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Size Effect in Concrete Structures: Nuisance or Necessity?

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Dedicated to Hans W. Reinhardt at his 60th birthday

Summary

The lecture reviews the case for incorporating size effect into the code provisions of concrete structures and computational evaluation. After commenting on the long history of the problem beginning in the Renaissance, some recent major structural catastrophes in which the size effect must have played a role are discussed. The mechanism of the energetic size effect is explained and numerous formulae for the size effect that have recently been derived for different purposes from various theories or experimental evidence are reviewed. The possibility of extending the strut-and-tie (truss) models in a way that captures the size effect is emphasized. Although the size effect might be seen as a nuisance, spoiling the beauty of the theory of limit states, its incorporation into the codes cannot be avoided. It is a necessity.

Introduction

The size effect is the dependence of the nominal strength of the structure, σ_N, on the characteristic size (dimension) D of the structure when geometrically similar structures (with similar loading and similar failure modes) are compared. For three-dimensional similarity $\sigma_N = P/D^2$, and for two-dimensional similarity $\sigma_N = P/bD$ where P = load capacity and b = structure thickness in the third dimension. Elastic analysis, as well as plastic limit analysis according to the limit state concept, exhibits no size effect. However, when a significant range of sizes is considered, tests of brittle failures of concrete structures as well as structures made of all quasibrittle materials reveal a significant size effect [1,2]. Although this is not a great concern for ordinary structures, the size effect needs to be introduced into the design procedures when sensitive or large structures, or new types of structural systems, are considered. For such purposes, updates of the existing design codes are inevitable.

Interest in the size effect is older than mechanics of materials itself. Leonardo da Vinci [3] suggested that the strength of ropes is inversely proportional to their length. Such excessive size effect was rejected by Galileo [4]. Later in the 17th century Mariotte [5] advanced in qualitative terms the basic idea of the statistical size effect due to strength randomness, namely the fact that the probability to encounter in a structure an element of a certain low strength increases with the structure size. A mathematical formulation of this idea had to await the development of the statistical weakest link model by Fisher, Tippet and Fréchet in the 1920's, and the discovery of the proper extreme value distribution by Weibull in 1939 [6]. Since Weibull until recently, the size effect, if observed experimentally, was generally considered to be statistical, something to be relegated to statisticians and buried in the safety factors. In the mid 1970's however, researches at Lund [8,9] and Northwestern University [10-12] revealed that a large deterministic size effect in quasibrittle structures is caused by stress redistribution, strain localization, and the consequent energy release associated with large fractures or large cracking zones developing before the maximum load.

The theory of the energetic size effect has gone through rapid development and its basic aspects are today well understood [1,2]. Extensive experimental data have recently been accumulated in reduced-scale laboratory testing, and some large-scale tests of real structures have been carried out. A positive development in this decade has been that the need for introducing size effect into the design practice is no longer generally dismissed but taken seriously. Various size effect provisions are appearing in the codes of various countries.

Even though extensive experimental evidence for large structures is still lacking, it is now clear that it is imprudent to omit the size effect from the design of large or innovative structures.

As for the classical statistical size effect, it was shown to be minor and usually negligible whenever large fractures or large cracking zones develop prior to the maximum load. Still other explanations of the observed size effect have been recently proposed, particularly the fractal nature of crack surfaces and microcrack distributions, or the width and spacing of cracks. However, there are good reasons to conclude that such hypothetical mechanisms do not play a significant role [1,2,7].

A New Look at Structural Catastrophes in the Past

Since it is either too expensive or outright impossible to test large structures to failure, lessons regarding the size effect should be drawn from structural catastrophes that happened in the past. These are, for example, the following: (1) the sinking of Sleipner A oil platform in 1991 [13]; (2) the toppling of the Han-Shin freeway viaduct in Kobe earthquake in 1995; (3) the collapse of Cypress Viaduct on Nimitz Freeway, in Oakland, in Loma Prieta earthquake in 1989 [14]; (4) the collapse of St. Francis dam near Los Angeles in 1928 [15]; (5) the collapse of Malpasset arch dam in French Maritime Alps in 1954 [14]; and (6) the collapse of Schoharie Creek Bridge on New York Thruway in 1987. Although, with the exception of Schoharie, the investigating committees did not list the size effect among the causes, from today's perspective it is clear that it must have been a significant additional contributing factor.

