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IDENTIFICATION OF STRAIN-SOFTENING CONSTITUTIVE RELATION FROM UNIAXIAL TESTS BY SERIES COUPLING MODEL FOR LOCALIZATION

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ABSTRACT

Recent studies of path stability in inelastic bifurcation problems have shown that, even if the specimen is stable, strain softening in uniaxial tests must localize right after the peak stress state into the smallest length permitted by the material, which is approximately equal to the characteristic length of nonlocal continuum. Without knowing this length, the uniaxial stress-strain relation cannot be identified from uniaxial test data. The post-peak stress-strain relation is analyzed on the basis of a series coupling hypotheses. Van Mier's uniaxial compression test results for specimens of different lengths show this hypothesis to be valid.

Introduction

Identification of the stress-strain relation from tests of specimens whose load-deflection diagram exhibits post-peak softening is not an easy problem, and considerable debate has recently arisen on this subject. Strain softening causes bifurcation of the equilibrium response path, such that a state of uniform strain is no longer the only equilibrium solution. There exists another response path in which strain softening localizes into a certain zone within the specimen while the rest of the specimen undergoes unloading. Normally, the localized response path is the stable path, i.e., the path which must actually occur (1,2). A general three-dimensional analysis of such localizations is a difficult problem; however for many practical situations one can assume a series coupling model for which the analysis is relatively easy.

In the series coupling model, one assumes that the zone of continued loading (strain softening) is coupled in series to the zone of unloading (Fig. 1a), such that the forces carried by both zones are equal and their deformations are superimposed. The load-deformation curve in Fig. 1b on the left characterizes the softening zone alone, the curve in the middle of Fig. 1b characterizes the unloading path from the peak stress point, and the curve on the right characterizes the response of the entire specimen. Thermodynamic analysis showed (1,2) that localization of strain softening in the

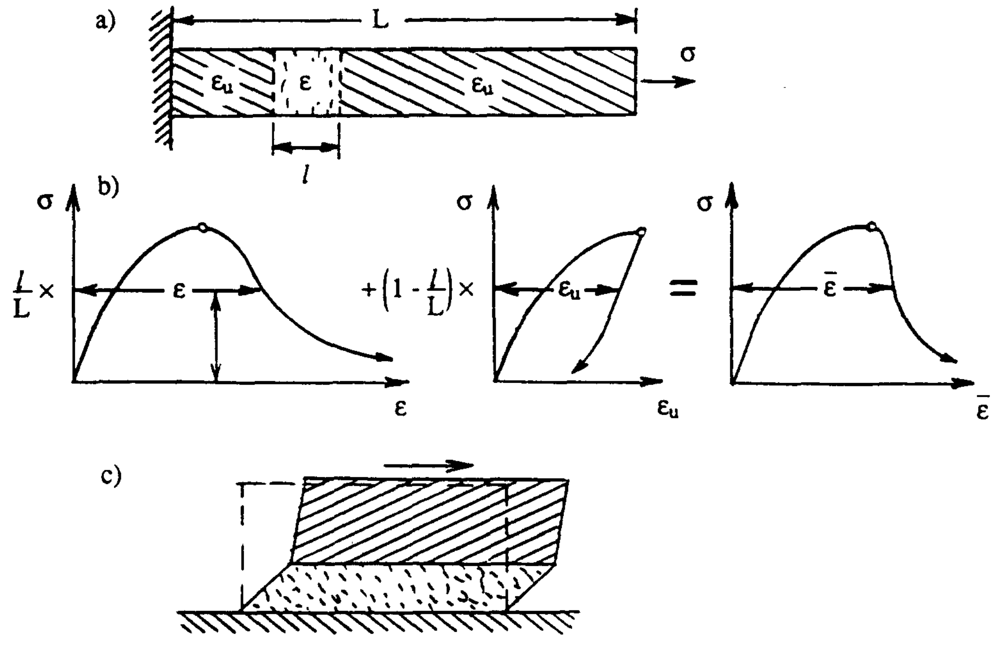

Fig. 1 Series coupling model for the effect of structure size on the post-peak load-deformation diagram.

