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# Improved prediction model for time-dependent deformations of concrete: Part 5 – Cyclic load and cyclic humidity

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*Cyclic variation of stress as well as environmental humidity causes an increase of creep. In this part of the present series, the prediction formulae presented in the original BP model are now adapted to the present improved creep prediction model. The increase of creep due to stress cycling is described as a deviation from the prediction according to the principle of superposition, and therefore is a non-linear phenomenon. The increase of creep due to humidity cycling is partially formulated on the basis of diffusion theory considerations. Comparisons with test data show an acceptable agreement.*

## 1. INTRODUCTION

When the principle of superposition is used to predict creep at variable stress, the creep under cyclic stress of many repetitions is significantly under-predicted. This part of the present series [1–4] presents modifications to the preceding formulae which are required to describe such situations. These modifications can be of importance for structures such as bridges carrying high traffic loads, structures carrying heavy vibrating machinery, ocean oil platforms and other structures.

We now consider uniaxial cyclic stress described as $\sigma = \sigma_0 + \frac{1}{2}\Delta \sin(2\pi\omega t)$ where $\sigma_0$ = mean stress, $\frac{1}{2}\Delta$ = cyclic stress amplitude, $\omega$ = circular frequency (we write the sine function only for convenience, since cyclic creep does not depend much on the shape of the time curve within the cycle). According to the principle of superposition, the creep after many cycles should be approximately equal to the creep at constant stress, equal to the mean value $\sigma_0$. Therefore, the cyclic creep is properly defined as the creep in excess of the creep caused by sustained constant stress $\sigma_0$. The excess creep is properly measured in each cycle at the point $\sigma = \sigma_0$. According to this definition, it appears that the cyclic component of load generally gives an increase of creep, for both sealed and drying specimens.

In what follows, the creep modification due to cyclic stress is expressed in the same manner as in the original BP model [5] but is now applied to the present form of the compliance function.

Like cyclic stress, the cyclic component of environmental relative humidity also causes an increase of creep. The original BP model was extended to cover this effect by Bažant and Wang [6], using a formulation which was partly empirical, and partly based on diffusion theory. An adaptation of this formulation is now presented in the present creep prediction model.



## 2. PROPOSED FORMULAE FOR CYCLIC BASIC CREEP

The compliance function representing the values of $\varepsilon/\sigma_0$, where $\varepsilon$ is the strain at the mid-points of cycles (i.e. at $\sigma = \sigma_0$), is calculated as follows:

$$J(t, t', \sigma) = q_1 + F(\sigma)C_{0c}(t, t') \tag{1}$$

$$C_{0c}(t, t') = q_2 Q(t_c, t') + q_3 \ln[1 + (t_c - t')] + q_4 \ln\left(\frac{t_c}{t'}\right) \tag{2}$$

in which

$$t_c = (t - t')[1 + k_\omega \omega \Delta^2 F^3(\sigma_{\max})] + t' \tag{3}$$

Here $\omega$ = frequency (Hz), $k_\omega = 15$ = empirical constant, and the function $F(\sigma_{\max})$ is the same as in Equations 4 of Part 2 [2], except that the value $f'_c$ must now be replaced by specimen strength, $f'$, at the age of loading.

Equation 7 reflects the fact that the shape of the creep curve under cyclic stress is about the same as the shape of the basic creep curve, and the effect of stress cycling consists essentially of time acceleration.

