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Structural Mechanics in Reactor Technology

VOLUME A

PRINCIPAL DIVISION LECTURES

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ASADOUR H. HADJIAN

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Modeling of the behavior of concrete. Formulation of constitutive laws for static and dynamic materials behavior, including temperature and time effects. Damage mechanics and life-prediction methods. Cyclic loading, creep, relaxation, fatigue and wave propagation problems. Experimental testing methods and data evaluation techniques. Specific aspects of constitutive laws of rocks, graphite, ceramics and composites involved in electrical equipment, including diffusion, permeability, strain-softening and fracture.

Advances in Material Modeling of Concrete

by

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Advances in Material Modeling of Concrete

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ABSTRACT

The paper gives a survey of the principal advances in material research in concrete presented at nine international conferences on Structural Mechanics in Reactor Technology since their inception in 1971. The survey focuses on nonlinear triaxial constitutive models, cracking and strain-softening damage, fracture, creep, hydrothermal deformations, heat transfer and moisture transfer. Although the survey is far from exhaustive, several fundamental advances are identified and discussed in some detail, although in nonmathematical terms. One hundred references are included.

1. INTRODUCTION

The tenth International Conference on Structural Mechanics in Reactor Technology (SMiRT) in Los Angeles is a milestone at which it is fitting to look back and appraise the evolution that has taken place in our understanding of the mechanical behavior of materials since the first conference in Berlin in 1971. The accomplishments, however, are too numerous, the subject too vast. Therefore, the present lecture, which is sponsored by Divisions Q and L of the Society, will focus on the mechanical behavior of concrete, and the broad and important field of metals will be relegated to a separate survey planned for the subsequent conference. The exposition which follows will include some general reflections on the historical evolution, as well as more detailed discussion of some selected highlights and interesting points dealing with nonlinear triaxial behavior, failure criteria, cracking, fracture, damage, creep, shrinkage, thermal strains, moisture and thermal effects, and diffusion. The references and bibliography will be limited to the contributfons presented at SMiRT conferences as well as the closely connected journal Nuclear Engineering and Design in which many SMiRT conference contributions were published in greater detail. No claim is made for exhaustiveness of this literature survey; in fact, those authors who are not quoted will still be in good company.

2. SOME REFLECTIONS ON THE EVOLUTION SINCE THE FIRST CONFERENCE

Many of us still vividly remember the feeling of excitement at the first conference in Berlin, 1971. Thomas Jaeger gathered there the most impressive assembly of experts, and it seemed that if only their knowledge of material behavior were translated into finite element programs which had just become available, all the mechanics problems of design could be readily answered. However, this did not turn out to be the case, mainly because of the complexities of material modeling. The pitfalls of material softening after passing the peak stress (or the so-called failure envelope) were as if they acted on the entire cross section of the structure instantaneously, neglecting diffusion, while for the purpose of shrinkage calculation the migration of moisture was described by linear diffusion theory. Effects of very high temperatures on concrete were ignored. The only size effect in structural response was thought to be statistical. We have indeed come a long way from these simplistic concepts, as the following discussion will try to document.

The advances of the research community in concrete were without delay reflected at the SMiRT conferences, as well as stimulated by them, although the important role of other meetings and especially workshops must not be overlooked. Throughout the 1970's, the progress was primarily driven by the needs of the expanding nuclear power programs, which justified lavish funding. Later, the thrust of research shifted from design for service conditions to safety evaluations. This forced serious attention to be paid to fracture mechanics aspects and post-peak energy dissipation, as well as the effects of very high temperature, exposure to liquid sodium or molten steel, etc. In recent years, despite the generally acknowledged necessity of nuclear power, and because of political pressures and choices made possible by very successful energy conservation, we have seen a retrenchment in public funding in nuclear reactor research. Fortunately, however, the community of researchers formerly supported by nuclear reactor programs continued to receive funds for basic and applied research in concrete which is useful to many technologies and objectives, nuclear power included. Thus, even at present we witness a rapid evolution in material modeling of concrete, which will make it possible to design better and safer structures for nuclear reactors when the need for further expansion becomes inevitable.

3. NONLINEAR TRIAXIAL BEHAVIOR AND FAILURE CRITERIA

The failure surface of concrete was relatively well known already at SMiRT1; e.g. Janda (1); formulas have been given for the rounded triangular shape of the deviatoric cross section, causing the tensile and compression meridians in the volumetric (Rendulić) cross section to be non-coincident (Fig. 1), as evidenced by the test results of Launay and Gachon (2). The biaxial failure envelopes at various values of the third stress were also known. However, it was not really known how to relate these failure surfaces to stress-strain relations and how to take into account the path dependence of concrete. Considerable complexities were later revealed by various new types of tests, e.g. on cubical triaxial specimens; Aschl and Moosecker (3); Valente (4); Robutti et al. (5); Ohnuma and Aoyagi (7); Watabe et al. (8), etc.

The first nonlinear incremental triaxial stress-strain relations were based on classical plasticity, especially the Drucker-Prager yield criterion, coupled with normality. However soon a variety of other approaches appeared. Description of a broad variety of test data was achieved in 1975 by Bažant (9) and coworkers, who developed for concrete the endochronic theory, extending ideas advanced previously for metals by Schapery and especially Valanis (Fig. 2). Other nonlinear triaxial models later appeared and achieved approximately the same closeness of fit of the existing data. These included the plastic-fracturing theory, which extended Dougill's previous idea for strain-space plasticity total strain models, and incremental orthotropic models. The latter ones however fell out of favor in recent years after a criticism of their problems with the form invariance restrictions. The incremental plasticity continued to be improved by refinements of the loading surfaces and the flow rule. An excellent description of a variety of test results has recently been achieved in the 1987 plasticity model by Chern et al., which introduced the volumetric cross section in the form of an expanding and shrinking slanted ellipse. In view of earthquake effects, and also in view of the shift of attention to the analysis of nuclear accidents, models for cyclic hysteretic stress-strain relations were formulated and rate effects introduced; e.g. Chen and Fardis (10); Soon, Buyukozturk and Einstein (11); Wang and Subia (12); Suaris and