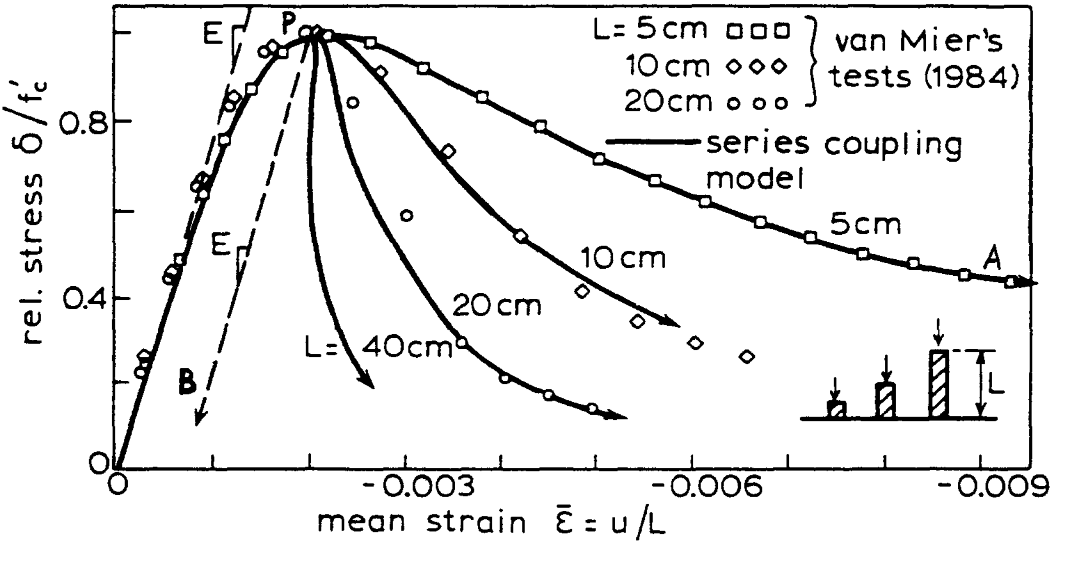

Fig. 2 Optimum fit of van Mier's uniaxial compression test results for specimens of different sizes by a series-coupling model with localized strain-softening zone.

series coupling model must begin right at the peak-stress state. The purpose of this paper is to show some implications of the series coupling model and check its validity.

Analysis of Uniaxial Tests Based on Series Coupling Hypothesis

In experiments, we are interested to find the actual strain, ε, in the strain-softening region, but we measure only the overall mean strain, $\bar{\varepsilon}$, on the basis of the displacements. Suppose the measurements are made on two specimens of the same cross section but different lengths L_1 and L_2. For the same chosen stress value σ, the corresponding post-peak mean strains are $\bar{\varepsilon}_1(\sigma)$ and $\bar{\varepsilon}_2(\sigma)$. According to the series coupling hypothesis, the compatibility conditions are

$$\ell\varepsilon(\sigma) + (L_1 - \ell)\varepsilon_u(\sigma) = L_1\bar{\varepsilon}_1(\sigma), \quad \ell\varepsilon(\sigma) + (L_2 - \ell)\varepsilon_u(\sigma) = L_2\bar{\varepsilon}_2(\sigma) \tag{1}$$

where $\varepsilon(\sigma)$ is the true strain at stress σ inside the strain-softening zone (Fig. 1b left), which we want to find; ε_u is the strain outside the strain-softening region in which the material unloads along an unloading branch emanating from the peak stress point (Fig. 1b middle); and ℓ = length of the strain-localization zone, which approximately coincides with the characteristic length of the material (5) and may therefore be considered constant and the same for any specimen length not less than ℓ.

For any given σ, Eqs. 1 could be regarded as a system of two linear equations for two unknowns $X = \ell$ and $Y = \ell\varepsilon(\sigma)$. Unfortunately, though, the matrix of this equation system is singular, i.e., the determinant is zero. Consequently it is impossible to determine ℓ from these test results. The value of ℓ must be found by some other suitable type of test. A rather simple test which yields ℓ was presented in Ref. 5.

If ℓ is known, strain ε may be solved from one of Eqs. 1. For specimens of length L_i we get

$$\varepsilon(\sigma) = \frac{1}{\ell}[L_i\bar{\varepsilon}_i(\sigma) - (L_i - \ell)\varepsilon_u(\sigma)]. \tag{2}$$

If the series coupling model were exact and the material exhibited no random scatter, Eq. 2 would have to yield the same result for any specimen length L_i. Since this is not so, one must take an average of the values obtained for different length L_i $(i = 1,\dots,n)$, i.e.

$$\varepsilon(\sigma) = \frac{1}{n}\sum_{i=1}^{n} \varepsilon_i, \quad \varepsilon_i = \frac{1}{\ell}[L_i\bar{\varepsilon}_i(\sigma) - (L_i - \ell)\varepsilon_u(\sigma)]. \tag{3}$$

One can also calculate the unbiased estimate of the corresponding coefficient of variation:

$$\omega(\sigma) = \{\frac{1}{n-1}\sum_{i=1}^{n}[\varepsilon_i - \varepsilon(\sigma)]^2\}^{\frac{1}{2}}. \tag{4}$$

Obtaining $\varepsilon(\sigma)$ for various stress levels $\sigma = \sigma_k$ $(k = 1,2,\dots,N)$, one may get the overall coefficient of variation

$$\omega = \{\frac{1}{N}\sum_{k=1}^{N}[\omega(\sigma_k)]^2\}^{\frac{1}{2}}. \tag{5}$$

If tests with significantly different sizes L_i are available, these equations may be used to check whether the series coupling model is valid. If it is, and if ℓ is known, then Eq. 3 may be used to obtain the strain-softening constitutive relation. The series coupling model may be assumed to be valid if the coefficient of variation, ω, is not significantly larger than the coefficient of variation that is inevitably observed in tests of many identical concrete specimens. The latter is typically 5 to 10%.

