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Creep and Shrinkage Prediction Model for Analysis and Design of Concrete Structures: Model B3—Short Form

by Z. P. Bažant and S. Baweja

Synopsis:

A simple model for the characterization of concrete creep and shrinkage in design of concrete structures is proposed. It represents a shortened form of model B3 which was presented in [2] (as and improvement of the original version [3]) and appears in this volume, and an update of a previous short form [4]. The main simplification compared to model B3 comes from the use of the log-double-power law as the basic creep compliance function. The B3 formulae for predicting material parameters in the model are simplified by dropping the dependence of these parameters on the composition of concrete mix, leaving only dependence on the strength and the specific water content of the concrete mix. The model is justified by statistical comparisons with all the data in the internationally accepted RILEM data bank. The differences between the present short-form and model B3 are discussed and limitations of the short form are compared to model B3 are noted. The model is suitable for design of concrete structures with the exception of highly creep-sensitive structures for which the full model B3 is necessary

Keywords: concrete; creep; design model; prediction model; shrinkage; viscoelasticity

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APPLICABILITY RANGE

The prediction of the material parameters of the present model from strength and composition is restricted to Portland cement concrete of normal weight with the following parameter ranges:

$$\begin{array}{lll} 2500 \text{ psi} \le \bar{f}_c \le 10{,}000 \text{ psi}, & 10 \text{ lb/ft}^3 \le c \le 45 \text{ lbs/ft}^3 & \text{in.-lb. system} \\ 17 \text{ MPa} \le \bar{f}_c \le 70 \text{ MPa} & 160 \text{ kg/m}^3 \le c \le 720 \text{ kg/m}^3 & \text{S.I.} \end{array} \tag{1}$$

$$0.35 \le w/c \le 0.85 \quad 2.5 \le a/c \le 13.5 \tag{2}$$

(the numbers 0.85 and 45 lbs/ft.^{3} or 720 kg/m^3 are of course outside the range of good concretes in today's practice). The formulae are valid for concretes cured for at least one day. Formulas predicting model parameters from the composition of concrete have not been developed for special concretes containing various admixtures, pozzolans, microsilica, and fibers. However, if the model parameters are not predicted from concrete composition and strength but are calibrated by experimental data, the model can be applied even outside the range given by Eq. (1) and (2), for example, to high-strength concretes, fiber-reinforced concretes, and mortars. Useful information for such calibration is compiled in a parallel ACI 209 Subcommittee 5 report.

The average compliance function for the cross-section of a long member, representing the sum of instantaneous deformation, the basic creep and the additional creep due to drying, is expressed as:

$$J(t,t') = q_1 + C_0(t,t') + C_d(t,t',t_0) \tag{3}$$

BASIC CREEP

Based on the log-double-power law [1], the basic creep compliance function is given as:

$$C_0(t,t') = q_0 \ln\{1 + \psi[(t')^{-m} + \alpha](t-t')^n\} \tag{4}$$

in which $m = 0.5$, $n = 0.1$, $\alpha = 0.001$, $\psi = 0.3$.

MEAN SHRINKAGE AND CREEP OF CROSS SECTION AT DRYING

The initial relative humidity in the pores of concrete is 100%. Subsequent exposure to environment causes a long-term drying process, which causes shrinkage and additional creep.

Shrinkage

Mean shrinkage strain in the cross section:

$$\epsilon_{sh}(t,t_0) = -\epsilon_{sh\infty}\, k_h\, S(t) \tag{5}$$

Time dependence:

$$S(t) = \tanh\sqrt{\frac{t-t_0}{\tau_{sh}}} \tag{6}$$

Humidity dependence:

$$k_h = \begin{cases} 1 - h^3 & \text{for} \quad h \le 0.98 \\ -0.2 & \text{for} \quad h = 1 \text{ (swelling in water)} \\ \text{linear interpolation} & \text{for} \quad 0.98 \le h \le 1 \end{cases} \tag{7}$$

Size dependence:

$$\tau_{sh} = 32D^2 \quad (D \text{ in inches}) \qquad = 4.9D^2 \quad (D \text{ in cm}) \tag{8}$$

where $D = 2v/s$ = effective cross-section thickness (in inches or cm).