Causes of Size Effect

The cause of size effect may be explained, without any calculations, by considering the mechanism of diagonal shear failure of reinforced concrete beams without or with stirrups (Fig. 1a). Prior to the maximum load, a long "shear" crack caused by diagonal tension develops, while significant compression stresses are transmitted along a so-called "compression strut" running parallel to the crack. For failure to occur, this compression strut must be crushed. Therefore, a crushing band (*B* in Fig. 1a), in which concrete undergoes axial splitting fractures, must propagate across the strut. If the area 12341 of this band covered the entire length of the strut (i.e., if the strut were failing simultaneously all along its length), there would be no size effect, and neither would if the width h of the band in the strut direction were proportional to the beam depth. But neither is the case.

The crushing localizes into the narrowest band possible, of a width equal to several maximum aggregate sizes, and so the width h of the band in the direction of the strut is approximately constant, independent of the beam size. Since the energy dissipation per unit area is also approximately constant, independent of beam depth d, the dissipation of energy in the crushing band is proportional to length c of the band, which in turn is approximately proportional to the beam depth d because the failures of small and large beams are known to be similar. Thus, the energy dissipated in the band is approximately proportional to beam depth d.

This energy must be supplied by a release of strain energy from the beam. The crushing band causes stress reduction in a strip of width c running all along the compression strut (area 67856 in Fig. 1a left, or 12561 in Fig. 1a right). The inclination of the strut being similar for various sizes, the area of this strip is proportional to cd, and since c is also proportional to d, this area is proportional to d^2. So the energy release is proportional to $(\sigma_N{}^2/E)\, d^2$. To preserve energy balance, this must be proportional to d, characterizing the energy consumed. Evidently, this is possible only if σ_N is proportional to $d^{-1/2}$.

Thus, the source of the energetic size effect is simply the fact that the energy consumed increases with size d linearly, while the energy released increases quadratically. This mismatch must be offset by a decrease of σ_N with increasing size. With this simple argument one can immediately realize, without any calculations, that there indeed must be a size effect, and that its source is energy release. This size effect would be avoided only if the failure occurred at the very inception of cracking (at which both the energy released and energy consumed are negligible). But ample experimental evidence shows that this is not the case.

