TRIAXIAL COMPOSITE MODEL FOR BASIC CREEP OF CONCRETE

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ABSTRACT: This paper shows how the mechanics of elastic composite materials can be adapted to predict the basic creep of concrete with aging due to hydration. The prediction is made on the basis of the given composition of concrete, the elastic constants of the aggregate, and the aging viscoelastic properties of the portland cement mortar. The triaxial action of the composite is approximated by Dvorak's transformation field analysis. To convert the aging creep problem to an elastic problem of a composite material with inelastic strains, Granger and Bažant's approach is used. This approach relies on Bažant's age-adjusted effective modulus method that reduces the integral-type stress-strain relation for linear aging creep with nonconvolution kernel to a quasi-elastic incremental stress-strain relation with inelastic incremental strain. Explicit expressions for the aging creep properties of concrete as a composite are deduced. The model is calibrated and verified by Ward et al.'s and Counto's test data. The predictions obtained are almost as close as those recently obtained by Granger and Bažant's model. While the present model has the advantage of describing the triaxial composite action in a rational manner, it does not yet capture the effect of the deviation of the aggregate configuration from the case of a contiguous aggregate skeleton of maximum possible compactness. Further refinements in this respect are needed. Another refinement might be needed to take into account possible enhancement of creep in the interface layers between the mortar and the aggregate.

INTRODUCTION

Effective physically based models have been developed for moisture diffusion and its effects on the long-term deformation of concrete (Bažant and Najjar 1972; Wittmann and Roelfstra 1980; Bažant and Chern 1985; Thelandersson et al. 1988; Bažant and Xi 1994), as well as for the phenomenon of aging (Bažant 1988; Bažant and Prasannan 1989; Carol and Bažant 1993; Bažant et al. 1997a,b). However, one effect has always been treated empirically—the triaxial effect of concrete composition on its long-term deformation. Yet, the composite materials literature offered a variety of rather rigorous approaches to the modeling of effective elastic constants of composites, based on known properties and volume fractions of the constituents (Hashin 1962; Hill 1965; Mori and Tanaka 1973; Boucher 1974; McLaughlin 1977; Christensen and Lo 1979; Benveniste 1987). Extensions to the modeling of inelastic, particularly viscoelastic, deformations, taking into account the inelastic behavior of one or more of the constituents, have also been made [e.g., Hashin (1962), Laws and McLaughlin (1978), and Dvorak (1992)].

Some early attempts were made to apply the theory of composite materials to the prediction of elastic modulus (Hansen 1960; Dougill 1962; Hirsch 1962; Counto 1964; Anson and Newmann 1965) or creep (Counto 1964; Popovics, 1986) of mortar or concrete composite. However, uncertainty about the elastic properties of the phases of the concrete composite (e.g., the fine aggregate in a saturated state), and even more the approximations of the triaxial action and geometry of the composite, led to errors that were greater than the errors of the classical empirical approaches developed for normal concrete with a particular type of constituents (aggregates within a certain range of elastic properties and cements of a certain type without any additives). Recently, there has been a revival of interest in micromechanical estimates of effective elastic properties (de Larrard and Le Roy 1992; Nilsen and Monteiro 1993; Simeonov and Ahmad 1995) and long-term deformations (Granger and Bažant 1995) of concrete. Such an approach became meaningful because the other aspects of the long-term deformation in concrete have become much better understood and because the empirical formulas developed for normal concretes have turned out to be inadequate for specialized concretes (Tighiouart et al. 1994).

The objective of the present study is to use micromechanics of composite materials to predict the basic creep of concrete composite from the known composition of concrete and the creep properties and volume fractions of the constituent phases. Only the basic creep is considered because the shrinkage of concrete and creep at drying are greatly influenced by moisture diffusion, which makes it difficult to separte the effects of composition alone. Previous studies of the problem include, notably, those of Counto (1964) and recently Granger and Bažant (1995). In both of these works, the composite action in concrete was modeled as a certain combination of series and parallel couplings of an aging viscoelastic matrix and an elastic aggregate. Although such simple uniaxial models cannot capture the multiaxial interactions of the phases, Granger and Bažant (1995) introduced a novel physically based concept—namely, the maximum compaction of a granular material—to determine the portions of the matrix that should be considered as coupled in series and in parallel. This concept endows the model with considerable predictive power. Nevertheless, the triaxial aspects of mechanics of composites are not utilized in this model.

In the present study, the triaxial interaction between matrix and aggregate is proposed to be taken into account by means of the recently developed method of transformation field analysis (Dvorak and Benveniste 1992; Dvorak 1992). For determining the effective elastic properties of the composite, which represents the elastic part of the overall composite response, and for carrying out some calculations of the transformation field analysis, we may choose from several methods available in the micromechanics literature. The present study employs the generalized self-consistent scheme due to Christensen and Lo (1979). Although the inclusion of the triaxial aspects precludes development of closed-form solutions, such as those obtained by Granger and Bažant (1995), a rather simple and

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computationally inexpensive numerical solution procedure can be developed.