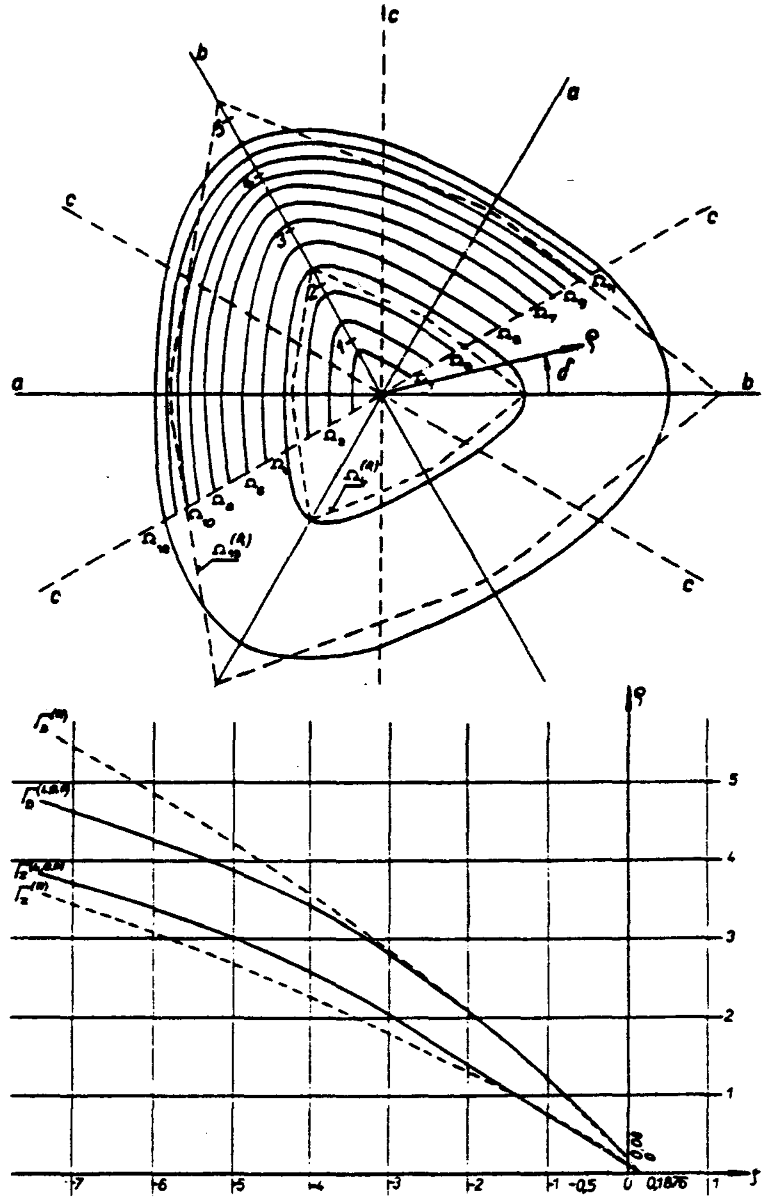

Fig. 1 Deviatoric (top) and volumetric (bottom) sections of the failure surface of concrete as presented by Janda (1).

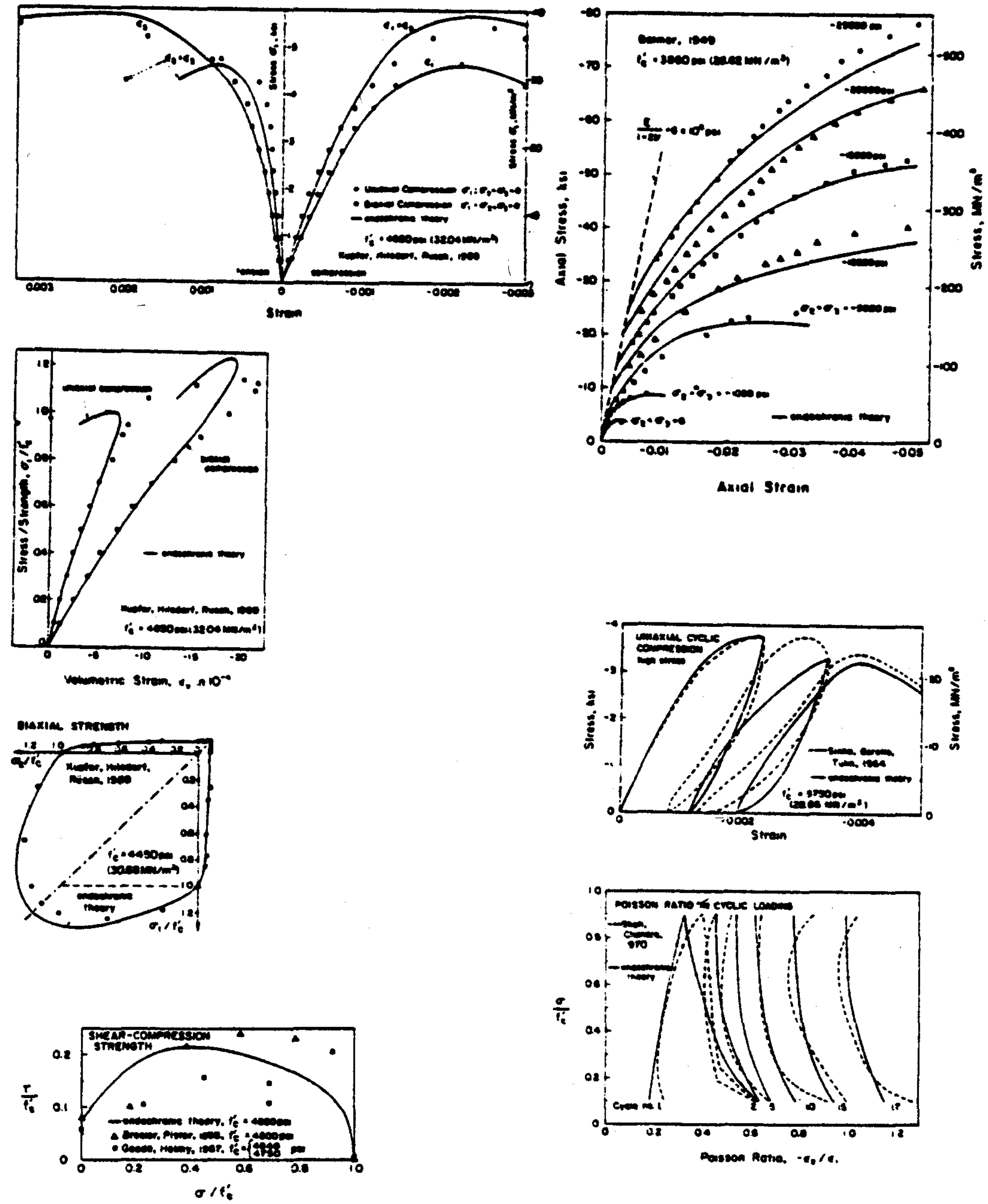

Fig. 2 Example of an early representation of a broad range of multiaxial stress-strain curves and failure envelopes by endochronic theory, presented in 1975 at SMiRT3 (9).

Shah (13); Eibl and Curbach (14); and Zeller (15). Numerical advantages and difficulties of various constitutive models were intensely discussed at SMiRT conferences. Problems related to stability, convergence and continuity of response were detected and partially handled first for the endochronic theory and later for plasticity without normality as well as other models.

After two decades of research, it seems that further improvements in the aforementioned nonlinear triaxial models, which may be needed especially if new test results appear for various nonproportional loadings, will be very difficult to achieve. This is probably true not only of classical plasticity but also of the plastic-fracturing models and the endochronic theory, as well as the continuum damage mechanics models, which appeared on the scene recently. The reason is that all these models are tensorial and essentially phenomenologic, ignoring the phenomena which cause inelastic behavior in the microstructure. Therefore, the promise of further progress probably lies in micromechanics modeling, which is simpler (nontensorial) and can take into account aggregate (particle) interactions and cracking, separating the phenomena occurring in the microstructure on planes of various orientations.