Additional Creep Due to Drying (Drying Creep)

$$C_d(t, t', t_0) = q_5 \left[e^{-3H(t)} - e^{-3H(t')}\right]^{1/2} \quad \text{for } t' \geq t_0 \tag{9}$$

in which

$$H(t) = 1 - (1 - h)S(t) \tag{10}$$

PARAMETER PREDICTION BASED ON STRENGTH AND WATER CONTENT OF CONCRETE MIX

Some formulae that follow are valid only in certain dimensions. These are given both in inch-pound system units (psi, in.) and in metric (S.I.) units (MPa, m). The units of each dimensional quantity are also specified in the list of notations (Appendix 1). In the following q_1, q_0 and q_5 are in the units of $(10^6 \text{ psi})^{-1}$ or $(\text{MPa})^{-1}$, $\bar{f}_c$ and E_{28} in the units of psi or MPa and $\epsilon_{sh\infty}$ in the units of 10^{-6}.

Basic Creep

$$\begin{array}{ll} q_0 = 200/\sqrt{\bar{f}_c}; \quad q_1 = 0.6 \times 10^6/E_{28}; \quad E_{28} = 57000\sqrt{\bar{f}_c} & \text{inch-pound system} \\ q_0 = 2408/\sqrt{\bar{f}_c}; \quad q_1 = 0.6 \times 10^6/E_{28}; \quad E_{28} = 4734\sqrt{\bar{f}_c} & \text{S.I.} \end{array} \tag{11}$$

Shrinkage

$$\begin{array}{lll} \epsilon_{sh\infty} = \alpha_1\alpha_2\left[26w^{2.1}(\bar{f}_c)^{-0.28} + 270\right] & (\text{in } 10^{-6}) & \text{inch-pound system} \\ \epsilon_{sh\infty} = \alpha_1\alpha_2\left[1.9 \times 10^{-2}w^{2.1}(\bar{f}_c)^{-0.28} + 270\right] & (\text{in } 10^{-6}) & \text{S.I.} \end{array} \tag{12}$$

where

$$\alpha_1 = \begin{cases} 1.0 & \text{for type I cement;} \\ 0.85 & \text{for type II cement;} \\ 1.1 & \text{for type III cement.} \end{cases} \tag{13}$$

and

$$\alpha_2 = \begin{cases} 0.75 & \text{for steam-cured specimens;} \\ 1.0 & \text{for specimens cured in water or 100\% relative humidity;} \\ 1.2 & \text{for specimens sealed during curing.} \end{cases} \tag{14}$$

Creep at Drying (happens to be the same in both inch-pound system and S.I. units)

$$q_5 = 6000/(\bar{f}_c) \tag{15}$$

Fig. 1 shows creep and shrinkage curves for typical parameter values. Fig. 2 shows comparisons of predictions of the model with some typical test data from the literature (for references to these data, see [2]). The scatter

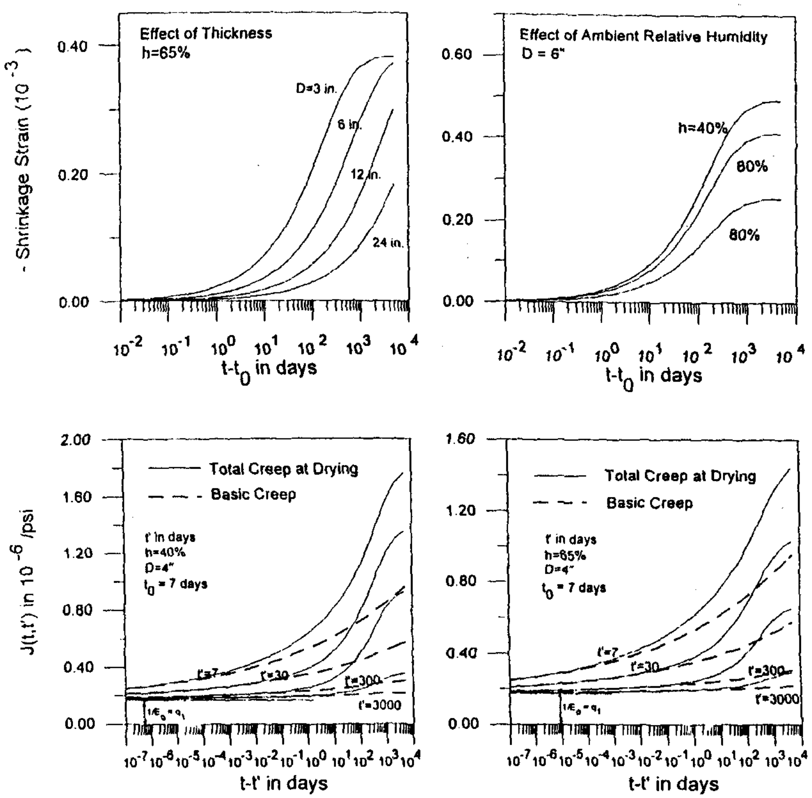

Figure 1: Typical shrinkage and creep curves given by the Model B3 (short form).