The previous model of an aging viscoelastic material (the hardened cement paste) in which the aging is modeled as the volume growth into the capillary pores (solidification) of a nonaging constituent (Bažant 1977; Bažant and Prasannan, 1989) also relied on a uniaxial model of a parallel coupling between the already solidified load-bearing portion and the solidifying, still stress-free, portion. This kind of model could also be converted to a model with triaxial composite action, but this is beyond the scope of the present study.

CONCRETE AS COMPOSITE MATERIAL

A realistic composite model would have to treat concrete as a hierarchy of several composites, because the length scales of the microstructures involved span nine orders of magnitude (from coarse aggregate particles of the typical diameter of a few centimeters to gel pores of a size less than a nanometer). Obviously, modeling the microstructure at all these scales would be extremely complex. Moreover, the information about the constituent properties and geometry required for such a modeling could not be obtained experimentally. The problem needs to be simplified to the extent that the properties of the constituent phases can be reliably determined experimentally or estimated theoretically.

It has been established experimentally that the source of creep in concrete and other cement-based materials is the hardened cement paste [e.g., Neville et al. (1987) and Bažant (1988)]. The aggregates typically used in concrete do not creep in the range of stresses normally encountered in service (Neville et al. 1987). It is therefore logical to treat concrete as a composite of an aging viscoelastic cement paste matrix and an elastic aggregate.

An important point for such composite modeling, however, is to ensure that the matrix, whose compliance properties and volume fractions are being used in the composite model, has the same constitutive properties as that which actually exists in the composite material. This might be a trivial point for many composite materials but not concrete. For example, it has been experimentally observed that the porosity of the hardened cement paste in the vicinity of aggregate particles is higher than the porosity of a pure paste having the same water-to-cement ratio (Perrin et al. 1972). There is also the problem of accurate determination or estimation of the elastic properties of fine aggregate in a saturated state in which it exists in concrete. Furthermore, due to the uncertainty of the procedure to determine moisture absorption by fine aggregates, the actual water-to-cement ratio existing in the concrete composite, after the water has been absorbed by the fine aggregate, cannot be accurately matched to the water-to-cement ratio of the concrete mix.

For all these reasons, it remains imperative to include some empirical parameters in any composite model that treats concrete as a composite (whether one- or two-level) of two phases—the hardened cement paste (the creeping phase) and the aggregate (the elastic phase). The purpose of adopting a triaxial composite materials approach in this study is to reduce the empiricism.

We will consider concrete as a composite of mortar and coarse aggregate. This reduces the aforementioned difficulties to a great extent. If the mortar (rather than the hardened cement paste) is treated as the matrix, the effect of increased porosity of the hardened cement paste near the surface of the aggregate is included (Perrin et al. 1972). This means that the mortar can reasonably be assumed to be the same as the matrix that exists in concrete. Furthermore, the uncertainty about the elastic properties of the fine aggregate is eliminated if the creep properties of the mortar matrix are measured. Also the effect of the changes in the water-to-cement ratio of the composite, due to the water absorption by the aggregate, is mostly suppressed because the water absorption by the coarse aggregate is significantly less than the absorption by the fine aggregate, which is treated as a part of the matrix. Thereby the composite model is simplified.

COMBINATION OF AGE-ADJUSTED EFFECTIVE MODULUS METHOD AND TRANSFORMATION FIELD ANALYSIS

The basic relation of the age-adjusted effective modulus method (Bažant 1972; Bažant 1988) reads

$$\Delta\varepsilon(t) = \varepsilon(t) - \varepsilon(t') = \frac{\Delta\sigma(t)}{E''} + \frac{\sigma(t')}{E(t')}\phi(t, t') \tag{1a}$$

$$E'' = \frac{E(t') - R(t, t')}{\phi(t, t')} \tag{1b}$$

in which E'' = age-adjusted effective modulus; $E(t')$ = initial elastic modulus; t = current time (age at loading); t' = age at first loading; $\sigma(t)$ = stress history; $\Delta\sigma(t) = \sigma(t) - \sigma(t')$; $\phi(t, t') = E(t')J(t, t') - 1$ = creep coefficient; $R(t, t')$ = relaxation function = uniaxial stress at age t caused by a unit strain introduced at age t'; and $J(t, t')$ = compliance function = strain at age t caused by a uniaxial unit stress applied at age t'. $R(t, t')$ can be calculated from $J(t, t')$ by solving numerically a Volterra integral equation. It can also be approximately calculated by the formula of Bažant and Kim (1979).

