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Nano-mechanics based modeling of lifetime distribution of quasibrittle structures

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ABSTRACT

The statistics of structural lifetime under constant load are related to the statistics of structural strength. The safety factors applied to structural strength must ensure failure probability no larger than 10^{-6}, which is beyond the means of direct verification by histogram testing. For perfectly brittle materials, extrapolation from the mean and variance to such a small tail probability is no problem because it is known that the Weibull distribution applies. Unfortunately, this is not possible for quasibrittle materials because the type of cumulative distribution function (cdf) has been shown to vary with structure size and shape. These are materials with inhomogeneities and fracture process zones (FPZ) that are not negligible compared to structural dimensions. A probabilistic theory of strength of quasibrittle structures failing at macro-crack initiation, which can be experimentally verified and calibrated indirectly, has recently been deduced from the rate of jumps of atomic lattice cracks governed by activation energy barriers. This paper extends this nano-mechanics based theory to the distribution of structural lifetime. Based on the cdf of strength and a power law for subcritical crack growth rate, the lifetime cdf of quasibrittle structures under constant loads is derived. The lifetime cdf is shown to depend strongly on the structure size as well as geometry. It is found that, for the creep rupture case, the mean structural lifetime exhibits a very strong size effect, much stronger than the size effect on the mean structure strength. The theory also implies temperature dependence of the lifetime cdf. For various quasibrittle materials, such as industrial ceramics and fiber composites, it is demonstrated that the proposed theory correctly predicts the experimentally observed deviations of lifetime histograms from the Weibull distribution.

1. Introduction

Many engineering structures, such as dams, nuclear structures and large bridges or buildings, must be designed against an extremely low failure probability under design load P during their service lifetime τ, i.e. $P_f(P,\tau) < 10^{-6}$ [17,30,33]. If such a low failure probability is required, it is impossible to determine the design lifetime by histogram testing. Therefore, it is imperative to develop a physically based probabilistic theory to predict the cumulative distribution function (cdf) of lifetime, so that it would be feasible to calibrate it experimentally.

The type of cdf of structural lifetime is well known for perfectly brittle structures, for which the failure is triggered by one negligibly small representative volume element (RVE) of material. In that case, the weakest-link model with an infinite number of links applies, and so the lifetime cdf must follow the Weibull distribution. This study focuses on structures consisting

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of quasibrittle materials, which include fiber composites, concretes, rocks, stiff soils, foams, sea ice, consolidated snow, bone, tough industrial and dental ceramics and many other materials on approach to nano-scale. These are with brittle constituents materials in which the inhomogeneities, and thus the RVE, are not negligible compared to the structure size. It has been demonstrated that the behavior of quasibrittle structures transits from quasi-plastic to brittle with increasing structure size [1,2]. Such a transition has serious consequences for structural reliability and lifetime prediction.

Tremendous efforts have been devoted to study the structural lifetime of many engineering materials in a deterministic framework. Tobolsky and Eyring [46] were first to adopt the concept of activation energy to make deterministic predictions of the mean lifetime of polymeric materials under constant loads. A similar deterministic model was developed by Zhurkov [50,51] to study the structural lifetime of polymers, alloys, and non-metallic crystals. This model neglects restorations of the ruptured interatomic bonds and thus gives unreasonably short lifetimes for failures at low stress [50], and for zero stress it even gives a finite lifetime. Hsiao et al. [22] employed a more general deterministic model which takes into account of the restorations of ruptured bonds. A comprehensive review of these deterministic models can be found in [21]. However, all these models derive the structural lifetime at the macro-scale directly from the frequency of rupture of interatomic bonds, and the multi-scale nano-macro transition is lacking. Furthermore, these deterministic models do not predict the type of lifetime cdf, which is essential for reliability-based design of engineering structures.

Meanwhile, significant advances have been made for various statistical models for structural lifetime of fibrous materials, unidirectional fiber composites and ceramics [15,16,45,37–39,31,29]. For unidirectional fiber composites, Coleman [15,16] first proposed a general lifetime distribution function to model the lifetime cdf of single fibers for a given loading history, which was based on the infinite weakest-link model. This model was subsequently adopted for the study of the lifetime statistics of fiber bundles and unidirectional fiber composites based on some chosen empirical rule of load redistribution among the fibers after breaks [37,38,45,23,29]. However, an infinite weakest-link model was used or implied, which was not physically justified because of non-negligible inhomogeneities, as evidenced by systematic deviations of measured histograms from the classical Weibull distribution [42,48,43]. Furthermore, simplified load sharing rules used for the fiber bundle model, such as the equal load sharing and the local load sharing [32,40], generally lack a physical basis and lead to questionable types of lifetime cdf. For ceramic materials, a more general approach has been adopted, in which the structural strength and lifetime are interrelated by the crack growth law [31,28]. In these models, the strength cdf was often assumed to follow the classical Weibull distribution, which inevitably leads to the Weibull distribution of structural lifetime. However, extensive experimental evidence shows both the strength and lifetime histograms of various ceramic materials to deviate from the Weibull distribution [27,34,31].

