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Keynote Lecture

SIZE EFFECT IN QUASIBRITTLE FRACTURE: SOME RECENT ADVANCES

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SUMMARY

Scaling, that is, the change of response due to similarity preserving changes of the size of a physical system, is the most fundamental aspect of every physical theory. If the scaling is not understood, the phenomenon itself is not understood.

The question of scaling of the failure of structures was studied already in the 16th and 17th centuries by Leonardo da Vinci and Galileo Galilei and the size effect in the failure of ropes was qualitatively explained in terms of strength randomness already in the 17th century by Mariotte. In the 20th century, while the problem of scaling has played a central role in fluid mechanics, it has been largely neglected in solid mechanics until about 1980. The reason is that the theories of structural failure that have prevailed for a long time exhibit no deterministic size effect. These are: (1) plasticity and other theories based on the concept of critical stress (strength) or critical strain, and (2) fracture mechanics applied to a critical flaw (crack) whose size at incipient failure is independent of the structure size D and negligible in comparison to D, as is typical of most metal structures embrittled by fatigue.

Therefore, the experimentally observed size effects were generally attributed to the randomness of material strength, as mathematically described in 1938 by Weibull, and their study was relegated to the probabilists. However, even though this explanation is realistic for metallic and other structures that fail at fracture initiation, before the crack reaches macroscopic dimensions, it does not work for quasibrittle structures.

Energetic Size Effect

The present keynote lecture deals with structures made of quasibrittle materials, such as concrete, rock, sea ice, toughened ceramics an composites. As a result of their heterogeneity and development of a large fracture process zone, these materials typically fail only after a large crack has grown in a stable manner.

The size effect is understood to be the change of σ_N as a function of D in geometrically similar structures with similar cracks (the effect of deviation of cracks from similarity is an effect of shape, which must be described separately from the size effect). The nominal strength σ_N of such structures exhibits a complex size effect, which has been explained by the release of stored release energy caused by fracture. While the energy dissipated in geometrically similar structures with similar cracks is proportional to the crack length and thus to the structure size D, the energy release caused by fracture in structures under the same nominal stress grows with D faster than proportionally—hence the size effect. But because of the large size of the fracture process zone, which releases additional energy in proportion to the fracture length, the overall energy release grows with D less than quadratically. This causes the size effect to deviate from the power law size effect ($\sigma \propto D^{-1/2}$) of linear elastic fracture mechanics (LEFM).

The lecture outlines a general asymptotic theory of scaling governing the quasibrittle size effect [1]. The energy release from the structure is assumed to depend on its size D, on the crack length, and on the material length c_f governing the fracture process zone size. Based on the condition of energy balance during fracture propagation and the condition of stability limit under load control, the large-size and small-size asymptotic expansions of the size effect on the nominal strength of structure containing large cracks or notches are derived. It is shown that the form of the approximate size effect law previously deduced by simpler energetic arguments can be obtained from these expansions by asymptotic matching (Fig. 1). This law represents a smooth transition from the case of no size effect, corresponding to plasticity, to the power law size effect of linear elastic fracture mechanics. Fig. 2 compares this law to some recent test data.

The analysis of size effect is then extended to deduce the asymptotic expansion of the size effect for crack initiation in the boundary layer from a smooth surface of structure. Furthermore, a universal size effect law which approximately describes both failures at large cracks (or notches) and failures at crack initiation from a smooth surface is derived by matching of the aforementioned three asymptotic expansions (Fig. 3).

Furthermore, the analysis is generalized to the case that a ductile failure mechanism operates simultaneously with crack propagation. A new approximate formula utilizing LEFM energy release functions is derived for the size effect arising when a residual cohesive stress is transmitted between the opposite crack surfaces. The logarithmic plot of this size effect (nominal strength versus size) exhibits a positive curvature for larger sizes, whose presence in test data was recently emphasized by Carpinteri.

