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u_{cy} – Porewater pressure induced by static undrained shear due to inertia force of dam

u_s – Porewater pressure induced by static undrained shear due to pond liquefaction

α – Coefficient for pore pressure build-up equation for cyclic triaxial test

α_1 & α_2 – Constants required to evaluate α

σ_{dcy} – Cyclic deviatoric stress applied in undrained condition for cyclic triaxial test

$\Delta\sigma_1=\sigma'_{1\ell}-\sigma'_{1c}$ – Change of major principal stress at failure surface due to pond liquefaction

$\Delta\sigma_3=\sigma'_{3\ell}-\sigma'_{3c}$ – Change of minor principal stress at failure surface due to pond liquefaction

σ'_{fc} – Effective normal stresses at failure surface after consolidation of dam

$\sigma'_{f\ell 1}$ – Effective normal stresses at failure surface after pond liquefaction and before allowance for u_s

$\sigma'_{f\ell 2}$ – Effective normal stresses at failure surface after pond liquefaction and after allowance for u_s

σ'_{1c} & σ'_{3c} – Effective major and minor principal stresses at failure surface after consolidation of dam

$\sigma'_{1\ell 1}$ $\sigma'_{3\ell 1}$ – Effective major and minor principal stresses at failure surface after pond liquefaction and before allowance for u_s

$\sigma'_{1\ell 2}$ $\sigma'_{3\ell 2}$ – Effective major and minor principal stresses at failure surface after pond liquefaction and after allowance for u_s

τ_{cy} – Cyclic shear stress on failure surface

τ_{fd} – Shear stress on failure surface after pond liquefaction and application of inertia force of dam

$\tau_{f\ell 1}$ – Shear stress on failure surface after pond liquefaction and before allowance for u_s

$\tau_{f\ell 2}$ – Shear Stress on failure surface after pond liquefaction and after allowance for u_s

ϕ' – Effective friction angle (Mohr-Coulomb strength envelope)

CONSTITUTIVE EQUATION FOR CYCLIC BEHAVIOR OF COHESIVE SOILS

by

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ABSTRACT

Endochronic theory is employed to develop a relatively general constitutive relationship to model the dynamic behavior of cohesive soils subjected to multi-dimensional stress or strain paths. The proposed constitutive law is capable of describing (a) strain softening and hardening, (b) densification and dilatancy, (c) frictional aspects, and (d) rate dependence of the stress-strain behavior; it also accounts for pore pressure response in undrained conditions by considering saturated soils as two-phase media. The theory is based on a series of new internal state variables that are defined in terms of semi-empirical intrinsic material relationships, and it is able to handle elastic and inelastic strain histories for rate-dependent materials from the very beginning of their cyclic stress-strain path. The intrinsic relations involve ten material parameters, including the initial elastic modulus, which must be determined from a quasi-static and cyclic tests. Although limited data preclude the development at this time of specific correlations for the material parameters in terms of soil characteristics, this model offers a significant improvement in the interpretation and analysis of the cyclic behavior of cohesive soils. The mathematical model is applied to describe data from low-frequency cyclic constant-strain-rate tests on undisturbed samples and low-frequency cyclic constant-load-amplitude tests on slurry-consolidated samples of kaolin; both types of test were conducted under undrained triaxial conditions with pore pressure measurements. Emphasis is directed toward the behavior of cohesive soils under low-frequency, large-strain cyclic conditions, such as those associated with earthquakes.

INTRODUCTION

In the absence of a representative constitutive relationship to describe the multi-dimensional stress-strain-time behavior of cohesive (as well as cohesionless) soils, various engineering tests have been

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used to simulate the response of a typical element in a soil mass for a specific loading pattern; then, the experimentally determined phenomenological parameters, tempered with personal knowledge and past experience, are used to solve field problems that involve general loading conditions. This study is directed toward outlining the basic framework, together with some preliminary results, for the development of a more realistic and physically sound constitutive formulation which, although macroscopic in the general sense, includes material parameters that relate the phenomenological behavior of a soil to the microscopic characteristics of its component parts.