The objective of this paper is to present a nano-mechanics based theory for lifetime distribution of quasibrittle structures. A constant sustained load (as in the creep-rupture test) is considered, but extensions to other monotonic loading histories would be straightforward. Attention is limited to the broad class of structures that fail as soon as one macro-crack initiates. The theory will be validated by the optimum fits of lifetime histograms of various quasibrittle materials such as fiber composites and industrial ceramics.

2. Review of strength distribution of one RVE

For concrete, industrial and dental ceramics, fibers and fiber composites, it has been observed that the strength histograms consistently deviate from the two-parameter Weibull distribution [27,34,31,35]. It has recently been found that the problem lies in the tacit assumption that the weakest-link model which underlies the Weibull statistics of strength has infinitely many links [8,9]. The strength cdf of quasibrittle structures of positive geometry (i.e. structures that fail at crack initiation) should rather be modelled by a weakest-link model with a finite chain of finite-size RVEs. The reason is that the size of the RVE, which roughly coincides with the width of the fracture process zone (FPZ) at crack tip, is not negligible compared to the structure size D. This is the salient feature of quasibrittle structures.

A probabilistic theory for the cdf of strength of the broad class of quasibrittle structures failing at macro-crack initiation has recently been derived on the basis of breaks of interatomic bond pairs [8,9]. It was further refined based on atomistic fracture mechanics [5,6]. In the refined theory, a nano-crack is considered to propagate by random jumps through either a regular atomic lattice or through a disordered nano-structure. These jumps are governed by the activation energy barriers separating a series of numerous metastable states on the surface of the free energy potential of the nano-structure.

When the nano-crack advances by one atomic spacing in the atomic lattice or by one nano-inhomogeneity in a disordered nano-structure, the energy release increment must correspond to the change of activation energy barrier. Applying the equivalent LEFM (linear elastic fracture mechanics) to the nano-crack propagation, the energy release increment can be expressed as a function of the remote stress applied on the nano-structure [5,6].

Since the crack jumps by one atomic spacing or one nano-inhomogeneity are numerous and thus very small, the activation energy barrier for a forward jump differs very little from the activation energy barrier for a backward jump. Therefore, the jumps of the state of the nano-structure, characterized by its free energy potential, must be happening in both directions, albeit with different frequencies. After a certain number of jumps of the nano-crack tip, the length of the nano-crack reaches a critical value at which the crack loses its stability and propagates dynamically, causing a break of the nano-structure.

Since, at nano-scale, it may generally be assumed that each jump is independent (i.e., the frequency of the jump is independent of the particular history of breaking and restoration sequences that brought the nano-crack to the current length) [25], the failure probability of the nano-structure is proportional to the sum of the frequencies of all the jumps that cause its

failure. The failure probability of the nano-structure has been shown to follow a power-law function of the remote stress with a zero threshold [5,6].

The multi-scale bridging between the strength cdf at the nano-scale and at the RVE scale may be statistically represented by a hierarchical model consisting of parallel and series couplings (Fig. 4e [9]). The parallel couplings statistically reflect the load redistribution mechanisms at various scales, especially the fact that a passage from one scale to the next higher scale involves strain compatibility conditions. The series couplings, represented by the weakest-link chain model, reflect the localization sub-scale cracking and slippage (or damage) into larger scale cracks or slips.