To apply the transformation field analysis, we need to introduce the stress, strain, and compliance matrices

$$\boldsymbol{\varepsilon}(t) = (\varepsilon_{xx}, \varepsilon_{yy}, \varepsilon_{zz}, 2\varepsilon_{xy}, 2\varepsilon_{yz}, 2\varepsilon_{zx})^T \tag{2a}$$

$$\boldsymbol{\sigma}(t) = (\sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \sigma_{xy}, \sigma_{yz}, \sigma_{zy})^T \tag{2b}$$

$$\mathbf{M}(t) = \frac{1}{E(t)}\mathbf{G}$$

$$= \frac{1}{E(t)}\begin{bmatrix} 1 & -\nu & -\nu & 0 & 0 & 0 \\ & 1 & -\nu & 0 & 0 & 0 \\ & & 1 & 0 & 0 & 0 \\ & & & 1+\nu & 0 & 0 \\ & & & & 1+\nu & 0 \\ \text{symm.} & & & & & 1+\nu \end{bmatrix} \tag{3}$$

where subscripts x, y, and z refer to Cartesian coordinates. Using the (6×6) matrix $\mathbf{G}$, one can convert (1) to a matrix form.

We may now write the strain hisory $\boldsymbol{\varepsilon}(t)$ of the concrete composite in response to the applied stress history $\boldsymbol{\sigma}(t)$ as a summation of elastic and inelastic strain histories

$$\boldsymbol{\varepsilon}(t) = \mathbf{M}(t)\boldsymbol{\sigma}(t) + \boldsymbol{\varepsilon}^{in}(t) \tag{4}$$

Similarly, the constitutive relations for the phases may be written as

$$\boldsymbol{\varepsilon}_r(t) = \mathbf{M}_r(t)\boldsymbol{\sigma}_r(t) + \boldsymbol{\varepsilon}_r^{in}(t) \tag{5}$$

where $r = m, a$ = subscripts referring to the mortar matrix and the aggregate. We assume the aggregate to be elastic; hence, $\mathbf{M}_a(t) = \mathbf{M}_a$ and $\boldsymbol{\varepsilon}_a^{in}(t) = 0$.

To calculate the overall strain response of the composite to a given stress history from the given constitutive properties and volume fractions of the matrix phase (mortar) and the inclusions phase (coarse aggregate), we consider a representative volume of the composite such that it has the same response as any larger volume. The volume fractions or concentrations of the aggregate (meaning the coarse aggregate only) and the mortar are denoted as c_a and c_m; obviously $c_a + c_m =$

1. The phases may be assumed to be isotropic, which is a good approximation.

The first step is to calculate the overall elastic properties. Several closed-form solution methods to calculate this for the case of spherical inclusions are available in the micromechanics literature. In the present study, the generalized self-consistent scheme (Christensen and Lo 1979) is used.

We introduce the concept of volumetric averaging as used in the micromechanics of composite materials (Hill, 1965; Hashin 1983). The average stress or strain ($\bar{\sigma}_{ij}$ or $\bar{\varepsilon}_{ij}$) in volume V of the composite is defined as

$$\bar{\sigma}_{ij} = \frac{1}{V}\int_V \sigma_{ij}\, dV; \quad \bar{\varepsilon}_{ij} = \frac{1}{V}\int_V \varepsilon_{ij}\, dV \tag{6a,b}$$

In the following, we also use the average stresses and strains in the constituent phases. All of the stress and strain tensors, which are written hereafter without the overbars, will be understood as the volumetric averages defined by (6).

A representative volume of the two-phase composite is assumed to be loaded by surface tractions derived from a uniform overall stress $\boldsymbol{\sigma}$. The local stress averages in the phases are

$$\boldsymbol{\sigma}_r(t) = \mathbf{B}_r(t)\boldsymbol{\sigma}, \quad r = a, m \tag{7}$$

where $\mathbf{B}_r(t) = (6 \times 6)$ mechanical stress concentration factor matrices. For a two-phase composite, they can be found in terms of the local (phase), overall compliances $\mathbf{M}_r(t)$ and $\mathbf{M}(t)$, and volume fractions c_r

$$\mathbf{B}_r(t) = [\mathbf{M}_r(t) - \mathbf{M}_s(t)]^{-1}[\mathbf{M}(t) - \mathbf{M}_s(t)]/c_r, \quad r, s = a, m \tag{8}$$

The subscript s denotes the phase other than the phase r. The overall $\mathbf{M}(t)$ may be estimated by any of the methods of micromechanics for calculating the effective elastic properties [reviewed, e.g., by Hashin (1983)].

