Printed with error in the title. The correct title should read: Justificiation of ACI-446 Proposal for Updating ACI Code Provisions for Shear Design of Reinforced Concrete Beams (Errata published in subsequent issue).

ACI STRUCTURAL JOURNAL TECHNICAL PAPER

Title no. 104-S57

Justification of ACI 446 Code Provisions for Shear Design of Reinforced Concrete Beams

by Zdenek P. Bažant, Qiang Yu, Walter Gerstle, James Hanson, and J. Woody Ju

Due to a relatively large amount of experimental evidence and recent scientific advances, it is now generally recognized that, to ensure adequate safety margins, the size effect for designing reinforced concrete beams against shear failure must be incorporated into ACI code provisions. A purely empirical approach is impossible because the available test data, mostly obtained on small beams, need to be extrapolated to much larger beams for which tests are scant or nonexistent. Arguments for an improved code formulation are summarized, and verification by a database compiled by Joint ACI-ASCE Committee 445 is reviewed.

Keywords: shear failure; size effect; reinforced concrete.

INTRODUCTION

ACI 318-05, Eq. (11-3), currently specifies the contribution of concrete to the cross section shear strength of reinforced concrete members as $V_c = 2\sqrt{f_c'}\, b_w d$ (valid in psi, lb, and in.), where f_c' is the required compression strength of concrete, d is the beam depth from the top face to the longitudinal reinforcement centroid, and b_w is the web width. This code formula was justified on the basis of a Joint ACI-ASCE Committee database,[1] which involved only small beams of average depth 13.4 in. (340 mm). This formula was set not at the mean of these data but near their lower margin, at a level that appears to be the 5% fractile (or probability cutoff) of the data if a Gaussian distribution is fitted to the data (refer to Fig. 1).

The code formula gives a size-independent average concrete shear strength, $v_c = V_c/b_w d$ (identical to the nominal strength in mechanics terminology). Compelling experimental evidence for size effect, however, has been gradually accumulated since 1962,[2-4] and some large-scale tests, particularly those in Tokyo[5-7] and in Toronto[8-11] showed the urgency of taking into account the size effect. Furthermore, recent analysis of some major structural disasters (for example, the Sleipner oil platform; a warehouse at Wilkins AF Base in Shelby, Ohio; and the Koror box girder bridge in Palau) indicated that the size effect must have been a contributing factor (and so it seems to have been for the overpass failure in Laval, Quebec, on September 29, 2006). A base of 296 data assembled at Northwestern University[2] and a recent larger database of 398 data compiled by ACI Subcommittee 445F (refer to Fig. 2 and 3), clearly show the current code to be unconservative for large beams. Especially, a large (6.2 ft [1.89 m] deep) and lightly reinforced concrete beam has been observed to fail at a load less than 1/2 of the required design strength V_u/ϕ (with ϕ from ACI 318-05).

The purpose of this paper is to summarize the justification of a revision[12] of Section 11.3 of ACI 318-05 (detailed arguments are presented separately[13,14]).

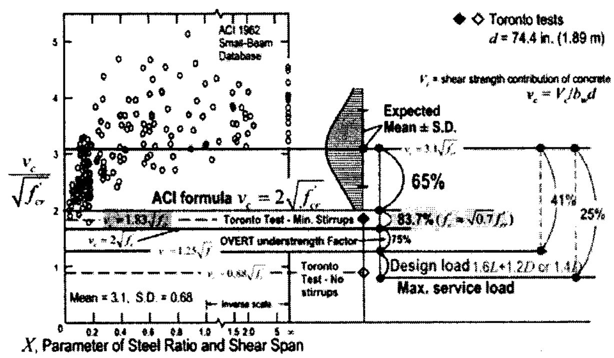

Fig. 1—Joint ACI-ASCE Committee 326[1] small beam database used to justify current ACI code formula for shear force capacity V_c due to concrete in reinforced concrete beams with and without stirrups, and reductions specified or implied by ACI 318-05 that were justified by this database (f_{cr}' = average compression strength of concrete from tests; $f_c' \approx 0.7f_{cr}'$ = required concrete strength, as defined in ACI 318-05).

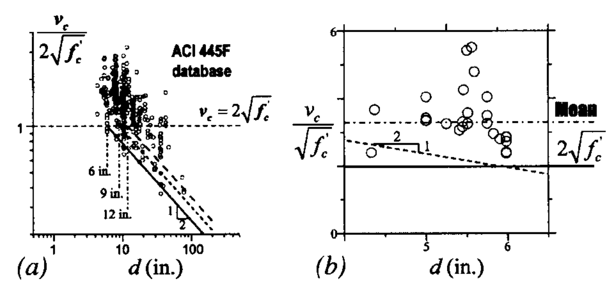

Fig. 2—Alternative simplified size effect formula compared with: (a) complete ESDB; and (b) small-size portion of that database in expanded scale.

RESEARCH SIGNIFICANCE

To make the risk of structural failure much smaller than various inevitable risks that people face, the tolerable failure probability is approximately one in 1 million.[15] This value agrees with experience for small beams, but not for large ones, for which it has been approximately one in 1000[16,17] (and could become one in 100 or higher as ever larger beams are built). Whether or not such intolerable risk will have to be tolerated depends largely on taking the size effect properly into account. This is an issue of paramount significance.

ACI Structural Journal, V. 104, No. 5, September-October 2007.
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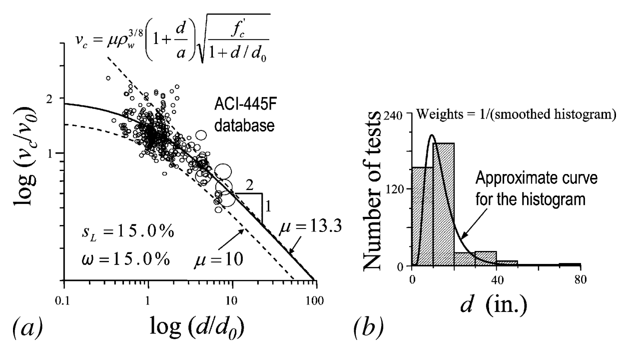

Fig. 3—(a) Comparison of proposed formula with ESDB; and (b) smoothing of histogram of beam depth in ESDB.