It has been analytically proven that the power-law tail of strength cdf is preserved through parallel and series couplings, in which the tail exponent increases in passing to higher scales until it becomes equal to Weibull modulus on the RVE scale [8,9]. The hierarchical model shows the strength cdf on the RVE scale to have an approximately Gaussian (or normal) distribution onto which a remote power-law tail with zero threshold is grafted from the left at the probability of about 10^{-5} to 10^{-3} [8,9]. The grafted probability density function (pdf) can be written as [9]:

$$\text{for } \sigma_N < \sigma_{N,gr} : p_1(\sigma_N) = (m/s_0)(\sigma_N/s_0)^{m-1} e^{-(\sigma_N/s_0)^m} = r_f \phi_W(\sigma_N); \tag{1}$$

$$\text{for } \sigma_N \geqslant \sigma_{N,gr} : p_1(\sigma_N) = r_f e^{-(\sigma_N-\mu_G)^2/2\delta_G^2} \Big/ \left(\delta_G\sqrt{2\pi}\right) = r_f \phi_G(\sigma_N) \tag{2}$$

Here, σ_N = nominal strength, which is a load parameter of the dimension of stress. If there is no stress singularity, σ_N may be chosen to be equal to the maximum principal stress in the structure; otherwise, and in general, $\sigma_N = P_{max}/bD$ or P/D^2 for two- or three-dimensional scaling (P_{max} = maximum load of the structure or parameter of load system, b = structure thickness in the third dimension, D = characteristic structure dimension or size). Furthermore, m (Weibull modulus) and s_0 are the shape and scale parameters of the Weibull tail, and μ_G and δ_G are the mean and standard deviation of the Gaussian core if considered extended to $-\infty$; r_f is a scaling parameter required to normalize the grafted cdf, such that $\int_{-\infty}^{\infty} p_1(\sigma_N)\mathrm{d}\sigma_N = 1$. Furthermore, continuity of the Gaussian and Weibull pdf's at the grafting point requires that $\phi_W(\sigma_{N,gr}) = \phi_G(\sigma_{N,gr})$.

3. Lifetime distribution ensuing from nano-mechanics

Consider load histories in which the stress is first raised rapidly to some value σ_0, then is held constant for various lengths of time, t_1, and finally is raised rapidly up to failure, occurring at random stress σ_1 (Fig. 1a). It appears logical that the failure stress σ_1 for all finite t_1 should follow the same kind of statistics as for $t_1 = 0$, except that the parameters will change with σ_1 and t_1.

For $t_1 \to \tau$ = lifetime (Fig. 1b), we have $\sigma_1 \to \sigma_0$. So, for the so-called creep-rupture test, in which the stress σ_0 is held constant until failure. For $t_1 = 0$ (Fig. 1b), $\sigma_1 = \sigma_N$, which is the strength test. Obviously, the statistics of strength σ_N and lifetime τ must be interrelated. We need to find this relationship.

Now consider an RVE containing a dominant subcritical crack of initial length a_0. Under a certain loading history, this crack grows to its critical value a_c, at which the RVE fails. This process is known to obey a power law [18,44,19,11,10,3,31]:

$$\dot{a} = C_0 e^{-Q_0/kT} K^n \tag{3}$$

where the superior dot refers to derivatives with respect to time t; C_0, n = positive constants; Q_0 = activation energy, k = Boltzmann's constant, T = absolute temperature, K = stress intensity factor. One may further write $K = \sigma\sqrt{l_0}k(\alpha)$, where $\alpha = a/l_0$ and l_0 = RVE size. A physical explanation of the power law for subcritical creep crack growth based on the nano-crack growth rate was presented in [6].

For the case of strength test, i.e. rapidly increasing stress, one has $\sigma = \lambda t$ (λ = loading rate). By integrating Eq. (3), one obtains:

$$\sigma_N^{n+1} = \lambda(n+1)e^{Q_0/kT} \int_{\alpha_0}^{\alpha_c} \frac{\mathrm{d}\alpha}{C_0 l_0^{(n-2)/2} k^n(\alpha)} \tag{4}$$

For the case of lifetime test, the lifetimes of interest are much longer than the duration of laboratory strength tests. Hence, the constant stress σ_0 is generally very low compared to the mean strength of the RVE. Therefore, the initial rapidly increasing portion of the load history has a negligible contribution to the overall structural lifetime. By integrating Eq. (3) at constant applied stress σ_0, one obtains for lifetime τ the relation:

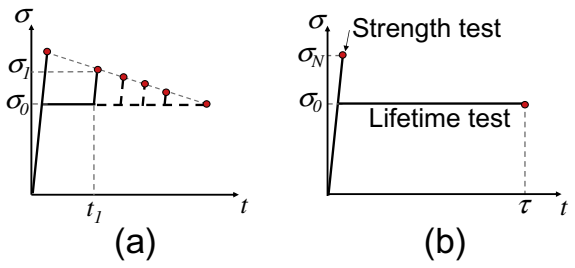

Fig. 1. (a) General loading history (b) loading histories for strength and lifetime tests.

$$\sigma_0^n \tau = e^{Q_0/kT} \int_{\alpha_0}^{\alpha_c} \frac{d\alpha}{C_0 l_0^{(n-2)/2} k^n(\alpha)} \tag{5}$$

By comparing Eqs. (4) and (5), one gets the following relationship between σ_N and τ:

$$\sigma_N = \beta \sigma_0^{n/(n+1)} \tau^{1/(n+1)} \tag{6}$$

where $\beta = [\lambda(n+1)]^{1/(n+1)}$ = constant.