## 3. PROPOSED PREDICTION FORMULAE FOR CYCLIC DRYING CREEP

The acceleration of the creep due to drying caused by the cyclic stress component, which is manifested as excess creep, appears to be much smaller than the acceleration of basic creep. Therefore different formulae are required, as shown by Bažant and Panula [5]:

$$J(t, t', \sigma) = q_1 + F(\sigma)[C_{0c}(t, t') + C_d(t_{dc}, t', t_0) + C_p(t_{dc}, t', t_0)] \tag{4}$$

in which $t_{dc}$ may be calculated as

$$t_{dc} = t' + (t - t')[1 + 10\omega^{1/4}\Delta^2 F^3(\sigma_{\max})] \tag{5}$$

in which $C_d(t, t', t_0)$ is given by Equation 2 of Part 3 [3] and $C_p(t, t', t_0)$ is the same as in Equation 4 of Part 3. The function $F(\sigma_{max})$ is defined by Equations 4 of Part 2 [2] in which, however, $\sigma/f'$ is replaced by $\sigma_{max}/f'$.

Due to the scarcity and limited scope of the existing test data for drying creep under cyclic stress, the evidence for Equation 5 is not very strong.

## 4. COMPARISON WITH TEST DATA

For the effect of stress cycling on basic creep, the data of Whaley and Neville [7] (all obtained for the frequency of 585 cycles $\min^{-1}$) are the only comprehensive data set available. Their comparisons with the present prediction formulae are shown in Fig. 1. Because the initial elastic strains were not reported, they had to be assumed, and so the comparisons are relevant only to the part of strain representing the creep increase due to stress cycling. The initial elastic strains (corresponding to $\sigma_0$ rather than $\sigma_{max}$) were assumed on the basis of the stress–strain relation of Wang *et al.* [8] (see Appendix).

For the effect of stress cycling on drying creep, five data sets from the literature [9–13] were used. Comparisons of the present formulae with these data are shown in Fig. 2. Since the strains at the mid-points of cycles ($\sigma_0$) were not reported in most cases, the stress peaks $\sigma_{max}$ were used in these comparisons. Thus, Fig. 2 gives the values of $\varepsilon_{max}/\sigma_0$, where $\varepsilon_{max}$ is the strain at the peak stress points ($\sigma = \sigma_{max}$), which is predicted as

$$J(t, t', \sigma)_{max} = J(t, t', \sigma) + \frac{\sigma_{max}}{E_{dyn}(\sigma_{max})\sigma_0} - \frac{1}{E_{dyn}(\sigma_0)} \tag{6}$$

Here $E_{dyn}$, representing the dynamic modulus, is assumed to be approximately equal to the slope of the cyclic stress–strain diagram. Approximately, $1/E_{dyn} \approx J(t, t', \sigma)/1.2$ for $t - t' = 0.1$ day. The values of $\varepsilon_{max}$ that yield the compliance function in Equation (6) have been normalized with respect to $\sigma_0$ rather than $\sigma_{max}$, because the creep under a constant stress equal to $\sigma_{max}$ is not equal to the creep predicted according to the principle of superposition.

In Fig. 2, the data of Suter and Mickleborough [12,13] show quite different values despite the use of a similar concrete. Perhaps this is just random scatter. However, the predictions would agree well with the basic trends of these data if the static drying creep were optimized (rather than predicted from composition and strength).

