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# Fracture Mechanics of Concrete Structures

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# Vertex Effect and Confinement of Fracturing Concrete via Microplane Model M4

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ABSTRACT: A newly developed powerful version of microplane model, labeled model M4, is exploited to study two basic phenomena in fracturing concrete: (a) The vertex effect, i.e., the tangential stiffness for loading increments to the side of a previous radial loading path in the stress space, and (b) the effect of confinement by a steel tube or a spiral on the suppression of softening response of columns. In the former problem, the microplane model is used to simulate the torsional response of concrete cylinders after uniaxial compression preloading to the peak compression load or to a post-peak softening state. Comparisons with new tests carried out at Northwestern University show the microplane model to predict the initial torsional stiffenss very closely, while the classical tensorial models with invariants overpredict this stiffness several times (in plasticity of metals, this phenomenon is called the 'vertex effect' because its tensorial modeling requires the yield surface to have a vertex, or corner, at the current state point of the stress space). In the latter problem, microplane model simulations of the so-called 'tube squash' tests are presented and analyzed. In these tests, recently performed at Northwestern University, steel tubes of different thicknesses filled by concrete are squashed to about half of their initial length and very large strains with shear angles up to about 70 degrees are achieved. The tests and their simulations show that, in order to prevent softening (and thus brittle failure and size effect), the cross section of the tube must be at least 16% of the total cross section area, and the volume of the spiral must be at least 14% of the volume of the column. When these conditions are not met, which comprises the typical contemporary designs, one must expect localization of damage and size effect to take place.

## 1. NATURE OF VERTEX EFFECT AND ITS THEORETICAL IMPLICATIONS

The normality rule implies that, for load increments that are parallel to the current yield surface (or loading potential surface), called the 'loading to the side', the response is purely elastic. However, testing of metals showed long ago that in fact the response is much softer than elastic (Bleich 1952; Gerard and Becker 1952, 1957; Phillips and Gray 1961). Gerard and Becker's tests were particularly simple and revealing; they tested axially compressed thin-walled cruciform steel columns that buckle in the plastic range by torsion. The critical load of such columns is proportional to the tangential inelastic stiffness for loading to the side, and it was found that it can be much smaller than the critical load for the elastic stiffness, as small as one half of the elastic critical load (Bažant and Cedolin 1991, Sec. 8.1; in detail Brocca and Bažant 2000).

The existence of inelastic strain increments for loading to the side implies that there must be a corner, or vertex, on the yield surface (or loading potential surface) at the current state point, traveling during loading with the state point as the material hardens or softens. Therefore, the phenomenon is called the 'vertex effect'. The vertex effect is the strongest for abrupt changes of loading direction in the stress space or strain space, but arises for all loading paths (even smooth paths) significantly deviating from proportional loading (radial loading in the stress or strain space).

## 2. MEASUREMENTS OF VERTEX EFFECT

To examine the vertex effect in concrete, one may use a loading path in which a uniaxial compressive strain is initially applied. No shear strains develop during this loading. A sudden imposition of incre-