According to the transformation field analysis (Dvorak 1992), the following expressions give the transformation concentration factors in two-phase systems:

$$\mathbf{F}_{ra}(t) = [\mathbf{I} - \mathbf{B}_r(t)][\mathbf{M}_a - \mathbf{M}_m(t)]^{-1}\mathbf{M}_a, \quad r = a, m \tag{9a}$$

$$\mathbf{F}_{rm}(t) = -[\mathbf{I} - \mathbf{B}_r(t)][\mathbf{M}_a - \mathbf{M}_m(t)]^{-1}\mathbf{M}_m(t), \quad r = a, m \tag{9b}$$

In the presence of phase inelastic strains that are subject to the additive decomposition implied by (4) and (5), the averages of the local stress fields in the phases are evaluated in the form (Dvorak 1990)

$$\boldsymbol{\sigma}_r(t) = \mathbf{B}_r(t)\boldsymbol{\sigma} - \sum_s \mathbf{F}_{rs}(t)\mathbf{L}_s(t)\boldsymbol{\varepsilon}_s^{in}(t), \quad r, s = a, m \tag{10}$$

Here, the first term represents the local stress caused in an elastic composite by the external stress $\boldsymbol{\sigma}$. The second term is a contribution caused by the inelastic strain in the mortar phase and $\mathbf{L} = \mathbf{M}^{-1}$.

The overall response of the concrete composite is found from the known overall elastic compliance $\mathbf{M}(t)$ and the inelastic strains $\boldsymbol{\varepsilon}_s^{in}(t)$ that have been evaluated for the local stress or strain histories (10) in the inelastic phases. At any given time t, the overall strain is found from (Dvorak 1992)

$$\boldsymbol{\varepsilon}(t) = \mathbf{M}(t)\boldsymbol{\sigma}(t) + \sum_s c_s[\mathbf{B}_s(t)]^T\boldsymbol{\varepsilon}_s^{in}(t), \quad r, s = a, m \tag{11}$$

As a very good approximation, the basic equation (1) of the age-adjusted effective modulus method may be used to calculate $\boldsymbol{\varepsilon}_m^{in}(t)$ in terms of the local stress average $\boldsymbol{\sigma}_m(t)$. This is substituted into (10) to determine the local stress states in the phases. The stresses are those used in (11) to find the inelastic strain $\boldsymbol{\varepsilon}_m^{in}$ in the mortar and the overall strain in the composite.

According to linear-aging viscoelasticity, the exact expression for the inelastic stress in the mortar is given in terms of the Stieltjes integral over the stress history (Bažant 1988)

$$\boldsymbol{\varepsilon}_m^{in}(t) = \int_0^t \mathbf{J}_m(t, t')\, d\boldsymbol{\sigma}_m(t') - \mathbf{M}_m(t')\boldsymbol{\sigma}_m(t') \tag{12}$$

Because of the nonconvolution kernel $\mathbf{J}_m(t, t')$, which is a function of two time variables t and t' rather than just the time lag $t - t'$, this Volterra integral equation cannot be reduced to an algebraic equation by means of the Laplace transform. Consequently, one needs to resort to numerical approximations based on subdividing time into small intervals, which would, however, be cumbersome, opaque, and unnecessarily complicated for composite material analysis.

Therefore, we adopt the approximate age-adjusted effective modulus method (Bažant 1972), which is used in structural creep analysis. This method provides a quasi-elastic relation with inelastic strain between the stress and strain changes from the moment of loading t_0 to the current time t. It greatly simplifies the calculations. This method is exact for stress histories that are linear functions of $R(t, t')$ or strain histories that are linear functions of $J(t, t')$. Such histories are good approximations for most situations with constant long-time loading, including the present problem (Bažant 1972). Hence, the error of the age-adjusted effective modulus method ought to be small.

NUMERICAL IMPLEMENTATION

This study's two-phase composite has one aging viscoelastic phase (mortar matrix) and one elastic phase (aggregate). The mortar matrix is characterized by the aging viscoelastic compliance function $J_m(t, t')$ and Poisson's ratio ν_m that is constant in time (Bažant 1988). The aggregate is considered to be isotropic and elastic, characterized by its elastic modulus E_a and Poisson's ratio ν_a. The compliance function for the mortar matrix may be assumed to be represented simply by the log-double power law (Bažant and Chern 1985). More accurately and with a rational physical basis, it can be represented by the following expression based on the solidification theory (Bažant and Prasannan 1989; Carol and Bažant 1993), used in the B3 model [Bažant and Baweja (1994), adopted recently as the standard recommendation of Réunion Internationale des Laboratoires d'Essais et de Recherches sur les Matériaux et les Constructions]:

$$J_m(t, t') = q_1 + q_2 Q(t, t') + q_3 \log[1 + (t - t')^{0.1}] + q_4 \log\left(\frac{t}{t'}\right) \tag{13}$$

where t and t' must be given in days; q_1, q_2, q_3, and q_4 = material parameters to be determined from test data; and function $Q(t, t')$ is given by the integral

$$Q(t, t') = \int_{t'}^{t} \left(\frac{1}{\tau}\right) \frac{0.1(\tau - t')^{-0.9}}{1 + (\tau - t')^{0.1}}\, d\tau \tag{14}$$

This is a binomial integral that cannot be evaluated in a closed form. It can be evaluated either numerically or by an approximate formula (Bažant and Prasannan 1989; Bažant and Baweja 1994). Although (13) was developed for concrete, the physical basis of the assumptions used in its derivation is also valid for mortar. Its suitability for mortar is confirmed by the present results.