EXPERIMENTAL DATABASE USED

Thousands of experiments have been conducted around the world to assess the shear capacity of concrete members, although only a small fraction of them were specifically aimed at the effect of size. ACI Subcommittee 445F extracted, from a collection of more than 1000 data, a new database of 398 data, called the Evaluation Shear Database (ESDB).[18] Only beams with no shear reinforcement, subjected to three-point or four-point loading, are included. All the beams have a rectangular cross section except that 24 are T-beams. The beam depth ranged from 4.33 to 78.74 in. (110 to 2000 mm) (with a mean of 13.6 in. [345 mm], which is nearly equal to the mean of 13.4 in. (340 mm) in the 1962 database, and a coefficient of variation [CoV] of 74%); the shear-span ratio (a/d) (with $a = M/V$) ranged from 2.41 to 8.03 (with a mean of 3.6 and a CoV of 26%); the compression strength f_c' of concrete of the beams ranged from 1828 to 16,080 psi (12.6 to 110.9 MPa) (with a mean of 6104 psi [42.09 MPa] and a CoV of 55%); the longitudinal steel ratio ranged from 0.14 to 6.64% (with a mean of 2.3% and a CoV of 52%); and the maximum aggregate size, known for only for 341 data points, ranged from 0.25 to 1.5 in. (6.35 to 38 mm) (with a mean of 0.71 in. [18 mm] and a CoV of 40%).

The ESDB has been adopted for the present studies in ACI Committee 446, even though the rationality and impartiality of the criteria used to select the data have been questioned.[19-23] For instance, the largest beams ever tested, up to 9.84 ft (3 m) deep[5-7] were excluded from the ESDB based on the fact that they were subjected to distributed load, a combination of which, with point loads in the same database, was thought to complicate interpretation. But this position disregards the fact that the code provision must apply to both. The reduced-scale beam tests at Northwestern University,[4] with an aggregate size of 0.19 in. (4.8 mm) and a beam width b_w of 1.90 in. (48 mm), were excluded with the explanation that, inexplicably, only beams with b_w greater than 1.97 in. (50 mm) were admissible; these tests, however, exhibited the most systematic size effect trend, had an exceptionally broad size range (1:16), and achieved the highest brittleness number[24] among all the available tests, thus mimicking the brittleness of very large beams (b_w equaled 10 maximum aggregate sizes in these tests, which is not only adequate but also, after a width increase by mere 4%, would have technically qualified these data for inclusion in the ESDB; the width increase would not have distorted interpretation because it is generally accepted that the effect of beam width on v_c is nil[8,20-23] if the width exceeds approximately four aggregate sizes).

While the size effect is of major concern for beams deeper than approximately 40 in. (1 m), 86% of the tests in the ESDB pertain to beam depths less than 20 in. (0.5 m), 99% less than 43 in. (1.1 m), and 100% less than 79 in. (2 m) (see the database histogram in Fig. 1 of Reference 13). The CoV or ω of the deviations of an empirical size effect formula derived directly from the ESDB will therefore be totally dominated by small size beams for which the size effect is unimportant. Thus, it is possible that some formula that gives the lowest ω for the ESDB could be completely wrong for large sizes while another formula that might give a higher ω could be much more realistic for large sizes. Obviously, a purely empirical extrapolation to large sizes cannot be trusted. A solid scientific basis is crucial. In the plot of $\log(v_c/\sqrt{f_c'})$ versus $\log d$ (refer to Fig. 2 in Reference 13) it is striking that, while the curves of various previously proposed formulas are very different, they all appear to be equally good (or equally bad) compared with the ESDB. The reasons are: 1) The size range covered by the database is not broad enough; 2) the scatter is enormous because the effects of concrete strength and type, longitudinal steel ratio, shear span, and aggregate size are not separated by a suitable choice of relevant regression variables; and 3) the ESDB database is biased by the fact that the interval averages of other influencing variables (ρ_w, a/d, f_c'), as well as the spread between the minimum and maximum interval values of each variable, vary strongly from one size interval to the next.

A serious obstacle to extracting a size effect formula purely empirically from the ESBD is the fact that the vast majority (more than 97%) of its 398 data points come from tests motivated by different objectives (such as the effect of concrete type, reinforcement, and shear span), in which the beam depth was varied only slightly or not at all. The effects of variables other than d exhibit enormous scatter, which masks the size effect trend. It is necessary to find regression coordinates that include the effects of influencing variables other than the size.

CHOICE OF BASIC SIZE EFFECT FORMULA

In view of the preceding arguments, it is necessary to establish the beam shear formula in two steps: 1) select the form of the formula on the basis of a sound theory and verify it by close fits of the available individual test series with geometrical scaling and a sufficiently broad size range; and 2) calibrate the selected formula using the whole ESBD. This procedure[12-14] led to the classical energetic size effect formula[25]

$$v_c = \frac{v_0}{\sqrt{1 + d/d_0}} \quad (1)$$

The first step shows that the choice of the form of size effect would not be contaminated by random variation of parameters other than size d. Because of high random scatter in beam shear tests, the size range should be at least 1:8 to obtain a clear size effect trend. Two data sets that closely approach these requirements are those obtained at Northwestern University (not included in the ESDB) and the University of Toronto (refer to Fig. 4), which shows that the fits by Eq. (1) are very close.

The salient property of this formula is that, for large sizes, it approaches an inclined asymptote of slope –1/2 in a doubly logarithmic plot, corresponding to a power law of the type $d^{-1/2}$. This property, which was endorsed as essential by a unanimous vote of ACI Committee 446 in Vancouver in 2003, is indeed verified by the available broad-range test series—Northwestern University tests (Fig. 4(a)), University of Toronto tests (Fig. 4(b)), and the record-size Japanese tests (Fig. 4(c) and (d)). It is not contradicted by any of the existing additional seven test series of a lesser but still significant size range[8,26,27] (refer to the plots in Reference 14).

The Japan Society of Civil Engineers (JSCE) pioneered the size effect for design code long ago. It adopted a power-law, $v_c \propto d^{-1/4}$, which was proposed by Okamura and Higai[28] already in 1980 before the energetic size effect was discovered and was motivated by the Weibull statistical theory, at a time when this classical theory was the only theory of size effect. A decade later it became clear that the Weibull theory applies only for structures failing right at the initiation of fracture growth from a smooth surface,[29,30] which is not the case for reinforced concrete beams, where a large crack or cracking zone develops before the maximum load is reached.[24,29-32] Besides, even if the Weibull statistical theory were the right explanation for the JSCE power law, its exponent would need to be changed from –1/4 to –1/12. The reason is twofold: 1) a realistic Weibull modulus for concrete is 24 rather than 12[14,33]; and 2) the fracture scaling must be considered two-dimensional ($n = 2$) because, in not too wide beams, the fracture must (for reasons of mechanics) grow over the whole beam width nearly simultaneously. But the exponent –1/12 would be far too small to describe the strong size effect evidenced by test data, including those of JSCE.

