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Stress Relaxation Testing of Plastics and Fibre-Glass

Plastics

/Měření relaxace plastických hmot a skelných laminátů/

Relaxaci plastických hmot a skelných laminátů lze jednoduše měřit tak, že rovný pásek se ohne do oblouku stažením konců k sobě a měří se pokles tahové síly, např. podle kmitočtu struny, napjaté mezi konci pásku, nebo i jednodušeji, ale méně přesně, podle tvaru, jaký pásek zaujme po uvolnění z podložky po jisté době. Byl propočten geometrický tvar oblouku podle teorie velkých deformací /je uvedena číselná tabulka/. Dále jsou odvozeny vzorce pro určení relaxačního modulu s korekcí /obecně Volterrova integrální rovnice/ a je uveden způsob odvození ostatních dat pro dotvarování. Provedení zkoušek pro stupňovitě rostoucí nebo klesající vzdálenost konců je možno ověřit rozsah platnosti principu superposice. Vzorky lze snadno přemísťovat do prostředí o různé teplotě, do vody, vetrometru apod. Zkouškou pevnosti po jisté době lze též sledovat dlouhodobý pokles pevnosti vlivem zatížení a vliv prostředí na dlouhodobou pevnost zatížených prvků /životnost/.

It is requisite in the design of structures made of plastics that due account be taken of the creep of these materials. Experimental data on creep can be obtained by e.g. the stress relaxation test in which the time variation of stress is measured at a prescribed deformation or possibly at additional prescribed changes of deformation.

The present paper describes the measurement of stress relaxation in bending easily effected by measuring the decrease

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in tensile force acting between the ends of a strip bent arc-wise. In the measurement proper, the sought-for curve is determined in the simplest way from changes of natural frequency of a steel string stretched between the ends of the strip, or from changes of the shape of the arc when the strip is removed from the backing. The measured specimens are easily placed in environment at different temperatures, in water, weatherometer, etc., thus enabling us to simultaneously follow the effect of the environment and of the load on the decrease in strength /life problem/, ascertain the modulus of elasticity, etc. Since we have just started with such tests at the Building Research Institute of the Technical University in Prague, we are not ready yet to present numerical results. Although the test is simple, the relationships for the evaluation of the results are somewhat involved, particularly as regard the shape of the arc; a pertinent numerical table is, however, presented herein. We shall also evolve formulae for computing the relaxation modulus with respect to the string elasticity, and give relations enabling the establishment of other creep data.

1. Measuring equipment. The measuring equipment is schematically shown in Fig. 1. Strips /a/ of plastics or fibre-glass reinforced plastics / strips of fibre glass plastics now under test are 2 x 40 x 360 mm and 2 x 70 x 360 mm/ are fitted with steel edges /b/ braced on adjustable jaws /c/ with steel string /d/ /0.2 mm dia., 200 mm long/ stretched between them. Electromagnetic pickup picks up the natural vibrations of the string; the tension of the string is best measured by electrically comparing its natural frequencies with those of a comparative string by means of Lissajous patterns displayed on oscilloscope /1/. The measurement can also be effected by application of additional weight AG to bring the tension of the string to a constant frequency.

2. Investigated effects. 2.1 The measurement is conducted at ambient temperature in exponentially increasing time intervals, starting with 1 to 5 min up to several years /in tests intended for the purposes of building practice/. Next

to standard relaxation tests made at various constant values of the strip arc height, stress variations at time changes of the arc deformation are also studied in order to verify the extent of the validity of the principle of superposition both at step-wise increase in the strip arc height /Fig.3a/, and at step-wise decrease in height of the arc whose shape can also assume opposite curvature /Fig.3c/. On free specimens removed from the backing we can also follow the reversibility of creep deformation, and on those with fixed ends, the reversibility /increase/ of stress.

2.2 The effect of the above factors may be studied on fibre-glass reinforced plastics with different kind of reinforcement /fabric with warp in the direction of or at 45° to the strip axis, glass matt/. The specimens can readily be placed in conditioning boxes at various temperatures /20, 50, 75° C/, water, weatherometer, etc.

3. Creep strength and life. Strength tests of strips after a certain period of relaxation enable us to follow the long-time decrease in strength due to the action of load. It is indicated that the effect of the environment on the mechanical properties of the material /life time/, i.e. the decrease in strength, modulus of elasticity, etc. must be studied on loaded rather than on unloaded specimens if the results are to be applicable to actual structures; this requirement is easily satisfied in this test.