By virtue of Eq. (6), one can obtain the lifetime cdf of one RVE from the strength cdf of the RVE (Eqs. (1) and (2)). The lifetime cdf can be expressed as follows:

$$\text{for } \tau < \tau_{gr} : P_1(\tau) = 1 - \exp\left[-(\tau/s_\tau)^{m/(n+1)}\right]; \tag{7}$$

$$\text{for } \tau \geqslant \tau_{gr} : P_1(\tau) = P_{gr} + \frac{r_f}{\delta_G \sqrt{2\pi}} \int_{\gamma \tau_{gr}^{1/(n+1)}}^{\gamma \tau^{1/(n+1)}} e^{-(\tau' - \mu_G)^2/2\delta_G^2} d\tau' \tag{8}$$

where $\gamma = \beta \sigma_0^{n/(n+1)} \tau_{gr} = \beta^{-(n+1)} \sigma_0^{-n} \sigma_{N,gr}^{n+1}$, and $s_\tau = s_0^{n+1} \beta^{-(n+1)} \sigma_0^{-n}$.

For one RVE, both the strength and lifetime cdf's have Weibull tails, but the Weibull modulus for lifetime distribution, $\bar{m} = m/(n+1)$, is found to be significantly lower than the Weibull modulus m for strength distribution. The grafting probability P_{gr} is found to be the same for both cdf's. In contrast to the strength distribution, the core of lifetime distribution does not follow the Gaussian distribution, although it can be analytically derived from the Gaussian core of the strength cdf by using Eq. (7).

In the context of softening damage and failure of a structure, the RVE must be defined as the smallest material volume whose failure triggers the failure of a structure (failing at crack initiation, which occurs if the geometry is positive [10,1,2]). Therefore, the structure can be statistically represented by a chain of RVEs (weakest-link model). Similar to the definition of nominal strength, we define σ_0 as the nominal stress, which is a load parameter ($\sigma_0 = P/bD =$ or P/D^2 for two- or three-dimensional scaling). According to the joint probability theorem, and under the assumption of independence of random strengths of the links in a finite weakest-link model, the lifetime cdf of a structure subjected to a nominal stress σ_0 can be calculated as:

$$P_f(\sigma_0, \tau) = 1 - \prod_{i=1}^{N} \{1 - P_1[\langle \sigma_0 s(\boldsymbol{x}_i) \rangle, \tau]\} \tag{9}$$

where $\sigma_0 s(\boldsymbol{x}, i)$ = maximum principal stress at the center of the ith RVE; $s(\boldsymbol{x}, i)$ = the dimensionless stress ratio which characterizes the spatial distribution of the stress, and $\langle x \rangle = \max(x, 0)$. As the structure size increases (i.e., as the number N of RVEs increases), the structure will fail at smaller and smaller τ. So, what matters for large structures, and not only for small but also large P_f, is the Weibull tail (or power-law tail) of the cdf of each RVE: $P_1(\tau) = (\tau/s_\tau)^{\bar{m}}$. Taking the logarithm of Eq. (9) and setting $\ln(1 - P_1) \approx -P_1$ for small P_1, we get for large size structures: $P_f(\tau) = 1 - \exp\{-\sum_{i=1}^{N} P_1[\langle \sigma_0 s(\boldsymbol{x}_i) \rangle, \tau]\}$. For $N \to \infty$, we obtain:

$$P_f(\tau) = 1 - \exp\left[-N_{eq}\left(\frac{\tau}{s_\tau}\right)^m\right] \tag{10}$$

where

$$N_{eq} = \int_V \langle s(\boldsymbol{x}_i) \rangle^{n\bar{m}} dV / l_0^{n_d} \tag{11}$$

where l_0 = RVE size = material characteristic length, n_d = number of dimensions in which the failure is scaled (1, 2 or 3). N_{eq} represents the equivalent number of RVEs, for which a chain of N_{eq} elements subjected to a uniform stress σ_0 gives the same lifetime cdf as Eq. (9) does. The concept of equivalent N_{eq} was introduced for strength distribution in [8,9], but it is sufficiently accurate only if $N_{eq} > 500$. Otherwise, one must use directly the joint probability theorem (Eq. 9) to calculate P_f.