The formula based on the crack spacing according to the modified compression field theory (MCFT) has the opposite problem of the JSCE formula. Its large-size asymptote is $v_c \propto d^{-1}$, while the exponent of the greatest thermodynamically possible magnitude is –1/2 (or else the energy flux into moving fracture front would be infinite[24,29,31]). Besides, the proposed justification of the MCFT formula[34] is unrealistic for two reasons[13,14]: 1) crack spacing is not uniquely related to energy release and depends also on other factors[35]; and 2) the crack-bridging tensile and shear stresses at maximum load are reduced to almost zero while the failure is caused mainly by near-tip compression stresses parallel to the diagonal shear crack. As for the CEB-FIP formula, it is purely empirical and thus cannot be trusted for large sizes for which data are scant or nonexistent.

The deceptiveness of a purely empirical power-law extrapolation of a combined database such as ESBD is illustrated in Fig. 5(a), (b), and (c). Suppose that the mean

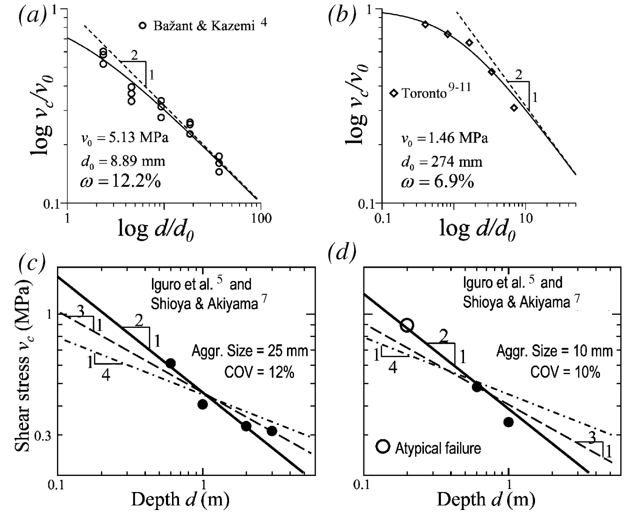

Fig. 4—Comparison of size effect Formula (1) to beam shear test series with greatest size range and with nearly geometrical scaling: (a) microconcrete beams tested by Bažant and Kazemi at Northwestern University in 1991[4] (not included in ESDB); (b) large size tests at University of Toronto reported by Podgorniak-Stanik[9] and Lubell et al.;[11] and (c) and (d) large beams under uniform loading in Tokyo.[5-7]

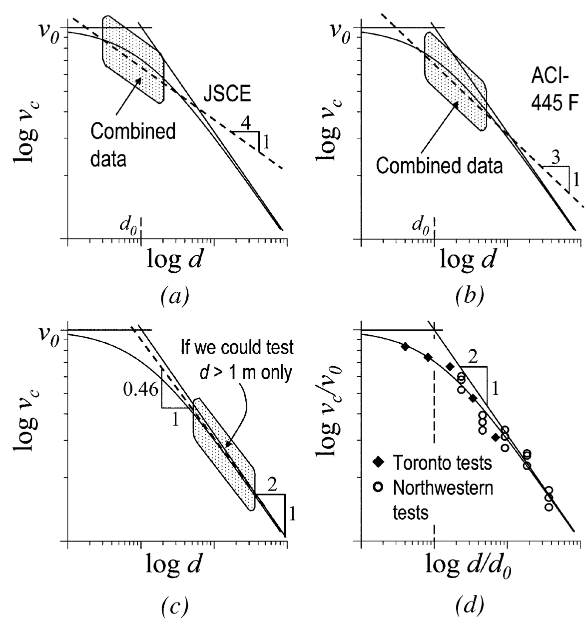

Fig. 5—Example of effect that choice of size range of highly scattered tests can have on regression result when straight line plot in log-scale in assumed.

size effect trend agrees perfectly with size effect Law (1), but different investigators choose different size effect ranges for testing. In view of scatter, each of them fits a power law to his data. The exponents of this power law will vary between 0 and –1/2 depending on the chosen size range. An unambiguous, purely experimental verification of Eq. (1) would require a very broad size range (Fig. 5(d)).

STATISTICAL CALIBRATION, VERIFICATION, AND EVALUATION OF PROPOSED FORMULA

The next step is to calibrate the size effect formula by proper statistical regression. Let $\overline{v}_i$ (i = 1, 2,...n) be the measured data points for sizes d_i and let v_i be the corresponding values of v_c calculated from the proposed formula. It turns out that the right approach is not to minimize the sum of squared errors (or residuals) $\Sigma_i(v_i - \bar{v}_i)^2$ because the variance of the data (precisely, conditional variance $\mathrm{Var}(v_c|d)$[36]) is heteroscedastic, that is, strongly decreases with the increasing size d. To minimize statistical bias, the statistical variable v_c should be transformed so as to make the variance, and thus the scatter band width, approximately uniform,[36] or homoscedastic. This is approximately achieved by the transformation $y = \ln v_c$. Thus, the objective of data regression is to minimize, in the scale of $\ln v_c$, the square of the standard error of regression s_L, the unbiased definition of which is $s_L^2 = \Sigma_{i=1}^n \ln(v_i/\bar{v}_i)^2/(n-p)$ where p is the number of free parameters in data fitting (because $(d\ln v_c)^2 = (dv_c)^2/v_c^2$), the transformation from v_c to y has a similar effect as applying weights proportional to $1/v_c^2$. In the linear scale of v_c, the corresponding CoV of regression is $\omega = (e^{s_L} - e^{-s_L})/2$ (which herein is almost equal to s_L).

According to the ACI code, the factored shear force V_u must not be greater than $\phi(V_c + V_s)$ where $\phi = 0.75$ is the understrength (strength reduction) factor and V_s is the yield shear force carried by shear reinforcement. The maximum shear force V_c that can be carried by concrete is proposed to be calculated as[14]

$$V_c = 10 b_w \rho_w^{3/8}\left(1 + \frac{d}{a}\right)\sqrt{\frac{f_c' d_0 d}{1 + d_0/d}}, \quad d_0 = \kappa f_c'^{-2/3} \tag{2}$$

$$\text{where, if } d_a \text{ is known, } \kappa = 3800\sqrt{d_a}\text{; if not, } \kappa = 3330 \tag{3}$$

where V_c is in lb, f_c' is in psi, ρ_w is the longitudinal steel ratio, and b_w and d are in inches. The expression for d_0 is empirical. Note that V_c increases continuously with d, but less than proportionately (because of size effect).

As seen in Fig. 2(b), for very small d, the V_c value according to the proposed Formula (2) is greater than predicted by the current Formula (4), $V_c = 2\sqrt{f_c'}\, b_w d$. This means that the current formula can be used safely within a certain range. The permissible safe range for Eq. (4) is $d \le 6$ in. (150 mm). This is ascertained from the ESDB plotted in Fig. 2, which reveals that for $d \le 6$ in. (150 mm), no beam test gave a shear strength less than the value given by Eq. (4).