From (13), the age-dependent elastic modulus of the material is obtained as $E(t) = J(t + 0.01, t)$, where t must be in days (Bažant 1988; Bažant and Baweja 1994). Substituting $\varepsilon(t') = \sigma(t')/E(t')$ we can rewrite (1) as

$$\varepsilon_m(t) = \frac{\Delta\sigma_m(t)}{E''_m} + \frac{\sigma_m(t')}{E_m(t')}\phi_m(t, t') + \frac{\sigma_m(t')}{E_m(t')} \tag{15}$$

in which $\phi_m(t, t') = E_m(t')J_m(t, t') - 1$ = creep coefficient of the mortar. Because $\varepsilon_m^{in}(t) = \varepsilon_m(t) - \varepsilon_m^{el}(t)$, and $\varepsilon_m^{el}(t) = \sigma_m(t)/E_m(t) = \sigma_m(t)J_m(t + 0.01, t)$, we may write

$$\varepsilon_m^{in}(t) = \varepsilon_m(t) - \sigma_m(t)J_m(t + 0.01, t) \tag{16}$$

Substituting (15) into (16), and in turn (16) into (10), we get a system of equations that may be written as

$$\boldsymbol{\sigma}_m(t) + \mathbf{F}_{mm}\mathbf{L}_m\{\mathbf{M}_m''(t)[\boldsymbol{\sigma}_m(t) - \boldsymbol{\sigma}_m(t')] + \mathbf{M}_m(t')\boldsymbol{\sigma}_m(t')[\phi_m(t, t') + 1] - \mathbf{M}_m(t)\boldsymbol{\sigma}_m(t)\} = \mathbf{B}_m\boldsymbol{\sigma} \tag{17a}$$

$$\boldsymbol{\sigma}_a(t) + \mathbf{F}_{am}\mathbf{L}_m\{\mathbf{M}_m''(t)[\boldsymbol{\sigma}_m(t) - \boldsymbol{\sigma}_m(t')] + \mathbf{M}_m(t')\boldsymbol{\sigma}_m(t')[\phi_m(t, t') + 1] - \mathbf{M}_m(t)\boldsymbol{\sigma}_m(t)\} = \mathbf{B}_a\boldsymbol{\sigma} \tag{17b}$$

The age-adjusted effective modulus E'' to be used here may be determined from the relaxation function $R(t, t')$ according to (1).

To calculate the relaxation function from the given compliance function, one may either solve it numerically from a Volterra integral equation or use the approximate formula of Bažant and Kim (1979). A recent slight modification of this formula given in Appendix I, which is more suitable for the solidification theory, has been used.

The solution proceeds in time steps as follows. A unit uniaxial stress, $\boldsymbol{\sigma} = (1, 0, 0, 0, 0, 0)^T$, is assumed to be applied to the composite at the given age at loading. Time steps whose durations increase in a geometric progression are then chosen. Based on $E_m(t) = 1/J_m(t + 0.01, t)$, ν_m, E_a, ν_a, and on the volume concentrations of the mortar and the coarse aggregate, the overall elastic modulus $E(t)$ is determined using the generalized self-consistent scheme due to Christensen and Lo (1979). The matrices $\mathbf{M}_m(t) = J_m(t + 0.01, t)\mathbf{G}$, $\mathbf{M}_a = \mathbf{G}/E_a$, and $\mathbf{M}(t) = \mathbf{G}/E$ are then used in (8) to calculate the mechanical stress concentration factors $\mathbf{B}_r(t)$. Eq. (9) is used next to calculate the transformation concentration factors. The system of equations (17) is solved at the end of each time step to evaluate the stress averages $\boldsymbol{\sigma}_m(t)$ and $\boldsymbol{\sigma}_a(t)$. Then, (15) and (16) are used to determine $\varepsilon_m^{in}(t)$, which is finally used in (11) to find the total strain in the composite at age t.

COMPARISONS WITH AVAILABLE TEST DATA

Although the effects of concrete composition on creep have been studied experimentally by many investigators, the only data sets that are sufficiently comprehensive for verifying a more general model appear to be those of Ward et al. (1969) and Counto (1964). The former consists of 14 tests of basic creep using the same constituents: two pastes, four mortars, and eight concretes (labeled P, M, and C), with or without entrained air (labeled with or without A). Table 1 shows the mix proportions for these concretes, mortars, and pastes.

For the purpose of this study, we need concretes whose matrix is one of the mortars separately tested. Among the eight concretes tested, five may be assumed to have the same matrix as one of the four mortars tested separately. This can be seen from Table 2, which groups together the mortars and the corresponding concretes having about the same water-to-cement ratio and the same fine aggregate-to-cement ratio (all of the aggregates in the mortars being the fine aggregate). The present creep model considers mortar M5 as the matrix in concrete C2, mortar M5A as the matrix in concrete C2A, mortar M6 as the matrix in concretes C3 and C4, and mortar M6A as the matrix in concrete C4A.