SCOPE OF STUDY

A considerable amount of research (mostly experimental investigations of material properties) on the cyclic behavior of cohesive soils has been conducted in the past two decades. However, much of this research has involved the use of different testing techniques and loading paths on specimens with a wide range of geometrical shapes and sizes In many cases the boundary conditions imposed on the specimen, as well as accuracy of the test equipment, are known to influence significantly the resulting response parameters. Although the above mentioned shortcomings exist for all soil experimentation, the effects of the testing technique and apparatus are much more pronounced for cyclic testing conditions. In general, cohesive soils exhibit the following important trends during deformation under any change in the stress state:

1. Recoverable and irrecoverable strains start from the very beginning of the stress and time paths.
2. Deformations depend significantly on the stress history and rate of loading.
3. The effect of the volumetric stress component is not negligible.
4. Total strains can be considered as a combination of the following three components, each of which have recoverable and irrecoverable portions;
   (a) Volumetric strains due to changes in volumetric stresses;
   (b) Volumetric strains due to changes in shear stresses (this will be called densification or dilatancy, depending on whether the initial volume decreases or increases);
   (c) Shear strains due to changes in shear stresses.
5. The moduli for recoverable parts are not constant, but dependent on (a) the accumulated irrecoverable strains and rate of loading and (b) the changes in the effective volumetric stress components (this is due to the frictional nature of soils).

The main emphasis in this work is aimed toward describing the behavior of saturated cohesive soils under low frequency, large strain earthquake loading conditions, and only the major microscopic properties that influence the response of a two-phase soil will be included in the formulation (the fluid phase is assumed to be natural water). The cyclic behavior of cohesive soils will be described mathematically within the framework of endochronic theory (Bažant and Bhat, 1976; Bažant and Krizek 1976; Valanis, 1971), and the applicability of the model will be demonstrated by using data from cyclic constant-strain-rate tests on undisturbed Newfield Clay (Sangrey, 1968) and low frequency cyclic constant-load-amplitude tests on slurry consolidated kaolin (Brewer, 1972); both types of test were conducted under undrained triaxial conditions with pore pressure measurements. The latter of these data sets (Brewer, 1972) is in the range of low frequency cyclic tests, and the rate dependence is introduced in the formulation to model these tests. However, since the former data set (Sangrey, 1968) was obtained under relatively slow axial strain rates, the rate dependence is neglected, as in the quasi-static case.

THEORETICAL FRAMEWORK

The basic concept underlying the proposed theory is based on the notion of intrinsic time, which has existed since the early 1950's, but Valanis (1971) was apparently the first to apply this concept to describe the inelastic, viscoplastic behavior of metals. The theory is called endochornic because stress, which is a functional of the strain history, is defined with respect to the intrinsic time scale, where the latter is a material property and depends on the deformation history. The notion of an intrinsic time scale is introduced to account for the dissipative effects of history-dependent materials. This intrinsic time parameter is a monotonically increasing function of strain and external time for rate dependent materials, such as clay soils. Assuming that the development of inelastic strain is gradual, the intrinsic time increment, dz, may be experssed as (Bažant and Krizek, 1976)

$$dz^2 = \left(\frac{d\zeta}{Z_1}\right)^2 + \left(\frac{dt}{\tau_1}\right)^2 \quad , \qquad (1)$$

where ζ is termed the rearrangement measure, t is external time, and Z_1 and τ_1 are material parameters. This formulation is complete in the sense that both the rate and microstructural dependence of the soil behavior are included; however, it is assumed that their coupled effect can be treated separately. The rearrangement measure, ζ, represents the effect of the soil structure and can be expressed as

$$d\zeta = F(\epsilon,\sigma,\zeta)d\xi; \; d\xi = \sqrt{J_2\ (d\underset{\sim}{\epsilon})} \quad , \qquad (2)$$

where F is a strain hardening-softening function, ξ is the distortion measure, σ and ε are the stress and strain tensors, and J_2 $(d\varepsilon)$ is the second invariant of the incremental deviatoric strain tensor.