As a simple and safe (though often uneconomical) alternative (Fig. 2(a)), the simple formulas

$$\text{for } d \le 6 \text{ in. (150 mm): } V_c = 2 b_w \sqrt{f_c'}\, d \tag{4}$$

$$\text{for } d > 6 \text{ in. (150 mm): } V_c = 5 b_w \sqrt{f_c'}\, d \tag{5}$$

can be used instead of Eq. (2). In Fig. 2, the solid inclined line represents Eq. (5). Note that if the small size limit were set at 9 or 12 in. (0.23 or 0.3 m), as shown by the other two dashed inclined lines, the design equation would not be safe.

Formula (2), as well as Formulas (4) and (5), are recommended for use regardless of whether or not there is shear reinforcement. For small beams, shear reinforcement appears to increase V_c appreciably. But this observation is based on only one large beam test, which is statistically insufficient, and the test shows that the size effect is only mitigated, but not eliminated, by shear reinforcement. Furthermore, finite element simulations at Northwestern University (based on nonlocal damage concept) show that, for large beams exceeding approximately 60 in. (1.52 m) in depth, shear reinforcement does not increase V_c and does not help against size effect. For very deep beams with strong shear reinforcement, these simulations indicate that not only is V_c not increased, but V_s at maximum load is much below the yield strength of stirrups $V_s = A_s f_y d/s$.

The general form of Formula (1) has been verified for many different structural geometries and many different quasibrittle materials. The analytical derivations (though not the numerical verifications) have been subjected to the hypothesis that a large crack or long band of cracking damage develops in a stable manner before the maximum load is reached and the failure modes of small and large structures are geometrically similar (experiments as well as finite element simulations document that this is approximately true for beam shear failures).

The current ACI code also involves corrections to the expression for V_c due to simultaneous action of compressive or tensile axial force, and for the calculation of the shear span ratio from the bending moment in the presence of axial force. The multiplicative factors for these corrections are applied to the present formula with no change.

The expressions for the parameters in Eq. (2) through (5) have been obtained by simplified mechanical considerations and calibrated by optimization of data fits.[14] The least-square fitting of the data, conducted in the plot of $\ln v_c$ versus $\ln d$, was a weighted regression. The weighting was necessary to counteract the subjective bias due to crowding of the data points in the small-size range; refer to Fig. 3 where the data points are represented by circles having areas proportional to the weight. A logarithmic scale of d needs to be used because, for example, the size effect from 11.8 in. (0.3 m) to 11.8 + 11.8 in. (0.3 + 0.3 m) is significant, but from 118 in. (3 m) to 118 in. + 11.8 in. (3 m + 0.3 m) insignificant. The optimum data fitting was accomplished by a standard library subroutine for the Levenberg-Marquardt nonlinear optimization algorithm. The heavy solid line in Fig. 3 represents the mean fit formula, and the dashed line represents the design formula, which is set at the lower 5% fractile of the scatter band width. The overall CoV or ω of the errors of Formula (2) calculated by the ESDB is 15%. The CoV of the errors for various size intervals of 10 in. (0.25 m) width are 18.8, 15.6, 11.6, 15.3, 14.5, and 15.7%, respectively (note that these values are approximately uniform, which conforms homoscedasticity, is required for a proper statistical approach and is achieved by transforming the regression variable from v_c to $\ln v_c$).

The reason why Eq. (3) gives two options for calculating d_0 is that sometimes the design needs to be made before the maximum aggregate size d_a has been decided. Both expressions for d_0 give the same value when $d_a = 0.77$ in. (≈20 mm).

REGRESSION OF DATA GROUPED IN EQUAL-RATIO INTERVALS

To minimize the size effect bias due to highly nonuniform distribution of data through the size range of interest, subdivide the range of beam depths d of the existing test data into five size intervals (Fig. 6). They range from 3 to 6 in. (76.2 to 152.4 mm), from 6 to 12 in. (152.4 to 304.8 mm), from 12 to 24 in. (304.8 to 609.6 mm), from 24 to 48 in. (609.6 to 1219.2 mm), and from 48 to 96 in. (1219.2 to 2438.4 mm).

Note that the borders between the size intervals are chosen to form a geometric (rather than arithmetic) progression because what matters for size effect is the ratio of sizes, not their difference (note that, for example, from $d = 4$ to 24 in. [100 to 600 mm], the size effect is strong and from 400 to 420 in. [10,160 to 10,668 mm], the size effect is negligible).

To filter out the effect of influencing parameters other than d, each interval of d must include only the data within a certain restricted range of ρ_w values such that the average $\bar{\rho}_w$ will be almost the same for each interval of d. Similarly, the range of a/d and d_a must be restricted so that the average $\overline{a/d}$ and $\bar{d}_a$ be approximately the same for each interval of d. Because, as generally agreed, the effect of the required concrete strength f_c' is adequately captured by assuming the shear strength of cross section v_c to be proportional to $\sqrt{f_c'}$, the range of f_c' does not need to be restricted and the ordinate $\bar{y}$ of data centroid in each interval may be obtained by averaging, within that interval, not the v_c values but the values of $y = v_c/\sqrt{f_c'}$ that fall into the aforementioned restricted ranges of ρ_w, a/d, and d_a.

As shown in Fig. 6, there are only three test data in the size interval 48 to 96 in. (1219 to 2438 mm), one of which has the longitudinal steel ratio of $\rho_w = 0.14\%$, the second is 0.28%, and the third is 0.74%. This extremely low ρ_w makes it impossible to find similar data in other intervals of d. For example, the minimum ρ_w is 0.91% within the first interval of d, and 0.46% within the third interval. Therefore, one may consider the size range from 3 in. (76 mm) to only 48 in. (1219 mm). After searching the ESDB, there are 7, 19, 25, and 36 data points within the admissible ranges for each interval of d (ideally, the number of data in each interval should be the same, and thus it is impossible to eliminate bias completely). For these restricted ranges, the mean values of ρ_w are 1.51%, 1.5%, 1.51%, and 1.5%; the mean values of a/d are 3.44, 3.25, 3.25, and 3.21, respectively; and the mean values of d_a are 0.66, 0.66, 0.68, and 0.65 in. (16.8, 16.8, 17.3, and 16.5 mm). Thus, data samples with minimum bias in terms of ρ_w, a/d, and d_a are achieved (a systematic computerized procedure toward this end is developed in Reference 37). The data centroids for each interval are plotted as the diamond points in the plot of $\log(v_c/\sqrt{f_c'})$ versus $\log d$ (Fig. 6(a))—on top they are shown together with all the data points of the database, and at bottom they are shown alone. Despite enormous scatter in the database (Fig. 6(a)[top]), the trend of these centroids is quite systematic.