Only a part of the test data reported by Counto (1964) is usable for the present study. Although these tests were carried out on concretes made with four different aggregates, the complete experimental creep curves were reported only for concretes made with two of these aggregates. These data (for concretes made with cast iron and polyethylene aggregates) are used herein. Two different volume fractions were selected for each aggregate. These data permit a limited verification of the model over a broader range of elastic moduli of the aggregate and aggregate volume fractions.

Fig. 1 shows the predictions of the creep of the five concretes tested by Ward et al. (1969) compared to their measured values. Fig. 2 shows the predictions of the model compared to the measured creep values of four concretes tested by Counto (1964).

In view of the scatter normally encountered in concrete creep measurements, and the fact that no empirical fitting parameter has been used in the calculations, the prediction may be regarded as satisfactory. The present study should pave the way for a more comprehensive experimental study designed to verify the applicability of the triaxial composite model to matrices that are used in modern concretes, as well as for a broad enough range of volume fractions and elastic properties of the aggregates used today.

The main advantageous feature of the present model is that it captures the triaxial interaction between two phases in the gradual transfer of stress from the matrix phase to the aggregate phase that takes place with time as the matrix creeps. The variation of the average stresses in the two phases with time is shown in Fig. 3 for a particular value of the elastic modulus of the aggregate and two different volume fractions of the aggregate.

TABLE 1. Composition of Concretes and Mortars Tested by Ward et al. (1969)

Mix number (1)	Strength at 7 days (MPa) (2)	w/c ratio (3)	Aggregate-to-cement ratio (4)	Fine-to-coarse aggregate ratio (5)	Air content (6)
M5	17.65	0.88	4.1	—	4.00
M5A	21.05	0.69	3.6	—	12.50
M6	40.00	0.60	2.6	—	3.75
M6A	39.80	0.45	1.5	—	11.0
C1	23.1	0.80	7.6	1:1.23	2.75
C1A	20.50	0.68	7.2	1:1.38	8.0
C2	17.4	0.88	8.9	1:1.16	3.25
C2A	19.5	0.70	8.4	1:1.33	8.00
C3	31.50	0.63	5.9	1:1.31	2.5
C3A	31.9	0.48	5.1	1:1.87	7.75
C4	34.15	0.62	5.9	1:1.28	3.00
C4A	36.6	0.47	4.9	1:2.12	8.00

TABLE 2. Groups of Mortar Matrix and Corresponding Concrete Composite Used in Calculations

Material (1)	w/c ratio (2)	Fine aggregate-to-cement ratio (3)
Matrix (mortar M5)	0.88	4.1
Composite (concrete C2)	0.88	4.12
Matrix (mortar M5A)	0.69	3.6
Composite (concrete C2A)	0.70	3.6
Matrix (mortar M6)	0.60	2.6
Composite (concrete C3)	0.63	2.55
Composite (concrete C4)	0.62	2.58
Matrix (mortar M6A)	0.45	1.5
Composite (concrete C4A)	0.47	1.57

PREPACKED CONCRETE: LIMITATION OF PROPOSED MODEL

There is one type of concrete that must be considered to lie beyond the range of validity of the present model (although an extension to that type should be possible): It is the prepacked concrete, which is produced by first placing the coarse aggregate into the form and then infiltrating the voids between

