

PROCEEDINGS OF THE EIGHTH INTERNATIONAL CONFERENCE ON CREEP, SHRINKAGE
AND DURABILITY OF CONCRETE AND CONCRETE STRUCTURES, ISE-SHIMA, JAPAN,
30 SEPTEMBER-2 OCTOBER 2008

Creep, Shrinkage and Durability Mechanics of Concrete and Concrete Structures

Edited by

Tada-aki Tanabe

Chubu Regional Institute of Infra-Technology Evaluation and Support (CRIIES), Japan

Kenji Sakata

Okayama University, Japan

Hirozo Mihashi

Tohoku University, Japan

Ryoichi Sato

Hiroshima University, Japan

Koichi Maekawa

The University of Tokyo, Japan

Hikaru Nakamura

Nagoya University, Japan

VOLUME 2



CRC Press

Taylor & Francis Group

Boca Raton London New York Leiden

CRC Press is an imprint of the
Taylor & Francis Group, an **informa** business

A BALKEMA BOOK

KEYNOTE LECTURE

Prediction of creep and shrinkage and their effects in concrete structures: Critical appraisal

Z.P. Bažant, G.-Hua Li & Q. Yu
Northwestern University, Evanston, USA

ABSTRACT: The use of a realistic prediction model for creep and shrinkage properties is an essential requirement for good design of structures sensitive to creep and shrinkage. With this aim, the lecture begins by discussing development of an extended, statistically unbiased, database on concrete creep and shrinkage, and of the statistical approaches to comparison and evaluation of various existing prediction models. The proposed statistical method separates the scatter in the database due to variability in concrete composition and curing from the scatter in the evolution of creep and shrinkage in time. The necessity of using bias compensating weights and statistics based on the method of least squares and on the criterion of maximum likelihood is emphasized. Statistically correct mutual comparisons of the existing prediction models are then presented. They conflict with the comparisons and model rankings in ACI new guide 209.2R-08, which are non-standard, statistically questionable and misleading. The importance of calibrating the prediction model by short-time measurements of creep, shrinkage and water loss measurements for the given concrete is underscored. The second part of the lecture attempts a critical appraisal of the structural analysis methods. The long-term prediction errors caused by ignoring the cracking damage, the nonuniformity of drying, and the statistical uncertainty, are highlighted. The usefulness and the misleading potential of accurate solutions according to the linear aging viscoelasticity is then appraised in this context. The statistical analysis of the effects of creep and shrinkage in structures is discussed, the need to calculate the 95% confidence limits of structural effects is emphasized, and conclusions are drawn. In the conference presentation, examples showing comparisons with various observations on structures, including the ill-fated world-record prestressed box girder in Palau, are presented.

STATISTICAL EVALUATION OF MATERIAL MODELS FROM A VERY LARGE DATABASE

A general prediction model for concrete creep and shrinkage, applicable to many different concretes and environmental conditions, must have a strong theoretical foundation (Bažant 2000), and must be validated and calibrated in an unbiased manner by a comprehensive database of the results of worldwide testing (Bažant & Li 2008, e.g.). The necessity of the former is clear from the fact that one must greatly extrapolate in time and in size, although hardly any attention has been paid to it in society committees formulating standard recommendations. The latter has recently received a great deal of attention in these committees but, unfortunately, various non-standard statistical indicators have been used, leading to statistically invalid comparisons and incorrect ranking of the existing models.

The histograms of the number of data available in subsequent time intervals show that their distribution in the database is highly nonuniform. This nonuniformity is unobjective. It results from human choice and introduces bias. For instance, if one experimenter crowds 300 readings into the time interval from 1000 to 2000 days and another experimenter takes just 3 readings in that same interval, the latter experimenter's data will have virtually no effect on unweighted error statistics. To counteract this bias, the data must be weighted. To this end, one may subdivide the load duration $t - t'$, age at loading t' , drying duration $t - t_0$, effective specimen thickness D (Bažant & Baweja 2000) and environmental humidity H into intervals of roughly equal importance, i.e., equal statistical weight ($t =$ time, representing the current age of concrete, $t' =$ age at loading, and $t_0 =$ age at the start of drying; all in days). The proper choice is to subdivide $\log(t - t')$, $\log t'$ and $\log(t - t_0)$ into equally

long intervals (Fig. 1) and give these intervals equal weights. The reason is that a time increment of, e.g., 10 days makes much difference when the test duration is 10 days but little difference when it is 1000 days. As for H , no good reason for its transformation is seen. But for D , the proper choice is to subdivide \sqrt{D} (rather than D) into equal intervals because, according to the diffusion theory (Bažant 2000, Bažant & Kim 1991, Bažant & Raftshol 1982), the diffusion half-time is proportional to D^2 ; e.g., changing D from 3 to 9 cm (or 3 + 6 cm) has about the same relative effect on shrinkage and drying creep as changing D from 10 to 100 cm (but far more than from 10 cm to 10 + 6 cm).

The intervals of $\log(t - t')$, $\log t'$, \sqrt{D} and H for creep, and $\log(t - t_0)$, $\log t_0$, \sqrt{D} and H for shrinkage, define four-dimensional boxes (or hypercubes). However, for the existing database (Bažant & Li 2008), many of them contain 0 or 1 data point. Consequently, such four-dimensional boxes (which were made by Bažant & Li 2008) are debatable. Hence, the present statistical comparisons will use only 1-dimensional intervals of time (Fig. 1a) and 2-dimensional boxes (or rectangles), particularly boxes of $\log(t - t')$ and H for creep, and of $\log(t - t_0)$ and \sqrt{D} for shrinkage (Fig. 1b) (for shrinkage, D is more important than for creep).

Other types of weights could conceivably also be applied, but their propriety is debatable and they have not been used in this study. An exception is the bias against concretes of higher strength. These concretes have a systematically lower creep and lower shrinkage, roughly proportional to $1/E \sim 1/\sqrt{f'_c}$ where f'_c = standard cylindrical strength. This gives them undeservedly lower weights in the database, which causes the statistics to be dominated by lower strength concretes. For an approximate compensation, all the measured compliances and shrinkage strains are scaled in proportion to $1/\sqrt{f'_c}$. In other words, the data ordinates are $y = \bar{J} = J(t, t')\sqrt{f'_c}/f'_c$, for compliance and $y = \bar{\varepsilon} = \varepsilon_{sh}\sqrt{f'_c}/f'_c$ for shrinkage, where $J(t, t')$ and ε_{sh} are the measured values; $f'_c = 5000$ psi (or 34.5 MPa) is a factor which is needed to get dimensionless ratios and may be chosen arbitrarily without affecting the statistical comparisons.

1.1 Evaluation by standard regression statistics

Based on the maximum likelihood requirement and the central limit theorem, the standard error s of a prediction model (representing the standard error of regression) is defined as follows (Haldar & Mahadevan 2000, Ang & Tang 1976, 1984, Bulmer 1979):

$$s = \sqrt{\frac{N}{N-p} \sum_{i=1}^n w_i \sum_{j=1}^{m_i} (Y_{ij} - y_{ij})^2} \quad (1)$$

Here m_i = number of data points in box number i , w_i = statistical weight assigned to this box, the same for each point in this box; $N = N \sum_{i=1}^n w_i = \sum_{i=1}^n m_i$ = number of all the data points in the database; y_{ij} = measured creep or shrinkage data; Y_{ij} = corresponding model predictions; $y_{ij} - Y_{ij} = \varepsilon_{ij}$ = errors of the predictions; and p = number of free input parameters of the model ($p = 12$ for model B3).

Let the intervals or boxes of data be labelled by one index, $i = 1, 2, \dots, n$, running consecutively through all the data sets in the database, as illustrated in Fig. 1a,b. To assign to every box of every data set the same weight, we set w_i of the data points in box i as inversely proportional to the number m_i of data points in that box. Normalizing the weights so that $\sum_{i=1}^n w_i = 1$, we have $w_i = 1/(m_i \bar{w})$, $\bar{w} = \sum_{i=1}^n (1/m_i)$ (the expression in Bažant & Baweja 2000 was $w_i = N/nm_i$; it might seem different but is in fact equivalent because $N/n = \text{constant}$).

