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CONTENTS

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Fracture energy tests of dam concrete with rate and size effects

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SUMMARY

This paper reports the results of an experimental study of the fracture mechanics properties of dam concrete. Large-scale wedge splitting compact tension specimens of dam concrete with 76 mm (3 inch) maximum size aggregate have been tested. Different specimen sizes and loading rates were used to investigate their influence on the fracture energy. The size effect method, proposed by Bažant [1990] and recently published as a new RILEM Recommendation, was used to compute the fracture energy. The results are:

1. The value of fracture energy of dam concrete is higher than for conventional concrete which has smaller aggregate.
2. The fracture energy increases when the loading rate increases.
3. As the loading rate decreases, the fracture response becomes more linear elastic.

Introduction

While the vast majority of the thousands of concrete dams worldwide have provided good service over their lives, many of them are severely cracked. Dam authorities are faced with difficult decisions regarding the potentially devastating consequences associated with failure and the economic implications associated with drawdown and rehabilitation.

Even though concrete dams are designed using a criterion of 'no tension' under normal loading, it has been established that this is not always a sufficient and conservative criterion to avoid crack growth and failure (Bažant, Z. P. [1990[1]]). When seismic loads are considered, the failure potential by cracking is obvious.

The present methods of analysis of dams subjected to earthquake excitation typically employ finite element analysis with smeared cracking based on strength criteria. The results of such analyses are not objective with respect to the analyst's choice of finite element size. In other words, by sufficient mesh refinement of elements at a crack tip, one can always obtain crack tip stresses which exceed the strength limit. Fracture mechanics has not been used in practice because methods of fracture mechanics analysis have not been sufficiently developed, and especially, fracture properties of dam concrete have not been experimentally determined.

The mass concrete which is typically used in dam construction is unreinforced, has very large maximum aggregate size (usually 76 mm (3 inch) or more), low water-cement ratio to improve strength, and low cement content to reduce thermal cracking and shrinkage during curing. Because of these characteristics, dams are fracture-sensitive structures. The large size aggregate used in dam concrete necessitates the use of very large fracture specimens. Data on fracture of dam concrete are presently quite limited. The work done at Swiss Federal Institute of Technology, Lausanne (Brühwiler, E., and F. H. Wittmann [1988]), and at University of Colorado, Boulder (Saouma, V. E., J. J. Broz, E. Brühwiler, and H. L. Boggs [1991]), represents most of the research that has been reported to date. This paucity of information represents a serious obstacle to the establishment of fracture mechanics as an effective criterion for dam failure and safety analysis. Previous testing of dam concrete concentrated on determining the value of the fracture energy or fracture toughness, which represents one essential piece of information for carrying out fracture analysis of a dam. The influence of time, or the rate effect, is also important. Cracks in dams often grow very slowly, over the span of many years. Then, at one instant, they may suddenly become unstable and propagate rapidly, even in the absence of dynamic loads (as observed from acoustic records, Lombardi, Dam fracture Workshop [1990]). Moreover, a dynamic event, such as an earthquake, may cause an existing stable or slowly growing crack to suddenly propagate. Based on knowledge of general brittle materials, fracture propagation is known to be rate sensitive. The rate sensitivity is expected to be particularly pronounced for concrete, because this material is known to exhibit pronounced creep. The rate effect, however, is likely caused not only by creep in the bulk of the material, but also by the rate-sensitivity of the process of bond ruptures. Even ceramics or rocks which exhibit no creep have rate sensitive fracture properties.

For these reasons, fracture properties governing slow crack growth over many years in dam concrete would be expected to be quite different from those governing rapid crack growth during events such as an earthquake. Indeed, preliminary tests of small fracture specimens, reported in Bažant, Z. P., and Getta, R. [1989], revealed such effects for microconcrete, and dam concrete should be similar in this respect. Ideally, it is of interest to determine the fracture properties of dam concrete for times of maximum load that can range from milliseconds to 50 years, representing a range of 12 orders of magnitude. Time and equipment constraints limited the present tests of dam concrete to approximately 5 orders of magnitude[1].

The rate effect in concrete failure has been studied extensively (see Rate effect bibliography). However, previous work has been limited to the dynamic loading range, has not addressed dam concrete, and much of it has focused on strength rather than fracture properties.

[1] We do not specify rates in terms of strain because the strain rate varies widely throughout the specimen and its maximum value in the fracture process zone is difficult to quantify.

In this paper, fracture behavior of dam concrete with maximum aggregate size of 76 mm is investigated with emphasis on determination of size-independent fracture properties, including the effects of loading rate. A preliminary report of these results was presented at a recent conference (Bažant, Z. P., He, S., Plesha, M. E., Getta, R., and Rowlands, R. E. [1991]). Tests of very large fracture specimens, up to 1.8 m (72 in.) in dimension, and with time to peak load ranging from 1 s to 72,000 s are reported. The specimens were similar to (but larger than) those used by Brühwiler and Wittmann [1988] and Saouma *et al.* [1991]. The size effect method, recently published as a RILEM Recommendation (RILEM Recommendation (TC89-FMT) [1990]), is used to determine the fracture energy. This method permits determination of fracture properties from smaller specimens than are required by other methods.

