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RILEM TECHNICAL COMMITTEES
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TC 107: CREEP AND SHRINKAGE PREDICTION MODELS: PRINCIPLES OF THEIR FORMULATION

Guidelines for characterizing concrete creep and shrinkage in structural design codes or recommendations

Guidelines and recommendations, which have been formulated by Subcommittees 1 and 2 of RILEM TC 107, are reported. These guidelines reflect the recent advances in experimental and theoretical research. They are formulated with computer analysis in mind and with a view towards the increased long term safety and durability of concrete structures.

The experimental results accumulated during the last several decades, as well as theoretical and numerical studies of the physical phenomena involved in the creep and shrinkage of concrete, such as moisture and temperature diffusion, have led to the following guidelines for creep and shrinkage prediction models to be used in design codes (or design recommendations). These guidelines, which evolved from RILEM TC 69 conclusions [1] and were produced by RILEM Committee TC 107 (chaired by Z. P. Bažant) in collaboration with RILEM Committee TC 114 (chaired by I. Carol), include a number of requirements and consistency conditions which should be satisfied by any creep or shrinkage prediction model in order to avoid conflict with experimental evidence, agree with solidly established theoretical concepts, and achieve mathematical consistency.

1. The creep of concrete in the service stress range can be characterized in terms of the compliance function. Its use is made possible by the fact that the creep of concrete (in contrast to the creep of metals, ice or clay) can be considered as approximately linear with regard to stress, following the principle of superposition. The principle of superposition agrees with test results very well if there is no drying. At drying, and especially if cracking takes place, there are appreciable departures from the principle of superposition, but they have to be neglected in simple design code formulations because of their complexity.

2. The design codes should specify the compliance function $J(t, t')$ rather than the creep coefficient ϕ, where ϕ is the ratio of creep to the initial 'elastic' strains, t is the current age of concrete, t' is the age at loading. For structural creep analysis, of course, it is often more convenient to use the creep coefficient, but its value can always be calculated easily from the compliance function specified in the code ($\phi = EJ - 1$, where E is the conventional 'elastic' modulus, characterizing the truly instantaneous deformation plus short-time creep). One reason for preferring J is that the E values specified in the codes are not defined on the basis of the initial strains measured in typical creep tests. A more profound reason is that concrete creep in the range of short load durations from 0.1 s to 0.1 day is already quite significant, which means that $1/E$ is inevitably an arbitrarily chosen point t_0 on the smoothly rising creep curve for unit stress. Depending on t_0, the corresponding E values vary widely (and different creep data correspond to very different choices of t_0). But what matters for the results of structural creep analysis is the values of J, not ϕ and E. If the creep coefficient ϕ is given to the structural analyst, there is always the danger that he might combine it with some non-corresponding value of E, which then implies an incorrect J. When J is specified, this kind of mistake is prevented.

3. Considering concrete as a homogeneous material, creep and shrinkage should be considered, strictly speaking, as phenomena associated with a point of that continuum. The evolution of such 'intrinsic creep' and 'intrinsic shrinkage' is affected by factors such as the specific moisture content in the pores or temperature, which can vary from point to point in a cross-section of the structure. Therefore, the compliance function and shrinkage in general will be

non-uniform throughout the cross-section, and the intrinsic creep will not be distributed linearly. If, however, the cross-section remains plane (which usually is true for long prismatic members such as beams or columns), some internal stresses will be generated to make the strain at each point of the cross-section conform to a linear strain distribution.

For most practical purposes, however, the mean (or average) creep and shrinkage can be defined for the cross-section as a whole. They have the meaning of the average cross-section compliances or average shrinkage strains, regardless of the associated internal stress and the inherent cracking at each particular point. Although, strictly speaking, such average creep or shrinkage depends on the type of loading (i.e., the ratio of the bending moment to the normal force), approximate average properties can be established for all types of loading. This is in general the meaning of the formulas characterizing creep and shrinkage in contemporary design codes. This type of formula is useful, but it must be emphasized that creep and shrinkage cannot be considered as material properties unless members in a sealed condition and at constant uniform temperature are considered. Rather, they are the properties of the cross-section. Consequently, the formulas depend on the properties of the cross-section, such as its size and shape, and are influenced by the non-homogeneity of creep and shrinkage within the cross-section. Inevitably, therefore, good prediction formulas are much more complicated than the constitutive law for a material point.