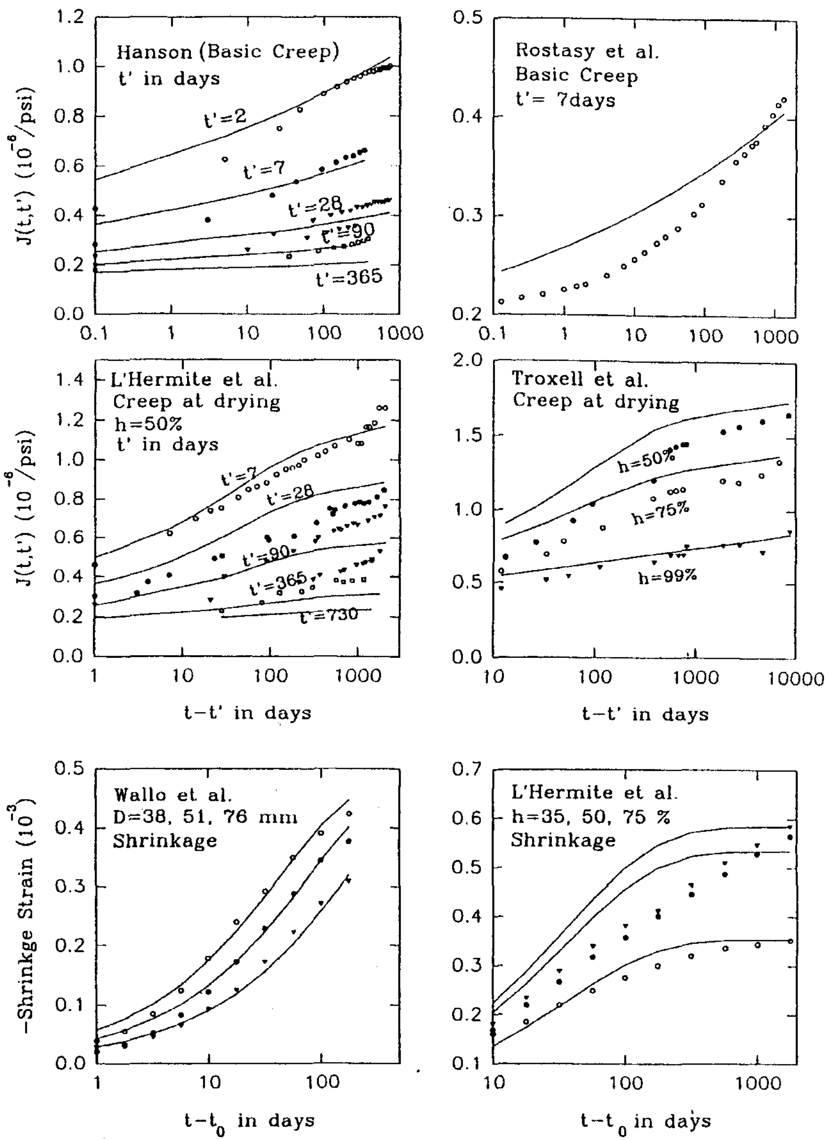

Figure 2: Comparison of model predictions to some typical test data from the literature.