4. Simplified test. The measured strips can be braced more simply yet in the shape of an arc by setting them between projections of a rigid backing; in this case the strips need not be provided with metal edges /Fig.4/. After a predetermined period of time, the strips are removed and measurements made of the height of the strip arc /or the distance between the ends/ and of the modulus of elasticity of the strip which is again most conveniently effected by measuring the force necessary for pressing the strip ends together, and the corresponding change of the arc height /the rigidity of the dynamometer used for the purpose may be very small/. Such a test would be very simple to carry out at the manufacturers' plants

and should be specified as routine test and standardized. A shortcoming of the test lies in that a temporary slacking of the strip to a certain extent affects /raises/ the stress in subsequent stress relaxation. Neither it is possible to measure during it the stress relaxation for short time periods.

5. Shape of the curved strip. Bending moments and hence also stresses in the bent strip vary along the strip length and are proportional to distance y from the chord, i.e. to deflection. The suggested method of stress relaxation testing is feasible thanks to the fact that stress relaxation and creep of plastics as well as ^{of} fibre-glass plastics appear linear in stress, so that the stresses change in equal proportion at all points of the strip and the shape of the strip arc does not change if the distance between the ends is constant. Appearance of a change in the arc height would prove the existence of a nonlinear component of the creep; then the test would afford some mean value of the relaxation modulus for a given domain of stresses, and the nonlinear component could be approximately ascertained from measurements made at various heights of the arc.

Let us assume that the strip deforms only by flexure while Navier's hypothesis of preserving plane sections applies, and moreover, that flexural rigidity EJ is the same and the creep homogeneous along the whole length of the strip. Although we need not know the shape of the strip for determination of the values of stress, we must ascertain the curvature $1/\rho$ if we wish to determine the relaxation modulus. Rather than to measure it directly, it is easier to compute it either from height f at the arc top, or from distance l between the strip ends.

The strip arc curvature $1/\rho$ is proportional to the bending moment and hence also to distance y from the chord. To express the curvature we must make use of the theory of large deflections; the curve given by this condition is termed "elastica" /for small arc heights when $1/\rho \approx d^2y/dx^2$, the curve is approximately sinusoidal/. It is obviously the same curve as that arising when a bar buckles ([5], pp.76-81) first

solved by Legendre; our solution can be carried out in an analogous way, except that our problem is opposite - to find the curvature from a given deflection / as against that of computing the deflection/. We shall, therefore, indicate only the principal points of the solution.

For the notation of Fig. 2, the strip curvature is $-d^2\vartheta/ds$ and hence it holds that $-d^2\vartheta/ds = Cy$. Introducing curvature at the arc top $1/\rho$ and arc height f at the top, we obtain relation $-d^2\vartheta/ds = y/f\rho$. Differentiating it with respect to s ($dy/ds = \sin\vartheta$), we get for the sought-for curve the following differential equation of the second order

$$11/ \quad \frac{d^2\vartheta}{ds^2} = - \frac{\sin\vartheta}{f\rho}$$

with the initial conditions $d\vartheta/ds = 0$ and $\vartheta = \vartheta_0$ for strip end $s = 0$. Multiplying eq. 11/ by $2 d\vartheta/ds$ and integrating with respect to s we obtain after rearrangement the first integral of the equation as follows

$$12/ \quad \frac{d\vartheta}{ds} = - \sqrt{\frac{2}{f\rho}} \sqrt{\cos\vartheta - \cos\vartheta_0} = - \frac{2}{\sqrt{f\rho}} \sqrt{\sin^2\frac{\vartheta}{2} - \sin^2\frac{\vartheta_0}{2}}$$

where ϑ_0 is the angle between the strip and the chord at the strip end. In this equation we can separate the variables and express the solution of ϑ by quadrature. If we wish to compute deflections y , we solve - instead of eq. 12/ - an equation that will result from introduction of $ds = dy/\sin\vartheta$ and similarly, when computing ordinates x , an equation resulting from introduction of $ds = dx/\cos\vartheta$. Integration of each of these equations and rearrangement of the integrals with the aid of substitution $\sin\vartheta/2 = \varphi \sin\vartheta_0/2$ yields the following relations