Although some aspects of micromechanics were discussed since the first conference; e.g. Wittmann and Zaitsev (16), the subject did not take off until recently. Aided by a powerful computer, Roelfstra (17), and Wittmann and Roelfstra (18), achieved a much better understanding of the micromechanism of inelastic behavior by their finite element analysis of stresses and deformations in the microstructure. Considerable potential is no doubt offered by rigid particle models of the type introduced first by Cundall for sands, which were recently generalized for concrete; e.g. by Zubelewicz and Bazant.

As a modeling tool for finite element programs, the microplane approach seems to have a particular potential. This approach is based on a 1938 idea of Taylor according to which the material behavior may be specified by stress-strain relations for the components of stresses and strains acting on planes of various orientations in the microstructure, called the microplanes. These are then combined according to a weak variational principle into a macroscopic stress-strain relation. By virtue of the fact that the stress-strain relation on the microplane does not have to deal with tensor components and invariance, the number of variables is greatly reduced and conceptual simplicity achieved (the tensorial invariance requirements are automatically satisfied by combining the responses from the planes of all orientations). A microplane model of this type was first developed for tensile cracking in 1973 by Bazant and Oh and later extended by Gambarova and Floris (19). It was also shown capable of modeling the resistance of cracks to shear. Future development should probably also take into account recent successes in the modeling of microcrack arrays, although for concrete this would be compounded by the difficulties of the aggregate structure.

4. CRACKING

In the early studies, concrete was assumed to have no tensile resistance. This idealization conveniently circumvented all the difficulties associated with the stress drop after cracking. However, in many types of failure, generally all the so-called brittle failures of concrete structures, the concept of a no-tension material grossly underestimates the carrying capacity, and even more the energy absorption capacity. A pioneering idea occurred to Rashid (20) in his famous paper in Nuclear Engineering and Design, in which he proposed to consider cracking to be smeared and handle it by a reduction of material stiffness in the cracked finite element, coupled with a stress drop to zero. This idea was further extended in 1971 in Scanlon's dissertation, who recognized the need to provide for a gradual stress decrease, i.e. strain softening, and modeled it by successive drops in stiffness. Aside from the modeling of cracks as interelement cracks, introduced in 1967 by Ngo and Scordelis, the concept of smeared cracking became particularly popular in the analysis of nuclear reactor vessels and containments and led to many analytical successes.

These early successes can for example be documented by the analysis of the spread of cracking in the cross section of a prestressed concrete pressure vessel reported by Zienkiewicz, Owen and Nayak (21), see Fig. 3, or the dynamic analysis of the cracking sequence in a prestressed concrete reactor vessel for a sodium-cooled breeder reactor, obtained in a hypothetical core disruptive accident; Marchertas et al. (22), see Fig. 4. As another of the early successes of complex nonlinear finite element analysis, one should mention the work of Rebora, Zimmermann and Wolf (23), in which a hypothetical aircraft crash onto a containment shell was analyzed using a sophisticated plasticity model with noncircular deviatoric cross sections; see Fig. 5 (cf. also Zimmermann (100)).

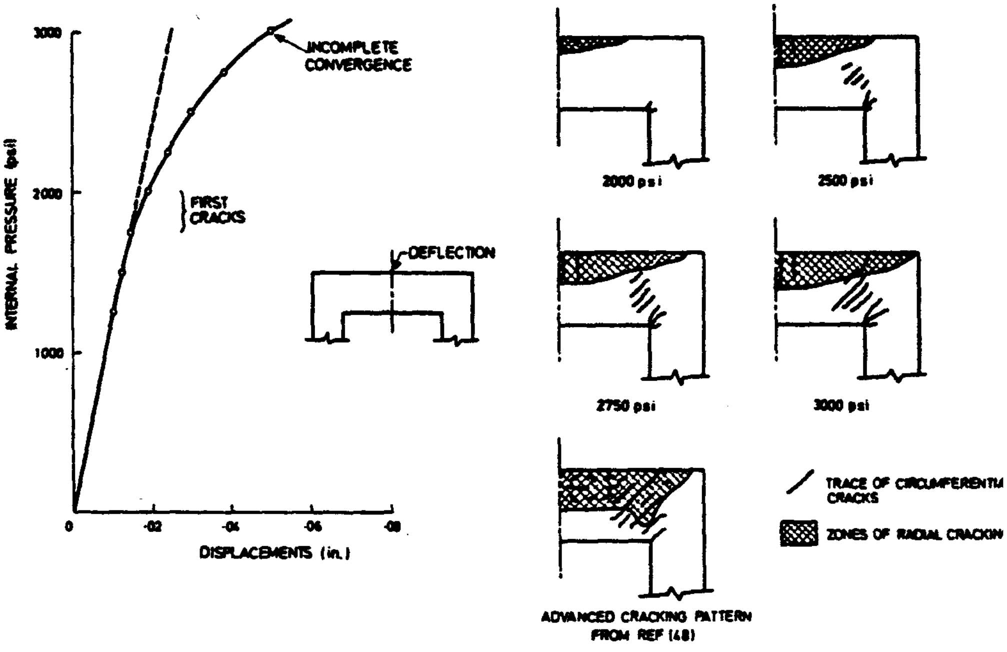

Fig. 3 Early finite results of Zienkiewicz et al. (21) showing the load-deflection diagram and the spread of cracking zones in a prestressed concrete pressure vessel.

The smeared cracking approach, however, was soon discovered to have some serious problems which may (although need not) make the solution invalid. As transpired already in the discussions between Willam and Bažant at SMiRT 3, London, the diagrams of load versus length of the cracking zone or load versus load-point displacements, and even more importantly the amount of dissipated energy, do not exhibit correct convergence as the mesh is refined. The results suffer from spurious mesh sensitivity. This is documented by Fig. 6 which shows the results of a rectangular panel in which a smeared cracking band propagates from an initially weaker element, the front of the cracking band being always localized to a single finite element. Subsequent years saw lively debates, first about the significance stress decrease, i.e. strain softening, and modeled it by successive drops in stiffness. Aside from the modeling of cracks as interelement cracks, introduced in 1967 by Ngo and Scordelis, the concept of smeared cracking became particularly popular in the analysis of nuclear reactor vessels and containments and led to many analytical successes.