Consequences of van Mier's Data and Discussion

At present there exist various test data (for both uniaxial tension and uniaxial compression) to which the foregoing equations could be applied. We choose the recent compression test results of van Mier (3,4), which were obtained under very carefully controlled conditions and include a sufficient range of sizes. These results are shown as the data points in Fig. 2. The specimens have the same cross section and lengths L_1 = 5 cm, L_2 = 10 cm and L_3 = 20 cm.

To fit the data it has been assumed that ℓ = 5 cm, which is a value that roughly agrees with the results of Ref. 5 (note, however, that any value of ℓ between 0 and 5 cm would yield the same optimum fits). The optimum fits that correspond to the expression $\bar{\varepsilon}_1(\sigma)L_i = \ell\varepsilon(\sigma) + (L_i - \ell)\varepsilon_u(\sigma)$ according to Eq. 2 are shown as the solid curves, and the unloading diagram assumed for the calculation is shown as the dashed line.

Fig. 2 also shows the response curve which would be theoretically predicted for L_4 = 40 cm. This curve reverses its downward slope from negative to positive, which means that such a specimen could not be tested because it would exhibit snapback instability even if the loading frame were infinitely stiff. (The other curves shown would indicate snapback instability only if the stiffness of the loading frame were less than L_i-times the magnitude of the maximum downward slope of the $\sigma(\bar{\varepsilon}_i)$ diagram; however, the stiffness of the testing frame can have no effect on the $\sigma(\bar{\varepsilon})$ diagram obtained, provided the series coupling hypothesis is valid.)

Now an important observation from Fig. 2 is that the data fit is very close. In fact, the overall coefficient of variation (Eq. 5) is only ω = 3.7%, which is surprisingly low. So the series coupling model is verified by these tests very well.

Instead of measuring specimens of various lengths, it is also possible to test only one very long test specimen and take measurements on various gage lengths, L_i (6). However, one must be sure that no strain-softening takes place outside any of the gage lengths. This is not easy to achieve.

During the 1960's and 1970's, many attempts were made to measure the so-called "complete" stress-strain curve in compression or tension (see, e.g., the review in Ref. 7). Although stiff loading frames were used to prevent instability, it is virtually certain that localization must have occurred in these tests. The frame stiffness controls stability of equilibrium but has no effect on the onset of stable localization (1,2). These old data could now be reevaluated to obtain the correct $\varepsilon(\sigma)$ diagram, but an estimate for ℓ for the concrete used would have to be made first.

As an extreme case of the present analysis, one might consider that $\ell \to 0$, in which case $\varepsilon \to \infty$ but $v = \ell\varepsilon$ = finite. Eq. 2 then yields the diagram of stress vs. opening displacement v for a line crack model (such as Hillerborg's); $v(\sigma) = \ell\varepsilon(\sigma) = L_i\bar{\varepsilon}_i(\sigma) - (L_i - \ell)\varepsilon_u(\sigma)$. Eqs. 3 - 5 may be adapted similarly.

The present method can be also applied to shear deformations in direct shear tests or torsional tests of hollow cylinders. As Fig. 1c illustrates, the series coupling model may again be applied.

Before closing, it must be admitted that a uniaxial analysis of localization in tension or compression (as well as shear) is too crude a simplification in general. Triaxial action may cause the effective value of ℓ to depend on the cross section size and shape, especially in compression. Compression softening is associated with volume expansion of the material due to axial splitting microcracks and cracks, or dilatant slip on an inclined shear band, or both in combination. A general realistic model must therefore be triaxial. But then the analysis is inevitably much more complicated than the present one; see, e.g., an elegant and rigorous recent study by Ortiz (8).

Van Mier's data nevertheless confirm the simple uniaxial series coupling model to be adequate with respect to the effect of specimen length. A more general but still simple model might be a combination of series and parallel coupling (9). Such a model might be necessary when both the length and width of the specimen are varied.

Conclusions

1. Van Mier's test data (3,4) indicate that series coupling is an acceptable hypothesis for the effect of specimen length on the post-peak diagram of stress vs. mean strain. This further justifies the use of the series coupling model in stability analysis of post-peak softening of uniaxially loaded specimens or structural elements.

2. The uniaxial stress-strain relation for strain softening can be identified from uniaxial tests only if the characteristic length of the material is determined by other means (5).

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