Comparison of models requires dimensionless statistical indicators of errors. Two are generally used: 1) the correlation coefficient, and 2) the coefficient of variation (C.o.V.) of regression errors. They characterize, respectively, the ratios of the regression error (or model error) to the scatter band width of data and to the weighted mean y of all the measured y_{ij} in the database. The latter is more informative for our purpose, and is defined as

$$\omega = \frac{s}{\bar{y}}, \quad \bar{y} = \sum_{i=1}^n w_i \sum_{j=1}^{m_i} y_{ij} \quad (2)$$

Figs. 2 and 3 present the results of optimum fitting of the NU-ITI database (Bažant & Li 2008). This database consists 621 creep tests and 490 shrinkage tests, and represents an enlargement of the RILEM database (Müller et al. 1999, Müller & Hilsdorf 1990, Müller 1993), which contained 518 creep tests and 426 shrinkage tests. The left side plots, which show the database in the coordinates corresponding to the column diagrams on the right, reveal the degree of heteroscedasticity. According to the regression theory, the ordinate should ideally be transformed so as to make the data as homoscedastic as possible (i.e., a uniform conditional variance). Homoscedasticity is nearly achieved by the logarithmic transformation of the relative compliance in Fig. 3c, but the scatter band is very broad. For shrinkage, unlike compliance, the transformation of ε_{sh} to $y = \ln \varepsilon_{sh}$ is unsuitable because shrinkage, unlike compliance, begins with zero. The relative shrinkage (Fig. 2b) is slightly less heteroscedastic than the actual shrinkage, and the transformation $y = \ln(\varepsilon/\varepsilon_0)$ is only marginally better. It seems impossible to achieve homoscedasticity of the shrinkage database by any transformation that does not

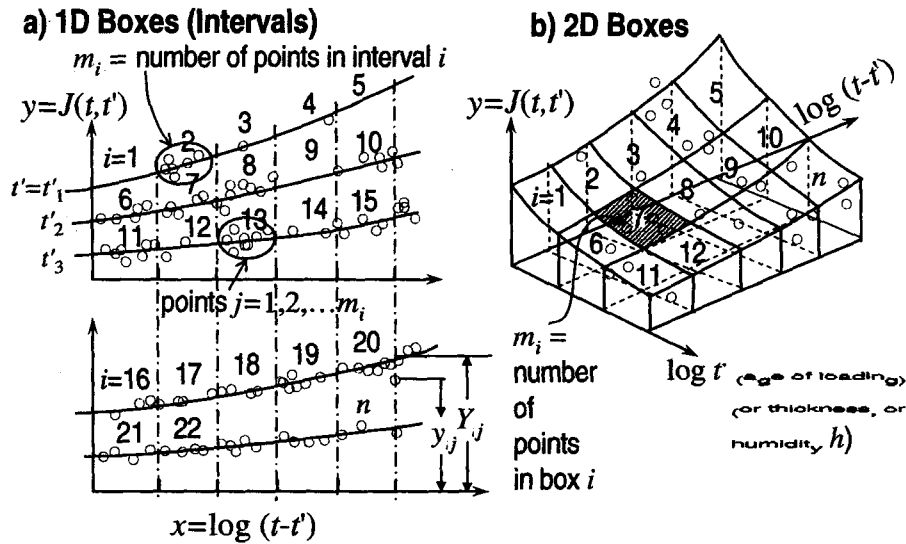


Figure 1. Sketches explaining subdivision of database variables into one-dimensional intervals and two-dimensional boxes of equal importance.

include some optimized constants. The outliers could be debated but have not been removed because they are relatively few, with little effect on the overall statistics.

One important point is to decide which errors to compare. Two kinds of error ought to be separated:

1. The error in predicting the effect of concrete type or composition (and strength) on the compliance and the shrinkage; and
2. the error in predicting the evolution of creep and shrinkage from short to long times, or extrapolating from small size D to large (i.e., from the lab to the field).

The error of the first kind is much larger but also much harder to master theoretically. It is now clear that an empirical approach will not succeed. What would be needed is a breakthrough in materials science and in scale bridging from nano to macro, which is not yet in sight.

It appears, though, that if the short-time creep (e.g. for $t - t' = 3$ days) and the short-time shrinkage with weight loss (e.g., for $t - t_0 = 28$ days) are measured, the error of the first kind gets essentially eliminated. What remains is only the second kind, which can be drastically reduced if the creep and shrinkage curves of the prediction model have the right shape to capture closely the relative changes of creep and shrinkage from short to long times. Model B3 is one that has such a property, as confirmed by experiments (Bažant & Baweja 2000, Navrátil 1998a,b, Granger 1995). The drastic reduction can be seen clearly in the individual

test curves but is masked in the database plots (Figs. 2 and 3) by the variation of many influencing variables.

One can, of course, benefit from this possibility of major error reduction only if short-time creep and shrinkage tests with weight loss measurements are carried out prior to the final design. Such tests should always be made for major structures highly sensitive to creep and shrinkage.

The relation of shrinkage to the simultaneous water loss is an essential feature. However, not every creep and shrinkage model is structured in a way that relates shrinkage and the drying part of creep to the water loss and allows easy and unambiguous calibration by short-time creep, shrinkage and water loss tests. Only model B3 has deliberately been formulated in a way allowing easy calibration by short-time tests and water loss tests, by simple linear regression (Bažant & Baweja 2000).

To appraise the error of the second kind, which is the only kind that is inevitable in the design of major creep-sensitive structures, Figs. 2 and 3 also compare the C.o.V. of the error of the relative shrinkage $y = \varepsilon(t, t_0)/\varepsilon_{28}$ and of the relative compliance $y = J(t, t')/J_0$ (the latter both in the scales of y and of $\ln y$); in the figures, $J_0 = J(28 + 3, 28) = 3$ -day compliance at 28 days of age, and $\varepsilon_0 =$ shrinkage of concrete cured for about 14 days and then exposed to 65% relative humidity for 28 days; J_0 and ε_0 are the same for all the tests of the same concrete, but different for different concretes.

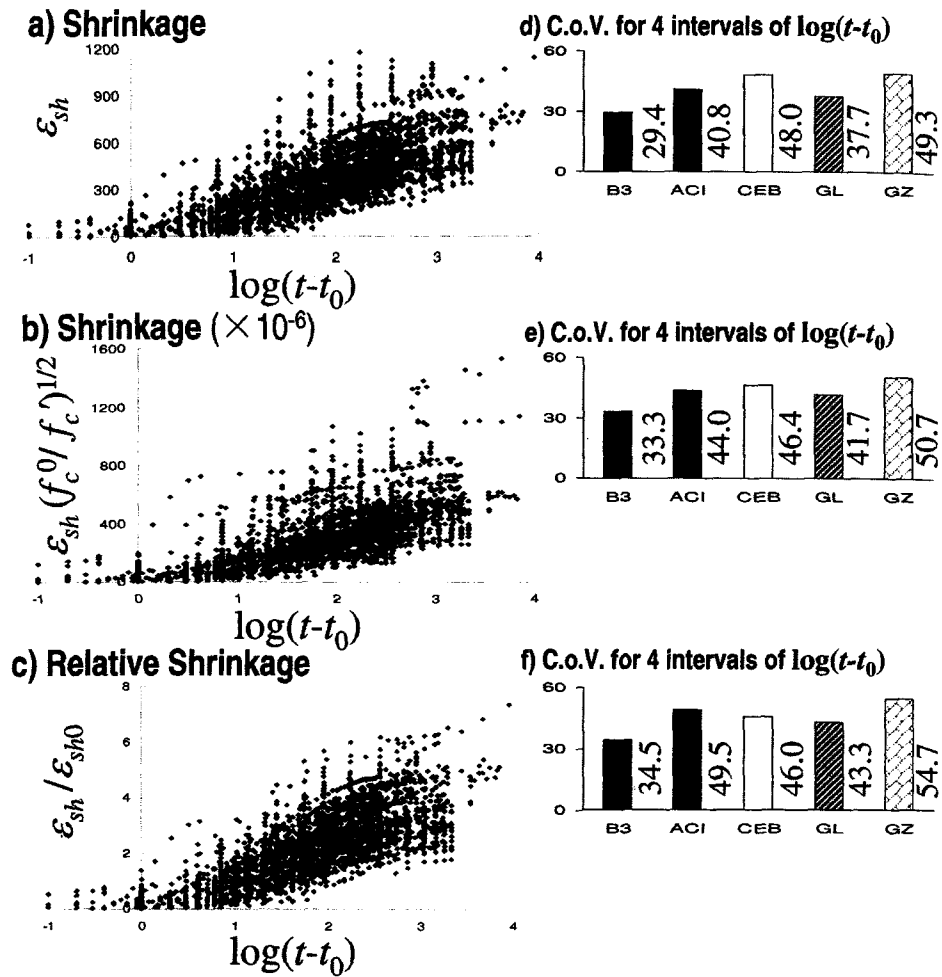


Figure 2. Use of NU-ITI database (left) for comparing 5 different shrinkage prediction models (right) in terms of coefficients of variation of regression errors for shrinkage (multiplied by $\sqrt{f_c^0/f_c'}$) and for relative shrinkage.

The column diagrams in Figs. 2 and 3 compare the C.o.V. ω and the standard deviation s of five models:

1. Model B3 (RILEM Recommendation 1995, slightly updated in 2000 (Bažant & Baweja 2000) (this is a refinement of model BP, Bažant & Panula 1978-1979, of Bažant et al. 1976, and of model BP-KX, Bažant et al. 1991-1992).
2. ACI209 model (labelled “ACI” in figures), 1972 (ACI Committee 209, 1972).
3. CEB Model 1990 (CEB-FIP Model Code 1990, Müller & Hilsdorf 1990), updated in 1999 (FIB 1999), co-opted in 2002 for Eurocode 2.

4. Gardner & Lockman’s (2000) model (labelled “GL”) (Gardner & Lockman 2001).
5. Gardner’s (1997) earlier model (labelled “GZ”) (Gardner & Zhao 1993).

Sakata’s model (Sakata 1993, Sakata et al. 2001), which has a more limited scope, is not evaluated here. Neither are numerous old models (Bažant 1975, 1982, Müller et al. 1999).