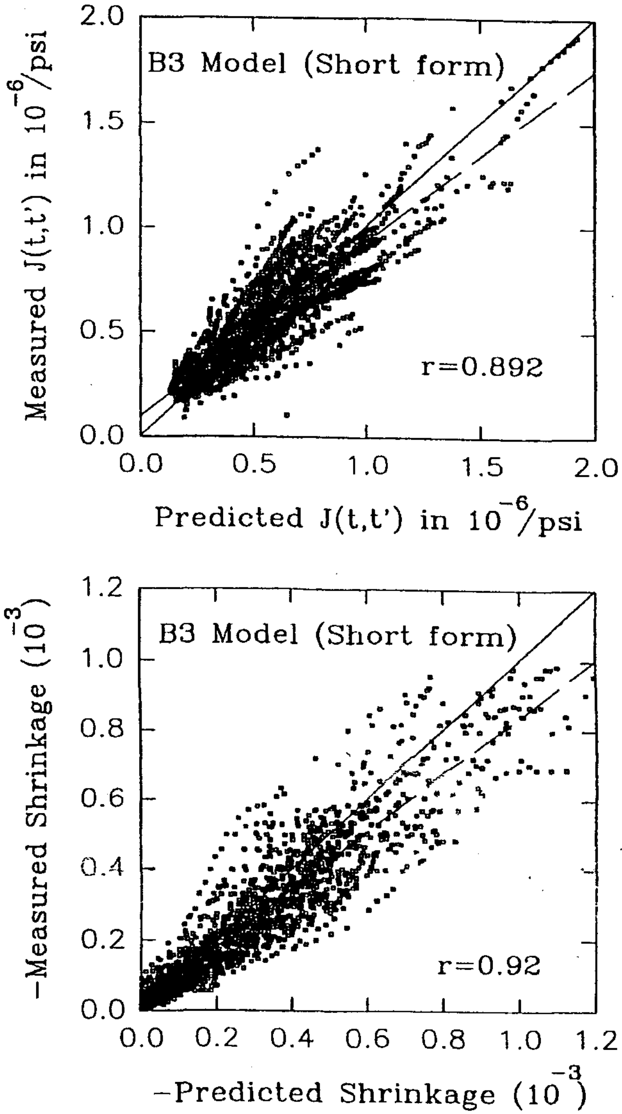

Figure 3: Scatter plots of measured versus predicted values of creep and shrinkage (dashed lines are regression lines).

Table 1: Coefficient of variations of errors (expressed as a percentage) of the basic creep predictions

Model	B3S
Test data	$\bar{\omega}$
1. Keeton	22.5
2. Kommendant et al	18.3
3. L'Hermite et al.	48.6
4. Rostasy et al.	16.4
5. Troxell et al.	9.8
6. York et al.	11.0
7. McDonald	8.4
8. Maity and Meyers	13.4
9. Mossiossian and Gamble	20.0
10. Hansen and Harboe et al. (Ross Dam)	18.2
11. Browne et al. (Wylfa vessel)	51.8
12. Hansen and Harboe et al. (Shasta Dam)	22.2
13. Brooks and Wainwright	15.4
14. Pirtz (Dworshak Dam)	9.6
15. Hansen and Harboe et al. (Canyon ferry Dam)	54.8
16. Russell and Burg (Water Tower Place)	30.2
17. Hanson	15.8
ω_{all}	26.9

plots of all the data in the data bank compared to the predicted values are shown in Fig. 3.

STATISTICS OF ERRORS COMPARED TO TEST DATA

The model is statistically evaluated in the same manner as previously described for model B3 [2, 3] and used in [4]. The coefficients of variation of errors in comparison to all the data from the RILEM data bank are tabulated in Tables 1–3. Fig. 3 shows the scatter plots comparing the model predictions to the measured data. As demonstrated by the comparisons in [2] and [4], these statistics and scatter plots are slightly worse than those for the full model B3 but are significantly better than those for the previous ACI 209 model (Chapter 2 in ACI R-92 [5]), and they are also better than those for the new CEB-FIP model [6] and the GZ model proposed to subcomittee 4 of ACI 209 [7] which also appears in this volume.

COMPARISON WITH MODEL B3

The full B3 model, presented in this volume and in a previous report [2] as a refinement of the original version in [3], is more detailed and rational than its present short form, which is more suited for simplified calculations of creep and shrinkage effects in concrete structures. Specifically, the following points must be mentioned when considering the relative merits of the two models.