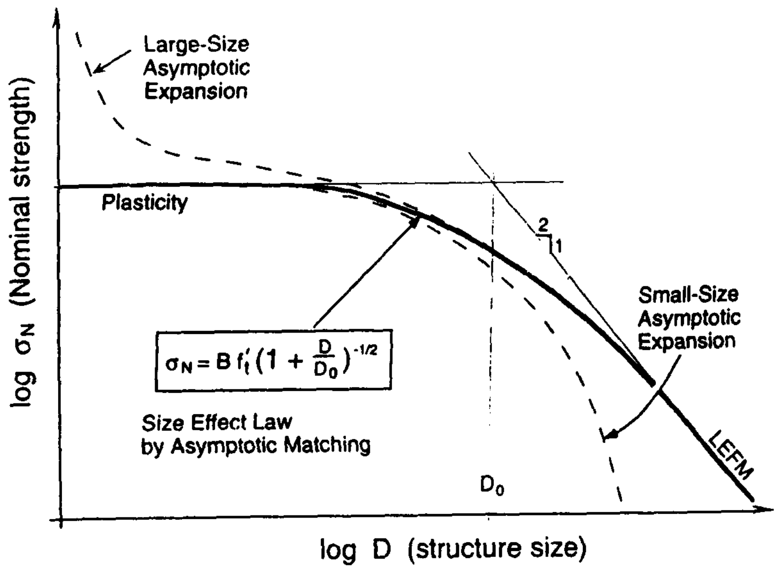

Figure 1: Size effect law based on energy release analysis (solid curve, Bažant 1984) as the asymptotic matching of the large-size and small-size asymptotic series expansions of size effect (dashed curves); B, f_t', D_0 = constants.

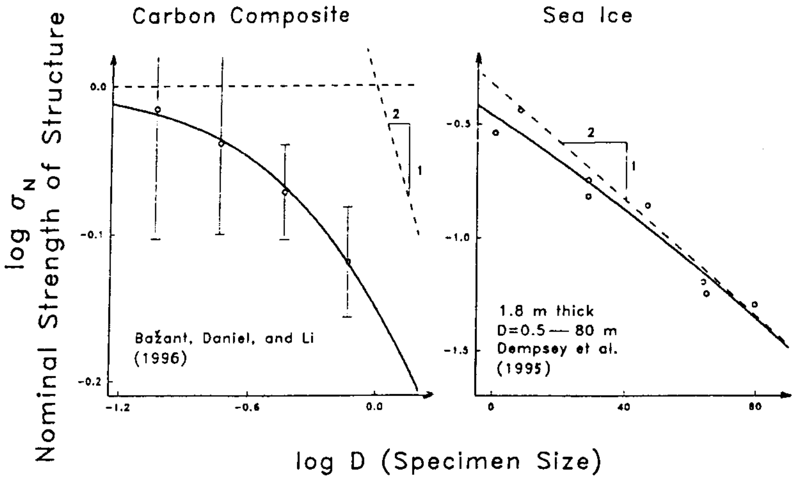

Figure 2: Size effects observed in fracture of carbon-epoxy fiber composites (Bažant, Daniel and Li, [2]) and of sea ice (tests of J. Dempsey's team [3] near Resolute in the Arctic Ocean, in which floating notched square specimens of thickness 1.8 m and sides ranging from 0.5 m to 80 m were broken in a computer-controlled manner), and comparison with the curves of the size effect law proposed by Bažant.