The comparisons show that, among all models, model B3, many of whose aspects are justified theoretically (Bažant 2000), gives the lowest prediction errors. Note that, since $d \ln y = dy/y$ (but $d \log y = dy/(y \log e)$), the standard deviation of very small

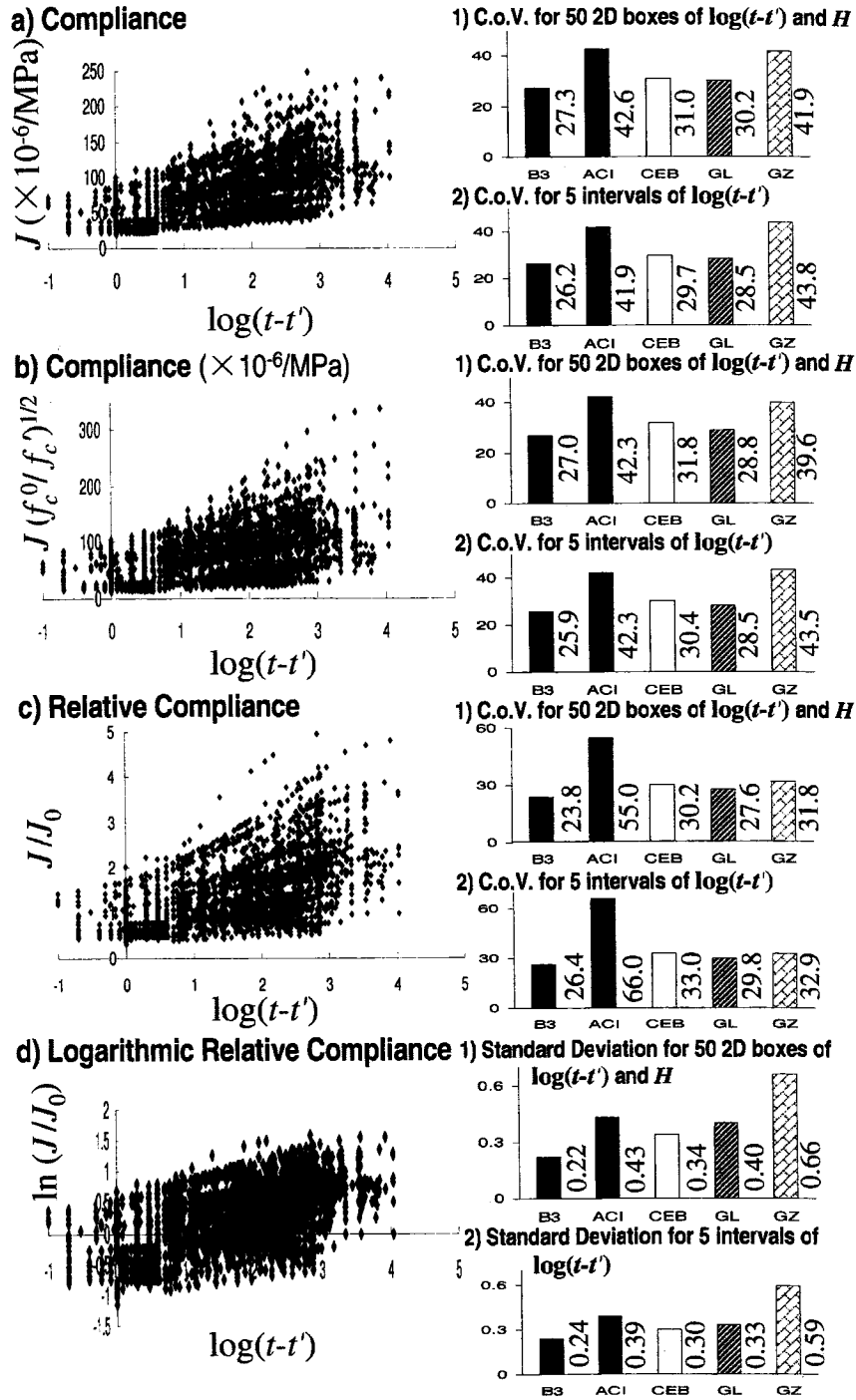


Figure 3. Use of NU-ITI database (left) for comparison of 5 different compliance prediction models (right) in terms of the coefficients of variation of regression errors for compliance (multiplied by $\sqrt{f_c^0/f_c}$) and for relative compliance, and of standard error of regression for logarithm of relative compliance.

errors in $\ln y$ (though not $\log y$) is equal to the C.o.V. of y . It is for this reason that the natural, rather than decadic, logarithm is used in the transformation of y , and that it makes no sense to calculate the C.o.V. of $\ln y$.

1.2 Problems with nonstandard statistical indicators used in other studies

Very different rankings of the models based on various non-standard statistical indicators were recently obtained in several recent papers (Gardner 2000, 2004, McDonald & Roper 1993, Al-Manaseer & Lakshmikanthan 1999, Al-Manaseer & Lam 2005) and in ACI report (ACI Committee 209 2008) [despite being listed among the authors (committee members), Bažant disagrees with the contents]. The problem with these indicators is that they are not based on the method of least squares and on maximum likelihood estimates. It was argued that the statistical approach was debatable, open to choice. True, and some aspects of the approach are indeed debatable, e.g., the weighting, data quality, relevance, omission of outliers, and certain aspects of sampling and transformation of variables. However, the method of least squares, based on the criterion of maximum likelihood, is not. So it may be useful to summarize the reasons, well understood from the work of Gauss (Gauss 1809).

Let $\varepsilon_k = y_k - Y_k =$ model errors, $k = 1, 2, \dots, K$ where $X_k =$ coordinates of data points, $Y_k = F(X_k) =$ predicted values, and function $F(X_k)$ defines the prediction model. The joint probability density distribution of all the data, called the likelihood function \mathcal{L} (Bulmer 1979), is $\mathcal{L} = f(y_1, y_2, \dots, y_k = \prod_{k=1}^K [\phi(y_k)]^{W_k}$ where $\phi(y_m) =$ probability density distribution (pdf) of measurements y_m , and $W_m =$ weight assigned to Y_k .

First assume the errors to be approximately normally distributed, i.e., $\phi(y_k) = \exp[-(y_k - Y_k)^2 / 2s_k^2] / (s_k \sqrt{2\pi})$, $k = 1, 2, \dots, K$, where $s_k =$ standard deviation of y_k , which is known (or knowable) a priori. The optimal data fit must maximize L (Bulmer 1979), which means minimizing $-\ln L$, i.e.,

$$-\ln \mathcal{L} = -\ln \left[e^{-\sum_k W_k (y_k - Y_k)^2 / 2s_k^2} / (s_k \sqrt{2\pi}) \right] \\ = \sum_k W_k (y_k - Y_k)^2 + C = \min \quad (3)$$

where $\omega_k = W_k / 2s_k^2 =$ new weights, and $C = \sum_k W_k \ln(s_k \sqrt{2\pi}) =$ constant. Eq. (3) represents minimization of the sum of squared errors. Hence, the method of least squares is a necessary consequence of the maximum likelihood criterion.

The histograms of the distribution of errors ε_{ij} are generally not broad enough to tell whether they are normal. So what if $\phi(y_k)$ of data y_k are not normal? In that case, the database may be subdivided into groups labelled as $r = 1, 2, \dots, N_g$, each group r containing a sufficient but not excessive number n_r of adjacent points located so closely that the statistical trends within each group are negligible ($n_r \approx 6$ is suitable). The variance s_r^2 of the mean of each data group is the variance of a sum of random variables, and according to the *central limit theorem* (Bulmer 1979, Bouchaud & Potter 2000), the distribution of this sum, and thus the group mean, converges to the normal distribution, albeit one with a scaled standard deviation. We may now assume that the best fit may be obtained as a fit of the group means, which have a Gaussian distribution. The remaining derivation up to Eq. (3) leads again to the same conclusion. Q.E.D.

It must be admitted that, aside from the normal regression problems of concrete design equations discussed here, there exist special problems where the least-square regression is insufficient or even inappropriate. One example is the extreme value statistics, leading to Weibull distribution of strength of brittle structures (Bouchaud & Potters 2000). Another is the extension of the least-square approach to Bayesian optimization, in which the posterior data are supplemented by some kind of prior information (e.g., Ang & Tang 1976). A third example is the case where the role of numerous outliers of heavily tailed non-Gaussian distributions is important, in which case the so-called 'robust regression' (Rousseeuw & Leroy 1987) may be used.

Four non-standard statistical indicators, which give deceptive or irrational ranking of models, may be mentioned:

1. **Gardner's Linear C.o.V.** (Gardner 2004). The logarithmic scales of load duration $t - t'$ and drying duration $t - t_0$ are divided into decades (or intervals), labelled as $j = 1, 2, \dots$. The overall mean of data is obtained as $\bar{y} = (1/N) \sum_{i=1}^n \bar{y}_i$ where $\bar{y}_i = (1/m_i) \sum_{j=1}^{m_i} y_{ij}$; $\bar{y}_j =$ mean of the data in decade j ; and \bar{y} represents the standard expression for a weighted mean. However, Gardner uses a nonstandard expression for the overall C.o.V. of prediction errors, ω_G ,

$$\omega_G = \frac{s_G}{\bar{y}}, \quad s_G = \frac{1}{n} \sum_{i=1}^n s_i, \quad (4)$$

$$\text{where } s_i = \sqrt{\frac{1}{m_i - 1} \sum_{j=1}^{m_i} (y_{ij} - Y_{ij})^2} \quad (5)$$

This is nonstandard and not statistically correct because what is averaged are the errors s_i rather than their squares. The linear averaging of s_i violates the principle of maximum likelihood and is tantamount to denying the validity of the central limit theorem of the theory of probability, underpinning the Gaussian distribution.