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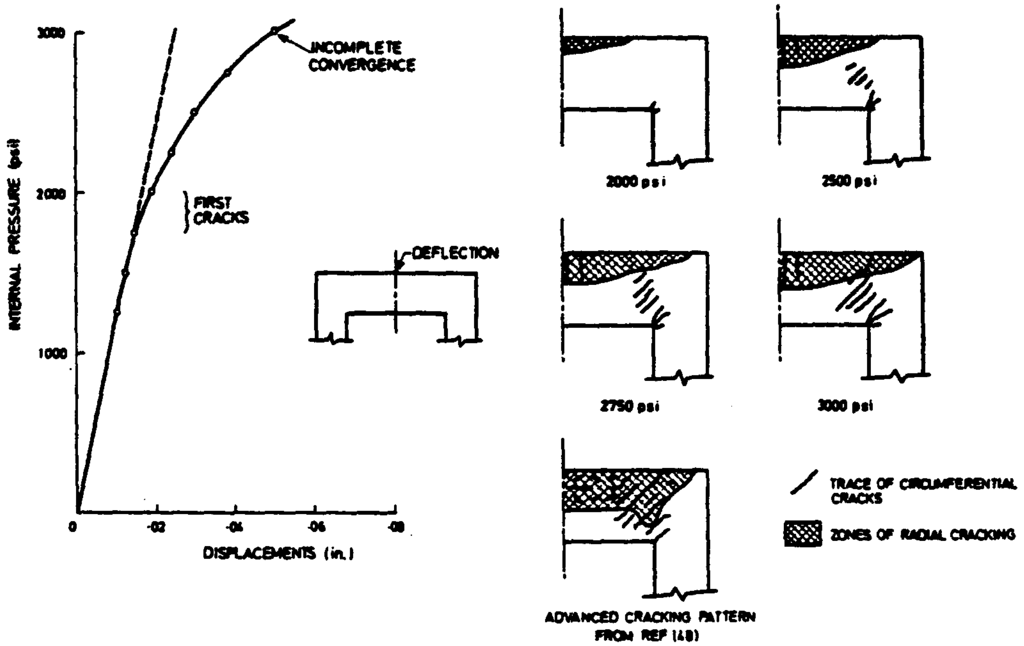

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5. FRACTURE, DAMAGE, AND STRAIN-SOFTENING

Fracture mechanics is a failure theory which (1) determines failure on the basis of energy criteria or combined energy and strength criteria, and (2) takes into account the fact that the failure is not simultaneous in various parts of the structure but propagates through the structure. From the practical viewpoint, the most important consequence of the fracture mechanics aspects of failure is the size effect, which is related to the spurious mesh sensitivity in finite element calculations already mentioned.

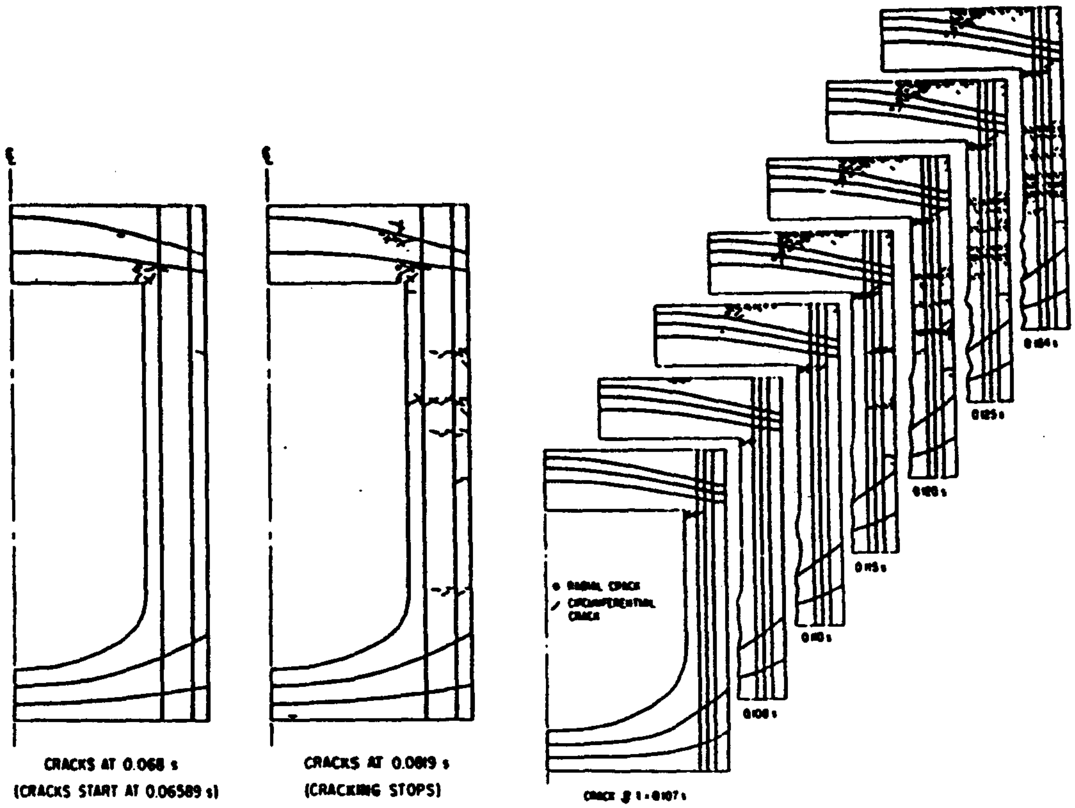

Fig. 4 Finite element results of Marchertas et al. (26) for the cracking pattern caused by a hypothetical core-disruptive accident in a prestressed concrete guard vessel for a sodium-cooled reactor.