Assuming the statistical weight of each size interval centroid in Fig. 6 to be the same, statistical regression is used to obtain the optimum least-square fit of these four centroids with the theoretically justified size effect law $v_c/\sqrt{f_c'} = C(1 + d/d_0)^{-1/2}$, where C and d_0 equal the free constants to be found by the fitting algorithm. The fit is seen to be good; it has a very small CoV of errors ($\omega = 2.7\%$), and the asymptotic slope –1/2 required by fracture mechanics[2,13,14,25] is seen to match the data trend well.

To increase the size range, consider now that one point from the largest size interval from 48 to 96 in. (1219 to 2438 mm), namely the Toronto beam with $\rho_w = 0.74\%$, is included; refer to Fig. 6 (admittedly, one data point is too few, but that is what must be accepted because of the cost of testing very large beams). Then the same procedure is followed as previously mentioned and, for the other four intervals of d, 1, 2, 4, and 15 data points are found for which the means of ρ_w in the interval of d are 0.91, 0.94, 0.92, 0.91, and 0.74%, while the mean of $a/d = 2.9$ and the mean maximum aggregate size $d_a = 0.39$ in. (10 mm) are the same

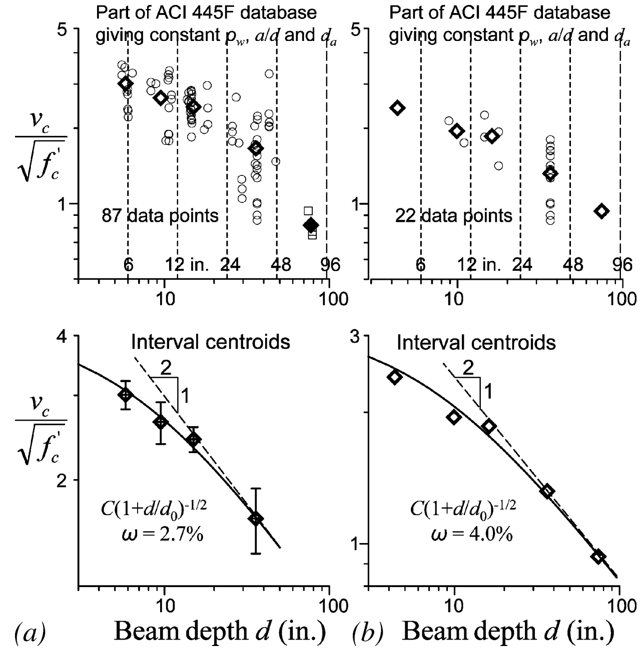

Fig. 6—ACI-ESDB and statistical regression of centroids of test data with intervals of equal width: (a) large-size interval not included; and (b) all intervals included.

for each interval. Again, the size effect trend is very clear, and agrees well with the asymptotic slope of –1/2. The CoV of errors is now $\omega = 4\%$.

The foregoing regression with minimized statistical bias lends no support for the previously proposed power laws $v_c/\sqrt{f_c'} = Cd^{-1/4}$ or $Cd^{-1/3}$. Neither does it lend any support to the asymptotic size effect $v_c/\sqrt{f_c'} = Cd^{-1}$ implied by an alternative model based on MCFT (an exponent magnitude greater than 0.50 is energetically as well as statistically impossible.[24,29,31]

EXCESSIVE FAILURE PROBABILITY CAUSED BY IGNORING SIZE EFFECT

Could the size limit of 6 in. (150 mm) in Eq. (4) be extended to 39.4 in. (1 m), as suggested by some researchers? No. To demonstrate it,[38] the data in the size range of d from 4 to 12 in. (101.6 to 304.8 mm), centered at 8 in. (203.2 mm), are isolated from the database (Fig. 7(a)). Within this narrow range, no size effect trend is discernible, and the data may be treated as a statistical population. Its mean and CoV are found to be $\bar{y} = \bar{v}_c/\sqrt{f_c'} = 3.2$ and $\omega = 25\%$ (this relatively high value of ω is the consequence of variability of many parameters in the database). The data in this range suffice to fix the probability density distribution function (pdf) for this range, which is assumed to be log-normal. The same pdf is compared in Fig. 7(a) with the series of individual tests of beams of various sizes made at the University of Toronto, which have been invoked by some engineers to claim that the size effect may be ignored for d up to 39.4 in. (1 m).

It should be noted that, for the type of concrete, steel ratio, and shear span ratio used in the Toronto tests, their shear strength value lies (in the logarithmic scale) at a certain distance a below the mean of the pdf. Because the width of the scatter band in Fig. 7(a) in logarithmic scale does not vary appreciably with the beam size, the same pdf and the same distance a between the pdf mean and the Toronto data must be expected for every beam size d, including the sizes

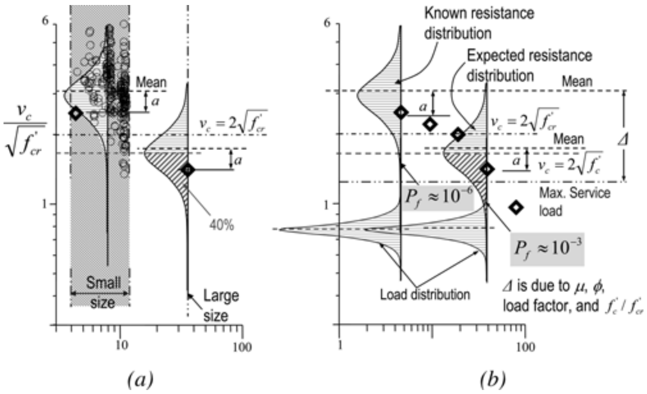

Fig. 7—(a) Probability distribution of shear strength of beams from 3.94 and 11.81 in. (10 to 30 cm) deep, based on ACI Committee 318-F database, compared with Toronto data; and (b) failure probability for small beam and 3.28 ft (1 m) deep beam.

of d = 39.4 and 74.4 in. (1 and 1.89 m) for which there is only one data point. In other words, if the Toronto test for d = 39.4 in. (1 m) were repeated for many different types of concrete, steel ratios, and shear span ratios, humidity and temperature conditions, etc., one would obtain a pdf shifted downwards, as shown in Fig. 7(a). According to the log-normal pdf shown, the proportion of unsafe 39.4 in. (1 m) deep beams would be approximately 40%, while for small beams, it is only 1%. This is intolerable. A design code known to have such a dangerous property is unacceptable.