The statistical indicator used to compare models must be such that its minimization yields the data fit of maximum likelihood. In the case of Eq. (4), the expression to minimize is:

$$s_G^2 = \frac{1}{n^2} \left(\sum_{i=1}^n \sqrt{\frac{1}{m_i - 1} \sum_{j=1}^{m_i} (y_{ij} - Y_{ij})^2} \right)^2 \quad (6)$$

which is not a quadratic expression. Even if the model were linear, the minimization of Eq. 6 would lead to a system of nonlinear equations, instead of linear regression equations (Bažant & Li 2007). Such a feature is unacceptable.

2. **CEB Coefficient of Variation of Errors** used by Müller & Hilsdorf (1990). it reads:

$$\omega_{CEB} = \sqrt{\frac{1}{n} \sum_{i=1}^n \omega_i^2}, \quad \bar{y}_i = \frac{1}{m_i} \sum_{j=1}^{m_i} y_{ij} \quad (7)$$

$$\omega_i = \frac{1}{\bar{y}_i} \sqrt{\frac{1}{m_i - 1} \sum_{j=1}^{m_i} (Y_{ij} - Y_{ij})^2} \quad (8)$$

(cf. Al-Manaseer & Lakshmikanthan 1999, Al-Manaseer & Lam 2005). This statistic is deficient in a different way: It ignores the statistical trend and applies what is, in effect, population statistics to a data set having a clear trend with respect to an influencing variable. Converting such a problem to a problem of population (or ensemble) statistics (Fig. 1e,f) is not a reasonable statistical approach. This conversion is effected as if the ω_i in the individual intervals were the coefficients of variation of different groups of realizations of one and the same statistical variable with no trend. Another objectionable feature is that, compared to the least-square statistical regression, the short-time data get overemphasized and the long-time data get underemphasized. This is caused by the appearance of \bar{y}_i (rather than \bar{y}) in the denominator of Eq. (8) before all ω_i are combined into one coefficient of variation. An interval with a nearly vanishing \bar{y}_i gives a very large ω_i and thus deceptively dominates the entire statistics.

3. **CEB Mean-Square Relative Error**. It has been defined as (Müller & Hilsdorf 1990):

$$\begin{aligned} S_{CEB} &= \sqrt{\frac{1}{n} \sum_{i=1}^n S_i^2} \\ S_i^2 &= \frac{1}{M} \sum_{j=1}^{m_i} \left(\frac{Y_{ij}}{y_{ij}} - 1 \right)^2 \\ &= \frac{1}{M_1} \sum_{j=1}^{m_i} w_{ij} (Y_{ij} - y_{ij})^2 \end{aligned} \quad (9)$$

where $M_1 = m_i - 1$ (cf. Al-Manaseer & Lakshmikanthan 1999, Al-Manaseer & Lam 2005). Unlike the previous indicators, this one is consistent with the method of least squares, but implies weights $w_{ij} = 1/y_{ij}^2$, which are unrealistic and inappropriate. The errors in the small compliance or shrinkage values get greatly overemphasized, and those in the large values greatly underemphasized. If anything, the opposite should be the case.

4. **C.o.V. of Data/Model Ratios**. For an ideal model, the ratios $r_{ij} = y_{ij}/Y_{ij}$ would be very close to 1. Motivated by this observation and a misguided desire to replace regression statistics by simpler population statistics, concrete researchers have often used the C.o.V. of r_{ij} . But this is not least-square regression. The variance S_R^2 of the population of $r_k = y_k/Y_k$ ($k = 1, 2, \dots, K$) is

$$s_R^2 = \sum_k w_k (r_k - \bar{r})^2, \quad \bar{r} = \sum_k w_k r_k \quad (10)$$

where $\Sigma_k = 1$, and \bar{r} = weighted mean of all r_k . Let now the model Y_k be multiplied by any factor c , i.e., $Y_k \leftarrow cY_k$. Then the variance changes from s_R to \bar{s}_R :

$$\bar{s}_R^2 = \sum_{k=1}^K w_k \left(\frac{y_k}{cY_k} - \sum_{m=1}^K w_m \frac{y_m}{cY_m} \right)^2 = \frac{1}{c^2} s_R^2 \quad (11)$$

So, the variance of the data/model ratios can be made arbitrarily small by multiplying the prediction formula by a sufficiently large number, and the corresponding C.o.V. is independent of c (Bažant 2004). Therefore, minimization of s_R^2 cannot be the objective of data fitting. Minimization of \bar{s}_R^2 also fails the test that, in the case of a linear model, linear regression must result. Another problem for creep is that the differences $1 - r_{ij}$ tend to be the greatest for short times and small Y_k , which thus dominate the statistics.

To check the soundness of the foregoing four non-standard indicators, one may consider the case of a linear model. In that special case, it is reasonable to require that the minimization of the indicator must reduce to the standard linear regression. But it does not, and the differences can be huge (Bažant & Li 2007).

The rankings of prediction models obtained with the aforementioned nonstandard statistical indicators are quite different from that obtained by from the standard least-square regression (Bažant & Li 2007, ACI Committee 209, 2008).

2 CRITICAL APPRAISAL OF STRUCTURAL ANALYSIS METHODS

2.1 Accuracy of principle of superposition and phenomena causing deviations

For basic creep, i.e., the creep at constant temperature and moisture content, there is no evidence of systematic (deterministic) deviations from the principle of superposition, i.e., from the solutions based on aging linear viscoelasticity). The relaxation tests of sealed specimen at constant temperature are predicted by the principle of superposition from the compliance data as closely as can be expected in view of the inevitable statistical scatter (RILEM 1988). However, major deviations from the principle of superposition, which are apparent rather than physically real, are caused by the neglect of the following three phenomena:

1. Random scatter of creep and shrinkage, which is of two kinds:
 - a. Intrinsic material scatter, whose C.o.V. is typically only about 8%.
 - b. Additional apparent scatter due to ignoring, or not knowing, randomly varying influencing variables, whose C.o.V. can exceed 50%.
2. Hygrothermal effects, including water diffusion, and time evolution of the distributions of pore moisture content and temperature.
3. Material damage, including distributed cracking and fracture, and also frictional micro-slips. In predicting the structural effects of creep and shrinkage, the result based solely on the principle of superposition can get changed by these phenomena by a factor of 2 or more.

Therefore, an analysis in which the equations of aging linear viscoelasticity are solved accurately makes sense only if the foregoing three phenomena are taken into account with comparable accuracy. If they are not, the prediction error is about the same whether or not the equations of linear aging viscoelasticity are solved accurately or with errors up to about

50%. In other words, if the foregoing three phenomena are neglected, it makes no sense to complicate analysis by striving for accurate solutions according to aging linear viscoelasticity. Even a crude and simple approximation is then sufficient.

The solutions of structural problems according to linear aging viscoelasticity cannot be simply adapted to obtain a realistic and accurate generalization for the effects of hygrothermal conditions, cracking, fracture and material randomness. Such a generalization must be made by replacing the linear viscoelastic constitutive law with a generalization taking these phenomena into account.

2.2 Methods of solving the linear aging viscoelastic problem

The Volterra integral equations of structural creep problems can be solved analytically, without a computer, only for some simple forms of the compliance function. Unfortunately, such forms (which have been proposed in earlier times by many researchers, e.g., Macmillan, Glanville, Dischinger, Arutyunian, Ulickii, Prokopovich, Aleksandrovskii, Růsch & Jungwirth, Illston, Nielsen) give only a poor representation of multi-year creep and creep for high ages at loading (Bažant & Osman 1975, Cinlar et al. 1977, RILEM 1988). These earlier forms, popular before the computer age, have by now been abandoned. A numerical solution can now be easily obtained for any form of the compliance function, as identified by experiments.

2.2.1 Accurate solution as a sequence of elasticity problems with initial strains

A completely general, accurate and most effective approach is to obtain first the incremental form of the aging linear viscoelastic constitutive relations for a small time step (Bažant 1967, 1975, 1982). There are two ways to do that:

1. One is an incremental form that ensues by replacing the integral over the past stress or strain history with a sum (Bažant & Najjar 1972, Bažant 1975, 1982). In that case one must store in computer memory the entire history of stress or strain at each point. This storage requirement used to be a computational burden, but for modern computers it matters only for systems with a huge number of unknowns.
2. Another way is to convert the integral-type creep law to a rate-type creep law with internal variables (Bažant 1971, 1982, RILEM 1988). In that case, the history need not be stored because it is implied by the current values of a few hidden variables. This approach allows a much more efficient computation (using the exponential algorithm; Bažant 1971, 1982, Jirasek & Bažant 2002). Further it allows separating the effects of variable pore humidity h or

temperature T on creep viscosity from those on the aging rate (Bažant 1994). The dichotomy of these effects can be realistically described only through the rate-type form.