The test data used included various load frequencies

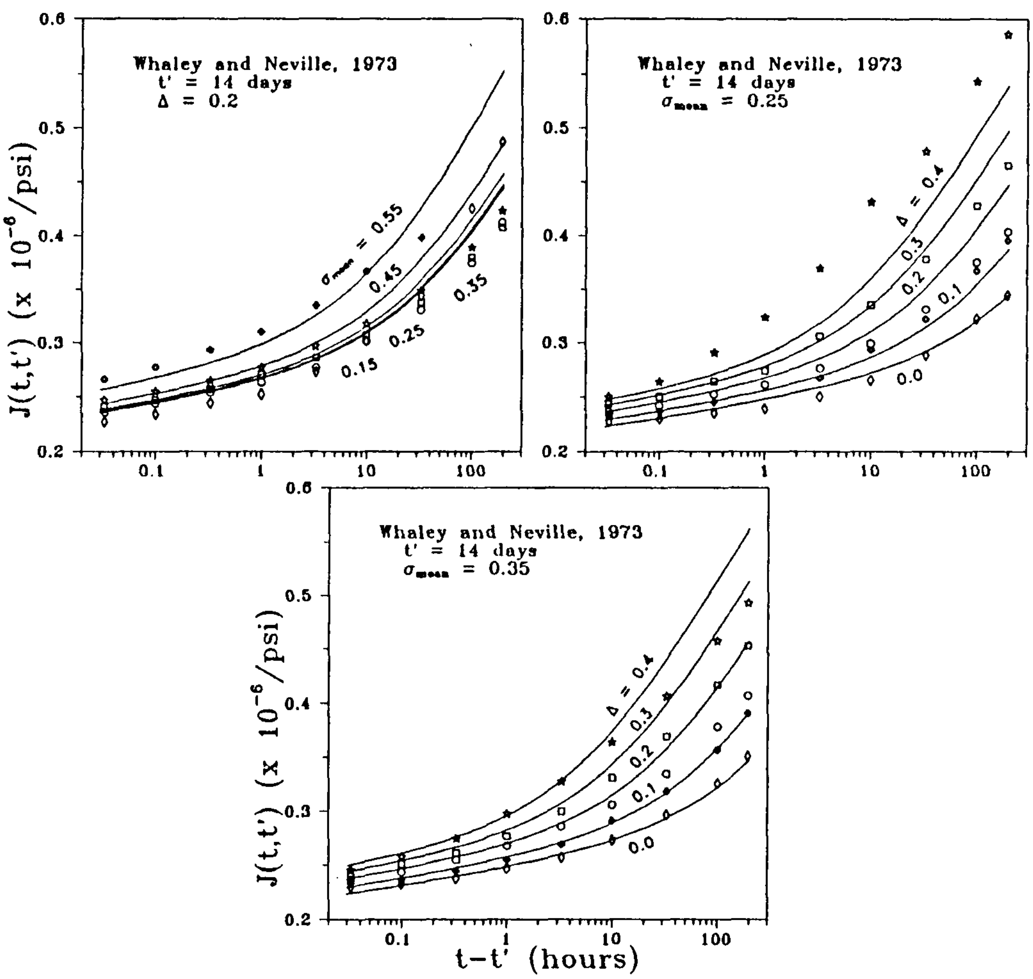


Fig. 1 Predictions of cyclic basic creep and data by Whaley and Neville [7].