1 Experiments

A series of wedge-splitting tests were conducted on compact tension specimens of three different sizes and at three different rates of loading. To obtain size independent material properties, the size effect method proposed in Bažant, Z. P., J. Kim, and P. A. Pfeiffer [1986] and standardized in (RILEM Recommendation (TC89-FMT) [1990]) was employed to compute the fracture energy. The shape and dimensions of the specimens are shown in Figure 1 and Table 1.

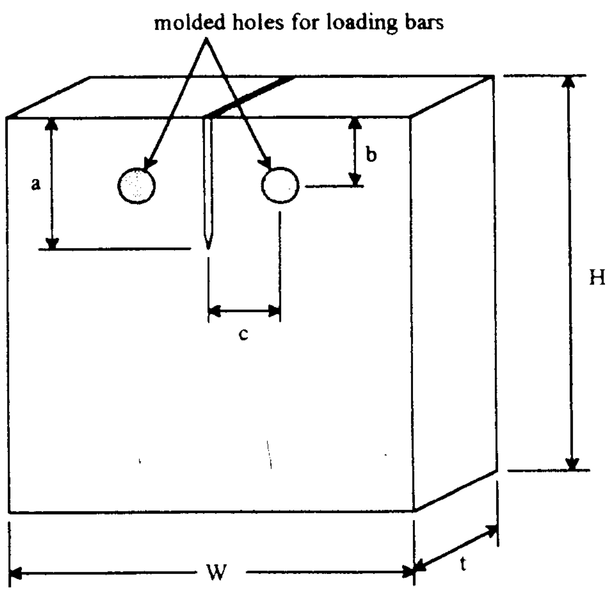

Figure 1. Specimen geometry

Concrete with maximum size aggregate (MSA) of 76 mm was supplied by a local ready-mix company. Three large, three medium and six small specimens were cast from each batch of concrete. Table 2 gives the mix proportions which are identical to those used for similar specimens by Saouma [1991]. A crack of depth a (shown in Table 1) was molded using a 6.45 mm-(1/4 in.)-thick greased steel plate. The specimen forms were typically removed after 5 days and the specimens were immediately wrapped in plastic for another 28 days (the total curing period was typically 33 days.) In the case of the batch 4 concrete specimens, difficulties encountered with the testing apparatus delayed their testing to 55 days age. However, an approximate correction of the results for the difference in age was made. All curing was indoors at room temperature (approximately 20 °C). Following curing, the specimens were tested under crack opening displacement control at three different rates labeled as 'fast', 'medium' and 'slow' and defined in Table 3. These rates were determined so that the load would reach maximum value after approximately 1 second, 200 seconds, and 20 hours, respectively, for any specimen size. These rates span almost 5 orders of magnitude and represent a range of loading from that typical of earthquake excitation to that typical of short-term creep loading. For some of the tests, load relaxation of partially fractured specimens under constant crack opening displacement (COD) was also measured.

Table 1. Specimen dimensions

Specimen	H (cm)	W (cm)	t (cm)	a (cm)	b (cm)	c (cm)
small	30	30	23	8.3	3.1	7.6
medium	76	76	23	21.6	5.7	7.6
large	182	182	23	45.4	15.2	6.4

Table 2. Dam concrete mix design (no admixtures or superplasticizers were used)

Maximum size aggregate (MSA) 76 mm no entrained air, w/c ratio = 0.66	
	Weight (kg/m^3)
water	123
Portland cement	187
sand	695
19 mm MSA	491
38 mm MSA	491
76 mm MSA	491
Total	2480

Specimens were tested laying flat on the testing floor as shown in Figure 2. A hydraulic actuator (MTS204) was used to generate the load. One end of the actuator was fixed to a specially designed loading frame that was bolted to the test floor. A wedge was attached to the other end of the actuator. The wedge contacted precision roller bearings that were fitted to steel bar stock that was inserted into the molded holes shown in Figure 1. An electronic controller (MTS406.11), a control unit (MTS436.11), and a digital function generator (MTS410.31) were used to control the actuator. All tests were conducted under crack opening displacement (COD) control. The COD was measured by an extensometer (MTS632.12B-20), and the load was measured by the actuator's built-in load cell. The loading and COD signals were recorded by an IBM PC containing a data acquisition board (DT2801 A/D). A computer software package DAS (data acquisition system) was developed to control the data acquisition board. The DAS displays the real time loading versus the COD curve on the computer screen and allows real time monitoring of the progress of the test.