Table 2: Coefficient of variations of errors (expressed as a percentage) of the shrinkage predictions

Model	B3S
Test data	ω
1. Hummel et al.	32.4
2. Rüsch et al.(1)	35.3
3. Wesche et al.	46.9
4. Rüsch et al.(2)	33.3
5. Wischers and Dahms	25.0
6. Hansen and Mattock	21.1
7. Keeton	47.1
8. Troxell et al.	69.6
9. Aschl and Stökl	31.2
10. Stökl	36.5
11. L'Hermite et al.	71.4
12. York et al.	57.5
13. Hilsdorf	15.8
14. L'Hermite and Mamillan	36.0
15. Wallo et al.	26.8
16. Lambotte and Mommens	44.8
17. Weigler and Karl	37.6
18. Wittmann et al.	42.7
19. Ngab et al.	25.9
20. McDonald	23.3
21. Russell and Burg (Water Tower Place)	42.3
ω_{all}	40.8

1. The compliance function for basic creep in model B3 has been derived from the solidification theory. It gives a simple formula for the time rate of compliance which is convenient for use in step-by-step computer analysis of structures. The expression for the compliance function itself is more complex for the B3 model than the log double power law used in the present short form.

2. The log-double-power law exhibits the phenomenon of divergence of creep curves and thus, in principle, violates one of the guidelines [8] by the RILEM TC107 for creep and shrinkage prediction models. However, the violation is never too pronounced and occurs only for short time periods. The violation may cause the phenomenon of stress reversal when creep recovery calculations are performed based on this formula

Table 3: Coefficient of variations of errors (expressed as a percentage) of the predictions of creep at drying

Model	B3S
Test data	$\bar{\omega}$
1. Hansen and Mattock	34.9
2. Keeton	4.4
3. Troxell et al.	14.7
4. L'Hermite et al.	27.2
5. Rostasy et al.	28.7
6. York et al.	34.1
7. McDonald	30.2
8. Hummel	28.9
9. L'Hermite and Mamillan	27.5
10. Mossiossian and Gamble	17.0
11. Maity and Meyers	71.3
12. Russell and Burg (Water Tower Place)	29.0
13. Weil	23.1
14. Hilsdorf et al.	23.3
15. Wischers and Dahms	25.9
16. Wesche et al.	34.9
17. Rüsch et al.	17.9
ω_{all}	30.8

using the principle of superposition. It may also cause long-time stress relaxation of concrete stressed at low age to reach into negative values. The B3 model, based on the solidification theory, is free from such problematic predictions. The problem is nevertheless not serious for normal applications.

3. The shrinkage formulation in the present short form, though essentially similar to model B3, does not include the influence of curing duration and specimen size on the final shrinkage.

4. A look at the values of coefficients of variation and the scatter-plots of measured versus calculated values of creep and shrinkage deformations [2, 3, 4] shows that the B3 model is overall distinctly more accurate than the present short form. The predictions of the present short form are better than the 1990 CEB-FIP model [6] for basic creep and shrinkage and comparable to it for creep at drying.

CLOSING COMMENT

Although the present model is less accurate than the full model B3 and does not always yield stress and strain histories of admissible form, it is simpler to

use and sufficient for structures that do not exhibit high sensitivity to creep and shrinkage.

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APPENDICES

Appendix 1. Notation

All notations introduced in presenting model B3 in this volume are retained. They are as follows:

t = time, representing the age of concrete, in days;
t' = age at loading, in days;
t_0 = age when drying begins, in days (only $t_0 \leq t'$ is considered);
$J(t,t')$ = compliance function = strain (creep plus elastic) at time t caused by a unit uniaxial constant stress applied at age t' (always given in 10^{-6}/psi, the S.I. version of the formulae give $J(t,t')$ in 10^{-6}/MPa, 1 psi = 6895 Pa);
$C_0(t,t')$ = compliance function for basic creep only;
$C_d(t,t',t_0)$ = compliance function for additional creep due to drying;
$\epsilon_{sh}, \epsilon_{sh\infty}$ = shrinkage strain and ultimate (final) shrinkage strain; $\epsilon_{sh\infty} \geq 0$ but ϵ_{sh} is considered negative (except for swelling, for which the sign is positive); always given in 10^{-6};
h = relative humidity of the environment (expressed as a decimal number, not as percentage) $0 \leq h \leq 1$;
H = spatial average of pore relative humidity within the cross section, $0 \leq H \leq 1$;
$S(t)$ = time function for shrinkage;
τ_{sh} = shrinkage half-time in days;
$D = 2v/s$ = effective cross section thickness in inches (in mm for the S.I. version, 1 inch = 25.4mm);
v/s = volume-to-surface ratio in inches (in mm for the S.I. version);
c = cement content of concrete in lb/ft^3 (in kg/m^3 for the S.I. version, 1 lb/ft^3= 16.03 kg/m^3);
w/c = water-cement ratio, by weight;
$w = (w/c)c$ = water content of concrete mix in lb/ft^3 (in kg/m^3 for the S.I. version;
a/c = aggregate-cement ratio, by weight; and
$\bar{f}_c$ = mean 28-day standard cylinder compression strength in psi (in MPa for the S.I. version, 1 psi = 6895 Pa) (if only design strength f'_c is known, then $\bar{f}_c = f'_c + 1200$ psi or $\bar{f}_c = f'_c + 8.57$ MPa);
$E(t)$ = Conventional Young's elastic modulus at age t;