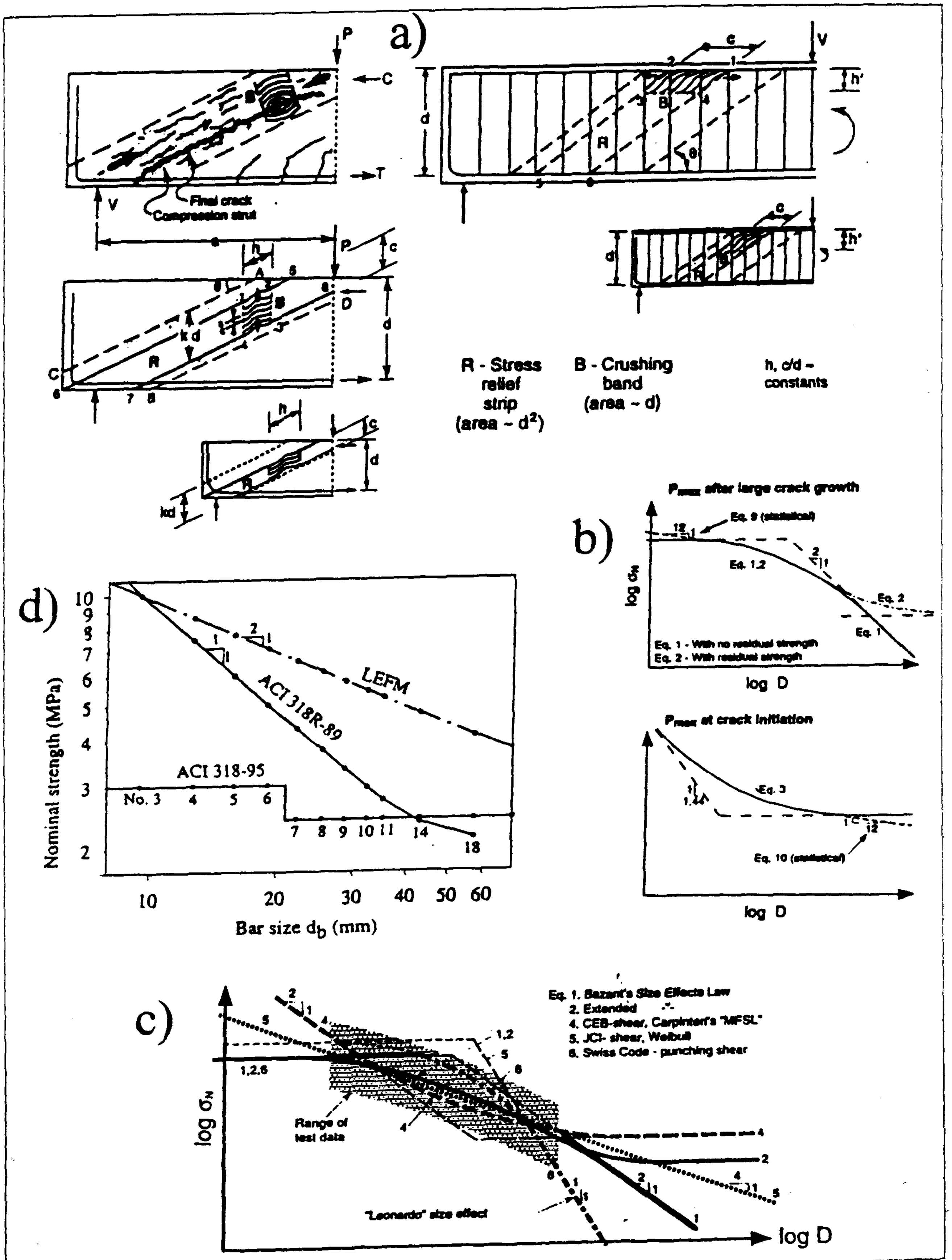

Fig. 1 a) Zones of energy and dissipation in a "compression strut".
b) Laws size effect for P_{max} (a) after large crack growth, (b) at crack initation.
c) Size effect curves based on various hypotheses or assumed empirically.
d) Simple size effects implied by two subsequent ACI specifications for splices.

$$\sigma_N = \frac{\sigma_0}{\sqrt{1+(D/D_0)}} \quad \text{(large crack) [1,2,12] where } D_0 = c_f\frac{g'}{g},\ \sigma_0 = \sqrt{\frac{EG_f}{g'c_f}} \quad (1)$$

$$\sigma_N = \frac{\sigma_0 + \sigma_R\sqrt{1+(D/D_1)}}{\sqrt{1+(D/D_0)}} \approx \frac{\sigma_0}{\sqrt{1+(D/D_0)}} + \sigma_R \quad [1,2] \text{ if } D_1 \approx D_0 \quad (2)$$

$$\text{where } D_1 = c_f\frac{\gamma'}{\gamma},\quad \sigma_R = \sigma_r\sqrt{\frac{\gamma'}{\gamma}}$$

$$\sigma_N = \sigma_0\left(1+\frac{rD_b}{D}\right)^{1/r} \quad \text{(crack initiation) [17] where } D_b = \frac{\langle -g''\rangle c_f}{4g'} \quad (3)$$

$$\sigma_N = \sqrt{A_1 + \frac{A_2}{D}} \quad \text{(Carpinteri's "MFSL", CEB—diag. shear)} \quad (4)$$

$$\text{where } A_1 = \frac{EG_f}{g'c_f},\quad A_2 = \frac{\langle -g''\rangle EG_f}{2g'^2} \quad \text{(crack initiation) [16]}$$

$$\sigma_N = C\,D^{-n/m} \quad \text{(Weibull, } n/m \approx \tfrac{1}{12}\text{. But JCI: } n/m = \tfrac{1}{4} \text{ for diag. shear)} \quad (5)$$

$$\sigma_N = \text{Min}\left(\frac{\sigma_0}{1+(D/D_0)},\ \tau_c\right) \quad \left\{\begin{array}{l}\text{Swiss Code for punching shear [18]}\\ \text{("Leonardo" asymptote)}\end{array}\right. \quad (6)$$

$$\sigma_N = C\,D^{-1/2} \quad \text{(LEFM, German and ACI codes for anchor pullout)} \quad (7)$$

$$\sigma_N = \text{Graph in Fig. 4 (ACI 318R-89 for splices, "Leonardo" size effect) [3]} \quad (8)$$

$$\sigma_N = \frac{\sigma_0}{\sqrt{(D/D_0)^{n/m} + (D/D_0)}} \quad \text{(large crack, statistical) [1, 19]} \quad (9)$$