For each batch of concrete, three standard cylinders with diameter × height dimensions of 25 cm × 30 cm (10" × 12") and three cylinders with dimensions 25 cm × 60 cm (10" × 24") were tested to determine the compressive strength f_c' and the tensile splitting strength f_t' according to the formulas: $f_c' = P/A$ and $f_t' = 2P/(\pi LD)$, respectively. Where P = maximum applied load, L = length of the cylinder, D = cylinder diameter, and A = area of the cross section.

Table 3. Load rates

Load rate	Time to reach P_{max}
slow	20 hr
medium	200 s
fast	1 s

2 Test results

The material properties determined from the tensile splitting test are listed in Table 4. The results show scatter from batch to batch, which will be magnified in the scatter of the maximum load values for specimens of different batches; but the properties within each batch were reasonably consistent. Some of the factors causing inconsistencies from batch to batch are:

1. slightly different water content since the concrete contractor, during batching, attempted to make corrections to the water added to account for differences in the moisture content of the sand and aggregate;
2. different ambient temperature at the time of mixing and casting; and
3. variability in the transit time from the batching plant to the laboratory.

Tests were performed on specimens cast from four batches of concrete. Three large, three medium, and six small specimens were cast from each batch. Three different loading rates were used for each batch. Results in Table 5 show that the magnitude of the maximum load for the fast loading rate is systematically greater than that for the slower load rates.

The results for the medium size specimen are shown in Table 6. The change of maximum load for the fast loading rate is not as significant as it is for the large specimen size (Table 5). The results for the small size specimen are shown in Table 7. The data of Table 7 show that the influence of the loading rate upon the maximum load is significant.

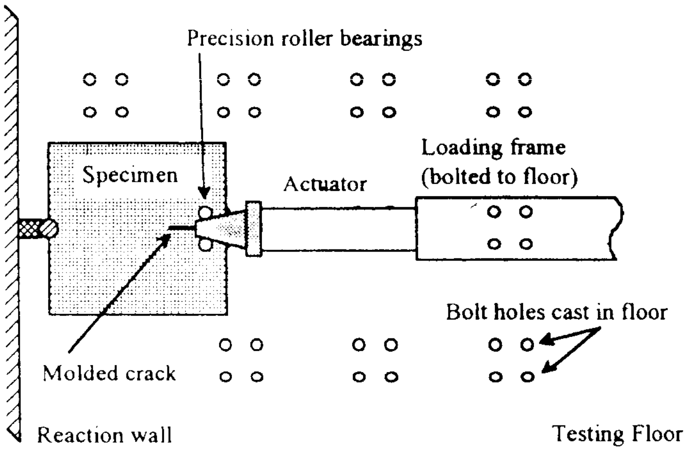

Figure 2. Plan view of the experimental setup on the testing floor

Table 4. Tensile strength from Brazilian splitting tests and compression strength

	Batch 1	Batch 2	Batch 3	Batch 4
f_t (MPa)	1.49	2.50	3.15	2.00
f'_c (MPa)	12.73	17.25	18.66	16.92

Table 5. Test results for large size specimen

Loading Rate	Batch	P_{max} (kN)	T@P_{max}	COD@P_{max} (mm)
fast	1	96.45	1.08 s	0.21
	2	106.74	0.68 s	0.32
	3	117.85	1.08 s	0.26
	4	129.68	2.88 s	0.15
moderate	1	63.47	60 s	0.17
	2	88.75	84 s	0.20
	3	112.82	108 s	0.25
	4	86.00	148 s	0.15
slow	1	65.74	15.8 hr	0.41
	2	95.73	11.8 hr	0.31
	3	----	----	----
	4	85.43	8.72 hr	0.23

Table 6. Test results for medium size specimen

Loading Rate	Concrete Batch #	P_{max} (kN)	T@P_{max}	COD@P_{max} (mm)
fast	1	30.64	1.02 s	0.20
	2	46.20	0.98 s	0.16
	3	50.50	0.98 s	0.19
	4	46.85	2.16 s	0.19
moderate	1	30.64	184 s	0.19
	2	33.56	204 s	0.20
	3	39.07	232 s	0.24
	4	40.28	180 s	0.18
slow	1	28.69	20.7 hr	0.23
	2	34.69	13.0 hr	0.14
	3	40.04	23.4 hr	0.25
	4	42.96	17.6 hr	0.18

Table 7. Test results for small size specimen

Loading Rate	Specimen #	P_{max} (kN)	T@P_{max}	COD@P_{max} (mm)
fast	1	31.94	1.12 s	0.13
	2	25.05	3.12 s	0.14
moderate	3	22.00	224 s	0.13
	4	21.64	184 s	0.11
slow	5	21.72	26.3 hr	0.14