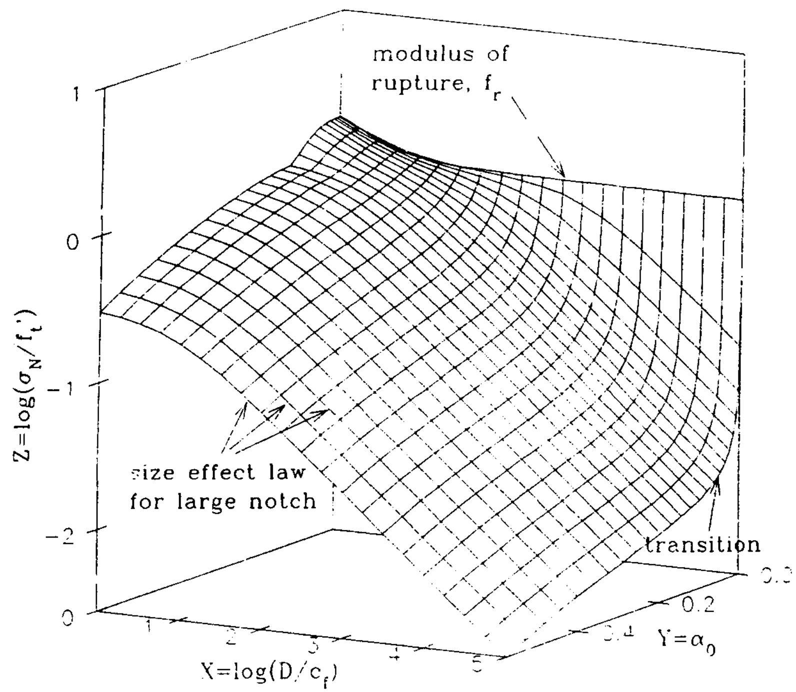

Figure 3: Universal size effect law for quasibrittle failures [1], having correct asymptotic behaviors for large and small sizes D and for large or zero relative length $\alpha_0 = a_0/D$ of traction-free crack at maximum load (f_t' = tensile strength of the material, c_f = effective length of the fracture process zone).

The Question of Size Effect Caused by Fracture Fractality

Recently an alternative suggestion has been made by A. Carpinteri—namely that the cause of the observed size effect on nominal strength of concrete structures might be the fractal nature of crack surface or microcrack distribution.

It is of course true that recent observations by many researchers have demonstrated that, within a certain range of scales, the fracture surfaces in many materials, especially brittle heterogeneous materials such as rock, concrete, ice, tough ceramics and various composites, exhibit partly fractal characteristics. Considerable advances in the study of the fractal aspects of crack morphology and energy dissipation by fractal cracks have been made by Mandelbrot, Brown, Mecholsky, Mackin, Cahn, Hornbogen, Peng and Tian, Saouma, Bouchaud, Chelidze and Gueguen, Issa, Long, Måløy, Mosolov and Borodich, Borodich, Lange, Xie, Carpinteri, Feng and others. A correlation between the fractal dimension of the crack surface (observed over a limited range of scales) and the fracture energy or toughness of some brittle materials has been detected. However, the connection between the fractal nature of cracks on the microscale and the scaling law on the macroscale has so far been based merely on intuitive analogy and geometric arguments. It has not been solidly established in terms of mechanics.

The lecture proceeds to outline an analysis of this connection on the basis global energy balance and asymptotic matching, generalizing the nonfractal analysis. The modifications of the scaling law caused by invasive fractality of the crack surface are derived, both for quasibrittle failures after large stable crack growth and for failures at the initiation of a fractal crack in the boundary layer near the surface. Subsequently, attention is focused on the hypothesis that lacunar fractal characteristics of the distribution of microcracks cause the size effect. This hypothesis is shown to lead to an analogy with Weibull's statistical theory of size effect due to material strength randomness [4] (Fig. 4).

The predictions ensuing from the fractal hypothesis, either invasive or lacunar, are shown to disagree with certain experimentally confirmed asymptotic characteristics of the size effect in quasibrittle structures. It is also pointed out that considering the crack curve to be a self-similar fractal (such as the von Koch curve) conflicts with the kinematics of crack opening. This can be remedied by considering the crack to be an affine fractal.

It is concluded that the fractal characteristics of either the fracture surface or the microcracking at the fracture front cannot have a significant influence on the law of scaling of failure loads, although they can affect the fracture characteristics such as the fracture energy value, and may have to be taken into account in micromechanical prediction of fracture energy.

Scaling of Compression Fracture

Furthermore, the lecture outlines a simplified fracture-mechanics-based model of compression failure of centrically or eccentrically loaded quasibrittle columns [5], which can predict the size effect on the nominal strength of a column. Failure is modeled as lateral propagation of a band of axial splitting cracks, in a direction orthogonal

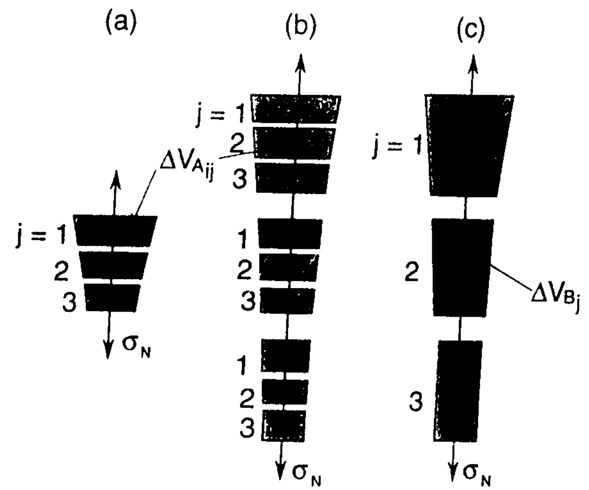

Figure 4: Idea behind consistent Weibull-type analysis of the hypothesis of size effect due to lacunar fractality of microcracks: Although small and large structures may be subdivided into small and large elements exhibiting different fractal dimensions (a and c), the use of the same material also requires that the large elements of the large structure be further subdivided into the small elements exhibiting the same fractal dimension as in the small structure (b) [4].