$$\sigma_N = \sigma_0(D_b/D)^{n/m}\left[1 + r(D/D_b)^{1-rn/m}\right]^{1/r} \quad \text{(crack initiation, statistical)} \quad (10)$$

$$\sigma_N = \sigma_0\left(1+\frac{D}{D_0}\right)^{-1/2}\left(1+\frac{D}{D_{s0}}\right)^{-1/4} \quad \text{(composite beams, studs scaled) [20]} \quad (11)$$

$$\sigma_N = C\,D^{-2/5} + \sigma_0 \quad \text{(compression fracture, large scale) [1]} \quad (12)$$

$$\sigma_N = C'\,D^{-3/8} + \sigma_0 \quad \text{(thermal bending of sea ice plate) [1]} \quad (13)$$

$$\sigma_N = \sigma_0 D^{(\delta-1)/2}[1 + (D/D_0)]^{-1/2} \quad \text{(fractal, large crack) [2]} \quad (14)$$

$$\sigma_N = \sigma_0 D^{(\delta-1)/2}[1 + r(D_b/D)]^{1/r} \quad \text{(fractal, crack initiation) [2]} \quad (15)$$

Tab. 1 Size effect formulae obtained from different theories or from experience, used for different purposes, some incorporated in codes.

Size Effect Formulae

While until almost the end of the 1980's, no size effect provisions were present in the codes, a number of them have been introduced for various types of failures in the codes of various countries. This is a healthy trend, however, what is striking is the variety of formulae and the underlying theories. A point to be noted in this regard is that the energetic size effect is inevitably present if the failure does not occur at the initiation of cracking. This means that if some other theory is assumed it could only come on top of the energetic theory, but not without it, not as a replacement.

Most of the existing formulae for size effect are listed in Table 1 in which D is the characteristic structures size; E, G_f, c_f, m, σ_r, r, d are material constants; σ_0, D_0, D_1, σ_R, D_b, A_1, A_2, n, C, τ_c, D_{50} are structural constants depending on geometry; $g = g(\alpha)$ = energy release function of relative crack length . based on linear elastic fracture mechanics (LEFM), and $g' = dg/d\alpha$. Eqs. 1, 2 and 3 are based on the energetic theory. Eq. 4 (curve 4 in Fig. 1c), called by Carpinteri et al. [16] the MFSL ("multi-fractal" scaling law), can also be justified by the energetic theory [17] (being a special case of Eq. 3), although it was originally proposed on the basis of geometrical (non-mechanical) arguments relying on fractal aspects of fracture geometry (the partly fractal nature of crack surfaces and microcrack distributions in concrete is not questioned, only its role in the mechanics of size effect is). Unlike the fractal hypothesis, the energy release analysis provides the geometry dependence of the coefficients of MFSL (Eq. 4). The fracture mechanics expressions in Table 1 using LEFM functions of g and g' are useful only if the effective fracture situation of a very large structure at maximum load is known, which is a difficult problem.

Eq. 1 (Fig. 1b left, curve 1 in Fig. 1c) is the original simple size effect law derived by Bažant [12], applicable to brittle failures occurring after large stable crack growth, which is typical of reinforced concrete. Eq. 2 is its modification applicable when there is a significant residual stress σ_r transmitted across the cracking band, which is important for compression fracture. Eq. 3 (curve 3 in Fig. 1c) is applicable to failures occurring at fracture initiation, which is typical of plain concrete and is exemplified by the modulus of rupture test. Eqs. 1-4 have been justified in a number of ways – by simple analysis of energy release zones, by asymptotic expansions of J-integral, by equivalent LEFM analysis based on asymptotic matching, and by numerical simulations with nonlocal finite elements or with discrete elements (random particle method). They have been verified experimentally for many types of brittle failures, including diagonal shear of beams, punching shear, torsion, bar pullout, anchor pullout, splice failure, slender column failure, and failure of steel concrete composite beams due to failure of connectors.

Eq. 4 (curve 4 in Fig. 1c) for MFSL is a special case of Eq. 3 for $r = 2$, however, the value $r = 1.44$ has been found optimal by comparisons with many test data on the modulus of rupture (in collaboration with Drahoslav Nov▬k, Brno). Fracture analysis indicates Eq. 4 to be applicable to failure at crack initiation, yet this equation has been proposed for the diagonal shear failure even though this failure occurs after large fracture growth (German and European codes).