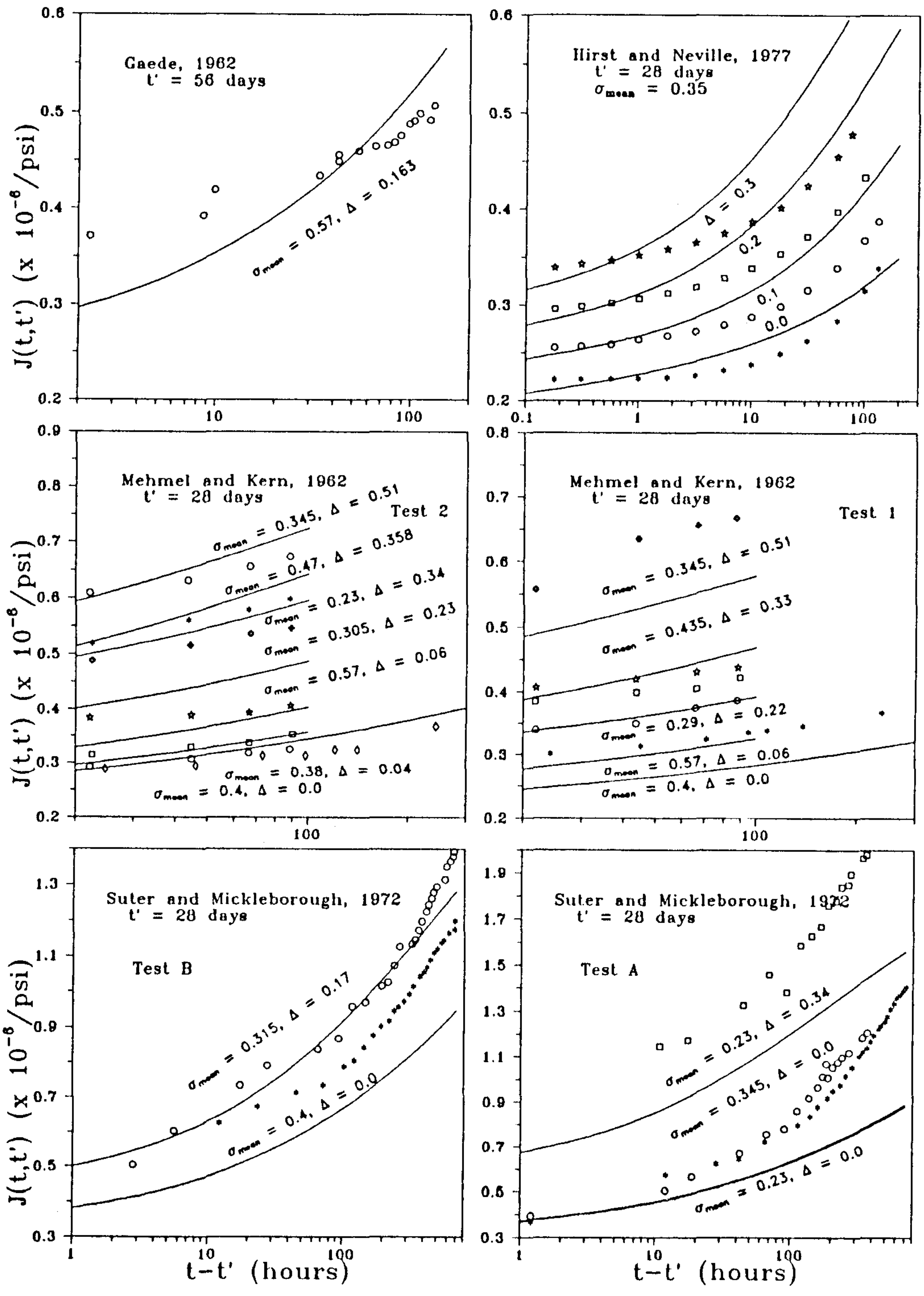


Fig. 2 Predictions of cyclic drying creep tests by Gaede [9], Hirst and Neville [10], Mehmel and Kern [11], and Suter and Mickleborough [12,13].

ranging from 380 cycles $\min^{-1}$ [11] to 3000 cycles $\min^{-1}$ [10]. These data provided the basic information for determining the coefficient $k_\omega$ (Equation 3). The values of $J(t, t', \sigma)$ at 20 cycles of Mehmel and Kern's tests [11] were excluded from analysis since they appeared to be inconsistent, being much too high. Some data of Gaede [9] were also excluded due to high scatter and probable lack of precise humidity control (for which the range 55–70% was indicated). These data involved relatively short test periods ($t - t' \leq 4$ days). In the data of Hirst and Neville [10], the initial elastic strain at $\sigma_{max}$ was not reported and had to be assumed (see Appendix).

Table 1 gives the values of the coefficient of variation of the deviations of the predictions from the hand-smoothed measured creep curves. The overall coefficient of variation is also listed.

Table 1 Coefficients of variation for deviations of formulae from hand-smoothed data

| Test data | $\bar{\omega}$ |
|---|---|
| *Cyclic basic creep* | |
| Whaley and Neville (a) [7] | 6.6 |
| Whaley and Neville (b) [7] | 6.3 |
| Whaley and Neville (c) [7] | 6.7 |
| All data[a] | $\bar{\omega}_{all} = 6.5$ |
| *Cyclic drying creep* | |
| Gaede [9] | 11.9 |
| Hirst and Neville [10] | 11.9 |
| Mehmel and Kern [11] | 10.7 |
| Suter and Mickleborough [12,13] | 24.8 |
| All data | $\bar{\omega}_{all} = 15.9$ |

[a] (a): $\sigma_{mean} = 0.35f'_c$, (b): $\sigma_{mean} = 0.25f'_c$, (c): $\Delta = 0.2f'_c$.