For cohesive soils it may simply be assumed as a first order approximation that the resistance to shear stress depends mainly on the effective normal stress and the interparticle distance, the latter of which is directly related to the void ratio and dependent in some manner on stress history and the applied stress path. The definition of strain hardening-softening enables the influence of these factors on the rearrangement measure to be included. Since strain hardening and strain softening are coupled in cohesive soils, $F(\underset{\sim}{\epsilon},\underset{\sim}{\sigma},\zeta)$ will be treated as a single composite function (not separate strain softening and strain hardening functions) which can be defined as

$$F(\underset{\sim}{\varepsilon},\underset{\sim}{\sigma},\zeta)\ d\xi = \frac{d\eta}{f(\eta)} \qquad (3)$$

and

$$d\eta = F_1\ (\underset{\sim}{\varepsilon},\underset{\sim}{\sigma})\ d\xi \qquad , \qquad (4)$$

where η is an auxiliary variable that accounts for continuous softening. For the purpose of establishing the necessary internal relations with respect to the governing factors, the function F_1, may be analyzed in the form

$$F_1\ (\underset{\sim}{\varepsilon},\underset{\sim}{\sigma}) = F_{11}\ (I_1^{\ \varepsilon})\ F_{12}\ (I_1^{\ \sigma'})\ F_{13}\ (J_2^{\ \varepsilon})\ , (5)$$

where F_{11} represents the effect of the volumetric strain in terms of the first strain invariant, $I_1^{\ \varepsilon}$, F_{12} represents the effect of the effective stress state in terms of the first effective stress invariant, $I_1^{\ \sigma'}$, and F_{13} represents the effect of shear stress and strain in terms of the second invariant of the deviatoric strain tensor, $J_2^{\ \varepsilon}$.

Most soils also manifest inelastic volumetric strain, termed densification or dilatancy, as a result of shear. This phenomenon is handled by considering another internal variable, λ, called the densification-dilatancy measure. It is assumed that the inelastic volume change is due only to the existence of shear, and inelastic volumetric strains due to changes in the hydrostatic stress (that is, consolidation) are excluded from the formulation. The densification-dilatancy measure, λ, can be expressed in the following differential form as a function of the stress and strain invariants and the accumulated value of λ:

$$d\lambda = L\ (\underset{\sim}{\varepsilon},\underset{\sim}{\sigma},\lambda)\ d\xi \qquad (6)$$

Although the volume change, as treated thus far, is due only to shear stresses, the function L will be formulated to include the relatively small volume change resulting from the volumetric stress that exists concurrent with the deviatoric stresses in multi-axial stress conditions. The densification-dilatancy function, L, is similar to the strain softening-hardening function, F, and it is related to the clay type, stress history, and stress path. These factors can be handled by the use of subfunctions which can be expressed in terms of the stress and strain invariants as

$$L = L_1\ (I_1^{\ \varepsilon})\ L_2\ (I_1^{\ \sigma'})\ L_3\ (J_2^{\ \varepsilon})\ L_4\ (\lambda) \quad . \qquad (7)$$

The rate dependence of the inelastic volume change can be included in the formulation as in the case of intrinsic time, thereby leading to

$$(d\bar{\lambda})^2 = (d\lambda)^2 + (\sigma_c\ dt/\tau_2)^2 \qquad (8)$$

where τ_2 is a constant value material parameter and σ_c is the consolidation stress.

Stress-Strain Relations

In the process of adaping endochronic theory to cohesive soils, the soils are assumed to be statistically homogeneous on a sufficiently large scale and incrementally isotropic. The assumption of isotropy is hardly ever true in the strict sense, but the effect associated with anisotropy is neglected at this stage to reduce the complexity of the mathematical formulation. As a consequence, the incremental stress-strain relations may be expressed separately in terms of the deviatoric and volumetric components of the stress and strain tensors as

$$de_{ij} = \frac{ds_{ij}}{2G} + \frac{s_{ij}}{2G}\ dz \qquad (9)$$

and

$$d\varepsilon = \frac{d\sigma'}{3K} + d\lambda \qquad , \qquad (10)$$

where $e_{ij} = \varepsilon_{ij} - \delta_{ij}\varepsilon$ is the deviator of the strain tensor with $\varepsilon = (1/3)\varepsilon_{kk}$ being the volumetric strain, $s_{ij} = \sigma'_{ij} - \delta_{ij}\ \sigma'$ is the deviator of the effective (as well as total) stress tensor with $\sigma' = (1/3)\sigma'_{kk}$ being the effective volumetric stress, and G and K are the elastic shear and bulk moduli, respectively. Equations (9) and (10) are both expressed as a sum of recoverable and irrecoverable components. The dependence of the recoverable component on the clay type, stress history, present state of stress, and change in the stress and strain states is achieved by defining the shear modulus as a function of the stress and strain invariants and the initial shear modulus. The irrecoverable strain component is treated as a product of the hypothetical recoverable strain corresponding to the present deviatoric stress level with the present magnitude of the shear modulus and intrinsic time. The hypothetical recoverable strain accounts for the effects of the applied stress level and the changes in the elastic properties on the irrecoverable strain, and the intrinsic time reflects the effects of the clay type, stress history, and variation in the states of stress and strain.