Formulae of the type of Eq. 1 and 2 are proposed as size effect factors to be incorporated into the code formulae for most types of brittle failure (diagonal shear, torsion, punching shear, anchor pullout, bar pullout, splice failure, stud failure in composite beams and failure of slender columns).

Eq. 5 (curve 5 in Fig. 1c) represents the size effect obtained from Weibull statistical theory; m = Weibull modulus of the material (widely considered as 12, but better taken as 24 according to the latest studies), and n = 2 or 3 for two- or three- dimensional similarity. This formula is in theory applicable only when the failure occurs at fracture initiation. However, based on the results of the largest-scale tests so far, conducted at Shimizu Corp. in Japan, this formula was introduced in JCI for the diagonal shear failure of beams, with the value $m/n = 1/4$. Another weakness of Eq. 5 is that a power law implies the structure to possess no characteristic dimension (complete self-similarity), yet a characteristic dimension must exist due to the size of aggregate as well as the spacing and size of reinforcing bars.

Eq. 6 (curve 6 in Fig. 1c) is an interesting formula introduced into Swiss code SIA 162 to describe the size effect in punching shear; σ_0, D_0 and τ_c = constants. The formula was based strictly on test results, but its form is theoretically objectionable. For sufficiently large sizes D it gives an impossibly strong size effect. It approaches the "Leonardo" size effect [3], namely σ_N being inversely proportional to D, which is thermodynamically impossible. Nevertheless, the Swiss code deserves praise for being the first to accept that there indeed is a strong size effect in punching shear.

Eq. 7 (asymptote of curve 1 in Fig. 1c) represents the size effect of LEFM, which was introduced into German and ACI Code Recommendations, based mainly on the tests of Eligehausen. This is a strong size effect which is excessive for anchors that are small, but such anchors might not be of great concern.

There are other provisions in various codes which imply a size effect although this is not stated explicitly. For example, the code ACI 318 R-89 implied a huge size effect for the failure of splices, shown graphically Fig. 1d. This was in fact the "Leonardo" size effect of slope 1, which is thermodynamically impossible. Fig. 1d shows for comparison also the size effect in ACI 318 R-95, in which the discontinuous jump is objectionable (note that these diagrams are plotted assuming the cover thickness to be proportional to the bar diameter, or else one could not speak of size effect). The enormous sudden change between the two ACI plots in Fig. 1d, a "U-turn" made in absence of any new revolutionary finding, is striking.

Eqs. 9 and 10 represent statistical generalizations of Eqs. 1 and 3. However, the additional Weibull-type statistical effect, given by the terms with exponents n/m, is very small. The fit of size effect test data with these formulae is not any better than that with the deterministic ones.

Eq. 11 represents the size effect in steel-concrete composite beams that fail due to shear failures of studs. The studs are not all failing simultaneously; rather their failures propagate along the beam, which is a behavior similar to crack propagation. In this problem there are two size effects superimposed on each other: (1) the size effect in stud failure, and (2) another size effect due to propagation of the stud failures through the steel concrete interface in the beam as a whole. Due to combination of these two, the compound size effect given by Eq. 11 can be stronger than in LEFM [20], provided that the studs are scaled with the beam. If they are not, then the size effect given by Eq. 1 applies.

Eqs. 12 and 13 represent special size effects applicable to compression fracture of concrete propagating laterally to the direction of the axial splitting cracks, and to thermal bending fracture of floating sea ice plate. Finally, Eqs. 14 and 15 represent the size effects derived on the basis of energy balance under the assumption that the crack surface or the microcrack distribution has a fractal geometry and that the fracture energy can be treated as fractal [25, 2]. They differ from Eq. 4, which was derived strictly geometrically, without any mechanical analysis. Still another size effect is obtained by a numerical calculation proposed by M. Collins for the diagonal shear failure (see Ref. [17]).

Among the formulae listed, Eqs. 1-3 have the strongest theoretical and experimental support and appear to be appropriate for the design code. A difficult problem is a theoretical prediction of geometry dependence of constants D_0 and σ_0. Formulae for this purpose need to be worked out for many cases. In absence of a theoretical prediction model, these coefficients can be determined empirically for each type of failure.