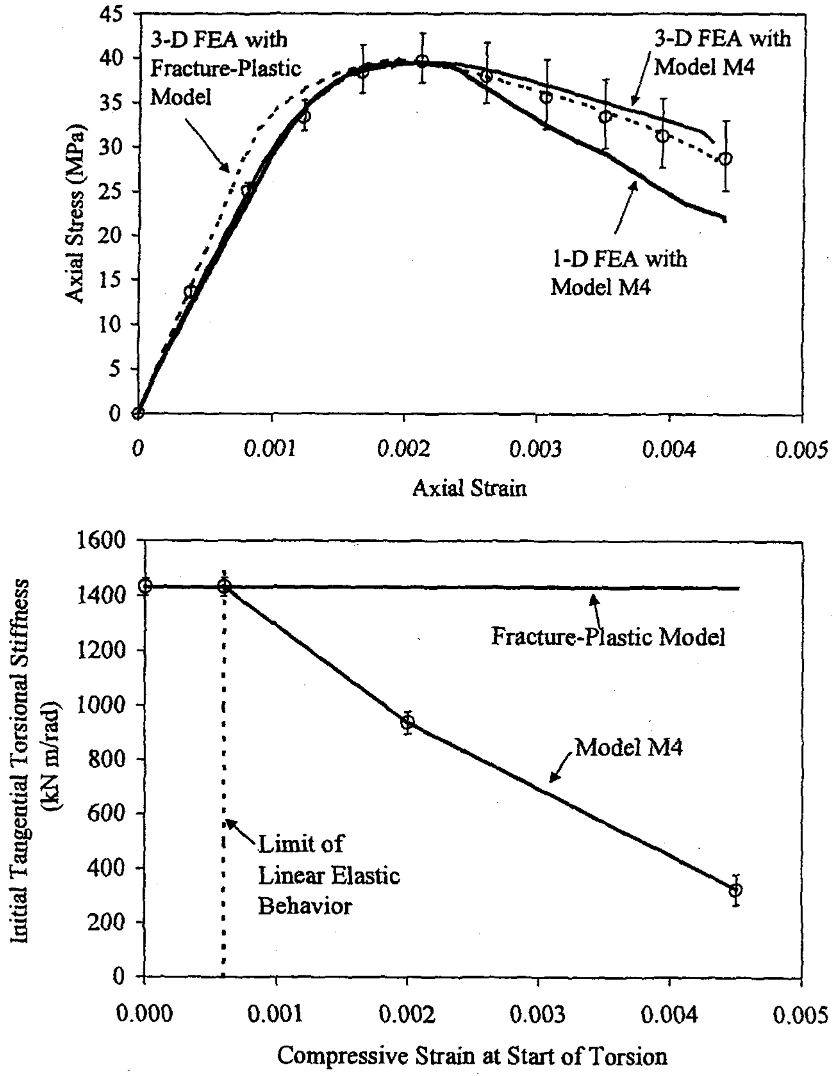


Figure 1. Plot of initial incremental torsional stiffness when the torsional loading starts, divided by the elastic torsional stiffness (test data points, prediction of microplane model M4, and prediction of damage-fracturing model based on tensorial invariants and loading potentials.

mental shear strain, introduced by torsion, represents 'loading to the side' (relative to the current loading surface). The experiments were designed for cylindrical specimens so that they can be tested in an axial-torsional testing machine. The load path is prescribed in the space of axial displacement versus rotation, and the response in the post-peak regime is of main interest. The loading path includes a sharp corner. A rounded corner could be avoided thanks to the availability of a state-of-the-art testing machine. The specimen was compressed under uniaxial stress (at zero rotation) to strain $\epsilon = 0.2\%$ (which is the axial strain at maximum uniaxial stress $\sigma = f_c'$ for a typical concrete). This was followed by concentric rotation at constant axial strain $\epsilon = 0.2\%$ until the peak torque is reached. Cylindrical specimens of concrete with diameter $D$=4 in (101.6 mm) and height $H$=8 in (203.2 mm) were used.

When a sudden switch from uniaxial compressive loading to torsional loading was made at prepeak stresses up to about 80% of peak, the incremental response was nearly elasttic and nearly path-independent. Therefore the vertex effect was tested only in peak and postpeak regions. Concrete was compressed axially at zero rotation, which was followed by concentric rotation.

Fig. 1 shows a plot of the initial incremental torsional stiffness when the torsional loading started, divided by the elastic torsional stiffness. The results are compared to the prediction of the microplane model M4 (whose parameters were not adjusted to optimize the fit). To get realistic simulations, three-dimensional finite element analysis of the test cylinder was carried out using microplane model M4 (Bažant et al. 2000a). For the sake of comparison, the finite element analysis with one of the sophisticated advanced damage-plasticity models for concrete formulated in the classical way, in terms of stress and strain tensors and their invariants (Červenka et al. 1998). A mesh of 2484 elements and 3000 nodes was used, covering only the top half of the specimen since symmetric behavior of the bottom half may be assumed. The boundary conditions at the top of the cylinder were prescribed as the axial displacements and horizontal displacements due to rotation of the platens considered as a rigid body. In each loading step, the boundary displacement and rotation were adjusted so a to match the relative displacements and rotations recorded during the tests at the attached steel rings carrying the LVDT's.