## 5. EFFECT OF ENVIRONMENTAL HUMIDITY CYCLING

As shown previously [6], the correction of creep prediction equations to take into account the effect of cyclic component of environmental humidity can be partly derived from diffusion theory. It is found that the correction must depend on the cross-section thickness as well as the period of environmental humidity. This dependence can be introduced by means of the well-known expressions for the drying penetration depth [14], and must be such that the correction disappears for a very thick cross-section, for a very short fluctuation period, and for a very small humidity amplitude. To take this phenomenon into account, the drying creep term $C_d(t, t', t_0)$ must be magnified applying the correction factor $\kappa$ as follows:

$$J(t, t', \sigma) = q_1 + F(\sigma)[C_0(t, t') + \kappa C_d(t, t', t_0) + C_p(t, t', t_0)] \quad (7)$$

According to diffusion theory [6], the correction must depend on the drying penetration depth calculated as

$$D_p \approx (6C_1 T_h)^{1/2} \quad (8)$$

in which $T_h$ is the period of humidity cycles, and $C_1$ is the drying diffusivity of concrete at the start of exposure to the environment. In the absence of any information on its value, one may assume $C_1$ to be approximately equal to 0.1 cm$^2$ day$^{-1}$. Equation 8 is true despite the dependence of diffusivity on pore relative humidity (see Part [1]) but it neglects the dependence of diffusivity on the age of concrete. As shown [6],

$$\kappa = 1 + \kappa_1(\Delta_h) \frac{D_p}{D_p + 0.5D} \quad (9)$$

in which $\kappa_1(\Delta_h)$ is a certain empirical function of the amplitude $\Delta_h$ of environmental relative humidity $h$. It was shown that this function may be approximately expressed as [6]

$$\kappa_1 = 2.5\Delta_h(1 - e^{-(t-t')/10})(1 - e^{-T_h/5}) \quad (10)$$

in which $t$, $t'$ and $T_h$ must be given in days. The dependence on the load duration $t - t'$ is transitional, having influence only at the beginning of exposure. For load durations $t - t' \gg 10$ days, one has $\kappa_1 \approx 2.5\Delta_h(1 - e^{-T_h/5})$. If the period of humidity cycles, $T_h$, is longer than about 20 days, $\kappa_1$ becomes approximately a constant.

The experimental results on the effect of cyclic humidity are rather scanty. The dashed lines in Fig. 3 show comparisons of the present prediction formulae with three data sets, namely those of Bernhardt [15, 16], Al-Alusi *et al.* [17] and Hansen [18]. Given that these are predictions, not optimum fits, the agreement is acceptable. The solid lines in these figures show predictions when the creep strains are adjusted by a constant multiplicative factor such that the initial measured strain is fitted accurately. These lines fit the measured data quite well; this demonstrates that the error of the dashed lines is due merely to an error in predicting the basic creep, while the effect of humidity cycles is captured satisfactorily.

In data fitting, the greatest emphasis has been placed on the data of Bernhardt [15,16] which are most comprehensive. Bernhardt conducted his tests at four different stress levels up to 0.372 of the compression strength. The present analysis employs the average of the compliances measured for these stress levels. Al-Alusi *et al.* [17] used two different types of strain gauge which showed considerable discrepancies. Their average is used in the present analysis. While Bernhardt and Al-Alusi *et al.* measured creep under uniaxial compression, Hansen [18] measured bending creep deflections. To evaluate these data, the creep compliance was assumed to be the same at all the cross-sections of the beam and on the tensile and compression sides of the cross-section. The fit of these data is poor, which is probably due to the fact that for bending the compliance expression should in principle be rather different from that for axial loading (which is one factor that is neglected in the entire present series of papers since a great increase in complexity would otherwise arise).