To calculate P_f directly from stresses according to Eq. (9), one needs to subdivide the structure into equal-size elements, having approximately the same size as the RVE. However, such a subdivision is possible only for rectangular boundaries. For general geometry, a nonlocal boundary layer approach has been recently proposed to deal with arbitrary boundaries and at the same time avoid the subjectivity of subdivision [6].

In this approach, a boundary layer of thickness $h_0 \approx l_0$ along all the surfaces is separated from the structure. For the boundary layer, one only needs evaluate the stress for the points of the middle surface Ω_M of the layer. For the interior domain V_I, the conventional nonlocal continuum approach [26] can be adopted. Eq. (9) may be rewritten as:

$$\ln[1 - P_f(\sigma_0, \tau)] = h_0 \int_{\Omega_M} \ln\{1 - P_1[\sigma(\boldsymbol{x}_M), \tau]\} dV(\boldsymbol{x}_M)/l_0^3 + \int_{V_I} \ln\{1 - P_1[\overline{\sigma}(\boldsymbol{x}), \tau]\} dV(\boldsymbol{x})/l_0^3 \tag{12}$$

Here, $P_1(\sigma, \tau)$ is the lifetime cdf for one RVE subjected to applied stress σ (Eqs. (8) and (7)). For very large structures, the boundary layer becomes very thin compared to the structure size (i.e. the first integral becomes negligible), the nonlocal stress in the domain becomes the local stress, and Eq. (12) eventually leads to Eq. (10).

4. Optimum fits of lifetime histograms

Extensive histogram testing has been aimed at the lifetime distributions of various quasibrittle materials, especially ceramics and fiber composites [14,31]. Although the two-parameter Weibull distribution has often been adopted [31,20,28], systematic deviations have typically been observed.

Munz and Fett [31] studied the lifetime histograms of MgO-doped HPSN (hot-pressed silicon nitride), stressed at the temperature of 1100 °C, and Al_2O_3 subjected to constant stress while submerged in high-concentration salt solution of temperature 70 °C. Standard four-point-bend tests were used for both materials. The applied stress was around 50% of the mean short-time strength.

Chiao et al. [14] investigated the lifetime histograms of organic fiber (Kevlar 49) composites, and used elevated temperature (100—120 °C) to accelerate the failure. Bar-shaped specimens were subjected to constant uniform uniaxial tensile stress which was about 70% of the mean short-time strength.

Fig. 2 shows the experimentally observed lifetime histograms of above-mentioned structures [14,31]. The solid and dashed lines represent the optimum fits by the present theory and by the two-parameter Weibull distribution, respectively. It is clear that, on the Weibull plot, the observed lifetime histograms do not appear as a straight line. Rather, they exhibit a kink separating two segments, of which the lower part is a straight line and the upper one deviates from the straight line to the right. Such deviations have also been found in the strength histograms of various other quasibrittle materials such as concrete [49], fiber composites [42,47] and ceramics [34,27]. Obviously, the two-parameter Weibull distribution cannot fit the two segments simultaneously, and the existence of the kink cannot be explained by the Weibull theory. The present theory gives an excellent fit of the entire lifetime histogram, with both segments and the kink locations matched well.

Within the framework of the present theory, the kink of the histogram corresponds to the grafting point of the distribution. The kink is a natural consequence of the quasibrittleness of the structure.

By the optimum fits of lifetime histogram, the Weibull modulus of lifetime distribution $\bar{m}$ can be obtained: For MgO-doped HPSN and for Al_2O_3, $\bar{m}$ is about 1–1.4, while for organic fiber (Kevlar 49) composites, $\bar{m}$ is about 2.4–3. These values are significantly lower than the Weibull modulus m of strength distribution of these materials, which is typically in the range of 30–50 [37,31].

5. Effect of temperature on lifetime distribution

The proposed theory takes into account the effect of temperature on the lifetime cdf, which is of the Arrhenius type. This temperature dependence stems from the thermal effect on the crack growth rate (Eq. (3)). The Arrhenius type of temperature dependence of the crack growth rate at macroscale has been successfully applied to predict the effect of the temperature on the fracture energy of concrete [11].

It has been recently shown that the crack growth rate at all the scales, from the nano-scale to the RVE scale, must follow the same temperature dependence [6]. Since the crack growth rate at nanoscale is proportional to the frequency of bond breaks, whose temperature dependence is of the Arrhenius type [5,6,41], the temperature effect on the crack growth rate on RVE scale must follow the same form, $e^{-Q_0/kT}$.