## 3. OBSERVATION AND CONCLUSIONS FROM VERTEX STUDY

1. By using a state-of-the art testing machine capable of a sudden switch from compression to torsion and on-specimen gauges with a fast feedback, the vertex effect in the response of concrete to nonproportional loading paths in the stress space has been documented experimentally.

2. At peak compressive load, the vertex effect is strong, and in post-peak very strong. Compared to the elastic torsional stiffness, the initial torsional stiffness after a sudden switch from compression to torsion is reduced to 65% when the torsion begins at compression load peak, and to 23% when the torsion begins at a postpeak state at which the axial load has decreased to 70% of the peak load.

3. The experimental data obtained are modeled using two state-of-the-art but conceptually completely different models. One is microplane model M4, and the other is a fracture-plastic model, a state-of-art tensorial model based on invariants.

4. The initial torsional stiffness after a sudden switch from compression to torsion is predicted by microplane model M4 quite accurately, and without any adjustment of the material parameters previously calibrated by other tests. This demonstrates the microplane model M4 can predict the vertex effect, and does so correctly.

5. The capability of predicting the vertex effect is due to fact that the model implies many simultaneous, independently activated (strain-dependent) yield surfaces on the microplanes and has also independent yield surfaces for volumetric, deviatoric and shear response on each microplane. It is the interaction of these surfaces that produces the vertex efect.

6. The classical invariant-based tensorial models employing only a few yield (or loading potential) surfaces, represented in the present simulation by an advanced model, the fracture-