$$13/ \quad l_0 = 2\sqrt{f\rho} \int_0^{\pi/2} \frac{d\varphi}{\sqrt{1 - \sin^2\frac{\vartheta_0}{2} \sin^2\varphi}}, \quad f = 2\sqrt{f\rho} \sin\frac{\vartheta_0}{2}, \\ l = 4\sqrt{f\rho} \int_0^{\pi/2} \sqrt{1 - \sin^2\frac{\vartheta_0}{2} \sin^2\varphi} d\varphi - l_0$$

where the first integral is a complete elliptic integral of the first kind, and the second, a complete elliptic integral of the second kind. Their values are tabulated [2].

(Since) Explicit computation of $1/\rho$ cannot be carried out, transcendental equations had to be solved for given f and l .

We have, therefore, prepared numerical Table 1 /computed by L. Dejnek and I. Folke, members of the Institute/ from which one can read to the measured arc height for the distance between the ends f , curvature $1/\rho$ at the strip top. So far as the accuracy of the measurement is concerned, the curvature may be determined from f for a range of about $0 \leq f/l_0 \leq 0.33$ or from l for about $0 \leq l/l_0 \leq 0.75$; at higher values, f and l change too slowly relative to curvature.

At an arbitrary point of the arc, the curvature can be determined by measuring its ordinate y ; evidently it equals $y/f\rho$.

6. Determining the modulus of elasticity. With changing distance l between the ends of the strip, bending moments and hence also the curvature of the strip change proportionally to the distance from the chord; consequently the new shape of the arc, as l_1 changes to l_2 is again curve "elastica" to which applies Table 1. On determining by measurement forces X_1 and X_2 acting on the ends of the strip /after deducting the weight of the jaws, edges, etc., if necessary/ and arc heights f_1 and f_2 , and reading for the latter values of f_1 or f_2 and l_1 or l_2 the corresponding l_0/ρ_1 and l_0/ρ_2 from Table 1, we obtain the change of bending moment at the strip top as $X_2 f_2 - X_1 f_1$. The modulus of elasticity corresponding to this change of distance between the ends is

$$14/ \quad E = \frac{l_0 (X_2 f_2 - X_1 f_1)}{J (l_0/\rho_2 - l_0/\rho_1)}$$

where J is the moment of inertia of the strip section ($J = \frac{1}{12} b d^3$).

7. Determining the relaxation modulus and corrections. 7.1

The braced strip forms a one-time statically indeterminate system /two-hinged arch/ with statically indeterminate force $P(t)$ acting on the string dependent on time t . Let us assume that the specimen is permanently freely suspended on one jaw /Fig.1/. Force $X(t)$ acting on one of the ends of the plastic strip proper is given by the condition of equilibrium as $X(t) = P(t) - G$ where G is the weight of the jaws and edges fitted to the strip plus 1/2 of the strip weight

on the average / and 1/2 of the string weight/.

Assuming the string to be absolutely rigid compared to the strip arc, the relaxation modulus $E_{rel}(t)$ defined as ratio $\sigma(t)/\epsilon(0)$ at constant deformation $\epsilon(0)$ is under linear creep /test under constant deformation/ given as

$$15/ \quad \frac{E_{rel}(t)}{E(0)} = \frac{X(t)}{X(0)}$$

where $t=0$ is the initial time of bending the strip and E - the modulus of elasticity, time variable ($E_{rel}(0) = E(0)$) /because of aging /degradation/ of the material /or possibly because of temperature, etc./.

If we wish to determine $E_{rel}(t)$ more accurately, we must consider the elasticity of the string which shortens with time because of stress release; this in turn raises the strip stress to a value higher than that corresponding to true relaxation. For the purposes of computation we must ascertain flexibility d_p of the strip arc for a small deformation under load induced by force X acting between the strip ends at a given arc height f , i.e. a change of distance between the strip ends corresponding to force $X=1$. For two adjacent values of l/l_0 and $l/l_0 + \Delta(l/l_0)$ in Table 1 we determine the respective change of curvature $\Delta(l_0/\rho)$ to which corresponds a change of bending moment $\Delta M = EJ \Delta(1/\rho)$ and to that a change of force $\Delta X = \Delta M / f$. Thus the following deformation corresponds to force $X=1$:

$$16/ \quad d_p = \frac{\Delta(l/l_0) f l_0^2}{\Delta(l_0/\rho) EJ}$$

For small arc height f for which the shape of the arc can approximately be taken as parabola, we would obtain $d_p = \frac{8 f^2 l_0}{15 EJ}$.