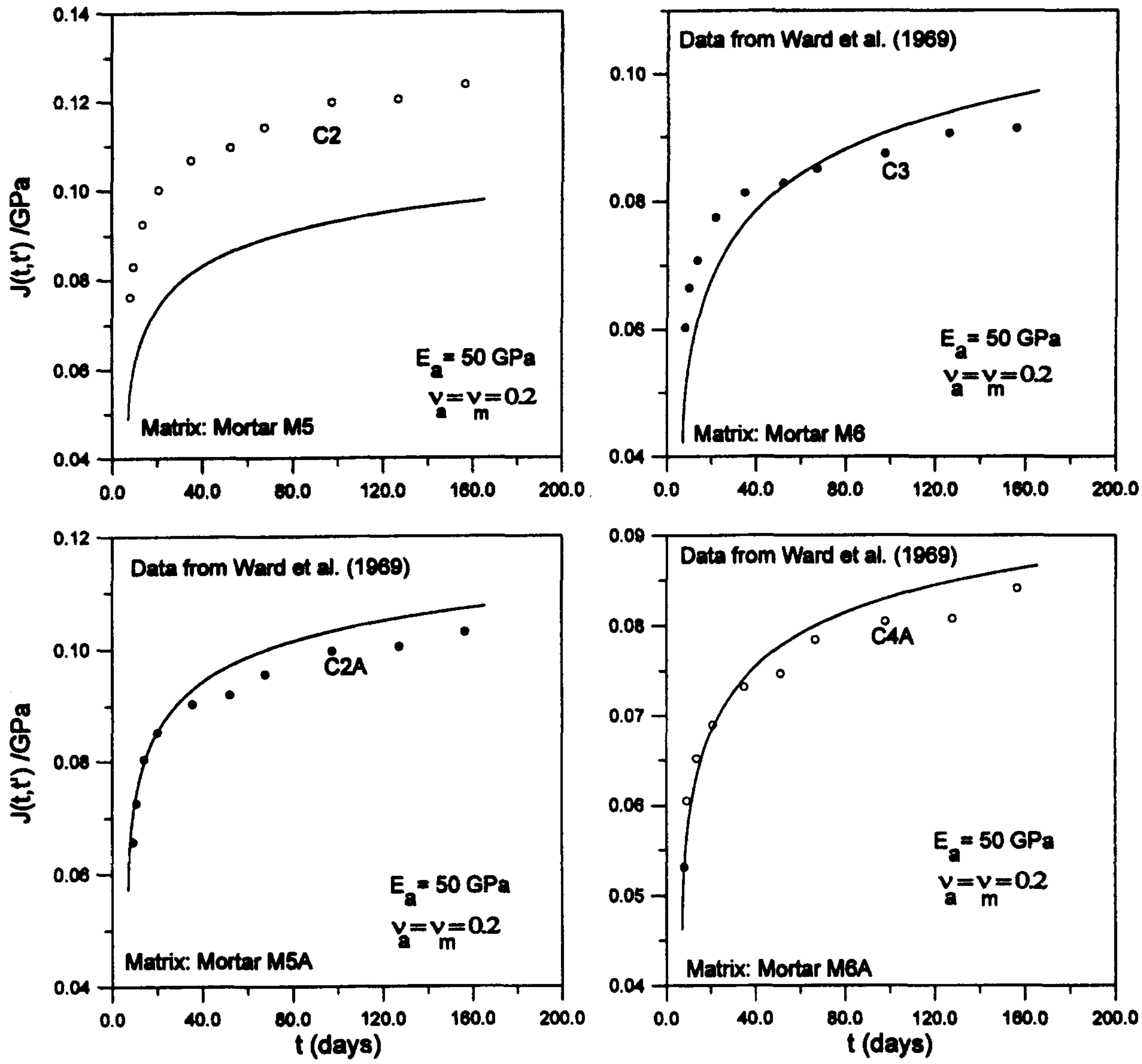

FIG. 1. Predictions of Creep of Five Concretes Tested by Ward et al. (1969) Compared with Their Measured Values

the aggregate pieces by a mortar. The basic difference is that the aggregate pieces in prepacked concrete are in contact, while in the normal (mixed) concretes they are not, being separated by a layer of mortar.

Although no hard data seem to exist in the literature, some engineers have commented that prepacked concrete exhibits very small creep and shrinkage [e.g., Neville et al. (1983)], which is apparently an order of magnitude smaller than normal concretes. The difference between the normal and prepacked concrete is geometrical in nature. It cannot be modeled in terms of volume fractions alone, and so it cannot be captured by the present model. Indeed, it is possible, at least in principle, to produce a prepacked concrete and normal concrete having the same aggregate volume fractions, provided the aggregate for the prepacked concrete is placed with significantly less than its maximum possible compactness.

The exclusion of prepacked concretes is apparently an inherent limitation for the previous model of Granger and Bažant (1995), because the maximum aggregate compactness is used as one basic parameter of that model. According to Granger and Bažant the part of mortar that would remain if the aggregate configuration were contracted to achieve a contiguous aggregate skeleton it assumed to act essentially in parallel coupling with the aggregate skeleton (having equal strains and additive stresses), while the additional part of mortar is assumed to act essentially in series coupling (having equal stresses and additive strains). In this view, it is logical to assume that, in a prepacked concrete, all of the mortar acts in parallel coupling with the aggregate and none in series coupling. The absence of series coupling yields, in that model, a greatly reduced creep.

One simple way to extend the present model to prepacked concretes might be obtained by considering the volume fraction of the mortar that would fill the voids at maximum aggregate compaction and the additional volume fraction of mortar as two separate parameters.