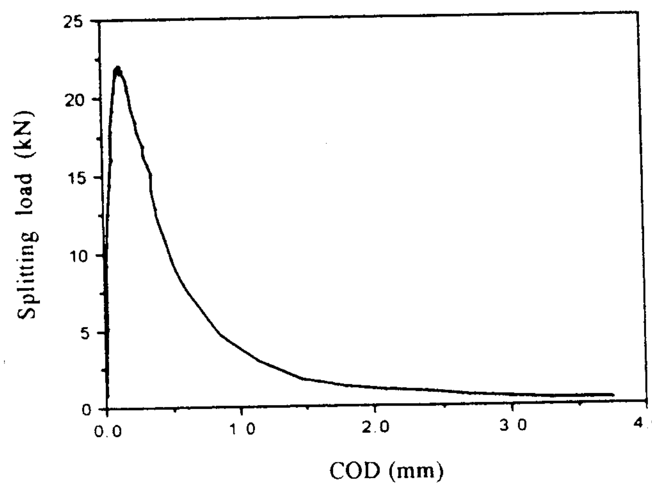

Figure 3. The loading versus COD curve of the small specimen

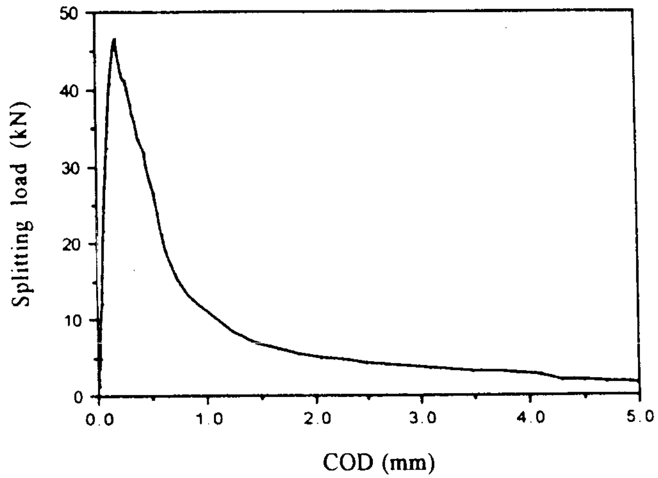

Figure 4. The loading versus COD curve of the medium specimen

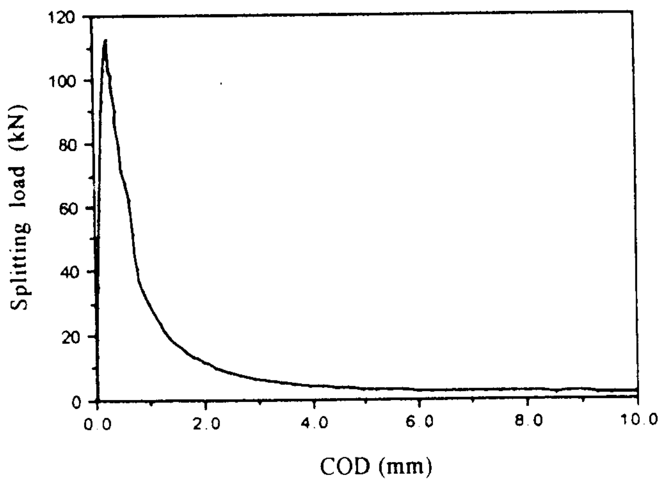

Figure 5. The loading versus COD curve of the large specimen

Typical load versus COD curves are shown in Figures 3 to 5, in which significant post-peak softening is apparent. For some specimens, a load relaxation test was carried out after the peak load had been passed. For these tests, the COD was held constant after the load decreased by about 20% from the maximum value, P_{max}. Figures 6 to 11 show the result of load relaxation tests.

The fracture energy was computed by employing the size effect law of Bažant [1986]. Only the maximum loads obtained from experiments are required for this computation rather than the complete load-COD curve. For the size effect method to be effective, all the specimens should be cast from the same batch of concrete and at least three widely different specimen sizes should be used. Because a difficulty was encountered, unreliable data for the small specimens cast from concrete batches 1 to 3 resulted in only the measured data for batch 4 concrete meeting this requirement. Thus, only the data from the batch 4 tests can be used to compute the constants in the size effect law, which has the form:

$$\sigma_N = Bf_t'(1+\beta)^{-1/2}, \qquad \beta = d/d_o \tag{1}$$

where $\sigma_N = P/td$ = nominal stress at maximum load P, t = thickness, β = relative size, d = characteristic dimension of the structure (in this case, $d = H - b$, see Figure 1), and f_t' = direct tensile strength (here the tensile splitting strength was used). B and d_o are empirical constants. To determine these two constants, Eq. (1) is transformed to the form of a linear regression, that is:

$$Y = AX + C \tag{2}$$

where $X = d$, $Y = \sigma_N^{-2} = (td/P)^2$, $A = C/d_o$, and $C = 1/(Bf_t')^2$.

Once the constants A and C are obtained by linear regression, the fracture energy G_f is then computed by

$$G_f = \frac{g(\alpha_o)}{AE_c} \tag{3}$$

where $g(\alpha_o)$ is the non-dimensionalized energy release rate, $\alpha_o = a_o/d$, and a_o is the traction-free crack length. The function $g(\alpha)$ can be obtained from handbooks (Tada, H., P. C. Paris, and G. R. Irwin [1985]) or from linear elastic fracture mechanics (LEFM) analysis. The parameter values for the size effect law are given in Table 8 for each of the loading rates studied here.