The incremental form represents a linear elastic stress-strain relation with initial strains (analogous to thermoelasticity), in which the elastic moduli and initial strains vary throughout the body, and from step to step. Thus the problem of aging linear viscoelasticity gets converted, already at the constitutive level, to a sequence of elasticity problems, as proposed in (Bažant 1967). Thus any elastic finite element program, used repeatedly in a loop, can be generalized for creep. This eliminates any need to ever formulate the Volterra integral equations governing structural variables.

2.2.2 Accurate solutions via equations for time evolution of structural variables

Mathematicians nevertheless prefer to have a set of equations defining each particular problem. It has perhaps been for this reason that an Italian school of researchers has worked on setting up the Volterra integral equations for the structural variables in various problems, and then conduct their direct numerical solutions. Such solutions may be obtained by approximating the history integrals with sums, which yields recursive matrix equations for structural variables (which are in fact equivalent to a sequence of elastic solutions). But although this alternative approach may be an interesting exercise, for an engineer it only represents an unnecessary complication.

2.2.3 Age-adjusted effective modulus method (AAEM)

The AAEM readily follows from the following fundamental theorem, found and proven by Bažant (1972, 1975) (see also p. 568, Jirásek & Bažant 2002).

If the strain history is $\epsilon = a + bJ(t, t')$, then the stress history is $\sigma = aR(t, t') + b$ where $a, b =$ constants and $R(t, t') =$ relaxation function corresponding to compliance function $J(t, t')$.

Matching the foregoing expressions for σ and ϵ to Hooke's law yields for AAEM an incremental elastic equation which allows approximate algebraic solution of creep problems. It converts the creep analysis to an elastic analysis with elastic moduli and initial strains which vary throughout the structure. It gives, in one large step, the change from the initial state after first applying the load to the final state at time t . While the classical effective modulus method is exact only if the stress is constant, the AAEM is exact whenever the stress histories at all points of the structure are linear transformations of the compliance function (or of the creep coefficient). This is usually a good approximation for stress variation under a steady load

(Bažant & Najjar 1973). If the loading history has a jump, as in the case of a change of structural system, two AAEM solutions may be superposed, to achieve good accuracy.

The AAEM could also be regarded as a rigorous reformulation and generalization of Trost's method (RILEM 1988). Both Trost and Bažant (as well as some Russian researchers) intended to correct the errors of the classical effective modulus method (RILEM 1988) by multiplying the creep coefficient with a corrective parameter less than 1. Trost, by semi-intuitive reasoning, proposed a corrective parameter attributed to stress relaxation, called the 'relaxation coefficient', which applied only for the case of non-aging elastic modulus. But Bažant (Bažant 1972) showed that the relaxation causes virtually no error, and that the error stems from aging in both the creep coefficient and the elastic modulus. Hence, the corrective parameter is called the aging coefficient.

2.2.4 Generalizations of AAEM by Lazić, Khazanovich, Bažant and Chiorino

Although the AAEM is often highly accurate (Bažant & Najjar 1973), there are some problems where it is not. One problem where the accuracy of AAEM is acceptable (Bažant & Najjar 1973) but not high is the composite cross section, for which AAEM has been generalized by Lazić & Lazić (1984, 1985). Another such problem may occur for large inhomogeneous systems with many statically indeterminate forces and, to deal with it more accurately, the aforementioned theorem underlying the AAEM has been generalized to a matrix form in which σ and ϵ are replaced by vectors of structural variables, and a and b by constant square matrices (Bažant 1987). The problems where the load changes abruptly, or where the structural system undergoes a sudden change, may be solved by superposition of AAEM (or generalized AAEM) solutions. For arbitrary time variations of load, the AAEM was generalized by Khazanovich (1990), using superposition of infinitesimal loading steps. Recently, Chiorino (2005), Chiorino et al. (2002) and Chiorino & Sassone (2008) independently obtained the same generalizations in an equivalent but somewhat different form. Chiorino with co-workers developed the so-called 'redistribution function' for changes of structural systems, which is a special case of the matrix generalization of AAEM and of Khazanovich's generalization to variable load.

However, all these generalizations of AAEM are quite a bit more complex than the original AAEM itself, yet hardly more realistic for practice, because they do not allow corrections for hygrothermal and diffusion effects, and for cracking damage, as explained later. Such corrections need to be introduced at the constitutive level of material, before structural analysis.

2.3 *Random scatter, uncertainty of prediction, and confidence limits*

The frequency of serviceability loss would doubtless be greatly reduced if structures were designed not for the mean prediction, but for the 95% and 5% confidence limits of prediction. Applying some uniform safety factor to the mean prediction is not a remedy, because these confidence limits may be very close to the mean for one kind of response, but far for another kind.

An effective method to estimate the confidence limits is to run a deterministic program for creep and shrinkage analysis about ten times, each time for different samples of random input variables, generated most effectively as the Latin hypercube samples (Bažant & Liu 1985, Xi & Bažant 1989, Teplý et al. 1996). The random input variables are the material parameters (e.g., the concrete strength and water-cement ratio), the environmental factors (the relative environmental humidity and temperature, and possibly amplitude and frequency of cycles), and the load. The computed values of each response are plotted (or considered as plotted) on the normal probability paper, and upon fitting them with the normal distribution one gets the confidence limits of each response. For a box girder bridge, this was demonstrated by Křístek & Bažant (1987) (who also showed that the hypothesis of cross sections remaining plane everywhere is often very poor for box girders and that the shear lag must be taken into account).

The uncertainty in predicting the structural effects of shrinkage and drying creep can be greatly reduced by adopting the Bayesian statistical approach, which allows improving the prediction based on prior data, e.g., the data from previous observations on similar concretes in the given geographical region; see Bažant & Chern (1984) on the material level, and Bažant & Kim (1989) on the structural level of a box girder bridge.

A further reduction of prediction uncertainty can be achieved by using spectral stochastic analysis to take into account the periodic fluctuation of environment (Bažant & Xi 1993, Bažant & Wang 1985). The spectral components with a short period of environmental humidity influence only a thin surface layer, while the components with a long period (annual period) can influence the core of cross sections.

The differences among different types of concrete are the greatest source of statistical scatter. Hence the uncertainty in long-time (say, 50-year) prediction of the effects of creep and shrinkage can be greatly reduced by conducting first, on the given concrete, short-time tests of 1 to 3 month duration, and then fitting the data by least-squares. Among the existing creep and shrinkage prediction models, B3 is the only one having a mathematical form for

which the least-square fitting of short-time tests can be converted to a linear regression, which is a simple task with unambiguous outcome. Even rescaling of model B3 according to the compliance measurement for short-time loading (~ 1 min. to peak) helps enormously.

A drastic reduction in the uncertainty of long-time prediction of shrinkage and drying creep can be achieved if the shrinkage test is accompanied by measurements of water loss and total evaporable water content (Bažant & Baweja 2000, Křístek et al. 2001). These procedures have been experimentally documented and applied in bridge designs to reduce uncertainty in creep and shrinkage effects; see Navrátil (1998a, b) and Fig. 1.4 in Bažant & Baweja (2000) based on data of Granger (1995).

Three kinds of statistical scatter in creep and shrinkage predictions may be distinguished:

1. *Intrinsic*;
2. *Extrinsic incognizable*, due to influences that are unknown; and
3. *Extrinsic cognizable*, due to knowingly ignoring the effects of randomly variable parameters that are already understood.

The first is what is observed in the testing of one and the same concrete (after separating measurement errors), and is quite small (generally under $\pm 8\%$; Bažant et al. 1987). The second appears mainly because the influence of concrete composition is not yet well understood. The third is caused by unrealistic oversimplifications of analysis (e.g., using a cross section compliance rather than the material constitutive relation, or the 'redistribution function'; while omitting a solution of cracking and size effects and of the variation of pore humidity distributions inside cross section, etc.). The second and third sources may cause a scatter exceeding $\pm 50\%$, even $\pm 100\%$. Of course, such uncertainty is often acceptable (e.g., for a small-span bridge insensitive to creep). However, to demonstrate that it is so, one must base the design on the 95% confidence limit rather than the mean prediction, in order to avoid trouble later.

Finally, while discussing scatter it must be admitted that creep and shrinkage are actually stochastic processes in time because the randomness is properly characterized by probability distribution of the deformation increments in time rather than of the total values. However, modeling of this aspect (Cinlar et al. 1977) is not yet ready for design applications and does not seem too important.

2.4 *Hygrothermal effects and cracking*

Aside from shrinkage, exposure to drying environment also causes a large increase of creep, called the

drying creep, or Pickett effect (Bažant 1975, 1982), or stress-induced shrinkage. This cross effect has two sources—one *apparent*, due to the cracking effect on shrinkage, and one *intrinsic*, due to micro-diffusion of water molecules through the nanopores of cement gel (Bažant & Xi 1994). The profiles of relative humidity in the pores of concrete are nonlinear and evolve with time according to a nonlinear diffusion equation (Bažant & Najjar 1972). Together with shrinkage, these phenomena produce self-equilibrated internal stresses (with an evolving nonlinear profile throughout the thickness of the cross section slabs), and cause microcracking and fracture (Bažant et al. 1992, Bažant et al. 2003).