As for the influence of humidity cycling on shrinkage, no significant trend has been detected in the existing experimental results as reported by Al-Alusi *et al.* [17] and Hansen [18].

## APPENDIX: BASIC INFORMATION ON THE TESTS USED

*Whaley and Neville* [7] – Cyclic basic creep. Specimens 76 mm × 76 mm × 203 mm cast vertically, fog-cured for 14 days at 20 ± 1°C. During the test the specimens were enclosed in polyethylene bags containing some water but water was not in direct contact with the specimen. The relative humidity of air in the bag surrounding the specimen was ≥98%, temperature 20 ± 2°C. The cyclic load varied sinusoidally at 585 cycles min$^{-1}$. Elastic strains were assumed to be 0.218, 0.221, 0.224, 0.230, 0.236 × 10$^{-6}$ psi$^{-1}$ (3.16, 3.20, 3.25, 3.34, 3.42 × 10$^{-11}$ Pa$^{-1}$) for $\sigma_{mean}$ = 0.25, 0.35, 0.45, 0.55, 0.65 × $f'$. 14-day strength of prisms (76 mm × 76 mm × 203 mm) = 39 N mm$^{-2}$; rapid-hardening Portland cement; max. size

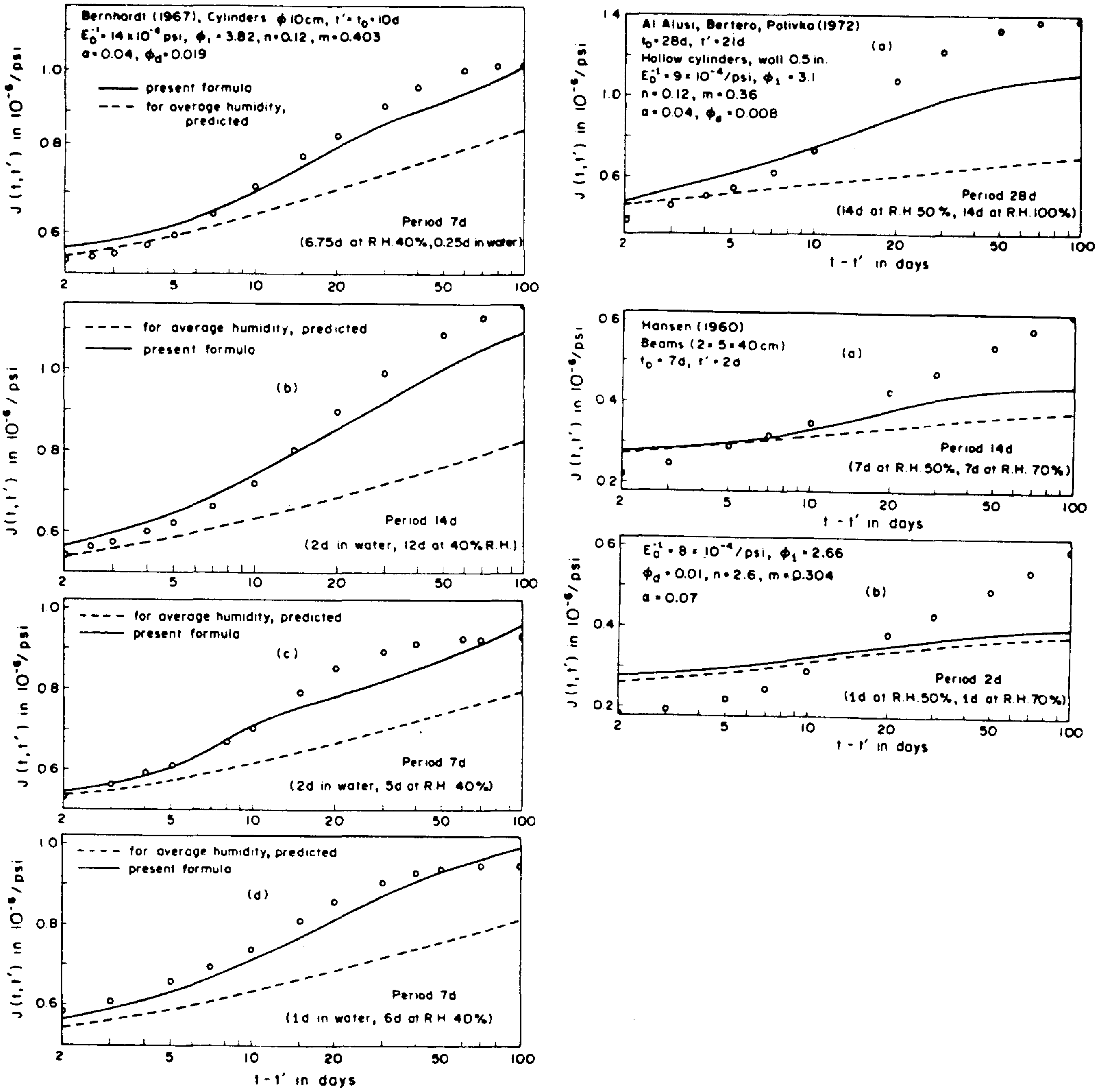


Fig. 3 Predictions (dashed lines) and adjusted predictions (solid lines) of tests of creep under cyclic humidity by Bernhardt [15,16], Al-Alusi *et al.* [17] and Hansen [18].

of aggregate 10 mm; water:cement:sand:coarse aggregate ratio = 0.5:1:2:4.