Normally there exists several activation energy barriers on the free energy potential of atomic lattice [25], and the dominant activation energy barrier, Q_0, depends on the temperature range. To avoid dealing with the transition rate theory having several activation energy barriers [24,36], we consider here a limited range of temperatures with one dominant activation energy barrier.

Consider that two RVEs are loaded by the same constant stress σ_0 but subjected two different temperatures T_0 and T_1. One may write Eq. (5) for each of these two cases, and by comparing the two equations, one gets:

$$\tau_1 = \tau_0 \exp\left[\frac{Q_0}{k}\left(\frac{1}{T_1}-\frac{1}{T_0}\right)\right] \tag{13}$$

where τ_i $(i = 0, 1)$ are the lifetimes of the RVE under stress σ_0 and subjected to temperatures T_i. Eq. (13) has two important practical implications, valid if the difference between T_1 and T_0 does not cause a change in the dominant activation barriers:

(1) If one knows the dominant activation energy barrier, then Eq. (13) makes it possible to predict the cdf of structural lifetime for temperature $T_1, P_f(\tau, T_1)$ based on the lifetime cdf for temperature $T_0, P_f(\tau, T_0)$, i.e.,

$$P_f(\tau, T_1) = P_f(C_T\tau, T_0) \tag{14}$$

where P_f can be calculated directly from Eq. (9) with the nonlocal boundary layer approach, and

$$C_T = \exp\left[\frac{Q_0}{k}\left(\frac{1}{T_1}-\frac{1}{T_0}\right)\right] \tag{15}$$

(2) If the lifetime cdf's of the same structure are obtained for two different temperatures T_0 and T_1, then the dominant activation energy barrier, Q_0, can be calculated based on the optimum fits of both histograms by Eqs. (12) and (14).