More seriously, a design code ignoring the size effect for beams of d < 39.4 in. (1 m) will cause the failure probability P_f of 39.4 in. (1 m) deep beams to be approximately 1000 times larger than that of small beams 8 in. (200 mm) deep. To demonstrate it, consider the pdf of the extreme loads expected to be applied on the structure, which is denoted as $f(y)$. Based on the load factor of 1.6 and the understrength factor of $\phi = 0.75$, the mean of the pdf of the extreme loads will be positioned as shown in Fig. 7(b). Assuming the individual loads to have the log-normal distribution, their pdf is as shown in Fig. 7(b). Based on the CoV of extreme loads, herein assumed as $\omega_L = 10\%$, the failure probability may now be calculated from the well-known reliability integral[36,39,40]

$$P_f = \int_0^\infty f(y)R(y)\mathrm{d}y \tag{6}$$

where $R(y)$ is the cumulative probability density distribution (cdf) of structural resistance. Upon evaluating this integral

for beams of 8 in. (200 mm) depth, $P_f \approx 10^{-6}$ (7)

for beams of 39.4 in. (1 m) depth, $P_f \approx 10^{-3}$ (8)

The failure probability of one in 1 million corresponds to what the risk analysis experts generally consider as tolerable,[15-17] but one in 1000 is intolerable.

SIZE EFFECT ON CONCRETE CONTRIBUTION V_c TO SHEAR STRENGTH OF BEAMS WITH STIRRUPS

Some researchers have recently voiced the opinion that shear failure of beams with minimum or heavier shear reinforcement exhibits no size effect. This opinion seemed to be reinforced by one recent test at the University of Toronto.[11] In this test, a beam 74.41 in. (1.89 m) deep, with approximately minimum stirrups, supported a shear force V exceeding the required nominal shear strength V_u/ϕ by 6% that is calculated according to ACI 318-05 (this observation was claimed to confirm safety, even though this test result is, in fact, 11% less than required if one notes that the design should be based on the required compression strength, that is, on $v_c = 2\sqrt{f'_c}$, rather than the average compression strength, that is, on $v_c = 2\sqrt{f'_{cr}}$).

A proper statistical analysis, however, reveals that this conclusion is incorrect. The correct interpretation of the Toronto test is that there is a size effect, and that the reduction of V_c caused by size effect is, for the Toronto test, approximately 41%, which is quite significant, though still much less than the 76.2% reduction observed in a companion beam without stirrups.[41] The reason is that, aside from the (overt) understrength factor $\phi = 0.75$, the shear design implies two covert understrength factors:

- Material understrength factor $\phi_m \approx \sqrt{0.7}$, due to the fact[41] that the design must be based not on f'_{cr} but on f'_c, which represents, on the average, approximately 70% of f'_{cr}; and
- Understrength factor ϕ_f due to the fact that the design formula has been set to pass at the margin (or fringe) of the experimental scatter band width rather than through its middle.

The situation is illustrated in Fig. 1. It shows all the points of the ACI (1962) database containing only small beams (accurately plotted from the table in the original source) and also shows the fit of the histogram of v_c data by a Gaussian distribution. This database still serves as the basis of the current ACI 318-05 shear design provisions. The ACI 318-05 formula for required average shear strength is shown by the horizontal line at $v_c = 2\sqrt{f'_c} = v_c = 2\sqrt{0.7f'_{cr}} \approx 1.67\sqrt{f'_{cr}}$.

The recent Toronto tests of two companion beams 74.41 in. (1.89 m) deep, one with and one without stirrups, are shown by the diamond points. The percentage strength reductions marked in Fig. 1 show that the creators of ACI Formula $2\sqrt{f'_c}$ considered it necessary, from the safety viewpoint, that their formula be set at approximately $\sqrt{0.7} \times 65\%$, that is, 54%, of the mean of their test database (note the separation of the horizontal line $2\sqrt{f'_c}$ and the line $3.1\sqrt{f'_{cr}}$ for the mean of database).

The Toronto test without stirrups represents 0.74/3.1 = 23.8% of the mean of the database, and so the strength reduction due to size effect is, for this test, 23.8%. But what strikes the eye immediately is that not only the point for the beam without stirrups, but also the point for the beam with minimum stirrups, lies far below the mean of the database, precisely at 1.83/3.1 = 59% of the mean. This indicates that the size effect reduced the strength of the Toronto beam with minimum stirrups to 59% of the average strength of the small-beam database—a reduction that is not negligible at all.

The benefit provided by the minimum stirrups in the Toronto tests was that the size effect reduction of V_c was mitigated from 23.8 to 59%. That is helpful, but insufficient for safety by far. Even with stirrups, the failure probability is several orders of magnitude higher than one in 1 million.

The aforementioned two covert understrength factors implied by the current ACI 318-05 code provisions are 65 and 83.7%, as shown in Fig. 1. If these factors were unnecessary, then the design formula would be $v_c = 2\sqrt{f'_c}/(0.65 \times \sqrt{0.7})$ = $3.68\sqrt{f'_c}$ instead of $v_c = 2\sqrt{f'_c}$, but this would, of course, be unsafe. Obviously, the same safety margin must be satisfied by any subsequent tests, such as the Toronto test.

These observations make it clear that stirrups do not eliminate the size effect. They only mitigate it. According to the theory,[42] the general size effect Formula (1) remains valid and the effect of stirrups is to increase the transitional size d_0. Avoidance of size effect would require elimination of post-peak softening on the load-deflection diagram, and this could be achieved only if the concrete were subjected to strong triaxial confinement (all the three negative principal stresses would have to exceed several times the uniaxial compression strength in magnitude[43]).

The crack band finite element model has been used at Northwestern University to check whether the shear failure of beams with minimum stirrups exhibits a size effect. The beam geometry is the same as in the Toronto tests,[10,11] except that the longitudinal steel ratio is slightly raised to 1%, to make sure that the beam would not fail by flexure. Computations are run for geometrically similar beams of depths 37.2 in. (0.945 m), 74.4 in. (1.89 m, the size tested in Toronto), and 148.8 in. (3.78 m). The fracture energy of the Toronto concrete is estimated from the empirical formula[44] as G_f = 60 J/m^2. The stirrups and longitudinal bars are assumed not to slip.

The mesh and the cracking pattern at maximum load are seen in Fig. 8(a), which shows the simulated dimensionless load-deflection diagrams for all the sizes. The diagram for d = 74.4 in. (1.89 m, the size tested in Toronto) shows the peak load of 340 kips (1513 kN), which is close to the measured value (despite a small increase of longitudinal steel ratio). Figure 8(c) shows the dependence of the average beam shear strength $v_n = V/b_w d$ on beam depth d, and Fig. 8(d) shows the same for the average shear strength $v_c = V_c/b_w d$ contributed by concrete ($V_c = V - V_s$, $V_s = A_s f_y d/s$ where A_s and s equal the stirrup area and spacing). These plots document the existence of a strong size effect. The asymptotic slope –1/2 of the size effect is also shown.