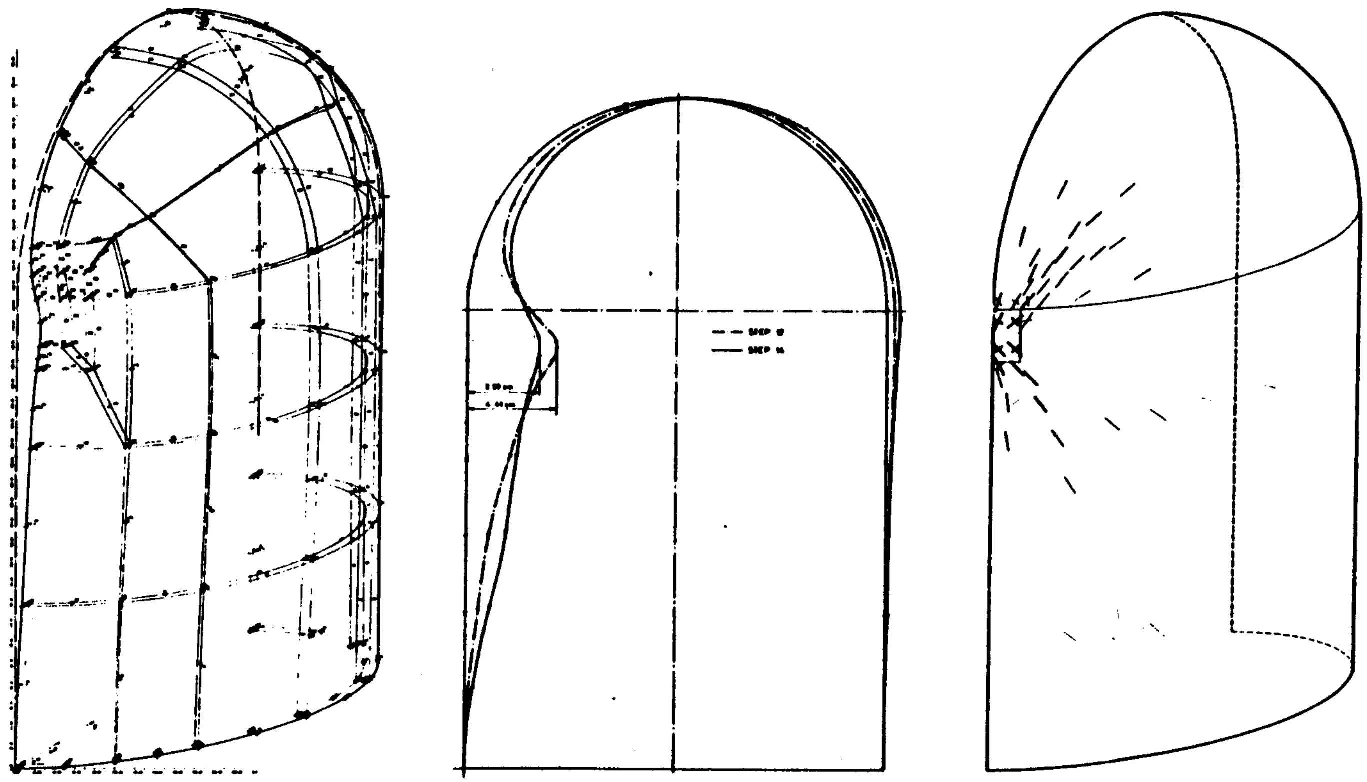

Fig. 5 Finite element results of Rebora, Zimmermann and Wolf (23) for impact of an aircraft into a containment shell; mesh (left), deflection profiles (middle) and cracking pattern (right).

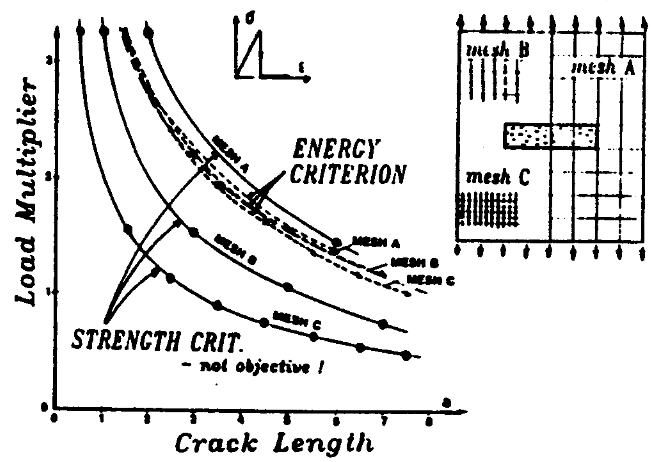

Fig. 6 Example of propagation of cracking band in a rectangular panel under tension, analysed by meshes A, B, C of sizes 4:2:1 (24).

The size effect is defined by comparing geometrically similar structures of different sizes. According to any theory with a strength criterion or a failure surface in the stress or strain space, such as the plastic limit analysis or elastic allowable stress design, geometrically similar structures of different sizes fail at the same value of the nominal stress, i.e. the failure shows no size effect. Not so in structures which exhibit fracture, either localized or distributed, the latter case being called the strain-softening.

The size effect began to be debated intensely at SMiRT conferences in the early 1980's, and a thorough discussion was presented at SMiRT 7 by Bazant (24); see Fig. 7, which shows a typical plot of the nominal stress at failure, N, versus the relative structure size, d, normalized with respect to the aggregate size, d_a, in logarithmic scales. As this plot reveals, the size effect is small for small structure sizes, and strength criteria for plastic limit analysis are then adequate. By contrast, for very large structure sizes, the size effect plot asymptotically approaches a straight line of slope -0.5, which is exactly the size effect of linear elastic fracture mechanics, a theory in which all the fracture process is assumed to be concentrated in a point and the rest of the structure to behave elastically. This size effect is the strongest possible. Real structures exhibit a transitional size effect between plastic limit analysis and linear elastic fracture mechanics.

The size effect can be exploited for determining the basic nonlinear fracture parameters, including the fracture energy G_f (32), the elastically equivalent size of the fracture process zone c_f, and the crack tip opening displacement, as well as the R-curve (Fig. 7 top right). Size effect measurements make it possible to determine the dependence of fracture energy on temperature and on moisture content.

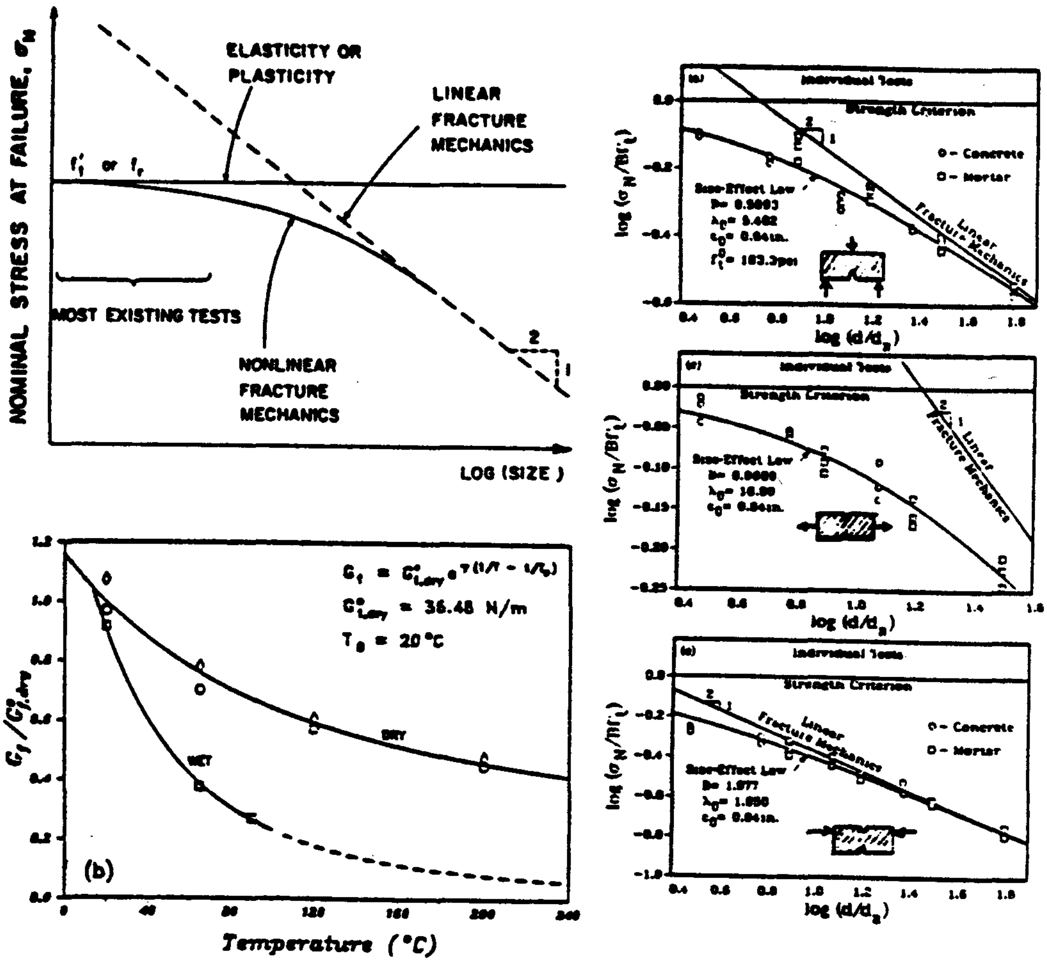

Fig. 7 The size effect law for blunt fracture, manifested by a dependence of the nominal stress at failure σ_N on size d of geometrically similar specimen, and test results (32) for specimens of various geometries, from which the fracture energy can be obtained - left; dependence of fracture energy on temperature at wet and dry states, obtained from size effect tests - right.