Table 8. Constants of size effect law obtained by linear regression of test results for batch 4 concrete

load rate	A	C (cm)	do (cm)	G_f (N/m)
fast	0.115E-5	0.975E-3	848.1	1313.7
moderate	0.995E-5	0.846E-3	85.1	152.0
slow	0.975E-5	0.846E-3	86.8	152.0

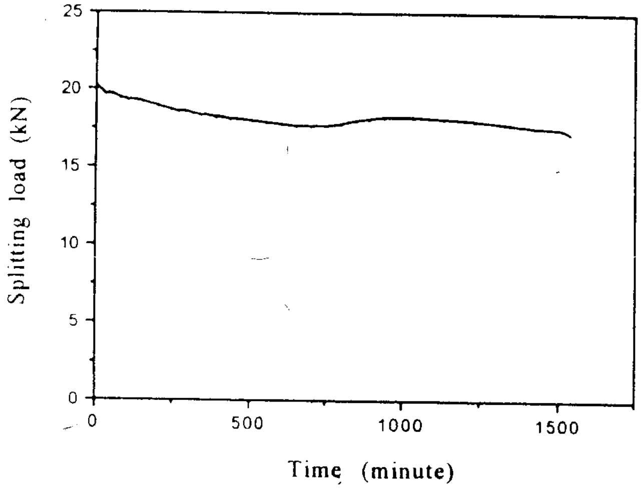

Figure 6. Load relaxation at constant COD for small specimen

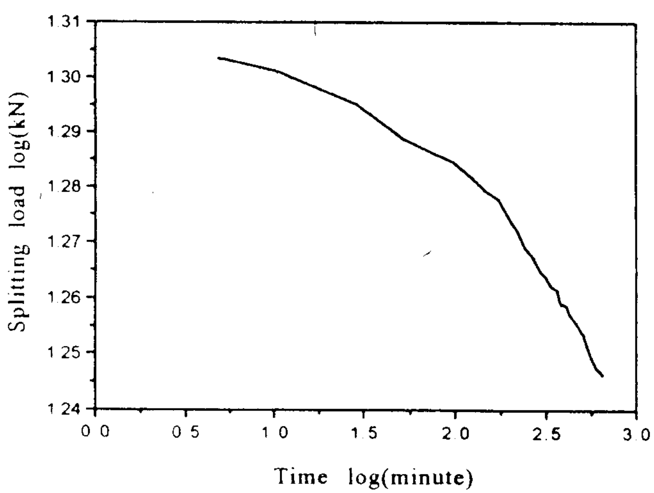

Figure 7. Load relaxation at constant COD for small specimen

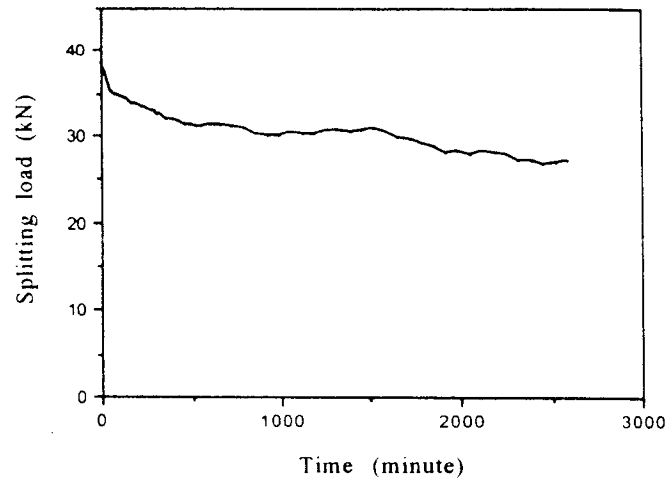

Figure 8. Load relaxation at constant COD for medium specimen

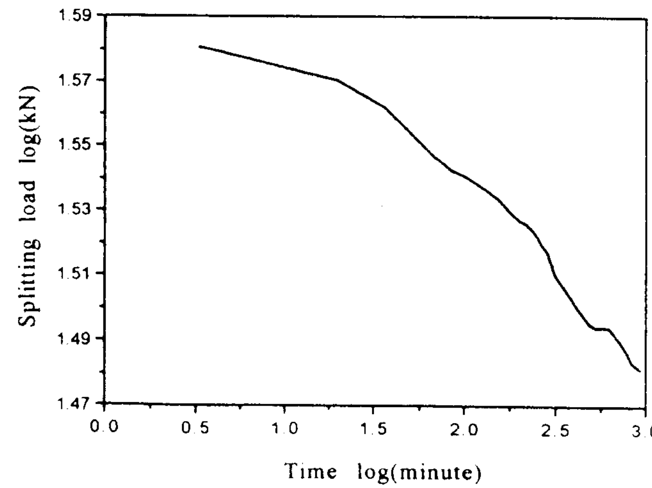

Figure 9. Load relaxation at constant COD for medium specimen

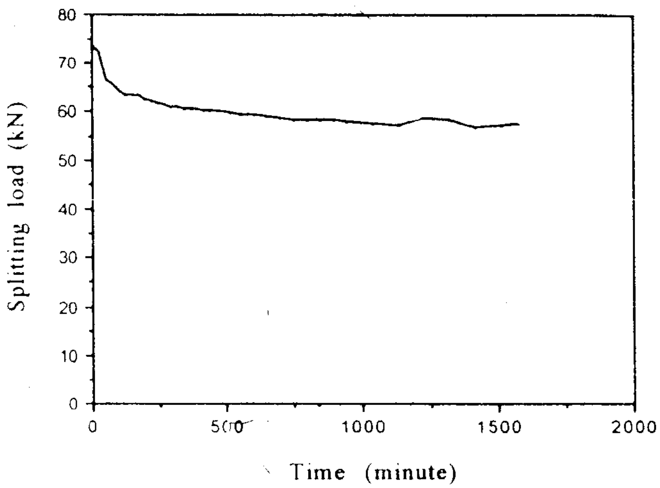

Figure 10. Load relaxation at constant COD for large specimen

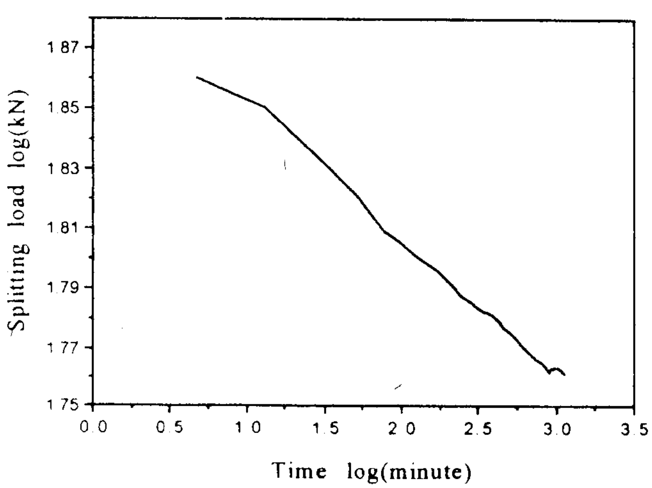

Figure 11. Load relaxation at constant COD for large specimen

In Tables 9 to 11, the characteristic specimen sizes as computed by the size effect law and the maximum loads are given for each specimen size and each loading rate. These data are plotted in Figures 12 to 14 along with the failure predictions according to strength criterion and linear elastic fracture mechanics. For comparison, all of the results for concrete batch 4 are shown collectively in Figure 15. The nominal stresses versus d/d_o for all loading rates of batch 4 concrete are shown in Figure 16. These plots show that the size effect law of Bažant [1986] agrees reasonably well with the test data for the slow and medium loading rates. However, for the fast loading rate, the discrepancy becomes larger, which might be caused by random errors.

As mentioned before, the material properties change some what from batch to batch, although the properties within one batch are consistent. In all cases, the data for the fast loading rate show the largest discrepancies from the size effect law.