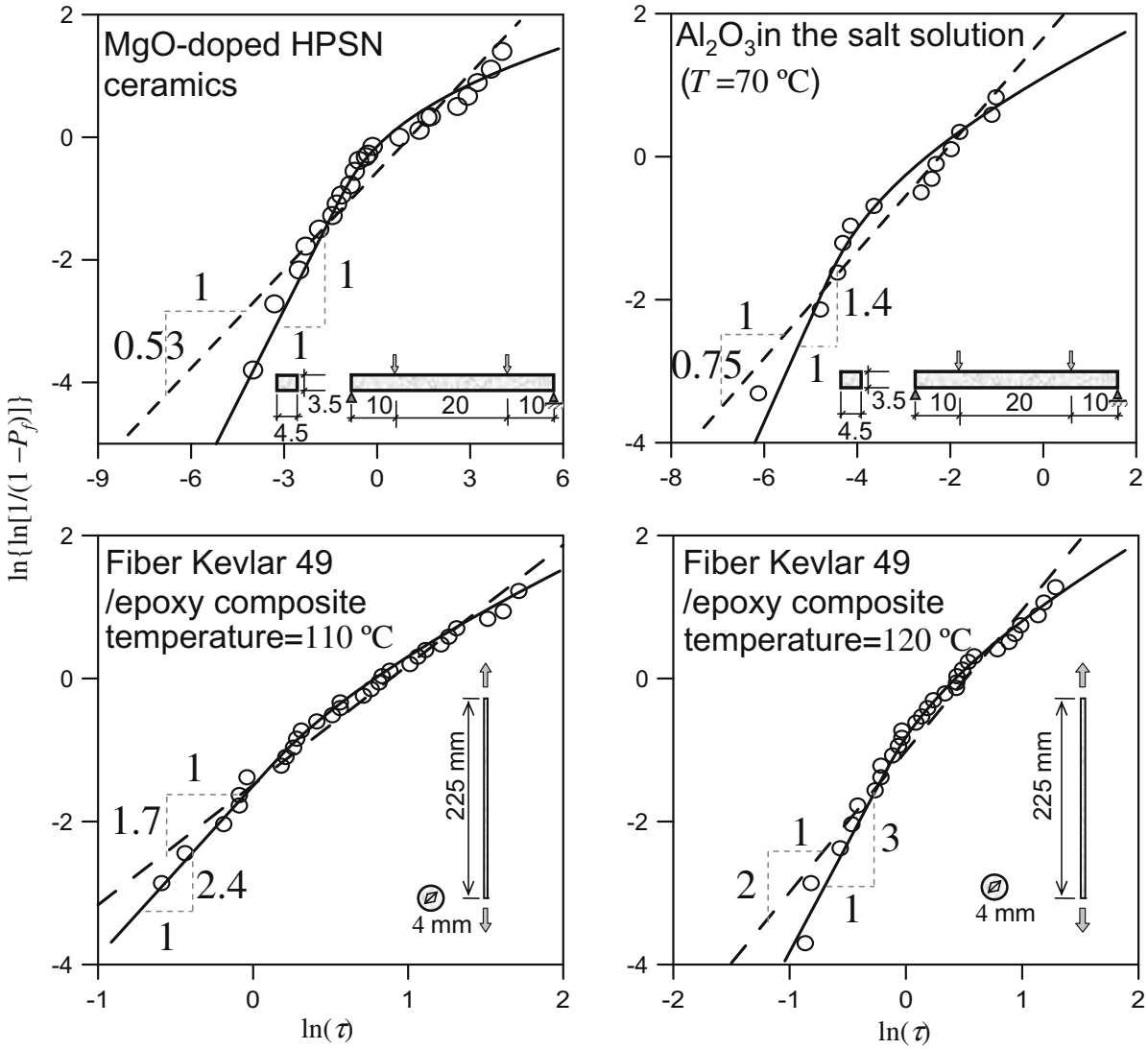

Fig. 2. Optimum fits of experimental lifetime histograms of ceramics and Kevlar 49 fiber composites by the present theory and the two-parameter Weibull distribution: (a) MgO-doped HPSN ceramics, (b) Al_2O_3 in the salt solution, (c) Fiber Kevlar 49/epoxy composite at 110 °C and (d) Fiber Kevlar 49/epoxy composite at 120 °C.

Eq. (13) also applies to the mean lifetime. Therefore, instead of testing the histograms, it is more effective to test the mean structure lifetime. Much fewer tests are needed. Thus it suffices to obtain the mean lifetime of the structure for two different temperatures, and Q_0 can then be calculated as:

$$Q_0 = k \log\left(\frac{\bar{\tau}_1}{\bar{\tau}_0}\right)\left(\frac{1}{T_1}-\frac{1}{T_0}\right)^{-1} \tag{16}$$

where $\bar{\tau}_0$ and $\bar{\tau}_1$ are the mean lifetimes of the structure subjected to a certain stress under different temperatures T_0 and T_1, respectively.

The present theory of lifetime is applied to the lifetime histograms tested on the organic fiber (Kevlar 49) composites [14] at elevated temperatures (100 °C and 110 °C). The specimens were subjected to constant uniaxial tension. The applied load was about 67% of the mean tensile strength of the structure. Eqs. (1), (2) and (12) are used to fit the lifetime histogram for the temperature of 100 °C, and the lifetime histogram for the temperature of 110 °C is fitted by extrapolating the calibrated cdf for 100 °C based on Eq. (14).

The Weibull plot in Fig. 3 shows that the fits of lifetime histogram at 100 °C and 110 °C are excellent. Note that the lifetime cdf at 110 °C can be obtained through a horizontal shift of the lifetime cdf at 100 °C by the distance of $Q_0/k(1/T_1 - 1/T_0)$. Based on the fitting, the dominant activation energy barrier of the organic fiber composite at temperature around 100 °C is obtained as $Q_0 = 0.79$ eV (and $Q_0/k = 9225$ °K).

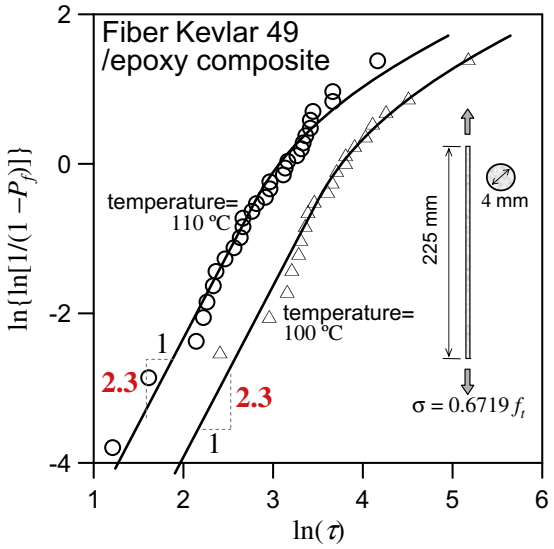

Fig. 3. Optimum fits of experimental lifetime histograms Kevlar 49 fiber composites tested at temperatures of 100 °C and 110 °C.