4. Drying plays a fundamental role in creep and shrinkage. It is the direct cause of shrinkage – if no drying occurs, no shrinkage occurs (except for autogenous shrinkage, which is of chemical origin and is usually negligible). Drying also affects creep, increasing the creep strain significantly with respect to a similar situation without drying. In a cross-section, the drying process is governed by the nonlinear theory of moisture diffusion through the pores in concrete. Its effects increase as the environmental relative humidity decreases. Of course, no drying occurs when the specimen is sealed. In the case of immersion in water (relative humidity 100%) there is swelling or negative shrinkage, which is usually rather small (the inbibition of water also causes some small non-uniformity in pore humidities throughout the cross-section, residual stress and possibly microcracking).

5. Basic creep is defined as the creep at no drying and at constant temperature. Under such conditions, the behaviour at all points in the cross-section is the same, and therefore the basic creep can be considered as an intrinsic material property. Drying creep is defined as the increase in the creep strain over the basic creep, when drying takes place. This component of creep vanishes for sealed conditions, which is approximately at 100% environmental relative humidity (the small humidity drop called self-desiccation, caused by hydration, is neglected). Therefore, under sealed conditions, the creep for any cross-section should be the same and equal to the basic creep regardless of cross-section size and shape.

6. Diffusion theory, linear as well as nonlinear, indicates that the drying times $\hat{t}$ required to achieve a similar degree of overall drying in a cross-section (i.e., the same relative water loss) increase with the square of the size. The shrinkage formulas must give shrinkage as a function of the relative time $\theta = \hat{t}/\tau_{sh}$ rather than the actual drying time $\hat{t}$ where τ_{sh} represents a coefficient with the meaning of the time necessary to achieve a certain percentage of the final shrinkage, e.g., the shrinkage half-time or the time necessary to achieve half of the final shrinkage; τ_{sh} should be proportional to the square of the cross-section size. Furthermore, τ_{sh} should depend on the shape of the cross-section as indicated by the solution of diffusion theory. The incorporation of τ_{sh} implies that, for a fixed duration of drying, the mean shrinkage in the cross-section should decrease with increasing size, with practically no shrinkage at all beyond a certain size.

7. Diffusion theory further indicates that, asymptotically for short drying times $\hat{t}$, the shrinkage strain ε_{sh} should grow in proportion to $\sqrt{\hat{t}}$. This property agrees well with tests. This is true, however, only for properly conducted tests, in which the first length reading should be taken right after the stripping of the mould, within no more than approximately 10 min. Unfortunately, in many measurements the first reading has been taken too late, even as late as several hours after the stripping. In that case, a significant initial shrinkage deformation (possibly 3% to 10% of the final shrinkage) has been missed. Such distorted data may then give the false impression that the initial shrinkage curve does not follow the $\sqrt{\hat{t}}$ law. Also, according to diffusion theory, the final shrinkage value ε_{sh_∞} should be approached asymptotically as an exponential of a power function of time, that is, the difference of the shrinkage strain from the final value should asymptotically decrease as $\exp(-ct^n)$ where c and n are constants and t is the time.

8. Drying creep (also called Pickett effect) is a consequence of the same diffusion process as shrinkage and so it should be described in relation to shrinkage. Thus, like shrinkage, the mean drying creep of a cross-section, for a fixed load duration, also should decrease with increasing size and almost vanish at a certain size beyond which only basic creep remains. It is reasonable that the decrease of shrinkage and drying creep with the cross-section size be governed by the same law, since both are caused by the same physical process, that is, the drying in the cross-section. Therefore, like shrinkage, the drying creep term should have the following properties: (a) it should be specified as a function of θ rather than $\hat{t}$; (b) asymptotically for short times, it should be proportional to $\sqrt{\hat{t}}$; and also (c) it should approach the final value asymptotically as $\exp(-kt^n)$ where k and n are constants.