It must be emphasized that the size effect shown in Fig. 7 is not always followed, and this comment is important to nuclear structures. Some failures are ductile in nature and lead to a single-degree-of-freedom mechanism at the maximum load. However, many failures of concrete structures are inevitably brittle, and the fracture mechanics size effect shown in Fig. 7 is then the dominant one. These include the diagonal shear failure of beams and one-way slabs, the punching shear failure of slabs or shell walls, the torsional failure of beams, the pullout failure of reinforcing bars, the failure of anchors as well as splices, and the ring and beam failures of unreinforced pipes. In the case of prestressed concrete pressure vessels, the size effect is likely to also occur for the cryptodome failure of the top slab due to internal pressurization. A typical characteristic of all of these failures is that the failure does not occur at the first cracking initiation, but only after a large cracking zone or fracture has already developed.

Through the papers and discussions at the last three SMiRT conferences (e.g. Pfeiffer et al. (25), Pan et al. (26), Bažant (24), Carpinteri and Spiga (27), Pan et al. (28), Saouma and Droz (29), Bažant, Pijaudier-Cabot and Prat (30), Roelfstra and Wittmann (31), Bažant, Prat and Pfeiffer (32), Brühwiller et al. (33), Willam and Sture (34), Bažant and Chang (35), Rokugo et al. (36), etc.), it became clear that the size effect is an essential feature which must be exhibited by the finite element code (see also Zimmermann). This can of course be achieved only by incorporating the basic features of fracture mechanics, which can be implemented in diverse ways.

Various types of fracture propagation criteria and testing methods for fracture properties of concrete were discussed at SMiRT 7, Chicago, by Shah (37). Basically, there are two different, but in principle equivalent, modeling choices: (1) either sharp interelement cracks or (2) smeared cracking concentrated in a band whose width is either a material property, as in the crack band model, or is determined from a characteristic length of the material, as in the recent nonlocal models; Bažant, Pijaudier-Cabot and Prat (30). The smeared cracking approach is probably simpler to implement in finite element codes and has traditionally been popular with numerical analysts. The simplest prototype of the fracture mechanics approach to smeared cracking, the crack band model, is illustrated in Fig. 8. This figure shows the fracture process zone as a zone of distributed cracking around the front of the final sharp fracture, and the softening stress-strain relation in the transverse direction which is assumed to govern the behavior in the fracture process zone, whose width is in this model assumed to be constant. In the more general nonlocal models, of which the nonlocal continuum with local strain (or nonlocal damage) is quite easy to implement in a finite element code, the width of the crack band front is not fixed but can vary, although it is essentially determined by the characteristic length of the material, a property which is related to the statistical characteristics of the heterogeneous microstructure.

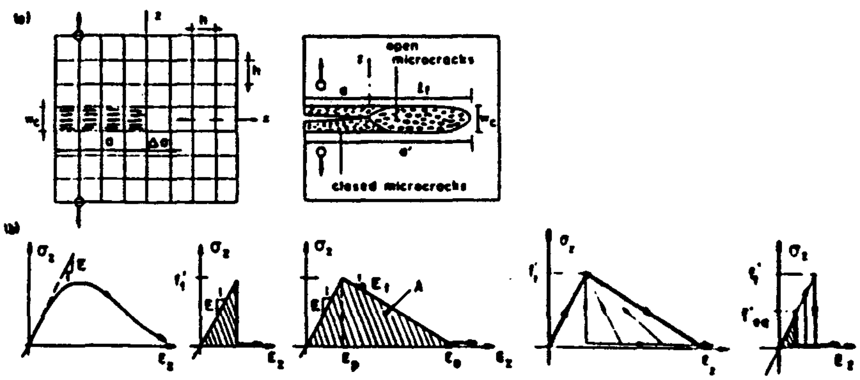

Fig. 8 Crack band model and strain-softening stress-strain relations used (24).

The physical reason for the transitional size effect between plasticity and linear elastic fracture mechanics is the development of a large fracture process zone. Microscopic observations of this zone are difficult and have been only partially successful (see e.g. the works of Shah, Cedolin, Darwin, etc.). Illuminating was the finite element simulation of fracture process zone evolution presented at SMiRT 9 by Roelfstra (17); see Fig. 9. Bazant and Lin recently obtained similar results with a finite element code based on nonlocal continuum with local strain.

Strain-softening which characterizes the fracture process zone is particularly important for dynamic analysis, see e.g. Isenberg and Richardson (38), since inertia tends to delay localization of the strain-softening damage.

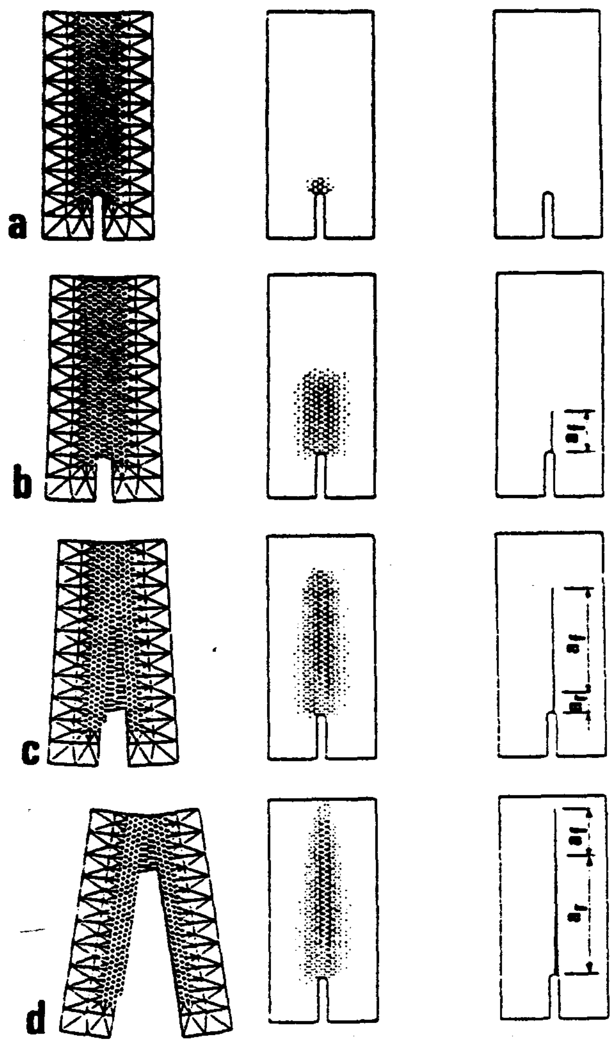

Fig. 9 Roelfstra's (17) finite element simulation of the evolution of fracture process zone in compact tension specimen.