6. Size effect on mean structural lifetime

Based on the finite weakest-link model and the grafted cdf of lifetime for one RVE, the cdf of lifetime of the structure must depend on its size and geometry. The mean lifetime for a structure with any number of RVEs can be calculated as follows:

$$\bar{\tau} = \int_0^\infty [1 - P_f(\tau)]\mathrm{d}\tau \tag{17}$$

Clearly, it is impossible to express $\bar{\tau}$ analytically. But its approximate form can be obtained through asymptotic matching. Such an approach has been used for structures failing at crack initiation to approximate the size effect on the mean structure strength [7,2,13]:

$$\bar{\sigma}_N = \left[\frac{N_a}{D} + \left(\frac{N_b}{D}\right)^{r/m}\right]^{1/r} \tag{18}$$

Here, D is characteristic structure dimension or size, m = Weibull modulus of strength cdf, and N_a, N_b, r are constants to be found from three asymptotic matching conditions. It has been shown that Eq. (18) agrees well with the predictions by many established mechanical models such as the cohesive crack model [1,10,12], crack band model [10], nonlocal damage model [4] nonlocal Weibull theory [13], and the finite weakest-link model [8,9]. Since the random strength is related to the random lifetime through Eq. (5), the mean strength and lifetime must be related by an equation of the same form. Therefore, the mean size effect on lifetime can be written as:

$$\bar{\tau} = \left[\frac{C_a}{D} + \left(\frac{C_b}{D}\right)^{r/m}\right]^{(n+1)/r} \tag{19}$$

where m = Weibull modulus of strength distribution, n = exponent of the power law crack growth rate, and $m/(n+1) = \bar{m}$ = Weibull modulus of lifetime distribution. Parameters C_a, C_b, r can be determined from three known asymptotic conditions for $[\bar{\tau}]_{D\to l_0}$, $[\mathrm{d}\bar{\tau}/\mathrm{d}D]_{D\to l_0}$, and $[\bar{\tau}D^{1/\bar{m}}]_{D\to\infty}$.

As seen from Fig. 4, the size effect on structural lifetime approaches the power-law size effect of Weibull theory for large structure sizes but deviates from it upward for small sizes. The cause is the finiteness of the FPZ.

Furthermore, the size effect on the lifetime of structures under constant loads is much stronger than the size effect on the mean structural strength. This phenomenon becomes physically plausible if the following two situations are compared: Let the mean nominal strength of a RVE of concrete or ceramic be $\bar{\sigma}_N$, and the mean nominal strength of a structure of large size D_L be $\bar{\sigma}_N/3$. If one applies nominal stress $\bar{\sigma}_N/3$ on two geometrically similar structures, one structure having the RVE size and the other the size D_L, one naturally expects the structure of size D_L to fail within the standard laboratory test period (about 5 min) and the small size structure to take many years to fail.

7. Concluding remarks

The present theory shows that the cdf of lifetime of quasibrittle structures are size- and geometry-dependent, in fact strongly so. This has important implications for the safety factors to be used in reliability assessments, for instance, in the

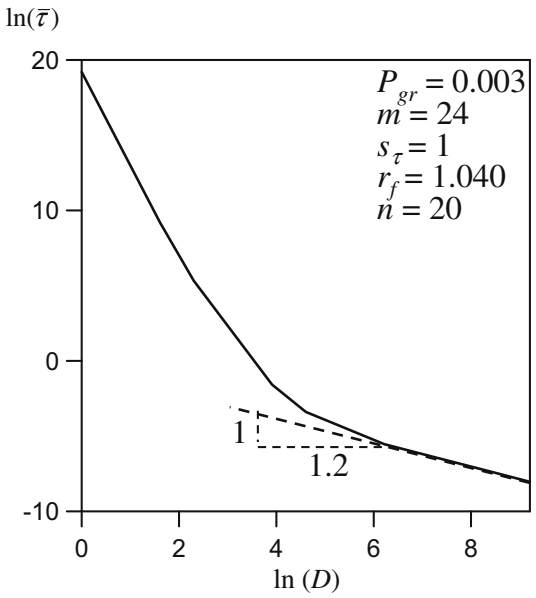

Fig. 4. Size effect on mean structural lifetime.

design of large prestressed concrete bridges, or large aircrafts and large ships made of fiber composites, or micro- and nano-scale devices. The size effects on the strength and lifetime cdf's indicate that the safety factors for reliability cannot be empirical, and cannot be constant—not only for strength but also for lifetime. They need to be calculated as a function of structure size, as well as shape.

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