficient. According to the size effect law [Eq. (1)],[16] Eq. (2) should be modified in the case of very different sizes as follows

$$v_u = C_1 k_1 \left(1 + \frac{d_b}{\lambda_0 d_a}\right)^{-1/2}$$
$$C_1 = \left(1.22 + 3.23\,\frac{C}{d_b} + 53\,\frac{d_b}{\ell_d}\right)\sqrt{f'_c} \qquad (3)$$

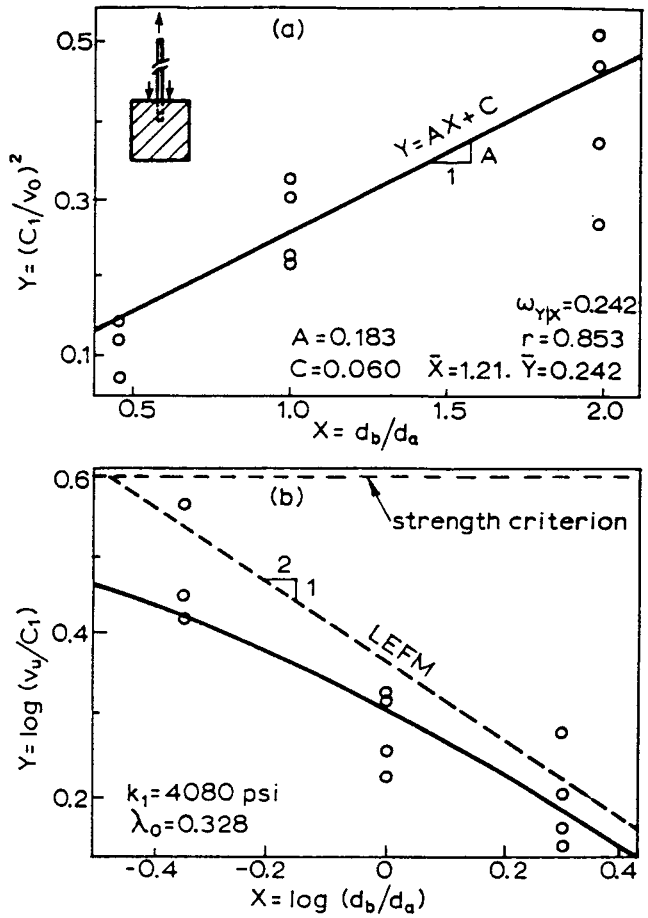

Fig. 4 — Measured size effect

This expression appears to yield better fits of the present test data than the expressions that use some other formulas instead of Eq. (2), e.g., the ACI 318-83 formula[19] or Aboona's formula,[21] which read $v_u = 9.5\sqrt{f_c'}/d_b$ and $v_u = 139 d_b \sqrt{f_c'}/\ell_d$. The nominal stress v_u at pullout failure is calculated from the equation

$$v_u = \frac{T}{s\ell_d} \tag{4}$$

in which T = tensile force in the bar and s = nominal surface area of the reinforcing bar embedded in concrete.

ANALYSIS OF PULLOUT TEST RESULTS

The size effect law [Eq. (1)] has the advantage that it can be algebraically rearranged to a linear form that makes linear regression possible

$$\frac{C_1^2}{v_u^2} = \frac{1}{k_1^2} + \frac{1}{\lambda_0 k_1^2}\frac{d_b}{d_a} \tag{5}$$

Plotting C_1^2/v_u^2 versus the relative size d_b/d_a, the data should ideally fall on a straight line of slope $1/k_1^2\lambda_0$ and vertical axis intercept $1/k_1^2$. The vertical deviations from this straight line, shown in Fig. 4(a), represent statistical errors.

The test results are summarized in Table 1. The regression plot is shown in Fig. 4 on the left, and the corresponding plot of $\log(v_u/C_1)$ versus $\log(d_b/d_a)$ is shown in Fig. 4 on the right. The parameters of the optimum fits are indicated in the figure, along with the coefficient of variation of the deviations from the regression line $\omega_{Y|X}$, the correlation coefficient r, and the coordinates X and Y of the data centroid.

Table 1 — Pullout test results

Beam No.	d_b, in.	l_d, in.	d_a, in.	f_c', psi	P_u, lb
A1	0.5	2.0	0.25	6650	5040
A2	0.5	2.0	0.25	6650	4801
A3	0.5	2.0	0.25	6650	5592
A4	0.5	2.0	0.25	6650	6624
A5	0.25	1.0	0.25	6650	1512
A6	0.25	1.0	0.25	6650	1800
A7	0.25	1.0	0.25	6650	1836
A8	0.25	1.0	0.25	6650	1560
A9	0.113	0.5	0.25	6650	502
A10	0.113	0.5	0.25	6650	468
A11	0.113	0.5	0.25	6650	656

Despite the scatter of test results, which is not abnormal for a material such as concrete, the plot in Fig. 4 confirms the existence of the size effect and shows that the size effect can indeed be approximated by the size effect law. The strength concentrations at the surface of the embedded bar in a plane concrete specimen are probably the reason that the scatter is larger than in other types of tests of reinforced concrete.

FAILURE MECHANISM

The failure occurred in two different modes. One mode consisted of splitting of the concrete surrounding the bar, and the other mode consisted of shearing of the reinforcement against the surrounding concrete. These types of failure are well known from previous studies.[18, 22]

The splitting failure is caused by the wedging action of the lugs on the bars. The wedging produces confining pressure from the surrounding concrete and is balanced by circumferential tensile stresses around the bar. These stresses cause formation of radial splitting cracks that lead to a sudden loss of bond resistance.[18]

The shearing failure occurs after the reinforcement lugs shear or crush the concrete in front of the lug, thus making a pullout along a cylindrical frictional surface possible. The splitting failure is obviously fracture dominated. Different though it might seem at first, the shearing failure is also of fracture mechanics type since it is propagating and progressive. The shearing failure starts from the loaded end and then propagates toward the free end as one lug after another shears or crushes the concrete in front of the lug. After the shearing has progressed over the entire length of embedment of the bar, the force drops and then the remaining pullout is resisted only by friction, which is nonsoftening (plastic) in nature but occurs at a force lower than its previous maximum. Nevertheless, due to the law of friction, the shearing failure is much less abrupt than the splitting failure,[18] which is almost purely of fracture mechanics type.

In the present test series, the splitting failures generally occurred in the medium and large specimens, while the shearing failures usually occurred in the small specimens. This behavior agrees with what one would expect according to the size effect law. This indicates that the behavior of a small specimen of a certain geometry should be closer to plastic limit analysis, while the behavior of a larger specimen should be closer to linear elastic fracture mechanics. A similar change of the failure type with increasing size was previously observed at Northwestern University in punching shear tests of slabs.[10]

CONCLUSIONS

1. The present test results on pullout of reinforcing bars from concrete confirms that a size effect is present, i.e., the nominal shear bond stress at failure decreases as the specimen size increases.

2. Although the scatter of the test results does not make it possible to verify the precise form of the size effect, it nevertheless appears that the results are consistent with Bažant's approximate size effect law for failures due to distributed cracking, as should be theoretically expected according to the known failure mechanism.

3. The experiments indicate that larger specimens, with larger bars, tend to fail in a more brittle, splitting mode, while smaller specimens, with smaller bars, tend to fail in a less brittle or more plastic shear-pullout mode. This transition in the type of failure as a function of specimen size is in agreement with the physical implications of the size effect law and supports its applicability.

4. In view of the limited scope and range of the present tests, further tests that would cover a broader size range and would use larger aggregate and bars are needed.

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