To change dX of force acting at the end of the strip at time $t - \tau$ during time $d\tau$ ($dX = \frac{dX(t-\tau)}{d\tau} d\tau$) corresponds string elongation $dX d_p$ accompanied by a change of force acting at the end of the strip $-dX d_p / d_p$ /condition of compatibility/, where $d_p = l_0 / E_s F_s$ is the string compliance. Assuming linearity, this force relaxes to time t to a value of $-dX \frac{l_0}{d_p} \frac{E_{rel}(\tau)}{E}$. On the assumption that the principle of superposition applies to such small changes and neglecting time changes of the modulus of elasticity, we

arrive at equation

$$17/ \quad X(t) = \frac{E_{rel}(t)}{E} X(0) + \frac{d_s}{d_p} \int_0^t \frac{E_{rel}(\tau)}{E} \frac{dX(t-\tau)}{d\tau} d\tau$$

This is Volterra's integral equation from which $E_{rel}(t)/E$ can be computed for given $X(t) = P(t) - G$. Eq. 17/ can be solved by successive approximations, etc. [3]. The value of d_s/d_p can be neglected in the majority of cases (it is usually less than 1/100 for the above mentioned dimensions of fibre glass plastics strips) by doing so we arrive at eq. 15/. Even in the case that a more flexible dynamometric element than a string is used, it is fully sufficient to introduce in integral 17/ $E_{rel}(t)$ instead of $E_{rel}(\tau)$ which gives

$$18/ \quad \frac{E_{rel}(\tau)}{E} = \frac{X(t)}{X(0) + \Delta_t} \quad \Delta_t = \frac{d_s}{d_p} [P(0) - P(t)]$$

The exact values of $E_{rel}(t)$ are larger than those according to 18/. On the other hand one can deduce from the convexity of function $E_{rel}(t)$ that the exact values are smaller than the expression resulting as $\frac{1}{2}[E + E_{rel}(t)]$ is substituted for $E_{rel}(\tau)$ in 17/, i.e.

$$18a/ \quad \frac{E_{rel}(\tau)}{E} = \frac{X(t) - \Delta_t/2}{X(0) + \Delta_t/2}$$

This also restricts the error to very narrow values.

7.3 If the measurement is carried out by the application of additional weight ΔG /Fig.1/ to bring the string tension to a constant frequency, we substitute in the above equations for $P(t)$ the string tension reduced by $\frac{\Delta G}{1 + d_s/d_p}$.

7.4 If the strips are mounted on a rigid backing /Fig.4/, $E_{rel}(t)$ can be found from the change of curvature of the arc from $1/\rho$ to $1/\rho'$ in time t after the strip has been removed from the backing. Prior to the strip removal the bending moment equals to $E(t)J(1/\rho - 1/\rho')$ in time $t = 0$, to $E(0)J/\rho$. Consequently

$$19/ \quad \frac{E_{rel}(t)}{E} = 1 - \frac{l_0/\rho'}{l_0/\rho}$$

modulus $E(t)$ must also be ascertained on the relieved speci-

men from eq. 14/. It is frequently quite sufficient to consider $E(t) = E(0)$.

8. Computing other creep data. Assuming the validity of the principle of superposition which assumption is well acceptable within certain limits [7], [8], see also paragraph 10/ we can compute from curve $E_{rel}(t)$ the behaviour at any arbitrary stress or deformation. Creep $\varepsilon(t)$ under constant stress $\sigma(t_0)$ can be described by creep modulus $E_{cr}(t)$ defined as ratio $\sigma(t_0)/\varepsilon(t)$. It is computed in the following way: To a small change of stress $d\sigma = \frac{d\sigma(t-\tau)}{d\tau} d\tau$ in time $t-\tau$ corresponds deformation $d\sigma(t-\tau)E_{cr}^{-1}(\tau)$ in time t . Superposition of the effects of all changes of stress $d\sigma$ during stress relaxation and introduction of $\varepsilon = 1$, $\sigma(t) = E_{rel}(t)$ leads to relation:

$$10/ \quad 1 = E E_{cr}^{-1}(t) - \int_0^t E_{cr}^{-1}(\tau) \frac{dE_{rel}(t-\tau)}{d\tau} d\tau \quad \left(= \frac{E_{rel}(t)}{E} + \int_0^t E_{rel}(t-\tau) \frac{dE_{cr}^{-1}(\tau)}{d\tau} d\tau \right)$$

which represents Volterra's integral equation for $E_{cr}^{-1}(t)$. The equation may be solved either by the method of successive approximations [3] ($E_{cr}(\tau) = E_{rel}(\tau)$ being best for the first approximation) or by using Laplace's transformation [4] or possibly Galerkin's method [3] or best of all, by numerical quadrature of the integral leading to recurrent algebraic equations.

For a limited time interval with limits of a not too different order, say from $0.1\tau_1$ to $10\tau_1$, creep of plastics and fibre glass plastics may be represented by simple Boltzmann's /standard/ model [7], [8], consisting of a spring (Fig.5) and a Voigt's unit (spring and dashpot coupled in parallel/coupled in series. For the stress relaxation curve within the limited interval [7], [8] it then follows that

$$11/ \quad E_{rel}(t) = E_{\infty} + (E - E_{\infty}) e^{-\frac{E}{E_{\infty}} \frac{t - t_0}{\tau_p}}$$

where $E_{\infty} = E_{rel}(\infty)$ characterizes the finite /asymptotic/ value of stress corresponding to this interval, and τ_p the rate of stress relaxation /retardation time/. For a required time interval (e.g. from 1 to 100 days, or from 10 days to 3 years, or from 1 hour to 4 days, etc.) E_{∞} and τ_p

can be determined by collocation of this curve with the measured data. Within the respective interval the curve of creep can be written out directly with the aid of ϵ_{∞} and τ_p as follows:

$$/12a/ \quad \epsilon_{cr}^{-1}(t) = \frac{1}{E} - \left(\frac{1}{E_{\infty}} - \frac{1}{E} \right) e^{-\frac{t-t_0}{\tau_p}}$$

and the differential equation of creep at generally variable stress and deformation as $\tau_p \frac{d\epsilon}{dt} + \epsilon = \frac{\tau_p}{E} \frac{dG}{dt} + \frac{\epsilon}{E_{\infty}}$ etc. [7], [8]. For a longer time interval we could analogically consider a model consisting of more elements [7], [8].

9. Note. Stress relaxation and creep in bending enable us to make a good enough estimate of the creep in tension and compression. Since the creep is linear, the two values are almost identical for unreinforced plastics; the same is true, except for a secondary disturbing effect of transverse fibres, of fibre-glass plastics so long as the glass reinforcement is distributed uniformly across the strip thickness. An approximate computation based on Boltzmann's model, suitable for non-uniform distribution is given in [7], [8].

10. Note. In the foregoing paragraphs we have assumed the principle of superposition to be valid for both stress relaxation and creep under constant stress. When the stress is suddenly released, the principle of superposition no longer applies in the case of glass-fibre plastics [7], [8]; the deformation and stresses are only partially reversible. It seems that this phenomenon can most simply be represented (Fig.5) by a rheologic model [7], [8], in which an additional Voigt's unit is coupled in parallel with the first Voigt's unit through a ratchet pawl opened for increasing and closed for decreasing creep deformations. When creep and stress relaxation under constant stress are involved, the ratchet pawl is continuously opened and the principle of superposition applies.

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Table 1 / Elastica /

l_0/g	f/l_0	l/l_0	α°	l_0/g	f/l_0	l/l_0	α°
0	0	1,000	0				
0,198	0,020		3,60°				
0,396	0,040		7,20				
0,595	0,060		10,85	3,801	0,322	0,680	67,18°
0,796	0,080		14,50	4,071	0,336	0,640	71,60
1,000	0,100	0,975	18,20	4,330	0,349	0,600	75,79
1,207	0,120		20,94	4,589	0,360	0,560	79,92
1,418	0,140		25,75	4,845	0,369	0,520	83,93
1,634	0,160		29,63	5,100	0,377	0,480	87,80
1,856	0,180		33,60	5,355	0,384	0,440	91,65
2,086	0,200	0,891	37,69	5,608	0,390	0,400	95,37
2,325	0,220		41,91	5,860	0,395	0,360	98,99
2,575	0,240		46,30	6,116	0,398	0,320	102,57
2,839	0,260		50,89	6,382	0,401	0,280	106,20
3,121	0,280		55,48	6,649	0,402	0,240	109,75
3,427	0,300	0,734	60,94	6,921	0,403	0,200	113,25
3,765	0,320		66,58	7,202	0,403	0,160	116,75
4,148	0,340		72,85	7,498	0,399	0,120	120,32
4,602	0,360		80,11	8,445	0,391	0,000	130,72