$E_{28} = E(t)$ for $t = 28$ days;
q_1, q_0, q_5 = empirical material constitutive parameters given by formulae based on concrete strength and composition;
$\phi(t,t')$ = Creep coefficient;
k_h = Humidity correction factor for final shrinkage;
k_t = Parameter used in calculation of τ_{sh}.

Appendix 2. Hypotheses and Explanations

The present prediction model is restricted to the service stress range for which creep is assumed to be linearly dependent on stress (generally up to about $0.5\ f'c$). This means that, for constant stress σ applied at age t',

$$\epsilon(t) = J(t,t')\sigma + \epsilon_{sh}(t) + \alpha\Delta T(t) \tag{16}$$

in which σ = uniaxial stress, ϵ = strain, $\Delta T(t)$ = temperature change from reference temperature at time t, α = thermal expansion coefficient. When stresses vary in time, the corresponding strain can be obtained from (3) according to the principle of superposition [9,10]. Simplified design calculations can be done according to the age-adjusted effective modulus method, which allows quasi-elastic analysis [9,10] of the structure.

The compliance function, giving the strain per unit stress, may further be decomposed as given by Eq. (3) in which q_1 = instantaneous strain due to unit stress, $C_0(t,t')$ = compliance function for basic creep (creep at constant moisture content), and $C_d(t,t',t_0)$ = additional compliance function due to simultaneous drying. For generalization to multiaxial creep, the creep Poisson ratio may be assumed to be constant and equal to the instantaneous Poisson ratio $\nu = 0.18$. (Tensile microcracking can cause the apparent Poisson ratio to be much smaller, but this is properly taken into account by a model for cracking.)

The instantaneous strain, same as in previous models [12,13], may be written as $q_1 = 1/E_0$ where E_0 = asymptotic modulus. The use of E_0 instead of the static elastic modulus E is convenient because concrete exhibits pronounced creep even for very short load durations (even shorter than 10^{-4} s). E_0 should not be regarded as the real elastic modulus but merely as an convenient parameter that can be considered age-independent. As a rough estimate, $E_0 \approx 1.5E$. The value of the usual static elastic modulus E normally obtained in tests and used in structural analysis corresponds approximately to

$$E(t') = 1/J(t' + \Delta, t') \tag{17}$$

in which the stress duration $\Delta = 0.01$ day gives values approximately agreeing with ACI formula, $E = 57,000\sqrt{f_c}$ in psi (or $E = 4734\sqrt{f_c}$ in MPa). The advantage of defining q_1 by extrapolation to extremely fast loading is that q_1 (or E_0) can be considered as age independent and equation (17) also gives the age dependence of the elastic modulus. The value $\Delta = 10^{-7}$ day gives approximately correct values of the dynamic modulus of concrete and its age dependence. The meaning of the value of $q_1 = 1/E_0$ is explained in Fig. 1, which also shows the typical curves of basic creep, shrinkage and drying creep according to the present model.

The creep coefficient, which represents the most convenient way to introduce creep into structural analysis, should be calculated from the compliance function, i.e.,

$$\phi(t,t') = E(t')J(t,t') - 1 \tag{18}$$

Note that for structural analysis it is not important which value of Δ corresponds to $E(t')$ in Eq. (18), and not even whether some other definition of E is used in Eq. (18). One can use the ACI formula, $E = 57000\sqrt{f_c}$ in psi (or $E = 4734\sqrt{f_c}$ in MPa), or Eq. (17) for any value of $\Delta \leq 0.1$ day. For the results of structural analysis of creep and shrinkage (for $t - t' \geq 1$ day), the only important aspect is that E and ϕ together must give the correct total compliance $J(t,t') = [1 + \phi(t,t')]/E(t')$, as defined by model B3.