or inclined with respect to the column axis. The maximum load is calculated from the condition that the energy released from the column due to crack band advance be equal to the energy consumed by the splitting cracks. The axial stress transmitted across the crack band is determined as the critical stress for buckling of the microslabs of material between the axial splitting cracks, and the work on the microslabs during postbuckling deflections is taken into account. The critical postbuckling deflection of the microslabs is determined from a compatibility condition. Under the assumption of small enough material inhomogeneities, the spacing s of the splitting cracks is calculated by minimizing the failure load and is found to decrease with structure size D as $D^{-1/5}$.

The size effect on the nominal strength of geometrically similar columns is found to disappear asymptotically for small sizes D, and to asymptotically approach the power law $D^{-2/5}$ for large sizes D (where D = cross section dimension). However, when the material inhomogeneities are so large that they preclude the decrease of s with increasing D, the asymptotic size effect changes to $D^{-1/2}$. The size effect intensifies with increasing slenderness of the column, which is explained by the fact that a more slender column stores more strain energy. The predicted size effect describes quite well previous tests at Northwestern University of reduced-scale tied reinforced concrete columns of different sizes (with size range 1:4) and different slendernesses (ranging from 19 to 53); Fig. 5.

Application to truss model for shear failures of concrete

Finally, the lecture outlines an application of the aforementioned approximate theory of the scaling of compression fracture. The classical truss model for shear failure of reinforced concrete beams (also called the strut-and-tie model) is modified to describe fracture phenomena during failure [6].

The failure is assumed to be caused by progressive compression crushing in the concrete strut during the portion of the loading history in which the maximum load is reached. The crushing is assumed to occur within a crushing band propagating across the strut. The width of this band is assumed to occupy only a portion of the strut length and to represent a fixed material property independent of the beam depth—features that inevitably lead to size effect (if the width of the compression crushing zone were proportional to the beam depth, there would be no deterministic size effect) (Fig. 6).

The energy release from the truss is calculated using two alternative approximate methods: (1) according to the potential energy change deduced from the concept of stress relief zones, and (2) according to the complementary energy change due to stress redistribution caused by propagation of the crushing band across the compressed concrete strut. Both approaches show that a size effect on the nominal strength of shear failure must exist and that it should approximately follow the size effect law proposed by Bažant in 1984.

The physical mechanism of the size effect is explained in a clear and simple manner. Further it is shown that the applied nominal shear stress that causes large initial

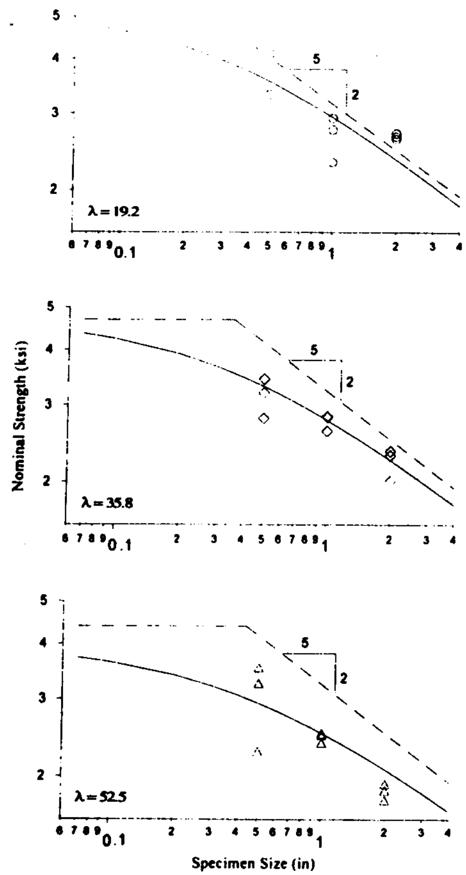

Figure 5: Data points obtained at Northwestern University by Bažant and Kwon in reduced-scale tests of tied reinforced concrete columns of different sizes and slendernesses λ, and comparison with the curves of size effect predicted by energy analysis of compression fracture [5].