The fracture energies were also computed by integrating the load versus COD curves, and then dividing this area by the ligament area (that is, by $(H-a)t$, Figure 1). The results for fracture energy computed in this manner are listed in Table 12 along with the test results of Saouma *et al.* [1991] who used similar concrete and specimens of similar size (that is, large and medium sizes of 1.5 m × 1.5 m × 41 cm (5 ft × 5 ft × 16 in.) and 91 cm × 91 cm × 41 cm (3 ft × 3 ft × 16 in.), respectively).

In Table 12, the data for batch 4 concrete are separated from those of the first three batches because batch 4 had a longer curing time (55 days versus 33 days). The results from the first three batches agree very well with those of Saouma *et al.* [1991] (whose curing time was 28 days). The results from batch 4 in particular suggest that the fracture energy computed by this definition does not show a strong size dependence.

Table 9. Characteristic specimen sizes and measured maximum loads for the slow loading rate tests

Specimen size	d (cm)	P_{max} (kN)
small	27.3	21.72
medium	70.5	42.96
large	167.6	85.43

Table 10. Characteristic specimen sizes and measured maximum loads for the medium loading rate tests

Specimen size	d (cm)	P_{max} (kN)
small	27.3	22.05
small	27.3	21.64
medium	70.5	40.28
large	167.6	86.00

Table 12. Fracture energy computed by integration of the splitting load versus COD response for medium loading rate

	Fracture energy G_f (N/m)		
Specimen	Batch 4	Batch 1-3	Saouma's
large	276.5	238.9	236.4
medium	263.2	204.4	225.9
small	259.5	---	---

Table 11. Characteristic specimen sizes and measured maximum loads for the fast loading rate tests