The size effects given by Eqs. 1, 2, 4, 5 and 6 are plotted in Fig. 1c. The difficulty in deciding which formula is appropriate is that the scatter of the existing data is too wide for the range of sizes tested. If the decision between various formulas should be made strictly theoretically, it would be necessary to greatly extend the test data into larger size ranges, and obtain a statistically significant number of test results for geometrically scaled structures. Unfortunately, most large-scale tests have been conducted in the past on structures that were not geometrically scaled (e.g., the bar sizes, cover thickness and bar spacing were not geometrically similar). The effect of the changes of shape (geometry) are known only crudely and introduce additional errors. Therefore, formulae that have the strongest theoretical support ought to be preferred. The theory itself may be verified by checks other than extension of the size range of the tests of the given particular failure. Unless such an approach is adopted, the choice between the formulae will be random, depending solely on voting of committee members.

Energetic Modification of Truss Model (Strut-and-Tie Model)

The strut-and-tie model, in which the action of reinforced concrete at maximum load is approximated by a statically determinate truss and the load capacity is calculated from equilibrium and compatibility at some assumed limit state, has gained enormous popularity and has unquestionably had considerable success. However, complacency has settled in as to its capability. The chief problem is that the compression struts exhibit strain softening. This causes the failure to localize and propagate. Thus the failure is not simultaneous as required by plastic limit analysis based on the limit state concept. The most important consequence of the progressive and localized nature of failure is the size effect, although this is not readily apparent from the existing comparisons with test data because of the lack of large-scale tests with proper geometrical scaling.

The strut-and-tie model does nevertheless capture well at least a part of the behavior of reinforced concrete at ultimate load, and therefore the model should not be scrapped but extended. The required extension has been worked out in detail for the diagonal shear failure [7], and can be applied in a similar way to all the other situations for which the strut-and-tie model has been used.

The equilibrium analysis of the strut-and-tie model can be retained, and so can the simplified concept of compression strut. The load capacity, however, needs to be calculated from energy balance during the propagation of a compression (or shear-compression) failure band (cracking zone, crushing zone) across the compression strut. From the forces determined by equilibrium analysis, the release of strain energy from the equivalent truss needs to be calculated and equated to the rate of energy consumed and dissipated by the failure band propagating across the strut. Such analysis inevitably provides a size effect, and the size effect generally has the form of Eq. 1 or 2 in Table 1 or Eq. 16 of Table 2, in which the structure size D is now represented by depth d to reinforcement: v_p, d_0 and v_r are structural constants taking into account the beam geometry – see Eqs. 17 and 18 of Table 2 in which c = width of the cracking band (crushing band) at maximum load, c/d = empirical constant, a = shear span, E_c = Young's modulus, G_f = fracture energy of the material, s_c = spacing of axial splitting cracks; h_0, w_0, s_c = constants; 2 = inclination of compression strut in a beam with stirrups, and v_r = residual strength calculated by plastic limit analysis from the residual compression strength of the strut (which might or might not vanish).

Fig. 2 shows the basic test data from the literature, as presented in [21].

$$\boxed{v_u = v_p[1 + (d/d_0)]^{-1/2}} \tag{17}$$

a) For beams without stirrups:

$$d_0 = w_0\frac{d}{c},\quad v_p = c_p K_c \left(\frac{a}{d} + \frac{d}{a}\right)^{-1},\quad K_c = \sqrt{EG_f},\quad c_p = k\sqrt{\frac{2h_0}{w_0 s_c}\frac{c/d}{a/d}} \tag{18}$$

b) For beams with stirrups:

$$d_0 = w_0\frac{d}{c},\qquad v_r = \frac{\sin 2\theta}{2}\sigma_r,\qquad v_p = K_c\sqrt{\frac{h_0}{2s_c w_0}\frac{c}{d}\sin 2\theta} \tag{19}$$

Tab. 2 Equations of the fracturing strut-and-tie model for diagonal shear failure of of reinforced concrete beams.

Nuisance or Necessity?

Until recently, the size effect was widely regarded as a nuisance foisted on the designers by some theoreticians. There were exceptions, though. Kani [22] in the mid 1960's carried out large-scale beam tests which clearly indicated the presence of size effect in shear of beams. Reinhardt [27] pointed out that the size effect may be due to fracture mechanics. Recent tests (e.g. [23]) clearly proved the existence of size effect for real size beams. A nuisance might be that the size effect formula cannot be established strictly experimentally because it is difficult to adhere to geometrical similarity in large-scale tests, and because large-scale experiments of a sufficient number to provide an adequate statistical basis are not in sight. This makes the use of a theory inevitable. The size effect cannot be avoided unless concrete could be made to behave perfectly plastically (which requires triaxial compression with the compressive principal stress of lowest magnitude exceeding the uniaxial compression strength [24]).