Note that significant errors would arise if a prediction model would specify ϕ instead of J. In that case the user would likely calculate J from E and *phi* values that are incompatible. The elastic deformation in the creep tests that established ϕ have often been obtained for very different load durations and load histories than those which correspond to the ACI formula (about 0.01 day). What matters for matters for predicting the long-time creep effects in structures is only the values of J, and not the values of ϕ and E that yield J. If different combinations of E and ϕ yield the same J, the predictions are about the same. Care in this regard must also be taken when updating the model parameters from some test data for which only the values of ϕ were reported. $J(t,t')$ cannot be calculated from such data using a definition of E, for example, $E = 57000\sqrt{f_c}$ in psi, which does not give values compatible with these ϕ values and gives $J(t,t')$ disagreeing with Eq. (18). Conversions of such data from ϕ to J-values must be based on the short-time strains actually measured on the creep specimens themselves, or else such data cannot be used.

The relative humidity in the pores of concrete is initially 100%. In absence of moisture exchange (as in sealed concretes), a subsequent decrease of pore humidity, called self-desiccation, is caused by hydration, but in normal concretes this decrease is small (to about 96%-98%). Exposure to environment causes a long-term drying process (described by the solutions of diffusion equation), which causes shrinkage and additional creep. This means that the normal strain $J(t,t')\sigma$, representing the sum of the elastic and creep strains, is measured by subtracting the deformations of a loaded specimen and a load-free companion. For shear creep this is not necessary because shrinkage is strictly a volume change.

In absence of drying there is another kind of shrinkage, called autogeneous shrinkage, which is caused by the chemical reactions of hydration. This shrinkage usually is small for normal concretes and can be neglected(but not for high-strength concretes). It does not occur if the relative humidity in the pores drops significantly below 100%. Further shrinkage (or expansion) may be caused by various chemical reactions, for example carbonation. But in good concretes, carbonation occurs only in a surface layer a few millimeters thick and can be neglected for normal structures. For concrete submerged in water ($h = 100\%$), there is positive ϵ_{sh}, that is, swelling, which is approximately predicted by the present model upon substituting $h = 100\%$.

Appendix 3. Parameter Uncertainties to Be Considered in Design

The parameters of any creep and shrinkage model must be considered as statistical variables. The preceding formulae predicting the creep and shrinkage parameters from concrete composition and strength give the mean value of $J(t,t')$ and ϵ_{sh}. To take into account statistical uncertainties, the parameters $q_1, q_2, q_5, \epsilon_{sh\infty}$ ought to be replaced by the values

$$\psi_1 q_1,\ \psi_1 q_2,\ \psi_1 q_5,\ \psi_2 \epsilon_{sh\infty} \tag{19}$$

Here ψ_1 and ψ_2 are uncertainty factors for creep and shrinkage, which may be assumed to follow roughly the normal (Gaussian) distribution with mean value 1. According to the statistical analysis of the data in the data bank [2, 3, 4], the following coefficients of variation of these uncertainty factors should be considered in design:

$$\begin{aligned} \omega(\psi_1) &= 31\% \quad \text{for creep, with or without drying} \\ \omega(\psi_2) &= 41\% \quad \text{for shrinkage} \end{aligned} \tag{20}$$

Other input parameters of the model are also statistical variables. At least, the designer should consider the statistical variations of environmental humidity h and of strength $\bar{f}_c$. This can be done by replacing them with $\psi_3 h$ and $\psi_4 \bar{f}_c$ where ψ_3 and ψ_4 are uncertainty factors having a normal distribution with mean 1. In absence of other information, the following coefficients of variation may be considered for these uncertainty factors [14]:

$$\begin{aligned} \omega(\psi_3) &\approx 20\% \quad \text{for } h \to \psi_3 h \\ \omega(\psi_4) &\approx 15\% \quad \text{for } \bar{f}_c \to \psi_4 \bar{f}_c \end{aligned} \tag{21}$$

Factor ψ_3 is statistically independent of ψ_1, ψ_2, and ψ_4 and all the factors may be assumed mutually statistically independent, as an approximation.