CONCLUSIONS

1. The main contribution of the present study is to apply an effective model for the triaxial action of a composite to the problem of prediction of basic creep of concrete, including its aging property, from the known elastic and creep properties of the portland cement mortar and the aggregate. This objective is met by applying Dvorak's (1992) transformation field analysis.
2. As in the previous model of Granger and Bažant (1995), the simplification that makes the triaxial action of the composite easy to analyze is the representation of the creep of mortar according to Bažant's (1972) age-ad-

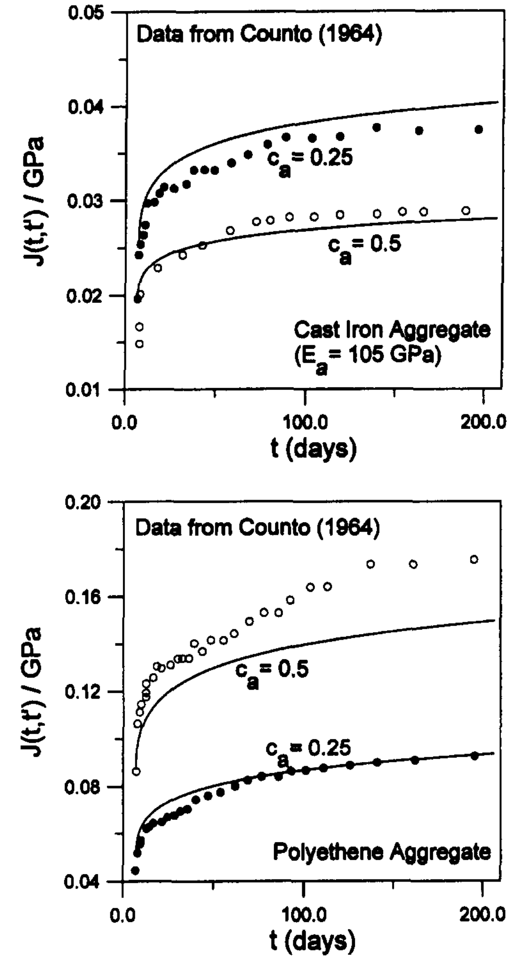

FIG. 2. Predictions of Model Compared with Measured Creep Values of Four Concretes Tested by Counto (1964)

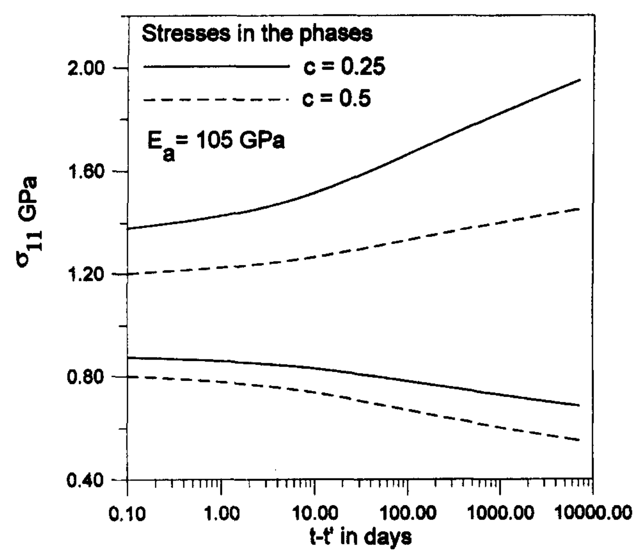

FIG. 3. Variation of Average Stresses in Two Phases with Time

justed effective modulus method. This method replaces the integral-type constitutive relation with nonconvolution kernel for the aging viscoelasticity of mortar by an approximate but rather accurate one-step incremental quasi-elastic stress-strain relation with inelastic strain. As a result, explicit expressions for the aging creep properties of concrete as a composite can be obtained.

3. Comparison of various methods for predicting the overall elastic moduli of composites shows Dvorak's (1992) transformation field analysis, applied to changes due to creep and combined with the generalized self-consistent scheme for the initial elastic deformations, to be an effective approach.
4. The model is calibrated and verified by the test data of Ward et al. (1969) and Counto (1964). The long-time predictions that can be obtained are almost as close to the measured data as those obtained recently with a simpler but more empirical model of Granger and Bažant (1995). In contrast to that model, the advantage of the present model is that it captures in a rational manner the triaxial composite action.
5. Further refinements of the present composite model for basic creep are desirable to take the following into account: (1) The geometrical packing of the aggregate configuration (especially the thickness of the contact layers between adjacent aggregate pieces manifested in the degree of deviation from the state of maximum possible aggregate compactness); (2) enhanced creep in the layers of mortar close to aggregate surfaces (important especially for porous aggregates); and (3) possible bond creep.

APPENDIX I. MODIFIED APPROXIMATE FORMULA FOR AGING RELAXATION FUNCTION

Bažant and Kim's (1979) approximate formula for converting the compliance function $J(t, t')$ of aging concrete into the relaxation function $R(t, t')$ has recently been slightly modified so as to make it optimum for the case of compliance function based on the solidification theory. The formula reads

$$R(t, t') = \frac{0.992}{J(t, t')} - \frac{0.115}{J_0}\left[\frac{J(t' + \xi, t')}{J(t, t - \xi)} - 1\right], \quad \xi = \frac{t - t'}{2} \tag{18}$$

in which the value $J_0 = J(t, t - 1)$ was used. The new empirical modification (Baweja and Bažant, unpublished internal research note, 1995) consists in using

$$J_0 = J(t' + \xi, t' + \xi - 1) \tag{19}$$

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APPENDIX II. REFERENCES

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