Pore Pressure Response

The undrained behavior of saturated cohesive soils can be conveniently analyzed within the framework of a two-phase medium (fluid and solid), because the compressibility of the minerals composing the soil grains is about thirty times less than the compressibility of water (Lambe and Whitman, 1969) and the assumption of incompressible soil grains will not introduce any significant error. The solid structure, on the other hand, is compressible in the sense that the size of the element may change, and it is capable of carrying both shear and volumetric stresses. The fluid phase is the pore water, which is treated as compressible. Although the bound water in a clay-water system has been reported (Mitchell, 1976) to have different properties than free pore water, the percentage of bound water to free pore water in most cases is very small, and its effect may therefore be neglected. Accordingly, it will be assumed that (a) the compressibility of free water is the same as that of bound water, (b) both are elastic, and (c) both are capable of carrying only volumetric stresses.

Conditions where the rate of loading is too fast to allow any drainage are treated as completely undrained cases. In such situations the mass of the fluid phase is constant, but the volume occupied will vary according to the pore pressure developed. Because the tendency of the solid structure to change its volume is coupled with the pore pressure, the solid structure will deform only as much as the pore water permits. Under undrained conditions the volume change of the solid structure will equal the volume change of the pore fluid, and the total volumetric strain can be given by Equation (10), where

$$d\varepsilon = \frac{1}{3}\frac{dV_s}{V_s} \quad , \qquad (11)$$

with V_s being the total volume (or the volume of the solid structure). Considering the porosity, n, of the soil-water element, the volume of the pore water can be taken as $V_f = nV_s$. Since the compressibility of water, C_w, is assumed to be constant, the pore pressure increment can be expressed as

$$du = \frac{C_w K}{nK + C_w}\left(\frac{d\sigma}{3K} + 3d\lambda\right) \qquad (12)$$

From Equation (12) it is seen that the assumption of an incompressible pore fluid would lead to an infinite pore pressure magnitude, and the use of this assumption would prevent any meaningful explanations for the pore pressure response.

Variation of Elastic Modulus

Previously it was suggested that the modulus for the recoverable portion of the strain be defined to account for the effects of the changing soil properties. The shear modulus, G, which has a major effect on the response, will be taken as the principal variable. Assuming that Poisson's ratio is constant for the elastic strains, the bulk modulus, K, can be determined for isotropic materials.

The initial values for the shear and bulk modulus of a cohesive soil are dependent on many factors, such as clay type, stress history, present effective normal stresses, void ratio, etc. However, since several of these factors vary along the stress path, the magnitudes of the moduli must also vary, and the resulting changes in modulus are described with respect to their initial magnitudes. Although any attempt to define these path-dependent moduli in terms of the many factors involved is not within the scope of current knowledge, the initial moduli may be determined with reasonable accuracy from certain tests or from empirical relations given in the literature.

The two most important factors that influence the value of the modulus along a given stress path are the void ratio and the effective normal stress, and the cumulative densification-dilatancy measure, λ, and the first stress invariant, I_1^σ, are adopted, respectively, to represent these factors. Then, the ratios of changes in these parameters to their initial values (as contrasted with ratios of initial to present values) are assumed to be linearly related to the shear modulus.

As a final step, it is necessary to express stress-strain relations for the three-dimensional situation. Due to the complex nature of the proposed set of differential equations, the stresses and strains are incremented in small steps and a step-by-step integration process is followed; in the initial step all incremental values are either estimated or taken as zero. The step-by-step integration is stable, and convergence (although better for the strain-controlled tests) is achieved after a couple of iterations. In the case of the strain-controlled test procedure the post-peak behavior can be determined, but the peak point can not be determined exactly in the stress-controlled tests and the post-peak behavior can not be predicted.