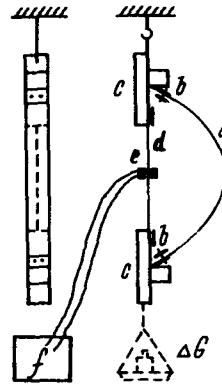


Fig. 1

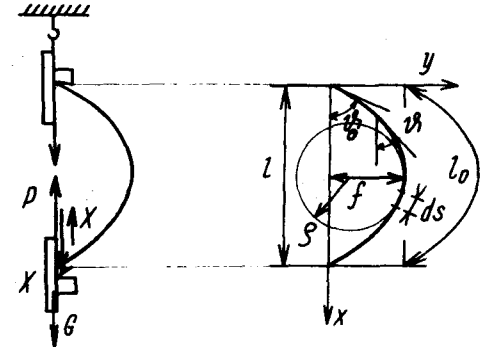


Fig. 2

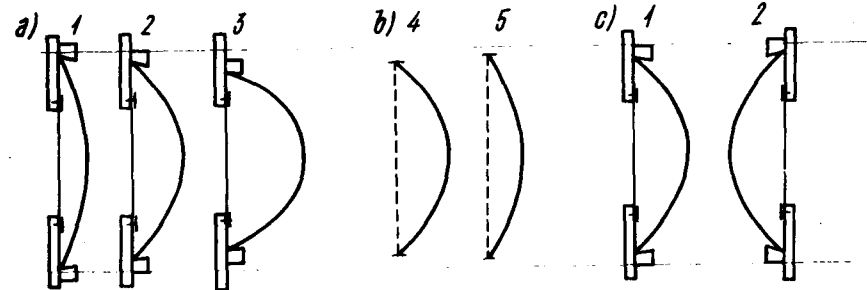


Fig. 3

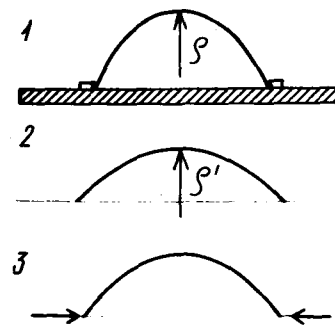


Fig. 4

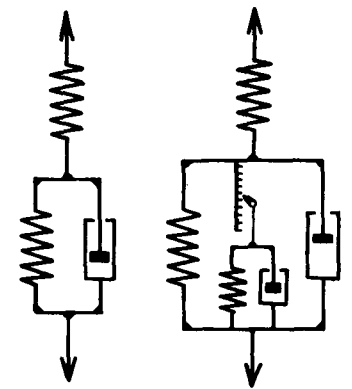


Fig. 5

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Installation for the Testing of Shell Models

Summary

In this paper, a special equipment is presented, for testing shell structures, consisting of three scales, three nets transmitting the load to the model and a metallic framework. The maximum available force of this equipment is 30 000 kg. The model surface can reach a maximum of $3 \times 9 = 27 \text{ m}^2$, meaning that its maximum load is $1\ 000 \text{ kg/m}^2$.

The surface of the model can be totally or partially loaded. The loads can be applied upwards or downwards on every direction so as to simulate the wind action.

Although a load could be effected in a few minutes, it can be maintained as long as it is necessary.

1. Introduction

One of the main problems arising from testing the big models of shells is the loading carrying out.

The device or equipment responsible for the load when testing a model is bound to fulfill the following conditions:

- a) to let the largest possible free portion of the model surface in order to permit observations and measurement during the test ;
- b) to allow quick and easy loading and unloading ;
- c) to allow for a great number of load hypotheses (symmetric load, non-symmetric load, partial load etc.) ;
- d) to permit the largest possible range of shell shape to be used with regard to the load application, as well as