Specimen size	d (cm)	P_{max} (kN)
small	27.3	25.05
medium	70.5	46.85
large	167.6	129.69

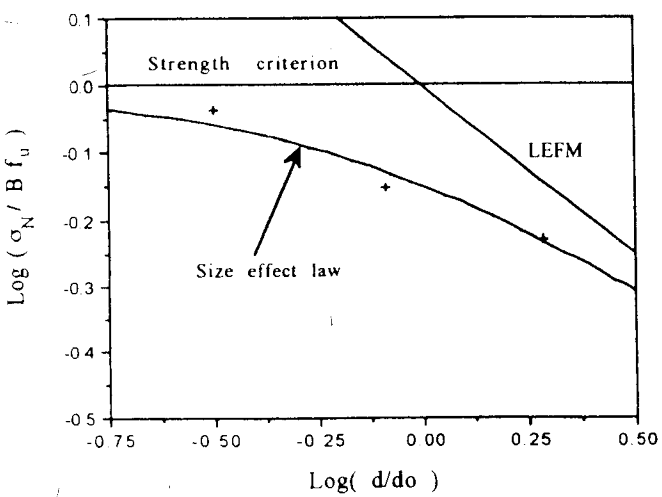

Figure 12. Size effect plot for slow loading rate response of batch 4 concrete

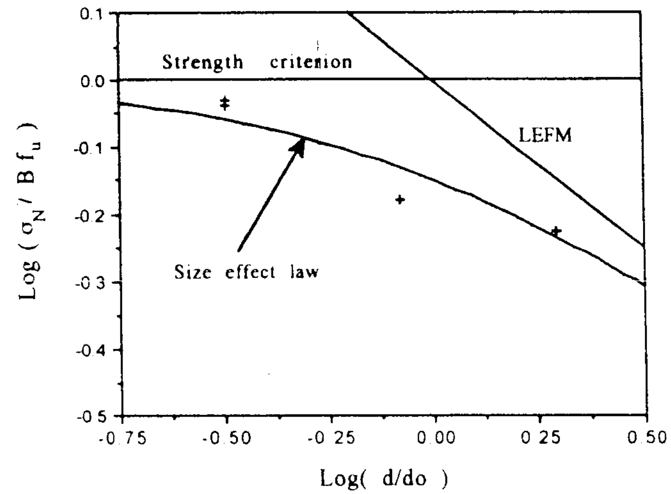

Figure 13. Size effect plot for medium loading rate response of batch 4 concrete

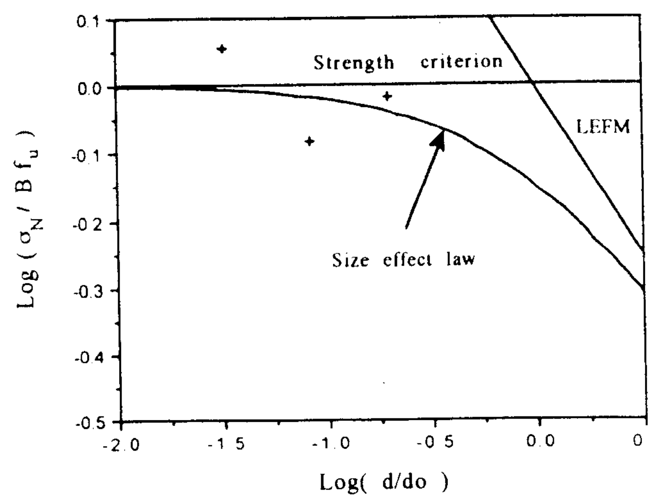

Figure 14. Size effect plot for fast loading rate response of batch 4 concrete

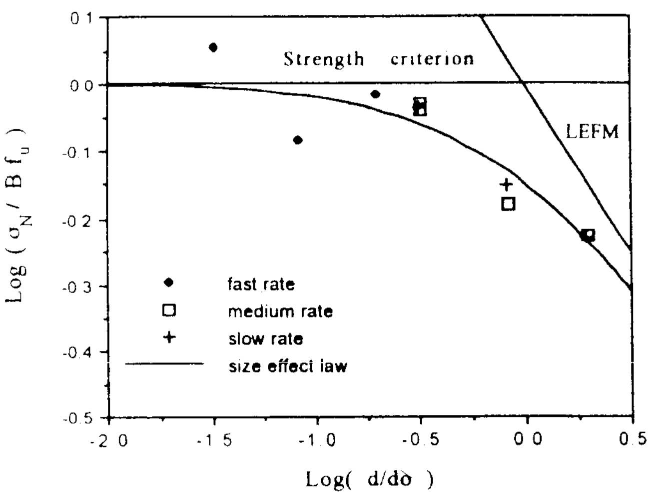

Figure 15. Size effect plot for all loading rates response of batch 4 concrete

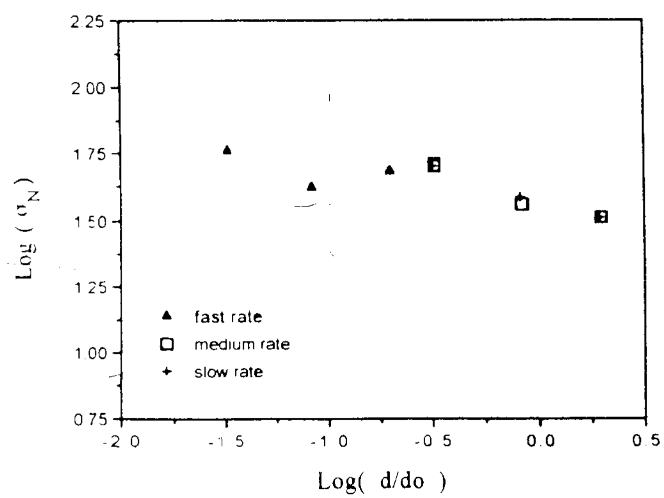

Figure 16. Data from batch 4 concrete of all rates on same plot

3 Further supporting evidence from small-scale tests

To shed more light on the rate effect, numerous smaller-scale specimens made of small-aggregate concrete of strength $f_c' = 37$ MPa were tested at Northwestern University. The specimens were tested by three-point bending tests on beams of cross section depths $d = 38$, 76 and 152 mm, and the maximum aggregate size was 13 mm. The results of these tests, which are being analyzed and published separately (Bažant, Z. P., and Kazemi, M. T. [1990[3]]), reinforce the present conclusions (a preliminary abbreviated presentation of these test results has already been given in Bažant, Z. P., and Getta, R. [1989] and Bažant, Z. P., and Getta, R. [1990[4]]). The trends of those tests are similar to those observed here. However, those tests exhibit less scatter as may be expected from small scale laboratory specimens. Figure 17 shows the measured average nominal strengths of groups of three specimens of three different sizes, tested at four different rates characterized by the t_p-values indicated in the figure. The agreement with the size effect law (the solid curve) is stronger than in the present large-scale tests, and the shift of response toward the LEFM asymptote with decreasing loading rate is now revealed unambiguously (Figure 17). For more details, see Bažant, Z. P., and Kazemi, M. T. [1990[3]].