Cracking is not only a consequence of shrinkage and nonuniform creep, but it is also the cause of apparent creep and rate effects. Fracture and distributed cracking are rate-dependent processes and are the cause, probably the only cause, of nonlinear dependence of apparent creep strain on stress at high stress levels (Bažant 1993, 1995, Bažant & Li 1997).

The effect of cracking may be to either increase or decrease the overall deformations due to creep and shrinkage. The latter occurs when the stresses can be transferred from concrete to reinforcement. This is the main reason why time-dependent deflections of reinforced concrete beams are much smaller than those of prestressed ones, as shown by Dischinger already in the 1930s (Bažant 1966). The increase occurs if such transfer cannot take place.

The half-times of drying and of heating or cooling vary in proportion to the square of the effective thickness D of cross section. For drying, they are about 1000-times longer than for heating or cooling (RILEM 1988, Bažant 2000, Bažant & Zebich 1983). The heating or cooling penetration front advances into a concrete halfspace as a square root of the duration of exposure and within the same time reaches about 30-times deeper than the drying front. Ditto for the depth affected by periodic environmental h and T . This means that the temperature problem can usually be uncoupled and solved in advance of the drying problem (Bažant et al. 2003).

When $J(t, t')$ is used to characterize, in the mean, the whole cross section (rather than a material point), the microcracking, and the diffusion and micro-diffusion effects, jointly cause major, though apparent, deviations from the principle of superposition.

If the drying-induced nonlinear stress distributions are ignored, as is typical of beam-type analysis, the stresses can have errors over $\pm 50\%$. Typically the humidity profiles due to drying are nonsymmetric with respect to the neutral axis, which may cause major errors in deflection predictions (Křístek et al. 2006). Then it makes no sense to seek solutions in which only the linear viscoelasticity is taken into account accurately.

In the simplest (but rather approximate) way, the nonuniformity of drying can be captured with a model such as B3 if applied to each slab of cross section separately. This way it has been shown (Křístek et al. 2006) that deflection predictions for a box girder bridge can change by several hundred percent and the predicted time evolution becomes drastically different (a very slow deflection growth for five to ten years, followed by a rapid catch-up). Also, it has been explained in this way, with the help of model B3, why excessive deflections, several times larger than predicted, have typically occurred in prestressed box girder bridges (Křístek & Víttek 1999, Křístek et al. 2006).

A more accurate analysis, however, requires subdividing the cross section into many finite elements. Such programs have been developed and demonstrated. In a simpler version, the time evolution of humidity profiles across the thickness of the slabs in the cross section may be obtained by simple one-dimensional analysis (based on a nonlinear diffusion equation; Bažant & Najjar 1972), and only the stresses are analyzed by two- or three dimensional finite elements. To simulate the distributed cracking, spurious damage localization must be avoided by using the crack-band (or nonlocal) model based on correct spacing of parallel cracks (Bažant et al. 2003, e.g.), where such analysis was demonstrated for the effect of creep on the stresses due to hydration heat in a nuclear containment.

Multi-dimensional finite element analysis necessitates a constitutive law, which must involve only local variables, such as pore humidity h and temperature T , but not the environmental conditions nor effective thickness D of the body. To deal with material damage, and with variable h and T , the local compliance function must be converted to a rate-type form. Only with the rate-type form it is possible to meet the thermodynamic restrictions and introduce physical concepts such as solidification and polymerization of C-S-H (Bažant & Prasanna 1989), the activation energies of creep and hydration reaction, and the relaxation of nano-scale microstress (i.e., the self-equilibrated stress field in the nanostructure of C-S-H) (Bažant et al. 1997, 2004).

The microstress relaxation is a physical concept that explains in a unified way the long-term aging (i.e., the aging after the chemical process of cement hydration has ceased), as well as the drying creep and the transitional thermal creep (i.e., creep intensification after a rapid change of h or T , or both). This concept has led to the rate-type constitutive law of the microstress-solidification theory (see Bažant et al. 1997, Bažant et al. 2004, and for an encyclopedic summary Bažant 2001). Together with the equations for heat transfer, moisture diffusion, sorption isotherms, and cracking damage, this theory provides a general material model for concrete creep

and shrinkage at variable humidity and temperature, necessary for accurate finite element computer simulations, requisite though for special sensitive structures only. This theory has, for instance, been used to analyze short-time creep and fracture in a novel procedure in which powerful microwave blasts are used to strip from concrete a thin surface layer contaminated by radionuclides (Bažant & Zi 2003).

The rate-type constitutive law, coupled with a damage law and based on extension of $J(t, t')$ into the dynamic range, was used in finite element predictions of impact (Bažant et al. 2000).

2.5 How useful are accurate viscoelastic solutions?

2.5.1 Effects of random scatter, hygrothermal effects and cracking

For these phenomena to be simulated realistically, they must be combined in the computational model with an accurate creep and shrinkage model and with a solution algorithm that accurately captures the linear viscoelastic part of the problem. This must be done at the constitutive level. It makes no sense to take a linear aging viscoelastic solution of structural system based on, e.g., Chiorino's redistribution function, and try to adapt it somehow for hygrothermal effects and cracking damage.

If these phenomena are either ignored or modeled in an oversimplified and poor way, errors of the order of 100% or more may occur. Then the accurate solutions of aging linear viscoelasticity bring about no benefit, and so it makes no sense to use them. The simplest approach, that is AAEM, is then sufficient.

The same is true for the more accurate generalizations of AAEM (Lazić & Lazić 1984, 1985, Bažant 1987, Khazanovich 1990, Chiorino 2005, Chiorino et al 2002), and particularly for the use of Chiorino's "redistribution function".

Ignoring the random scatter, hygrothermal effects and cracking is not the only situation that takes away the benefit expected from the more accurate linear viscoelastic approximations with the redistribution function. This function can be used only in structural parts that behave as homogeneous beams. But this is not the case when the age of concrete, or the thickness of the cross section slabs, varies significantly along the beams. Hence, there is hardly any practical application where the use of AAEM generalizations, including the redistribution function, would be more than a mere mathematical exercise.

2.5.2 Ignoring shear lag in box girders

Trying to predict the behavior of prestressed concrete box-girder bridges, which are notorious for strong shear lag, some Italian researchers have been applying accurate solutions of the linear viscoelastic Volterra

integral equations for statically indeterminate internal forces in a framed (beam) structure, or the corresponding solutions obtained by generalizations of AAEM ('redistribution function'). It must be kept in mind, though, that the validity of these solutions depends, aside from other aspects, also on the hypothesis of the cross sections remaining plane. Due to shear lag, however, this hypothesis is known to be grossly in error for typical highway box girders, especially when their width-to-span ratio is not small (Křístek & Bažant 1987). In such structures, the shear lag has a large effect on deflections and on the overall distribution of internal forces.

Unfortunately, the shear lag is not taken into account in Chiorino's redistribution function. Consequently, there must be a large error due to shear lag in these predictions. In that case, an accurate solution of aging linear viscoelasticity, or a generalization of AAEM (such as Chiorino's redistribution function, or matrix AAEM), would be a waste of effort for a design firm. It would not achieve any tangible improvement over the use of simple AAEM.

On the other hand, when the box girder is analyzed by 3D or shell-type finite elements, the shear lag is automatically accounted for. Then it does make sense to begin with a linear aging viscoelastic constitutive law in rate form, provided, of course, that it is extended at the material constitutive level to also take the three aforementioned phenomena into account.

3 CLOSING COMMENT

Creep and shrinkage have been a pervasive cause of damage and excessive deflections in structures, and long-time creep buckling has caused a few collapses. E.g., in the case of the Koror-Babeldaob Bridge in Palau, a prestressed box girder that had the world-record span of 241 m span when built in 1977, the long-time deflection reached 5 ft. (1.52 m) by 1996, which was 4 ft. (1.24 m) more than predicted. An ill-fated attempt to remedy it led to collapse, with two fatalities. Avoiding repetitions depends on using the most realistic material model and structural analysis method.