9. Because the basic creep and the drying creep have different properties, depend on different variables and originate from different physical mechanisms, they should be given by separate terms in the creep model. Therefore, the formula for the creep curves must contain a separate term for the drying creep, which is additive to the basic creep term. Although other more complicated possibilities might be conceivable, a summation of the basic and drying terms in the creep formula appears to be acceptable. Unlike the basic creep term, the drying creep should approach a final asymptotic value.

10. The model of basic creep should be able to fit the existing experimental data throughout the entire ranges of the ages at loading and the load durations. It should exhibit the following characteristic features. (a) According to test results, the creep curves do not possess any final asymptotic value. (b) A power function of the load duration, with the exponent around 1/8, fits very well the data available for short durations, while a linear function of the logarithm of the load duration, with a slope independent of the age of loading, fits very well the data available for long durations. The creep model should satisfy these properties asymptotically. (c) The transition from the short-time to the long-time asymptotic creep curves, which is quite gradual, should be centred at a certain creep value rather than at a certain creep duration, which means that for an older concrete the transition occurs later.

11. The ageing property of creep, that is the decrease of creep strain at a fixed load duration with the age at loading, is well described by a power function of age with a small negative exponent, approximately $-1/3$. This function satisfies the experimental observation that a significant ageing effect continues through a very broad range of ages at loading, certainly from the moment of set to over 10 years. This property contrasts with the age effect on strength or elastic modulus, which becomes unimportant after several months of age. For this reason, the functions suitable for describing those properties, e.g. $t/(t + \text{const.})$, do not work well for the age effect on creep.

12. Although not strictly necessary on a theoretical basis (i.e., thermodynamic restrictions), it is appropriate and convenient that the creep model exhibits no divergence of the creep curves for different ages at loading. There is no clear experimental evidence of such a phenomenon, and models showing such divergence can lead to various unpleasant difficulties when used for complex load histories in combination with the principle of superposition (for instance, giving a reversal of the creep recovery that follows load removal).

13. The creep formulation should also have a form suitable for numerical computation. In the interest of efficient numerical solution of large structural systems, it is necessary to expand the creep formula (i.e., the compliance function) into a Dirichlet series, which makes it possible to characterize creep in a rate-type form corresponding to a Kelvin or Maxwell chain model. Every compliance function can be expanded into Dirichlet series (or converted to a Maxwell chain model), but for some there are more difficulties than for others because the expansion process (a) requires a computer solution rather than a simple evaluation from a formula, (b) leads to an ill-posed problem with a non-unique solution, and (c) yields negative values of the chain moduli within short periods of time. Creep formulations that avoid these unpleasant characteristics should be preferred for the codes.

The source of the aforementioned difficulties is the age dependence of creep properties which, in general, requires considering the chain moduli to be age-dependent. These difficulties disappear with a creep formulation in which the chain moduli are age independent, just as in classical viscoelasticity. Such a desirable property of the creep model can be achieved if the age dependence is introduced separately from the chain model, by means of some transformations of time. At least two such time transformations are known to be necessary, and they should preferably be based on some reasonable physical model for the solidification process that causes the age dependence of creep.

14. An important requirement for any creep and shrinkage model is continuity in the general sense. Small variations in the dimensions, environmental conditions, loading times, etc., should lead to small variations in the creep and shrinkage predictions, without any finite jumps.

15. Because the available creep data do not, and cannot, cover all possible practical situations, it is important that even a simple prediction model be based to the greatest extent possible on a sound theory. In that case one has the best chance that the creep model would perform correctly in such experimentally unexplored situations. This means the model should be based on, and agree with, what is known about the basic physical processes involved, particularly diffusion phenomena, solidification process, theory of activation energy, etc.