The size effect is a necessity that concrete designers must learn to live with.

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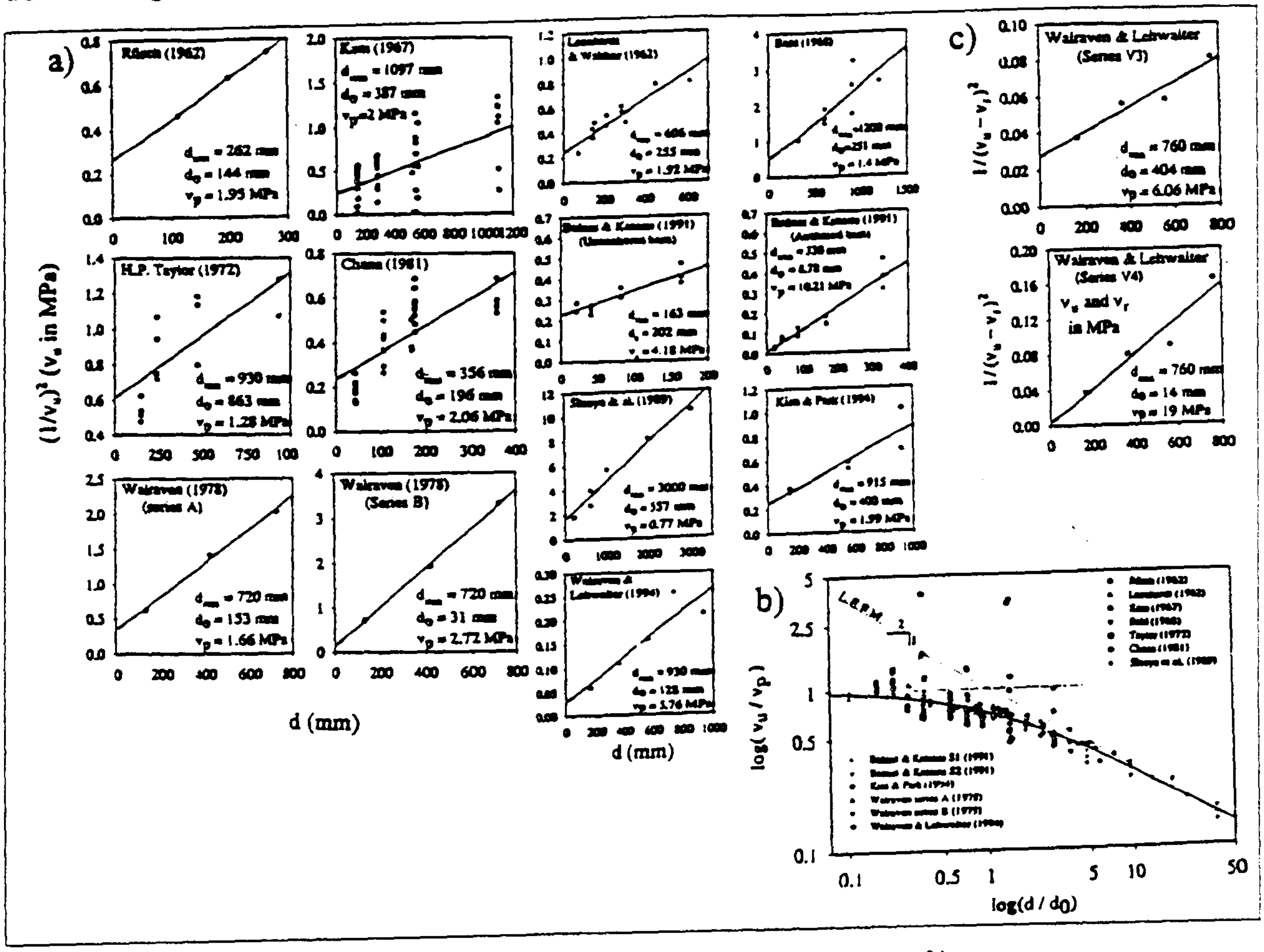

Fig. 2 Linear regression fits of various test data for diagonal shear failure of beams;
a), b) without stirrups, fit by Eq. 1,
c) with stirrups, fit by Eq. 2
(If no size effect existed, all these plots would have to be horizontal).

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