PROPOSED CONSTITUTIVE RELATIONSHIPS

The proposed mathematical approach, which is based on the definition of two internal variables (intrinsic time and densification-dilatancy measure), accounts for the effects of all major factors on the volumetric and shear response of cohesive soils along any stress or strain path. The variations in these two parameters, formulated on the basis of previous observations and experimental data by assuming linear relationships with respect to the factors considered in Equations (5) and (7), can be expressed in terms of material functions as

$$F_1 = a_1 + \frac{|1 - a_2\, I_1^{\,\varepsilon}|\,(1 + a_3\, J_2^{\,\varepsilon})}{(0.01 + a_4\, I_1^\sigma/p_A)} \quad , \qquad (13)$$

$$f(\eta) = 1 + \frac{\beta_1 \eta}{1 + \beta_2 \eta} \quad , \qquad (14)$$

and

$$L = \frac{C_1\,|1 + C_2\, I_1^{\,\varepsilon}|}{(1 + C_3\, I_1^\sigma/p_A)\,(1 + C_4\, J_2^{\,\varepsilon})\,(1 + C_5\,\lambda)} \quad , \qquad (15)$$

where a_1, a_2, a_3, a_4, β_1, β_2, C_1, C_2, C_3, C_4, and C_5 are material constants and p_A is atmospheric pressure (introduced to normalize the stress invariant). The first variable, intrinsic time, which includes Z_1 and τ_1 from Equation (1), is expressed in terms of eight material parameters, and the second variable, densification-dilatancy measure, is expressed in terms of six material constants, including τ_2 from Equation (8). All of these parameters can be related to soil characteristics, such as clay type, plasticity, etc., and correlations can be established with certain soil indicies. Insofar as possible with available data, the capabilities of the proposed model are demonstrated.

Rate-Independent Cyclic Behavior

Sangrey (1968) conducted cyclic constant-strain-rate tests on specimens of an undisturbed Newfield Clay consolidated under different effective stresses. The specimens were isotropically consolidated to the specified stress level in a triaxial chamber, and undrained tests with pore pressure measurements were performed under a uniform cyclic stress amplitude. Because of the relatively limited amount of data available, a trial-and-error procedure (rather than mathematical optimization) was used to obtain the fits when applying the model; one example of the resulting agreement between theoretical and experimental response is illustrated in Figure 1 for a case where the amplitude of the stress difference is 325 kN/m^2; in this case the values of the constants in Equations (1), (13), (14), and (15) are given in Table 1.

Table 1. Constants in Constitutive Relationship for Example Illustrated in Figure 1.

G_o/P_A	Z_1	a_1	a_2	a_3	a_4	β_1	β_2	c_1	c_2	c_3	c_4	c_5
216	0.0155	4	25	10	0.75	15	2	2.2	1500	0.25	4500	35,000

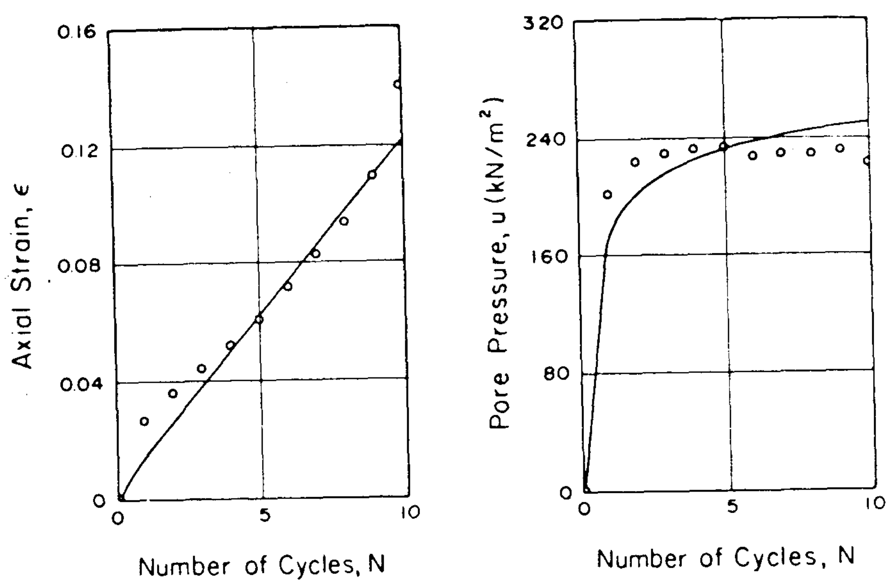

Figure 1. Constant Strain Rate Cyclic Test Data for New Field Clay (after Sangrey, 1968) with Theoretical Response (solid curves)