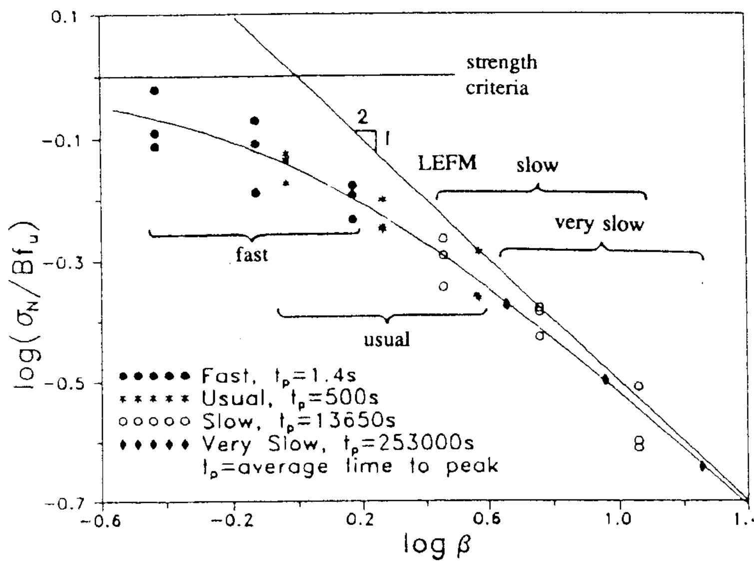

Figure 17. Small scale laboratory tests results

Conclusion

The size effect law proposed by Bažant was calibrated using the experimental data from dam concrete for three different loading rates spanning almost 5 orders of magnitude. The size effect law shows good agreement with the test data for the medium and slow loading rates. Agreement with the test data for the fast loading rate is poor. The data show (Figure 15) that slower rates of loading tend to shift the response closer to LEFM. Comparison with test results justifies the following conclusions:

1. The fracture behavior of dam concrete generally agrees reasonably well with the size effect law proposed by Bažant. Moreover, testing of geometrically similar fracture specimens of various sizes makes it possible to determine nonlinear fracture parameters of dam concrete quite easily, using specimens which are not as large as required for other LEFM methods.
2. The size effect law (1) appears to be valid over a wide range of loading rates, spanning over five orders of magnitude.
3. With decreasing rate of loading, the response shifts closer to LEFM. An important implication of this observation is that long-time fracture behavior is closer to LEFM than is short-time behavior.
4. There is strong load relaxation in the post-peak load range which is no doubt caused by ongoing creep in the fracture process zone.
5. There is strong interaction between fracture and creep in dam concrete, and long-time fracture predictions must take such interaction into account.

Acknowledgements

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