To explore the effect of longitudinal steel ratio ρ_w, the crack band finite element calculations are also run for increasing ρ_w values (and for fixed size d = 74.4 in. [1.89 m]) (refer to Fig. 8(b)). It transpires that an increase of ρ_w raises the shear capacity V of these beams, but only up to a certain critical value, $\rho_w \approx 0.9\%$. For a further increase in ρ_w (and up to 75% of the balanced steel ratio ρ_b), the shear capacity slightly decreases and then levels off.

The conclusion from these finite element simulations is that the shear reinforcement, whether minimum or heavier than minimum, is unable to suppress the size effect. It mitigates the size effect significantly, but not enough by far to make the size effect negligible.

CLOSING COMMENTS

At present, the concrete design experts are not yet in complete agreement. As pointed out, several alternative formulas for size effect, including those of JSCE, CEB-FIP, and ACI Subcommittee 445F, are being debated. They do not show major differences within the range of the existing database but give very different extrapolations to very large beams. The extrapolation according to Eq. (2) gives much smaller V_c values than the other formulas for beam depths of the order of 393.7 in. (10 m). Even if the present rational arguments are set aside, the prudent choice is the formula offering the safest extrapolation of the database to large sizes, which is Formula (2). If calibrated to the same database, this

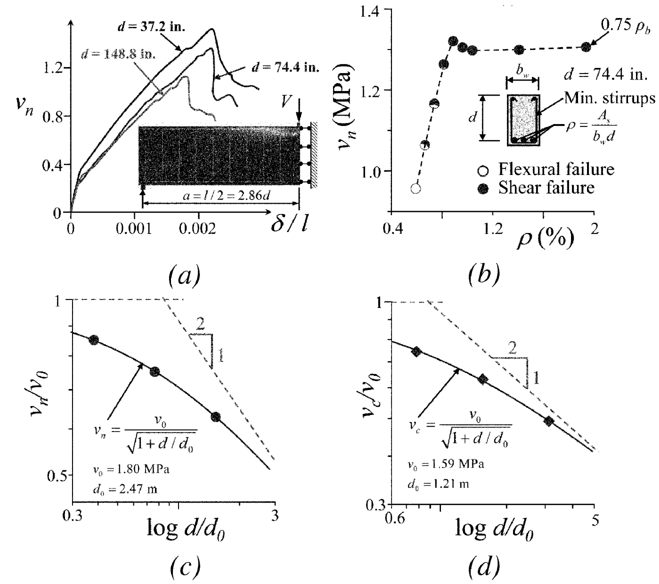

Fig. 8—Crack band finite element simulations for shear failure of beams with stirrups.

formula will always give, for sizes beyond the database range, lower values of V_c than the JSCE, CEB-FIP, and ACI Subcommittee 445F formulas.

In view of costs, real-size tests of extremely large beams are hardly feasible, and even moderately large beams cannot be tested in sufficient numbers (and for a sufficient range of shear spans, steel ratios, and concrete types), so as to provide statistically significant evidence for an empirical formulation. Some information, however, can be extracted from past structural disasters. Their recent studies show that the size effect must have been a contributing factor in many of them. The reason that this was not initially recognized is that the true overall safety factor (the ratio of the mean of test results to the unfactored design service load) is huge—approximately 3.5 to 7 for shear failures of the small laboratory-size beams,[41] and, even after taking the size effect into account, still approximately 1.7 to 3.5 for the largest.

Therefore, not one mistake, but typically two or more mistakes, are usually needed to cause shear failure of a reinforced concrete beam. Unfortunately, multiple mistakes can happen, and doubtless will. When they do, designing for size effect can make the difference between failure and survival.

ACKNOWLEDGMENTS

The work of the first two authors was supported by the Department of Transportation through the Infrastructure Institute of Northwestern University, under Grant No. 0740-357-A475.

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APPENDIX—DISCUSSION OF DESIGN SITUATIONS NOT COVERED BY EXISTING DATABASE

Beams with low longitudinal steel ratio ρ_w in Joint ACI-ASCE Committee 445 database

Low ρ_w was one point on which concern has been voiced. In the ESDB, ρ_w ranges from 0.14 to 6.64%, with a mean value at 2.3%, and among the 398 tests, only 58 had $\rho_w < 1\%$. Therefore the data for $\rho_w < 1\%$ are plotted separately in Fig. A(a) (in this and further figures with varying ρ_w, the size of each circle is proportional to ρ_w). As can be seen, the fit is just as good as that for the total ESDB, and so there is no problem in this regard.

To clarify the role of ρ_w further, 18 beams, with ρ_w ranging from 0.25 to 8%, have been simulated by a crack-band finite element code with the microplane model (refer to Fig. A(b)). In the computations, all the beams failed by shear. Again, the ACI Committee 446 formula is seen to give a good and safe estimate of shear strength for all the computer-generated data.

Design example: Fixed-end beam under distributed load

The ESDB is restricted to simply supported beams under three- or four-point loading. The proposed code revision, however, will, in practice, be applied also to redundant beams and distributed loading. Although the existing code specifications have, for a long time, been extended the same

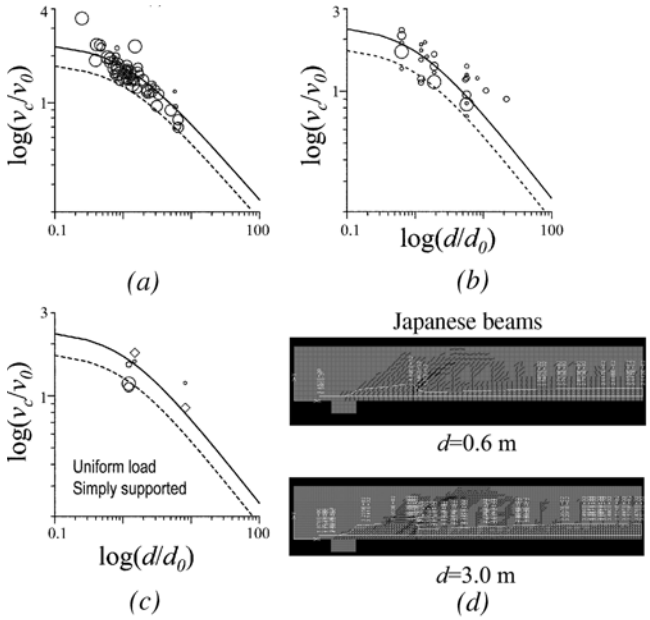

Fig. A—Test and simulations compared with proposed formula: (a) ESDB data with ρ_w < 1%; (b) simulations of beams with different ρ_w; (c) Japanese tests of simply supported beams under distributed loads and their simulations; and (d) crack patterns for Japanese beams at maximum load.