REFERENCES

- ACI Committee 209. 1972. "Prediction of creep, shrinkage, and temperature effects in concrete structure." ACI-SP-194; reapproved in 1982 (ACI-SP-76) and 1992 (ACI-209 T92).
- ACI Report 209.2R-08: Guide for Modeling and Calculating Shrinkage and Creep in Hardened Concrete, ACI, 2008.
- Al-Manaseer, A. & Lakshmikanthan, S. 1999. "Comparison between current and future design code models for creep

- and shrinkage". *Revue française de génie civil* 3 (3-4), 39-40.
- Al-Manaseer, A. & Lam, J.-P. 2005. "Statistical evaluation of creep and shrinkage models." *ACI Materials Journal* 102 (May-June), 170-176.
- Ang, A.H.-S. & Tang, W.H. 1976. *Probability concepts in engineering planning and design*. Vol. 1, Sec.7, J. Wiley, New York.
- Ang, A.H.-S. & Tang, W.H. 1984. *Probability concepts in engineering planning and design*. Vol II. Decision, Risk and Reliability. J. Wiley, New York.
- Bažant, Z.P. 1966. *Creep of Concrete in Structural Analysis* (in Czech). State Publishers of Technical Literature (SNTL), Prague.
- Bažant, Z.P. 1967. "Linear creep problems solved by a succession of generalized thermoelasticity problems." *Acta Technica SAV* (Prague), 12, 581-594.
- Bažant, Z.P. 1971. "Numerically stable algorithm with increasing time steps for integral-type aging creep." *Proc., First Intern. Conf. on Struct. Mech. in Reactor Technology*. (SMIRT-1, held in Berlin), ed. by T.A. Jaeger, Commission of Eur. Communities, Brussels, Vol. 4, Part H, 119-126.
- Bažant, Z.P. 1975. "Theory of creep and shrinkage in concrete structures: A precis of recent developments." *Mechanics Today*, ed. by S. Nemat-Nasser (Am. Acad. Mech.), Pergamon Press 1975, Vol. 2, pp. 1-93.
- Bažant, Z.P. 1982. "Mathematical models for creep and shrinkage of concrete." Chapter 7 in *Creep and Shrinkage in Concrete Structures*, Z.P. Bažant and F.H. Wittmann, eds., J. Wiley & Sons, London, 1982, 163-256.
- Bažant, Z.P. 1987. "Matrix force-displacement relations in aging viscoelasticity." *J. of Engrg. Mechanics*, ASCE 113 (8), 1235-1243.
- Bažant, Z.P. 1988. "Material models for structural creep analysis" Chapter 2 in *Mathematical Modeling of Creep and Shrinkage of Concrete*, Z.P. Bažant, ed., J. Wiley, Chichester & New York, 99-200.
- Bažant, Z.P. 1993. "Current status and advances in the theory of creep and interaction with fracture." *Proc., 5th International RILEM Symposium on Creep and Shrinkage of Concrete (ConCreep 5)*, held at U.P.C., Barcelona, September, Z.P. Bažant and I. Carol, eds., E & FN Spon, London, 291-307.
- Bažant, Z.P. 1994. "Creep and thermal effects in concrete structures: A conspectus of some new developments." *Proc., Computational Modelling of Concrete Structures (EURO-C)*, held at Innsbruck, Austria, March, Pineridge Press, 461-480.
- Bažant, Z.P. 1995. "Creep and Damage in Concrete." *Materials Science of Concrete IV*, J. Skalny and S. Mindess, Eds., Am. Ceramic Soc., Westerville, Ohio, 355-389.
- Bažant, Z.P. 2000. "Criteria for Rational Prediction of Creep and Shrinkage of Concrete, Criteria for rational prediction of creep and shrinkage of concrete." *Creep and Shrinkage-Structural Design Effects*, ACI SP-194, A. Al-Manaseer, ed., Am. Concrete Institute, Farmington Hills, Michigan, pp. 237-260.
- Bažant, Z.P. 2001. "Creep of concrete," in *Encyclopedia of Materials: Science and Technology*, K.H.J. Buschow et al., eds. Elsevier, Amsterdam, Vol. 2C, 1797-1800.
- Bažant, Z.P. 2004. Discussion of "Shear database for reinforced concrete members without shear reinforcement," by K.-H. Reineck, D.A. Kuchma, K.S. Kim & S. Marx, *ACI Structural Journal* 101 (Feb.), 139-140.
- Bažant, Z.P. 2006. "Critical Appraisal of Methods of Creep and Shrinkage Analysis of Concrete Structures." Report to ACI Committee 209, Northwestern University, Evanston.
- Bažant, Z.P. & Baweja, S. 2000. "Creep and shrinkage prediction model for analysis and design of concrete structures: Model B3." *Adam Neville Symposium: Creep and Shrinkage—Structural Design Effects*, ACI SP-194, A. Al-Manaseer, ed., pp. 1-83 (update of paper published in *Materials and Structures* Vol. 28, 1995, pp. 357-365, 415-30, and 488-95).
- Bažant, Z.P., Caner, F.C., Adley, M.D. & Akers, S.A. 2000. "Fracturing rate effect and creep in microplane model for dynamics." *J. of Engrg. Mechanics* ASCE 126 (9), 962-970.
- Bažant, Z.P. & Chern, J.-C. 1984. "Bayesian statistical prediction of concrete creep and shrinkage." *Am. Concrete Inst. Journal* 81, 319-330.
- Bažant, Z.P., Cusatis, G. & Cedolin, L. 2004. "Temperature effect on concrete creep modeled by microprestress-solidification theory." *J. of Engrg. Mechanics* ASCE 130 (6) 691-699.
- Bažant, Z.P., Hauggaard, A.B., Baweja, S. & Ulm, F.-J. 1997. "Microprestress-solidification theory for concrete creep. I. Aging and drying effects," *J. of Engrg. Mech.* ASCE 123 (11), 1188-1194.
- Bažant, Z.P., Kim, Jin-Keun & Jeon, Sang-Eun 2003. "Cohesive fracturing and stresses caused by hydration heat in massive concrete wall." *J. of Engrg. Mech.* ASCE 129 (1), 21-30.
- Bažant, Z.P. & Kim, Joong-Koo. 1989. "Segmental box girder: deflection probability and Bayesian updating." *Journal of Structural Engineering* ASCE, 115 (10) 2528-2547.
- Bažant, Z.P. & Kim, Joong-Koo. 1991. "Consequences of diffusion theory for shrinkage of concrete." *Materials and Structures* (RILEM, Paris) 24 (143), 323-326.
- Bažant, Z.P., Kim, Joong-Koo & Panula, L. 1991-1992. "Improved prediction model for time-dependent deformations of concrete." *Materials and Structures* (RILEM, Paris) Vol. 24 (1991), pp. 327-345, 409-21, Vol. 25 (1992), pp. 21-28, 84-94, 163-169.
- Bažant, Z.P., Křístek, V. & Vitek, J.L. 1992. "Drying and cracking effects in box-girder bridge segment." *J. of Structural Engineering* ASCE 118 (1), 305-321.
- Bažant, Z.P. & Li, G.-H. 2007. "Unbiased Statistical Comparison of Creep and Shrinkage Prediction Models." *Structural Engineering Report No. 07-12/A210u*, Northwestern University, Evanston (submitted to *ACI Materials Journal*).
- Bažant, Z.P. & Li, G.-H. 2008. "Unbiased Statistical Comparison of Creep and Shrinkage Prediction Models." *Structural Engineering Report 07-12/A210u*, Northwestern University, Evanston, Illinois.
- Bažant, Z.P. & Li, G.-H. 2008. "Database on Concrete Creep and Shrinkage." *Infrastructure Technology Institute (ITI)*, Northwestern University, Report (in preparation, to be posted on ITI website).
- Bažant, Z.P. & G.-H. Li 2008. "Comprehensive Database on Concrete Creep and Shrinkage." *Structural Engineering*