16. Even though the creep and shrinkage model for a design code must be sufficiently simple, it should be compared with all the test data that are relevant and exist in the literature. The model parameters should be calibrated on the basis of these data by optimum fitting according to the method of least squares. This task is made feasible by the computer and is greatly facilitated by the existence of a comprehensive computerized data bank for concrete creep and shrinkage (such a bank was compiled in 1978 by Bažant and Panula, was extended by Müller and Panula as part of a collaboration between ACI and CEB creep committees, and is now being further updated, extended and refined by a subcommittee of RILEM TC 107 chaired by H. Müller). No model, even

the simplest one, should be used or incorporated in a code without evaluating and calibrating it by means of such a comprehensive data bank.

17. Selective use of test data sets from the literature does not prove the validity of any model (unless a sufficiently large number of data sets were selected at random, by casting dice). Many examples of the deception by such a practice have been documented. For instance, as shown by Bažant and Panula (1978), by selecting 25 among the existing 36 creep data sets from the literature, the coefficient of variation of errors of the BP creep model dropped from 18.5% to 9.7%, and by selecting only 8 data sets, it further dropped to 4.2%; likewise, by selecting 8 among the existing 12 shrinkage data sets, the coefficient of variation dropped from 31.5% to 7.9% (no reason for suspecting the ignored data sets of being faulty could be seen from the viewpoint of experimental method). Another problem that requires careful attention is the weighting of the data used in the calculation of ω. For example, if most data refer to load durations under 700 days and loading ages from 10 to 100 days, gross misfits of a few available data for load duration 3000 days or loading age 300 days have little effect on the calculated ω unless these data are assigned larger weights.

18. Design codes should specify, as a matter of principle, the coefficient of variation ω of prediction errors of the model compared with all the data that exist in the aforementioned data bank. The values of ω should also be given separately for basic creep, creep at drying, and shrinkage. The value of ω results automatically from a computer comparison of the model with the data bank.

19. Design codes should require that creep sensitive structures be analysed and designed for a specified confidence limit, such as 95%. This means designing the structure in such a way that the probability of not exceeding a certain specified value of response (for example, maximum 50-year deflection, maximum bending moment or maximum stress) would be 95%. The response values that are not exceeded with a 95% probability can be calculated easily with a computer by the sampling method, in which the statistical information is obtained by repeatedly running a deterministic creep analysis program for a small number of randomly generated sets (samples) of the uncertain parameters in the creep prediction model.

20. The notoriously high uncertainty of long time creep and shrinkage predictions can be reduced significantly by recalibrating the most important coefficients of the model according to short time tests of creep and shrinkage of the given concrete, and then extrapolating to long times. This practice should be recommended in the code.

21. The unknown material parameters in the model should be as few as possible and should preferably be involved in such a manner that the identification of material parameters from the given test data (i.e., data fitting) be amenable to linear regression. This not only greatly simplifies the identification of material parameters and extrapolation of short time tests, but it also makes it possible to obtain easily the coefficients of variation characterizing the uncertainty of the material parameters and the predictions. However, this desirable property should not be imposed at the expense of accuracy of the model. It seems that for creep this property can be achieved without compromising accuracy, but for shrinkage this does not seem to be the case.

One important influencing factor, namely temperature, has been ignored in the foregoing guidelines. The same has been true of all the existing design codes or recommendations. Although the effect of constant and uniform temperature is known quite well (it can be described by two activation energies, one controlling the rate of creep and the other the rate of hydration), there is a problem in regard to the statistical variability of environmental temperature, and its daily and seasonal fluctuations. In the future codes and recommendations, however, the effect of temperature ought to be taken into account.

It is highly desirable that the models incorporated in design codes or design recommendations should satisfy the foregoing basic guidelines. Unfortunately, a close examination of the existing models shows that this is not the case. It is important that revisions that might be adopted in the future should adhere to these guidelines.

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REFERENCE

1. RILEM TC 69, Conclusions for structural analysis and for formulation of standard design recommendations, in 'Mathematical Modeling of Creep and Shrinkage of Concrete', edited by Z. P. Bažant, Chap. 6 (Wiley, New York, 1988); also *Mater. Struct.* **20** (1988) 395–398.