Another important aspect worth mentioning is the shear transmission capability of cracks, due to aggregate interlock; see e.g. the paper by Gambarova and Karakoc (39), extending the rough crack model previously formulated by Bazant and Gambarova, and the paper by Gambarova and Floris (40), which models crack shear resistance and the development of transverse normal stresses on shear cracks by means of a strain-softening band in which microcracking happens in an inclined direction. This model utilizes for the description of inclined microcracking a typical strain-softening fracture model, for which the microplane approach, previously formulated by Bazant and Oh, was adopted.

While only about five years ago there were considerable disagreements among experts as to how to deal with strain-softening damage and fracture blunted by a cracking zone, today the opinions display remarkable convergence. Some models thought to be cardinally different are now recognized to be only diverse but nearly equivalent manifestations of the same principles. The fundamental property appears to be the structural size effect. The question whether the typical transitional size effect is correctly exhibited should be adopted as the basic criterion of acceptability of a finite element code for the analysis of failures of nuclear concrete structures.

One intriguing but important aspect of fracture propagation is its direction. It has been widely assumed in calculations, including most computer codes, that fracture propagates in the direction of the maximum principal stress. However, this notion was dispelled by recent tests of fracture in shear, of which most indicative are perhaps those in mode 3, conducted on cylindrical specimens subjected to torsion; Bazant and Prat, (41), Fig. 10. In these specimens, weakened by a circumferential notch, the fracture does not form a spiral surface which corresponds to the principal stress criterion for the fracture direction, but propagates straight in the cross section, which is in the direction of maximum shear.

Interestingly, in other respects this type of fracture seems to behave in the same manner as the opening mode 1 fracture. For example, the shear fracture exhibits a similar size effect (see Fig. 10), which may be

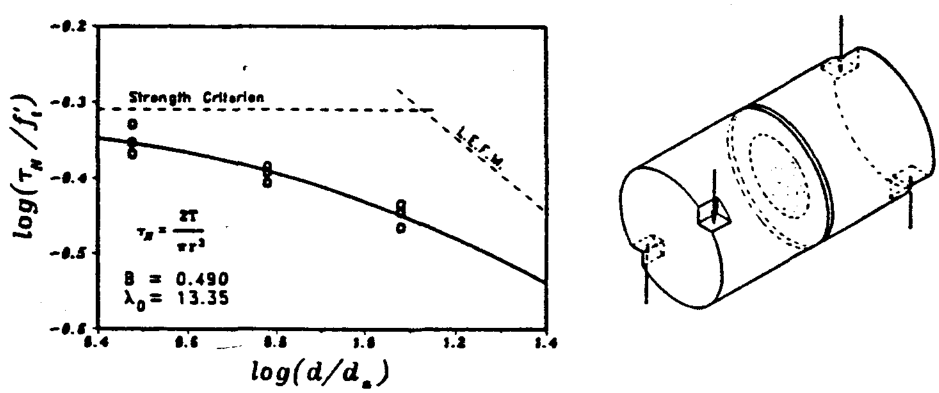

Fig. 10 Size effect observed in shear (Model III) fracture tests and the specimen used (41).

exploited for determining the shear fracture energy. However, in contrast to mode 1 (opening) fracture, the shear fracture energy is not a constant but is found to depend strongly on the transverse normal stress across the fracture plane.

6. CREEP AND HYGROTHERMAL STRAINS

Since the early days of prestressed concrete reactor vessels, these phenomena have been recognized to be of particular importance because the structures carry a high long-term load, due to prestress, and at the same time are exposed to elevated temperatures which intensify creep and hygrothermal deformations such as shrinkage. At SMiRT 1, several papers dealt with the mechanism of creep in concrete and its thermodynamic theory, e.g. Bažant (42, 43), structural analysis for creep, e.g. Stefano et al. (44), England and Allen (45), effective analysis algorithms, e.g. Bažant (46), and measurement of creep, e.g. Browne and Blundell (47), and Browne and Bamforth (48). Two important properties of concrete which complicate structural analysis were recognized to be: (1) the long term hereditary (memory) properties, characterized by a very broad relaxation spectrum with relaxation times ranging from a fraction of a second to 30 years, and (2) the strong effect of the age at loading on the subsequent creep. The latter effect precludes the use of standard linear viscoelasticity and Laplace transforms, and makes numerical finite element analysis inevitable. Formulation of the constitutive equation and its identification from test data is no easy matter and needs to be aided by theoretical concepts describing the creep mechanism and the process of aging (24).

One innovation which appeared at SMiRT 1 was the introduction of the rate-type model, in which the creep constitutive behavior is approximated by either the Kelvin or the Maxwell chain model. In contrast to standard (non-aging) viscoelasticity, the elastic moduli and viscosities of the units of the chain had to be considered as functions of age, modified according to the humidity and temperature history. The age dependence, coupled with approximate thermodynamic restrictions, was found to allow only certain forms of the stress-strain relation for the springs and dashpots in the chain. Initially a variety of models existed. They used or implied the age dependence to be manifested by a time variable elastic modulus which related either (1) the strain rate to the stress rate or (2) the total stress to the total strain. Later it was found that thermodynamic considerations permit only the latter form (24), although various models which ignore these thermodynamic restrictions have earlier been conceived, developed and used in large codes.

The transformation of the constitutive equation for concrete creep from a Voltera-type integral equation with a nonconvolution kernel to a system of first-order differential equations for internal variables corresponding to the Maxwell or Kelvin chain greatly simplified computer analysis. It eliminated the need for storing the history of stress for each finite element, effectively replacing the history by the current values of the internal variables.

In addition to this, the demands of the time integration process for computer time were greatly reduced by invention of the so-called exponential algorithms. These are based on an exact integral of the system of the first-order differential equations in time representing the constitutive equations, obtained under the assumption that the strain rate and the properties of the material during the time step remain constant. The algorithm of Bažant (46), extending a similar idea proposed previously for nonaging materials by Taylor, Pister and Goudreau as well as by Zienkiewicz and Watson, removes any restrictions on the length of the time step due to numerical stability considerations. It permits, for the case of permanent constant loads, the time step to be gradually increased from minutes at the time of loading to years as the end of lifetime is approached. Bažant's aging form of the Maxwell chain model along with his exponential algorithm was implemented in a large finite element code (NONSAP-C) by Smith, Cook and Anderson (49), later refined and extensively used by Anderson. Other large finite element codes of a similar nature were developed by Argyris, Pister, Szimmat and Willam (50) and Bažant, Rossow, and Horrigmoe (51). As an example of application of the latter code (24), Fig. 11 exhibits the results of creep analysis of the combined pressure and temperature effects in the corner of a prestressed concrete reactor vessel, showing the deflection curves at various times. Numerous large-scale finite element calculations of creep effects were also presented by Shioya et al. (52), Aoyagi et al. (53), Cheung and Koploy (54), Huterer et al. (55, 56), Rodriquez et al. (57), Ohnuma et al. (58), and Ducret et al. (59). Most of these codes also took into account cracking, Chern and Marchertas (60), although not yet in the fracture sense, thus missing possible size effects. A particularly versatile large code of this type, which also incorporated sophisticated thermal and moisture transfer analysis, was developed at Argonne by Marchertas, Kennedy, and Pfeiffer (61). The finite element analyses were supplemented by creep measurements on PCRV and other structures; e.g. Hornby (62), Sakuta et al. (63), etc.