- Report 02-08/A210u, Northwestern University, Evanston, Illinois.
- Bažant, Z.P. & Li, Yuan-Neng. 1997. "Cohesive crack with rate-dependent opening and viscoelasticity: I. Mathematical model and scaling." *Int. J. of Fracture* 86 (3), 247–265.
- Bažant, Z.P. & Liu, K.-L. 1985. "Random creep and shrinkage in structures: Sampling." *J. of Structural Engrg.* ASCE 111, 1113–1134.
- Bažant, Z.P. & Najjar, L.J. 1972. "Nonlinear water diffusion in nonsaturated concrete." *Materials and Structures (RILEM, Paris)*, Vol. 5, pp. 3–20 reprinted in "Fifty Years of Evolution of Science and Technology of Building Materials and Structures," F.H. Wittmann, ed., RILEM, publ. by Aedificatio Publishers, Freiburg, Germany, 1997, 435–56).
- Bažant, Z.P. & Najjar, L.J. 1973. "Comparison of approximate linear methods for concrete creep." *J. of Structural Division, Proc. Am. Soc. Civil Engrs.*, 99, ST9, 1851–1874.
- Bažant, Z.P. & Osman, E. 1975. "On the choice of creep function for standard recommendations on practical analysis of structures." *Cement and Concrete Research*, 5, 129–137; disc. & reply 5, 631–641; 6 (1976) 149–157; 7 (1977) 119–130; 8 (1978) 129–130.
- Bažant, Z.P., Osman, E. & Thonguthai, W. 1976. "Practical formulation of shrinkage and creep in concrete." *Materials and Structures (RILEM, Paris)*, Vol. 9, pp.395–406.
- Bažant, Z.P. & Panula, L. 1978–79. "Practical prediction of time-dependent deformations of concrete." *Materials and Structures* Vol.11, pp. 307–328, 415–34, Vol. 12, 169–183.
- Bažant, Z.P. & Prasannan, S. 1989. "Solidification theory for concrete creep: I. Formulation, and II. Verification and application." *J. of Engineering Mechanics ASCE* 115 (8) 1691–1703 and 1704–1725.
- Bažant, Z.P. & Raftshol, W.J. 1982. "Effect of cracking in drying and shrinkage specimens." *Cement and Concrete Research*, 12, 209–226.
- Bažant, Z.P. & Wang, T.S. 1985. "Practical prediction of cyclic humidity effect in creep and shrinkage of concrete." *Materials and Structures (RILEM, Paris)*, 18 (106), 247–252.
- Bažant, Z.P., Wittmann, F.H., Kim, Jenn-Keun & Alou, F. 1987. "Statistical extrapolation of shrinkage data—Part I: Regression." *ACI Materials Journal*, 84, 20–34.
- Bažant, Z.P. & Xi, Y. 1993. "Stochastic drying and creep effects in concrete structures." *J. of Structural Engineering*, ASCE, 119(1), 301–322.
- Bažant, Z.P. & Xi, Y. 1994. "Drying creep of concrete: Constitutive model and new experiments separating its mechanisms." *Materials and Structures* 27, 3–14.
- Bažant, Z.P. & Zebich, S. 1983. "Statistical linear regression analysis of prediction models for creep and shrinkage." *Cement and Concrete Research*, 13, 869–876.
- Bažant, Z.P. & Zi, G. 2003. "Decontamination of radionuclides from concrete by microwave heating. I. Theory, and II. Computations." *J. of Engrg. Mech.* ASCE 129 (7), 785–792 and 777–784.
- Bouchaud, J.P. & Potters, M. 2000. *Theory of financial risks: From statistical physics to risk management*. Cambridge University Press, Cambridge, UK.
- Bulmer, M.G. 1979. *Principles of Statistics*. Dover Publ., New York (chapter 12).
- CEB-FIP Model Code 1990. *Model Code for Concrete Structures*. Thomas Telford Services Ltd., London, Great Britain; also Comité euro-international du béton (CEB), Bulletins d'Information No. 213 and 214, Lausanne.
- Chiorino M.A. 2005. "A Rational Approach to the Analysis of Creep Structural Effects," in *Shrinkage and Creep of Concrete*, J. Gardner and J. Weiss, eds., ACI SP-227, pp. 107–141.
- Chiorino M.A., Lacidogna G. & Segreto A. 2002. Design Criteria for Long-term Performance of Concrete Structures Subjected to Initial Modifications of Static Scheme, in *Concrete Structures in the 21st Century, Proc., 1st fib Congress, Osaka*, pp. 285–294.
- Chiorino, M. & Sassone, M. 2008. "Time Dependent Effects in Concrete Structures." Preliminary Report, Politecnico di Torino, submitted to ACI 209 Subcommittee 6.
- Cinlar, E., Bažant, Z.P. & Osman, E. 1977. "Stochastic process for extrapolating concrete creep." *J. of the Engrg. Mech. Div., ASCE*, 103, 1069–088; Disc. 1979, 485–89.
- FIB 1999. *Structural Concrete: Textbook on Behaviour, Design and Performance, Updated Knowledge of the of the CEB/FIP Model Code 1990*. Bulletin No. 2, Fédération internationale de béton (FIB), Lausanne, Vol. 1, pp. 35–52.
- Gardner, N.J. 2000. "Design provisions for shrinkage and creep of concrete." *Adam Neville Symposium: Creep and Shrinkage—Structural Design Effects*, ACI SP-194, A. Al-Manaseer, ed., pp.101–104.
- Gardner, N.J. 2004. "Comparison of prediction provisions for drying shrinkage and creep of normal strength concretes." *Canadian Jour. of Civil Engrg.* 31 (5), Sep.-Oct., 767–775.
- Gardner N.J. & Lockman M.J. 2001. "Design provisions for drying shrinkage and creep of normal strength." *ACI Materials Journal* 98 (2), Mar.-Apr., 159–167.
- Gardner, N.J. & Zhao, J.W. 1993. "Creep and shrinkage revisited." *ACI Materials Journal* 90, 236–246. Discussion by Bažant and Baweja, (1994) *ACI Materials Journal* 91, 204–216.
- Gauss, K.F. 1809. *Theoria Motus Corporum Caelestium*. Hamburg (reprinted by Dover Publ., New York 1963).
- Granger, L. 1995. "Comportement différé du béton dans les enceintes de centrales nucléaires: analyse et modélisation." PhD Thesis, ENPC Paris; Research Report of Laboratoire centrale des ponts et chaussées, Paris.
- Haldar A. & Mahadevan, S. 2000. *Probability, reliability and statistical methods in engineering design*. J. Wiley, New York
- Jirásek, M. & Bažant, Z.P. 2002. *Inelastic Analysis of Structures*. J. Wiley & Sons, London and New York.
- Khazanovich, L. 1990. "Age-adjusted effective modulus method for time-dependent loads," *ASCE Journal of Engineering Mechanics*, Vol. 116, No.12, pp. 2784–89.
- Křístek, V. & Bažant, Z.P. 1987. "Shear lag effect and uncertainty in concrete box girder creep." *J. of Structural Engrg.*, ASCE 113 (3), 557–574.
- Křístek, V., Bažant, Z.P., Zich, M. & Kohoutková, A. 2006. "Box girder deflections: Why is the initial trend deceptive?" *ACI Concrete International* 28 (1), 55–63.

- Křístek, V., Petřík, V. & Pilhofer, H.-W. 2001. "Creep and shrinkage prediction on the web," *ACI Concrete International*, Vol. 23, pp. 38–39.
- Křístek, V. & Vitek, J.L. 1999. "Deformations of Prestressed Concrete Structures: Measurement and Analysis," Proc. fib Symposium 1999 Structural Concrete—The Bridge Between People, FIB and CBZ, Prague, 1999, pp. 463–69.
- Lazić, J.D. & Lazić, V.B. 1984. "Generalized age-adjusted effective modulus method for creep in composite beam structures. Part I—Theory." *Cement and Concrete Research* 14 (6), 819–832.
- Lazić, J.D. & Lazić, V.B. 1985. "Generalized age-adjusted effective modulus method for creep in composite beam structures. Part I—Application." *Cement and Concrete Research* 14 (6), 1–12.
- Mandel, J. 1984. *The statistical analysis of experimental data*. Dover Publications.
- McDonald, D.B. & Roper, H. 1993. "Accuracy of prediction models for shrinkage of concrete." *ACI Materials Journal* 90 (3), May–June, 265–271.
- Müller, H.S. 1993. "Considerations on the development of a database on creep and shrinkage tests." *Creep and Shrinkage of Concrete* (Proc., 5th Intern. RILEM Symp., Barcelona 1993), Z.P. Bažant & I. Carol, eds., E&F Spon, London, 859–872.
- Müller, H.S., Bažant, Z.P. & Kuttner, C.H. 1999. "Data base on creep and shrinkage tests." RILEM Subcommittee 5 report RILEM TC107-CSP, RILEM, Paris (81 pp.)
- Müller, H.S. & Hilsdorf, H.K. 1990. "Evaluation of the time-dependent behaviour of concrete: summary report on the work of the General Task Force Group No. 199." CEB (Cop-mite euro-internationale du beton), Lausanne (201 pp.).
- Navrátil, J. 1998a. "Updating of Concrete Creep and Shrinkage Prediction" (in Czech), *Stavebni obzor* (Prague), 2/98, pp. 44–50.
- Navrátil, J. 1998b. "The Use of the B3 Model Extension for the Analysis of Bridge Structures" (in Czech), *Stavebni obzor*, 4/98, pp. 110–116.
- RILEM Committee TC-69 1988. (Z.P. Bažant, Chairman and princ. author), "State of the art in mathematical modeling of creep and shrinkage of concrete," in *Mathematical Modeling of Creep and Shrinkage of Concrete*, Z.P. Bažant, ed., J. Wiley, Chichester and New York, 1988, 57–215.
- RILEM Committee TC-69 1988. "Creep analysis of structures" (principal authors Z.P. Bažant & O. Buyukozturk), in *Mathematical Modeling of Creep and Shrinkage of Concrete*, Z.P. Bažant, ed., J. Wiley, Chichester and New York, 1988, 217–273.
- Rousseeuw, R.J. & Leroy, A.M. 1987. *Robust Regression and Outlier Detection*. J. Wiley, New York.
- Sakata, K. 1993. "Prediction of concrete creep and shrinkage." *Creep and Shrinkage of Concrete* (Proc., 5th Intern. RILEM Symp., Barcelona 1993), Z.P. Bažant & I. Carol, eds., E&F Spon, London, pp. 649–654.
- Sakata, K., Tsubaki, T., Inoue, S. & Ayano, T. 2001. "Prediction equations of creep and drying shrinkage for wide-ranged strength concrete." *Creep, Shrinkage and Durability Mechanics of Concrete and Other Quasi-Brittle Materials* (Proc., 6th Intern. Conf., CONCREEP-6, Cambridge, MA), F.-J. Ulm, Z.P. Bažant & F.H. Wittmann, eds., Elsevier, Amsterdam, 753–758.
- Teplý, B., Keršner, Z. & Novák, D. 1996. "Sensitivity Study of BP-KX and B3 Creep and Shrinkage Models." *Materials and Structures*, Vol. 29, 500–505.
- Vitek, J.L. 1997. "Long-term Deflections of Large Prestressed Concrete Bridges," CEB Bulletin d'Information No. 235 Serviceability Models—Behaviour and modelling in serviceability limit states including repeated and sustained load, 1997, CEB, Lausanne, pp. 215–227 and 245–265.
- Xi, Y. & Bažant, Z.P. 1989. "Sampling analysis of concrete structures for creep and shrinkage with correlated random material parameters." *Probabilistic Engineering Mechanics* 4 (4), 174–186.