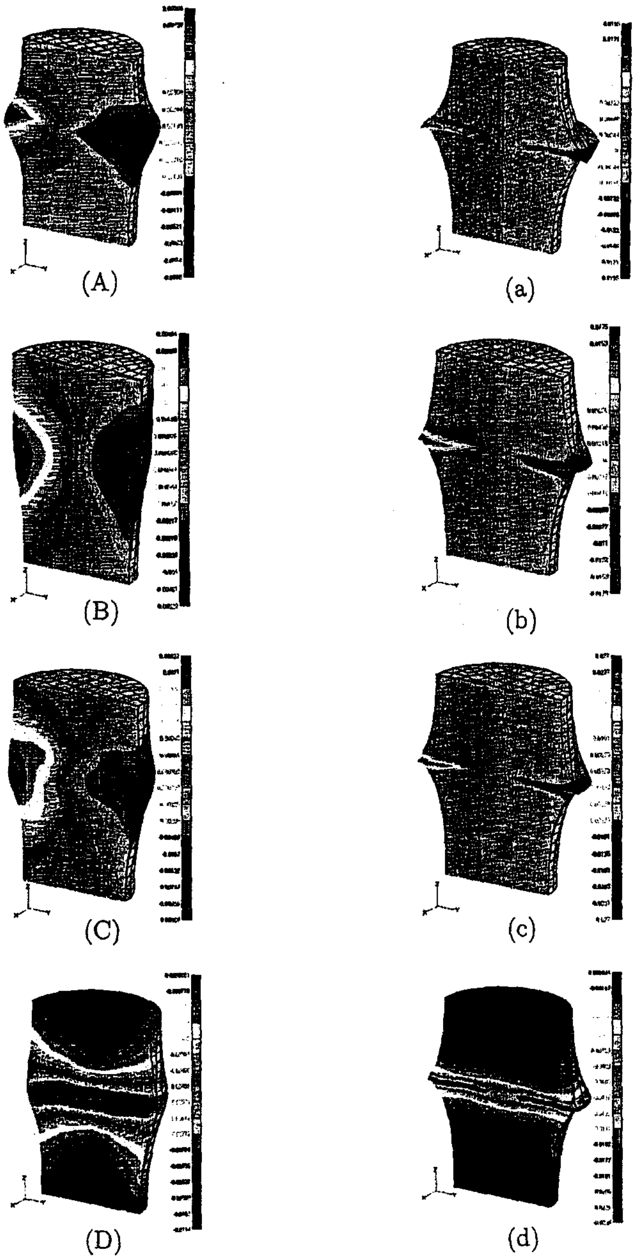


Figure 2. Deformed shapes obtained using model M4 (on the left) and fracture-plastic model (on the right). The loadings paths are proportional (A, a); vertex at peak (B, b) and vertex in the postpeak (at $\epsilon_{zz} = 0.45\%$) (C,D,c,d). Strain distributions shown are $\gamma_{xz}$ (A,B,C,a,b,c) and max. principal strain $\epsilon_I$ (D, d).

plastic model, are inherently incapable of simulating the vertex effect, and more generally the response to highly nonproportional loading paths. This is documented by the fact that they incorrectly predict the initial torsional stiffness after the switch to be the elastic torsional stiffness.

7. The microplane model prediction of response to torsional loading after a sudden switch from compression to torsion is less accurate but better than the fit with the plastic-fracture model. It must be emphasized that the former is a true prediction, with no adjustment of material parameters previously calibrated by other tests, while the latter is a fit obtained after adjusting some material parameters in the fracture-plastic model.

## 4. NATURE OF CONFINEMENT EFFECT ON DUCTILITY AND THEORETICAL APPROACH

The way to achieve ductility of concrete is lateral confinement by steel (e.g. Schneider 1998; Roeder et al. 1999; van Mier 1987). Ductility should be understood as a plastic behavior, or absence of brittleness. In mechanics terms, the response of a structure is brittle when the tangential stiffness $K_t$ becomes negative, in other words, when the structure undergoes softening (decrease of load at increasing deflection). More precisely, the ductility of a structure is lost when the tangential stiffness matrix $\boldsymbol{K}_t$ ceases being positive definite. The material at a point of the structure looses ductility (plasticity) and becomes brittle (or quasi-brittle) locally when strain softening begins, i.e., when the tangential moduli matrix $\boldsymbol{E}_t$ ceases being positive definite. It is well established that when the material is softening at some point of the structure, the inelastic deformation localizes and plastic limit analysis is inapplicable because the material strength is not mobilized simultaneously at various points of the structure; rather, localized damage zone propagates during loading, which eventually leads to fracture.

More seriously, when strain-softening develops and damage propagates, a deterministic (energetic) size effect is always present. Thus the failure loads seen in reduced-scale laboratory tests do not scale up for full-size real structures according to material failure criteria expressed in terms of the stress and strain tensors and a fracture mechanics type energy criterion of failure must be used. The larger the structure, the steeper its post-peak softening and the more prone the structure is to explosive dynamic failure driven by a sudden release of its stored energy. The post-buckling response of columns then becomes dynamic and much more sensitive to imperfections, which calls for using higher safety factors for larger structures.

## 5. TUBE SQUASH TESTS AND THEIR FINITE ELEMENT EVALUATION

A recent experimental and theoretical investigation at Northwestern University (Caner et al. 2000b) attempted to clarify the minimum confinement by steel that is necessary to completely prevent softening, and thus damage localization fractures and size effect. The recently developed tube-squash tests (Bažant Kim and Brocca 1999) have been conducted on concrete-filled tubes of various thicknesses and analyzed with a large-strain finite element code. A realistic finite element analysis has been made possible by employing microplane constitutive model M4 (Bažant et al. 2000a,b; Caner and Bažant 2000), which was verified by numerous test data and was originally developed for simulating the penetration of missiles into concrete walls and ground shock effects on buried structures (Bažant et al. 2000c). The steel in the tube was simulated by a microplane model (Brocca and Bažant 2000a,b) calibrated so as to be equivalent to the classical hardening $J_2$ plasticity for the special case of proportional (radial) loading paths. The large (finite) strains that occur in the steel tube are handled in step-by-step loading by the updated Lagrangian approach (Crisfield 1997, Zienkiewicz and Taylor 1991). Microplane model M4, with a special finite strain formulation combining non-conjugate Green's Lagrangian strain and back-rotated Cauchy stress, was used for concrete (Bažant et al. 2000c) (despite lack of cojugacy, non-negativeness of energy dissipation was ensured). The crack band model was used to avoid spurious mesh sensitivity.