*Gaede* [9] – Cyclic drying creep. Prisms 100 mm × 100 mm × 500 mm, 22 h in mould, then in water until the age of 7 days. Thereafter specimens in cellar at 55–70% relative humidity and temperature 15–19°C. Relative humidity assumed to be 63%. Specimens loaded at the age of 65 days. Frequency of sinusoidal loading 665 cycles $min^{-1}$. Cement type PZ425, content 345 kg $cm^{-3}$. 28-day cube strength = 507 kg $cm^{-2}$; water:cement:sand:coarse aggregate ratio = 0.585:1:1.986:3.017.

*Hirst and Neville* [10] – Cyclic drying creep. Prisms 76 mm × 76 mm × 203 mm, fog-cured at 20 ± 1°C. Specimens loaded at the age of 28 days and allowed to dry at 50% relative humidity. Cyclic load sinusoidal, frequency 50 Hz (3000 cycles $min^{-1}$). 28-day strength of prism = 46.5 N $mm^{-2}$; rapid-hardening Portland cement; water:cement:sand:coarse aggregate ratio = 0.5:1:2:4. Initial elastic strains assumed to be 0.224, 0.245, 0.295, 0.340 × $10^6$ $psi^{-1}$ (3.25, 3.55, 4.28, 4.93 × $10^{-11}$ $Pa^{-1}$) at $\sigma_{mean} = 0.35f'$ and $\Delta = 0$, 0.1, 0.2 and $0.3f'$, respectively.

*Mehmel and Kern* [11] – Cyclic drying creep. Cylinders 150 mm × 600 mm, 1 day in mould, then 6 days curing in water. At the age of 7 days, specimens removed to test environment, 50–60% respectively, and temperature 20–21°C. Relative humidity assumed to be 60%. Frequency of sinusoidal loading 380 cycles $min^{-1}$. Cement type I PZ275. In test series 1 and 2, average 28-day cube strengths = 488 and 483 kg $cm^{-2}$; water:cement:sand: coarse aggregate ratios = 0.44:1:1.51:3.01 and 0.47:1: 2.7:4.81; cement contents = 400 and 270 kg $cm^{-3}$, respectively.