Appendix 4. Prediction Improvement Based on Short-Time Tests

The large uncertainty in the prediction of creep and shrinkage of concrete, reflected in the values of the coefficients of variation in Eq. (20), is caused mainly by the effect of the composition and strength of concrete. This effect is very complicated and not sufficiently understood in quantitative terms. At present, the only way to reduce the uncertainty is to conduct short-time tests and use them to update the values of the material parameters in the model. This approach is particularly simple for creep but is more difficult for shrinkage [2, 3]. A method to improve the prediction based on short-time shrinkage tests coupled with measurements of water (weight) loss is described in [2, 3]. This method can be applied to the present short form.

Appendix 5. Levels of Creep Sensitivity of Structures and Type of Analysis Required

Accurate and laborious analysis of creep and shrinkage is necessary for some type of structures but not for others. That depends on the sensitivity of the structure. Although more precise studies are needed, the following approximate classification of sensitivity levels of structures can be made on the basis of general experience.

Level 1. Reinforced concrete beams, frames and slabs with spans under 65 ft (20 m) and heights of up to 100 ft (30 m), plain concrete footings, retaining walls.

Level 2. Prestressed beams or slabs of spans up to 65 ft. (20 m), high-rise building frames up to 325 ft (100 m) high.

Level 3. Medium-span box girder, cable-stayed or arch bridges with spans of up to 260 ft (80 m), ordinary tanks, silos, pavements.

Level 4. Long-span prestressed box girder, cable-stayed or arch bridges; large bridges built sequentially in stages by joining parts; large gravity, arch or buttress dams; cooling towers; large roof shells; very tall buildings.

Level 5. Record span bridges, nuclear containments and vessels, large offshore structures, large cooling towers, record-span thin roof shells, record-span slender arch bridges.

For the type of model and analysis, the following recommendation can be made:

1. *The use of a model as realistic and sophisticated as B3:* recommended but not strictly required for level 3, mandatory for levels 4 and 5. For levels 1 and 2 simpler models are adequate. Such a model ought to be always used for structures analyzed by sophisticated computer methods, including two or three dimensional finite elements (because it makes no sense to input inaccurate material properties into a very accurate computer program for the analysis of stresses and deflections).

2. *Method of structural creep analysis:* The age-adjusted effective modulus method is recommended for levels 3 and 4. The effective modulus method suffices for level 2. For level 1, creep and shrinkage analysis of the structure is not needed but a crude empirically based estimate is desirable. Level 5 requires the most realistic and accurate analysis possible, typically a step-by-step computer solution based on a constitutive law, coupled with the solution of the differential equations for drying and heat conduction.

3. *Statistical analysis with estimation of 95% confidence limits:* (a) mandatory for level 5; (b) highly recommended for level 4; (c) for lower levels desirable but not necessary, however, the confidence limits for any response X (such as deflection or stress) should be considered, being estimated $\bar{X} \times (1 \pm 1.96\omega)$ where $\bar{X}$ = mean estimate of X and ω = coefficient of variation of X.

4. *Analysis of temperature effects and effects of cycling of loads and environment:* must be detailed for level 5 and approximate for level 4. It is not necessary though advisable for level 3 and can be ignored for levels 1 and 2 (except for heat of hydration effects).

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Design Provisions for Shrinkage and Creep of Concrete

by N. J. Gardner

Synopsis:

This paper presents a simple design-office procedure for calculating the shrinkage and creep of concrete using the information available at design; namely the 28 day specified concrete strength, the concrete strength at end of curing or loading, element size and the relative humidity. The method includes strength development with age, relationship between modulus of elasticity and strength, and equations for predicting shrinkage and creep. The only arbitrary information are the factors appropriate to the cementitious material, which can be improved from measured strength age data. At the most basic level the proposed method requires only the information available to the design engineer. The prediction values can be improved by simply measuring concrete strength development with time and modulus of elasticity. Aggregate stiffness can be taken into account by back calculating a concrete pseudo strength from the measured modulus of elasticity. Measured short term shrinkage and creep values can be extrapolated to obtain long duration predictions for similar sized elements. The predictions are compared with experimental results for seventy nine data sets for compliance and sixty three data sets for shrinkage. The comparisons indicate shrinkage and creep can be calculated within +/- 25%. The method can be used regardless of what chemical admixtures or mineral by-products are in the concrete, casting temperature or curing regime.

Keywords: concrete; creep; modulus of elasticity; shrinkage; strength development