Quasi-Static Behavior

Cohesive soils, due to their relatively impermeable nature and the viscous effect of their bound water, usually exhibit rate-dependent properties; accordingly, changes in their microstructure are not instantaneous, but time dependent. Under relatively slow load or deformation rates, the imposed microstructural changes can respond to the applied external excitation, but this is not true for faster rates. In modelling the quasi-static behavior of cohesive soils under relatively slow strain

rates, the rate effect will be ignored by neglecting the true (second) term in Equations (1) and (8). The capability of the theory is demonstrated in Figure 2, which shows the predicted and measured response of consolidated undrained triaxial compression tests on kaolinite (Brewer, 1972). In this case it was possible to express Equations (13), (14), and (15) in terms of only three material dependent parameters (a_3, β_2, and c_1); the parameters listed in Table 2 are the same for all three tests.

Table 2. Constant Parameters for Examples Illustrated in Figure 2

a_1	a_2	a_4	β_1	c_2	c_3	c_4	c_5
4	500	0.75	5	2500	0.25	1000	9000

The complete constitutive relationship was expressed with a total of five material parameters, including Z_1 (Equation 1) and the initial shear modulus, G_o. The values of the material parameters given in Table 3 were determined mathematically by using a nonlinear least squares algorithm to obtain the best fits.

Table 3. Material Parameters for Examples Illustrated in Figure 2

Consolidation Stress (kN/m^2)	G_o/P_A	Z_1	a_3	β_2	c_1
400	120	0.0232	79.5	30.3	1.0
200	70	0.0131	138	10.3	0.637
100	34	0.0196	257	3.81	0.184

Rate-Dependent Cyclic Behavior

Brewer (1972) observed that (a) there was no phase lag between stress and deformation, (b) the measured deformation was only a certain percentage of the total deformation corresponding to a specified stress level, and (c) this percentage decreased with increasing strain rate or frequency. The influence of the rate dependence of the deformation (or microstructure) is reflected in the pore pressure response, and, depending on the type of clay and the void ratio, there appears to be a critical zone of strain rates over which the rate dependence starts to play a significant role. This phenomenon can best be observed in the pore pressure response under various strain rates. In the case of applied strain rates that are lower than critical, the pore pressure increases quickly in the first couple of cycles and approaches an asymptotic value, as shown in Figure 1. If the strain rates are faster than critical, the pore pressure increases almost linearly with each cycle, as shown in Figure 3.

The introduction of two material parameters, τ_1 and τ_2, in Equations (1) and (8) is sufficient to account for rate dependent

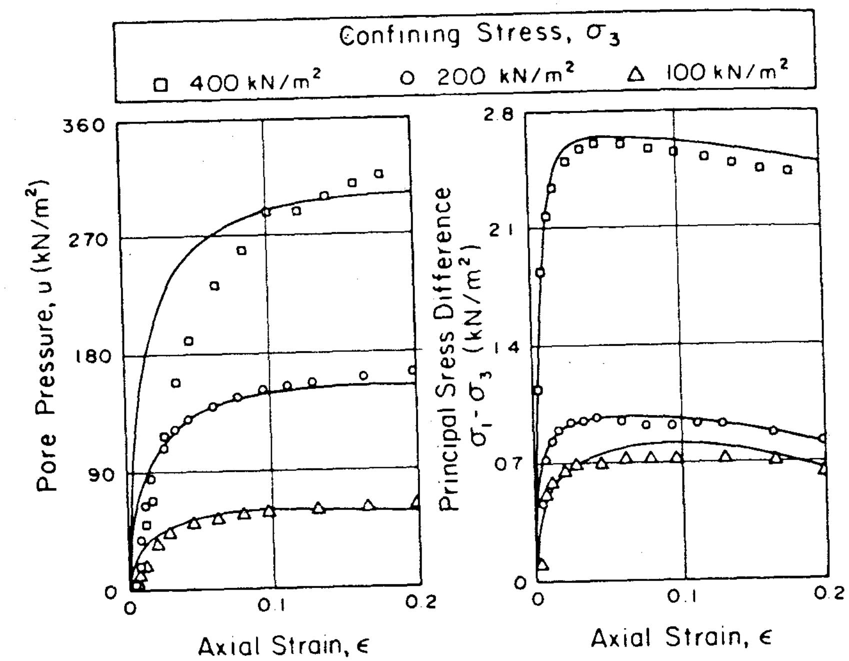

Figure 2. Stress-Strain-Pore Pressure Data for Kaolinite (after Brewer, 1972) with Theoretical Response (solid curves)