Tubes of thicknesses $t = 3/16$ in $= 4.7625$ mm ($\rho = A_s/A = 36.0\%$; specimen type no.1) and $t = 1/16$in $= 1.5875$ mm ($\rho = A_s/A = 14.8\%$; specimen type no.2) made of a highly ductile steel alloy ASTM No.1020 with Young's modulus $E = 6800$ ksi $= 46852$ MPa and Poisson's ratio $\nu = 0.25$ were filled with concrete and cured in a fog room for 28 days (Caner et al. 2000b). All the tubes had the same inner diameter $D = 1.5$ in. $= 38.1$ mm and the same length $L = 3.5$ in. $= 88.9$ mm. Normal strength concrete was cast into the tubes; it had a maximum aggregate size of 0.375 in. = 9.52 mm, uniaxial compressive strength $f_c' = 6$ ksi $= 41.37$ MPa, and Young's elastic modulus $E = 3500$ ksi $= 24115$ MPa (Poisson's ratio was taken as $\nu = 0.18$).

The concrete-filled tubes were compressed axially under displacement control in a servo-controlled closed-loop MTS testing machine until the steel

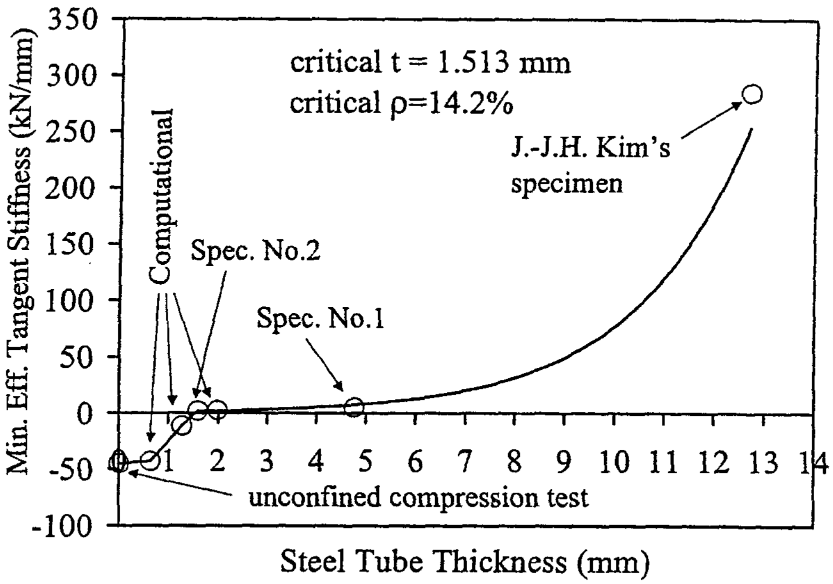


Figure 3. Determination of critical wall thickness of steel tube using the experimental data points in the min. effective tangential stiffness in axial direction vs axial displacement space.