way, it is proper to check some cases. One case of concern is an example of wide beam (slab) design presented in 2004 at the ACI Committee 446 meeting in Washington, D.C., which seemed to cast doubt on the present proposal. A fixed-end beam with a span of 20 ft (6.1 m), under an 11 ft (3.35 m) overburden of soil, was considered and it was found that the beam depth of d = 14 in. (356 mm) with ρ_w = 1.14% is required according to the current ACI code, and the depth of d = 34 in. (864 mm) with ρ_w = 0.13% would apparently be required by the present code proposal. Due to negative bending moment at ends, this is a case for which no test data exist. Therefore, extensive simulations have been undertaken using a crack-band finite element code to clarify the perplexing conclusion (note that regular commercial finite element codes lacking a nonlocal or crack-band concept cannot be used because they cannot capture the size effect, as a matter of principle).

The simulations of this loading, which is not covered by the current ESDB, include two classical Japanese tests of two beams 23.62 and 118.11 in. (0.6 and 3.0 m) deep,[7] and further three beams, all of them 14 in. (0.36 m) deep, with ρ_w = 1, 2, and 3%. The results are shown by circles in Fig. A(c), where the two Japanese tests are displayed as diamonds. The simulations agree well with the Japanese tests, and also with the proposed formula. The agreement with the Japanese tests verifies the correctness of the finite element simulation and confirms that the size effect is reproduced. Figure A(d) further documents that the crack patterns at maximum load simulated for the Japanese beams are quite realistic. Figure A(d) also shows the simulated stress distribution along the longitudinal steel bar, in which it should be noted that the longitudinal steel bar does not yield at failure. For the distributed load, the shear span is defined as $a = M/V$, and it needs to be noted that it exceeds 2.5 for all the beams considered herein. This means that these beams fit within the range of validity of the current and proposed ACI specifications.

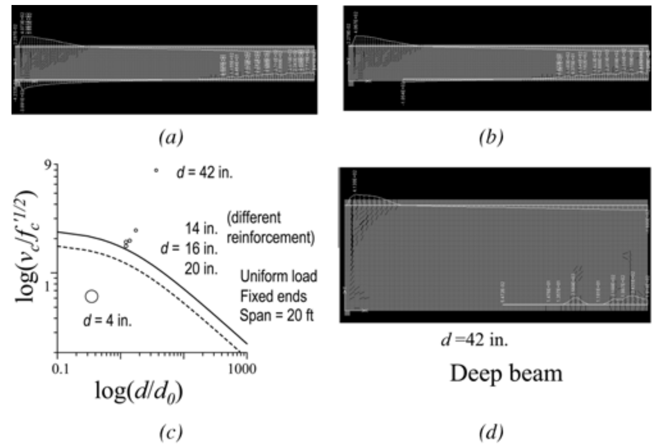

*Fig. B—Finite element simulations for Bentz's slab: (a) 14 in. (356 mm) deep slab with steel bar at bottom face across whole span; (b) 14 in. (356 mm) deep slab with steel bar at bottom face terminated 1.5*d *away from support; (c) simulations of fixed-end wide beams of different thicknesses, of sizes within and outside the range of proposed formula; and (d) 42 in. (1.07 m) deep slab showing deep beam behavior.*

The proposed calculation suggested that an incredibly deep beam with incredibly low ρ_w might be required if the present code formula is used. Test data for this situation are lacking. Because of negative bending moment at beam ends, the effect of longitudinal steel entering the compression zone needs to be simulated. Two beams shown in Fig. B(a) and (b) were considered, both with d = 14 in. (356 mm), l = 240 in. (6.1 m), and $a/d = M/Vd$ = 2.86. In the beam on the left, the longitudinal bars at the bottom face run through the whole span, and in the beam on the right, the longitudinal bars terminate at distance 1.5d from the supports. All the simulated beams fail by shear (that is, the longitudinal steel does not yield) and exhibit a clear diagonal shear crack at peak load. The beam on the left of the figure has a shear strength higher by 9% than the beam on the right. This result confirms that the shear strength prediction is conservative when there is steel bar in the compression zone. This is not surprising because all finite element simulations show that the shear strength is controlled by compression failure of the concrete above the tip of the diagonal shear crack caused by compression force parallel to the crack.

Although the design strength for both simulations is close to the present proposal (Fig. B(c)), this proposal gives, for d = 14 in. (356 mm), a design strength slightly less than the factored load. This is what motivated the proposal to Joint ACI-ASCE Committee 445 to calculate how much the beam depths need to be enlarged to satisfy the present code proposal. The calculation indicated that d = 34 in. (864 mm) was needed if the present proposal were used. It was overlooked in this calculation, however, that the present code proposal, as well as the current code, becomes invalid once d exceeds 16 in. (406 mm). The reason is that the beam becomes a deep beam, which is defined as a beam with $a/d \leq 2$ and requires a different design procedure, based on the strut-and-tie model. Using this procedure, one finds that the necessary depth in the proposed example is d = 20 in. (508 mm), and not 34 in. (864 mm).

This conclusion cannot be checked by the ESDB because of its limitation to beams with $a/d \geq 2.5$. Therefore, for further clarification, four other beams are simulated by a computer program. All parameters are the same except that d = 4, 16, 20, and 42 in. (100, 406, 508, and 1067 mm) at constant beam span L = 20 ft (6.1 m). The ratio a/d decreases with increasing d, and this is seen to increase the shear strength rapidly. The crack propagation and stress distribution along the steel bar in the beam of 42 in. (1067 mm) depth are plotted in Fig. B(d). A typical short beam failure is clearly seen, and the steel bar yields at peak load. Formula (2) gives good predictions for d = 14, 16, and 20 in. (356, 406, and 508 mm) even though it is supposed to apply only for $d \leq$ 16 in. (406 mm) (which corresponds to $a/d \geq 2.5$). For unusually small depths, however, $d < 4$ in. (100 mm, $a/d > 10$), the simulated shear strength is much less than predicted, which suggests that an upper bound, $a/d \approx 8$, might be considered for adoption, with a different formula for higher a/d. The reason is that, in very slender beams, the region having, at maximum load, very high compressive stress (close to f_c') is found to be much more elongated than for normal a/d, and this apparently promotes crushing of concrete. Such inferences cannot be checked with the ESDB, however, in which the maximum a/d is 8.03. To cover a large a/d, which is not included in the ESDB, the parameter a/d will have to be included in Eq. (2).