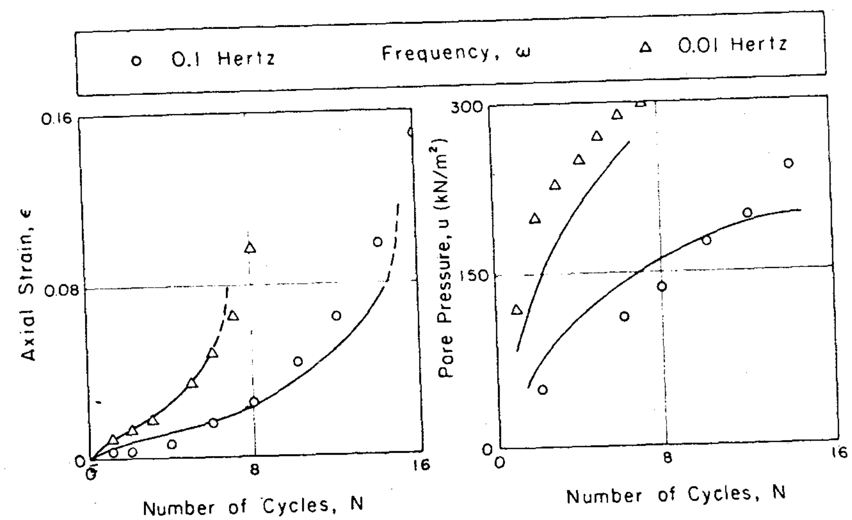

Figure 3. Cyclic Constant Load Amplitude Triaxial Data for Kaolinite (after Brewer, 1972) with Theoretical Response (solid curves)

behavior and thereby achieve a realistic model. The proposed constitutive relationship was applied to model the results from two cyclic triaxial tests conducted at different frequencies (0.01 and 0.1 Hertz) on kaolinite specimens (Brewer, 1972), and the results are shown in Figure 3. Both specimens were slurry consolidated, rebounded, and then reconsolidated in a triaxial chamber to the same effective stress (400 kN/m^2). The theoretical curves shown in Figure 3 were obtained by using the same intrinsic relations given by Equations (13), (14), and (15). As before, the limited amount of data dictated that a trial-and-error procedure (rather than mathematical optimization) be used to determine the necessary parameters, which are given in Table 4; the values of the parameters not listed in Table 4 were the same as those determined for the quasi-static tests. Ideally one set of parameters should be sufficient to model the response for both static and cyclic tests; however, at this stage of development it appears necessary to vary some of the parameters.

Table 4. Material Parameters for Examples Illustrated in Figure 3

Frequency (Hertz)	a_3	β_1	β_2	c_1	c_4	c_5	Z_1	τ_1	τ_2
0.1	1200	0.01	35	0.1	10,000	0.1	0.0213	900	30,500
0.01	20,000	0.03	400	0.1	10,000	8000	0.0800	110	26,000

SUMMARY AND CONCLUSIONS

An endochronic constitutive relationship was applied to describe the response of cohesive soils under cyclic loads. Data from rate-independent and rate-dependent cyclic tests, as well as quasi-static compression tests, were modelled, and the intrinsic relationships were cast in exactly the same form for all cases; however, a few of the coefficients in some cases had to be considered as variables that depend on the type of clay and strain rate.

The proposed constitutive law with the indicated extension to account for rate dependence offers signficant improvements over constitutive models reported in the literature; in particular, strain softening and hardening, frictional aspects, densification and dilatancy, and different strain rates and loading patterns can be handled in a more fundamental manner. The intrinsic relations developed on the basis of limited test data involve five material parameters for the quasi-static case and ten material parameters, including the initial elastic modulus, for the cyclic case. However, the scarcity of data and the inherent shortcomings of cyclic testing procedures preclude the establishment of specific correlations for these material parameters. Nevertheless, endochronic theory has been demonstrated to provide a powerful means for interpreting and analyzing the seismic behavior of cohesive soils.

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