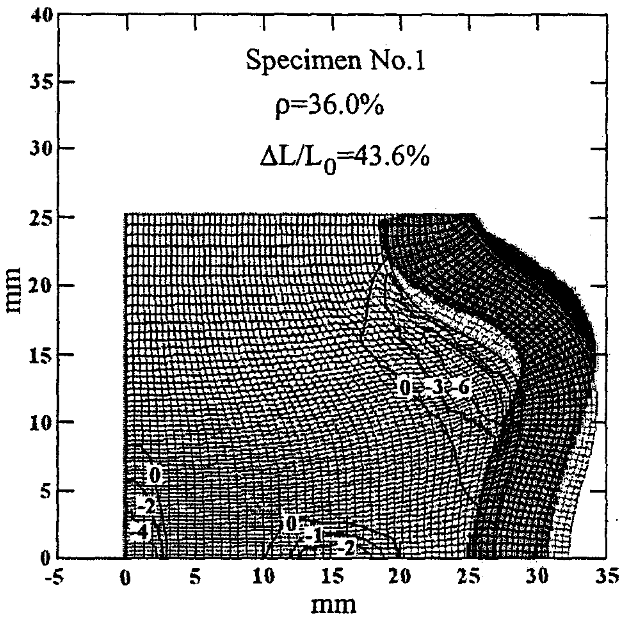


Figure 4. The experimental deformed shape of a typical specimen type no.1 and its prediction by the finite element analysis. Also shown are the contours of equal $K_t \leq 0$ normalized by Young's modulus $E$ of concrete.

tube fractured, which happened when the tube length was reduced to about one half. Shear angles over 70° and axial compressive strains of the order of 50% are achieved in concrete in these tests (Bažant et al. 1999). Based on visual inspection on cuts made after the test, the specimens with the thickest tube showed no visible damage.

Fig. 4 shows that the computed deformed shapes were quite accurate. These figures also show the contours of equal effective tnagent modulus $E_t \leq 0$ normalized by Young's modulus $E$ of concrete.

To ensure perfect ductility of concrete-filled tubular columns, the thickness of the steel tube must exceed a certain critical (or minimum) value. It was assumed that this value should be such that the effective tangential stiffness $K_t$ along the loading path of the column would always remain non-negative.

Fig. 3 shows the minimum values of $K_t$ determined by tests as a function of steel tube thickness $t$. The plot also includes a point with negative $K_t$ which corresponds to the standard (unconfined) compression test of a cylinder ($t = 0$). It may be mentioned that the experimentally determined critical thickness of the tube matches the value predicted numerically using the microplane model M4, without any adjustment in the prediction model.

## 6. OBSERVATIONS AND CONCLUSIONS FROM CONFINEMENT INVESTIGATIONS

1. The series of tube-squash tests on concrete-filled tubes of different wall thicknesses showed, and finite element computations confirmed that, for the normal concrete used, a fully ductile inelastic response can be ensured, and the size effect avoided, only if the ratio of cross section area of steel to the whole cross section area exceeds the critical value of about

   14%

2. For spirally reinforced columns, finite element computations showed about the same value of the critical steel ratio.locally.

3. The aforementioned minimum steel ratio found necessary to completely prevent softening response, i.e., to achieve plastic behavior, is rather high—significantly higher than the steel ratios currently used in design. The implication is that plastic limit analysis is not an adequate design concept for the currently used column dimensions. Therefore, if the current values of steel ratios are not increased, one must pay attention to the localization of softening damage and accept the size effect engendered by it. Because of the brittleness of failure and size effect caused by every strain-softening behavior, very large columns are of particular concern. The safety advantages of moving toward columns with stronger steel confinement should be explored.

## 7. CONSEQUENCES FOR SIMULATIONS OF MISSILE IMPACT AND PENETRATION

Numerical simulations of the impact and penetration of missiles represent a problem in which it is very important to capture realistically both the plastic ductile response and the fracturing response of concrete. Under the nose of the missile, concrete is exposed to enormous confining pressures and behaves plastically, dissipating a large amount of energy. Capturing this dissipation accurately is important for correct prediction of the depth of penetration into a concrete wall or the exit velocity of the missile. Farther away from the missile the impact waves cause fracturing, which also dissipates much energy, and produce the entry and exit craters. During the impact events, concrete is subjected to highly nonproportional loading path, for which the vertex effect must be captured realistically in order to guarantee correct tangential stiffness and dissipation.

Large-scale impact simulations have been conducted by M.D. Adley and S.A. Akers at Waterways Experiment Station in Vicksburg, using the finite-strain microplane model M4 developed at Northwestern University, which is capable to capture all the aforementioned phenomena. The details are presented in Bažant et al. (2000b).

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