*Suter and Mickleborough* [12,13] – Cyclic drying creep. Cylinders 30.5 mm × 76 mm, 1 day in mould, then cured in tank until 8th day and removed to test room of 50% relative humidity and temperature 24°C. Frequency of sinusoidal cyclic load 1500 cycles $min^{-1}$. Specimens loaded at the age of 28 days. Cement type III Canada HES, content 214 kg $cm^{-3}$. 28-day average cylinder strengths (30.5 mm × 76 mm) in test series A and B = 2520 and 2580 psi (17.4 and 17.8 MPa), respectively; water:cement:aggregate ratio = 0.9:1:9.0; max. size of aggregate = 6.4 mm; the fine aggregate followed ASTM C 33 and CSA A 23.1 grading limits.

*Al-Alusi* et al. [17] – Cyclic humidity effect. Hollow cylinders of diameters 5 and 6 in. (127 and 152 mm) and length 40 in. (1.02 m) cured at 100% relative humidity, and temperature 22.8°C for 21 days. Period $T_h$ = 28 days, of which 14 days was wetting at 100% relative humidity, 14 days drying at 50% relative humidity. Loading at the age of 21 days. Content 12 sacks per cubic yard (0.76 $m^3$); 21-day cylinder strength 3600 psi (24.8 MPa); modulus of elasticity 3.43 × $10^6$ psi (23.6 GPa); water:cement: aggregate ratio = 0.58:1:2.

*Bernhardt* [15,16] – Cyclic humidity effect. Cylinders 10 cm × 28 cm, cured at 100% relative humidity and temperature 20°C. Specimens loaded at the age of 9–11 days. Four test series: I. The period was 14 days, of which 2 days in water and 12 days in air, applied stress, was $0.367f'_c$; II, III, IV. The period was 7 days for all but different wetting and drying intervals: 2, 1, 0.25 days in water and 5, 6, 6.75 days in air, applied stress was 0.372, 0.348, $0.340f'_c$, respectively. The relative humidity of the air ranged from 35 to 55%. Cement type B250, 28-day average cylinder strength 200 kg $cm^{-2}$ (2824 psi); water: cement:sand:coarse aggregate ratio = 0.67:1:2.87:4.3.

*Hansen* [18] – Cyclic humidity effect. Beams 2 cm × 5 cm × 40 cm cured 1 day under wet burlap at 100% relative humidity, then 6 days in water at 20°C and 21 days in air at 70% relative humidity and 20°C. Loaded at age 28 days. Two test series used: (1) $T_h$ = 2 days (of which 1 day at 50% relative humidity, 1 day at 70% relative humidity); (2) $T_h$ = 14 days (of which 7 days at 50% relative humidity, 7 days at 70% relative humidity). Standard Portland cement, content 850 kg $cm^{-3}$. Water: cement ratio = 0.35:1; volume concentration of cement paste 0.59. The stress in the extreme fibres of each beam was 32 kg $cm^{-2}$.

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## RESUME

**Modèle amélioré de prédiction des déformations du béton en fonction du temps: 5ème partie. Chargement et humidité cycliques.**

*La variation cyclique de la contrainte et l'humidité ambiante déterminent un accroissement du fluage. Dans ce chapître de la série en cours, on adapte maintenant les formules de prédiction présentées dans le modèle original BP à l'actuel modèle de fluage BP perfectionné. L'accroissement du fluage dû à l'alternance de la charge est décrit comme un écart de la prédiction selon le principe de superposition; par conséquent, c'est un phénomène non linéaire. La formulation de l'accroissement du fluage dû au cycle d'humidité s'appuie en partie sur des aspects de la théorie de la diffusion. Des comparaisons avec les données d'essai montrent une concordance acceptable.*