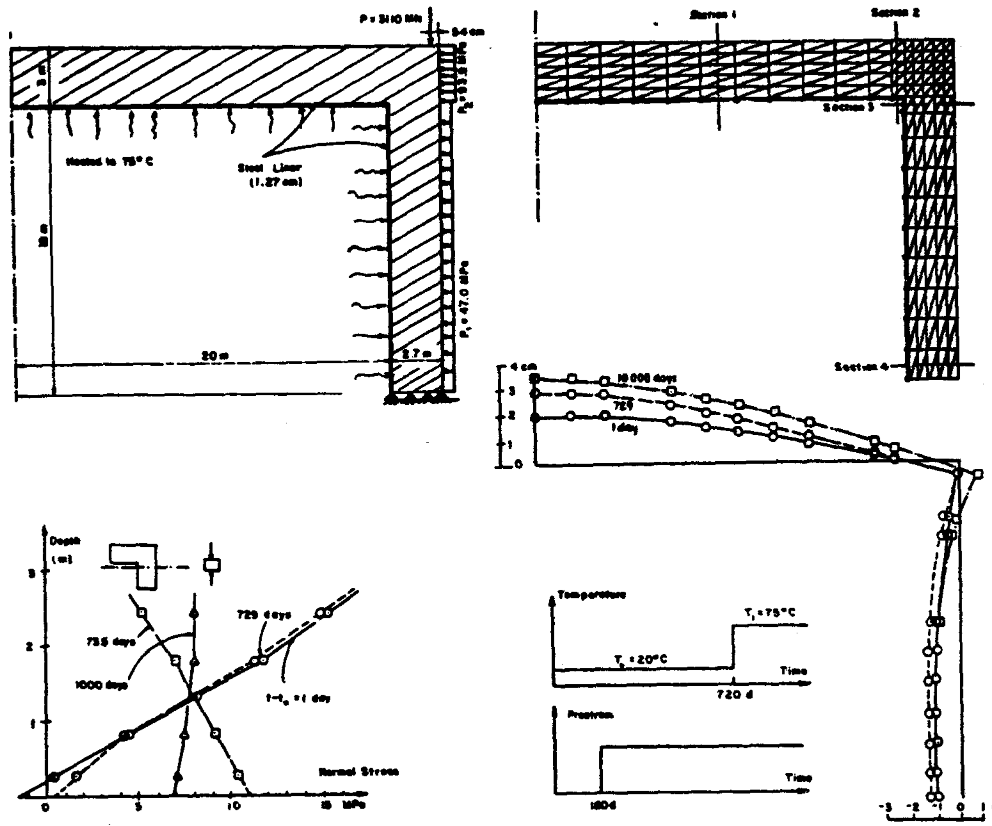

Fig. 11 Example of a finite element creep analysis of a heated PCRV with age and temperature dependent creep properties (51) (deflection profiles at various times - bottom right, and stress distributions across the wall thickness at various times - bottom left).

One aspect particularly important to nuclear structures is the thermal effect. Extensive early data were presented already at SMiRT 1 by Browne and Blundell (47) and later extended by Browne and Bamforth (48); see the diagrams of the compliance (strain due to unit sustained stress) as a function of the stress duration and the age of loading, for room and elevated temperatures, in Fig. 12, in which the long-term rise of the creep curves, the large effect of the age at loading, and the strong effect of temperature are to be noted. Such and other properties were subsequently codified in the BP model, and a package for the prediction of concrete constitutive properties, called MATPAR, was later developed and refined to the greatest extent at Ontario Hydro (Huterer et al., 1985). The temperature effects become considerably more complicated at temperatures beyond 100°C, which are important not only for the analysis of accidents but also for some proposed new designs, such as the hot linear vessels; e.g. Witt, Zemann and Scheifer (64), Bazant and Fistedis (65), etc.

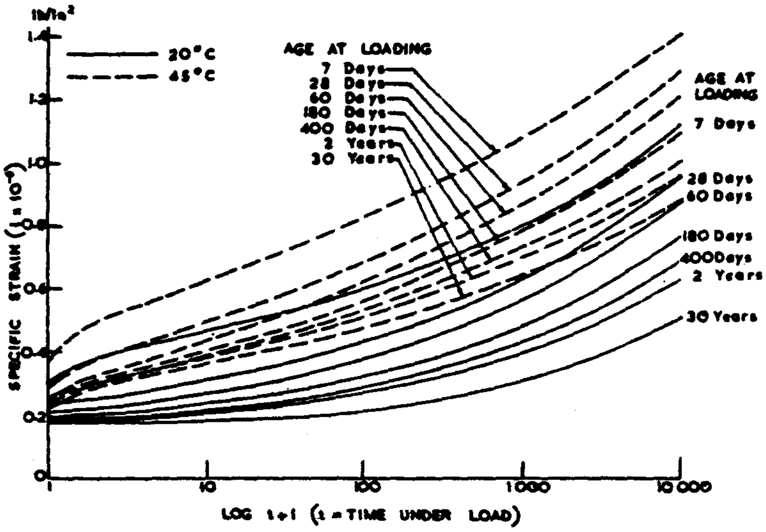

Fig. 12 Long-term creep curves of concrete for unit constant stresses applied at various ages; after Browne and Blundell (47).

The strong effect of temperature on creep seems to be simple for a perfectly dried concrete, and follows quite well the theoretical concept of activation energy theory (rate process theory), as established by the time of SMiRT 1 in the preceding researches of Maréchal and others. The behavior is unfortunately (for the analyst) much more complicated in the presence of moisture. Changes of temperature disturb the balance of the Gibbs free energies (chemical potentials) between the gel water or adsorbed water in the micropores and the liquid or capillary water in the macropores of the cement paste, producing microdiffusion fluxes which are suspected of promoting large additional deformations. Furthermore, macroscopic diffusion of water through the pores of concrete is caused by temperature gradients. This makes experiments difficult to conduct and interpret.

As transpired over the last two decades, temperature changes simultaneous with creep produce additional strains beyond the steady-state creep, alternatively called the transitional thermal creep and stress-induced thermal strain. A similar effect, discovered earlier, is exerted by simultaneous changes of moisture content or pore relative humidity in concrete. These effects are so large that they can change the creep strains by more than 100%. Pioneering work in the modeling of this phenomenon was that of Thelandersson (1982), who not only presented a fairly complete picture of the surface of creep strains and unit stress as a function of

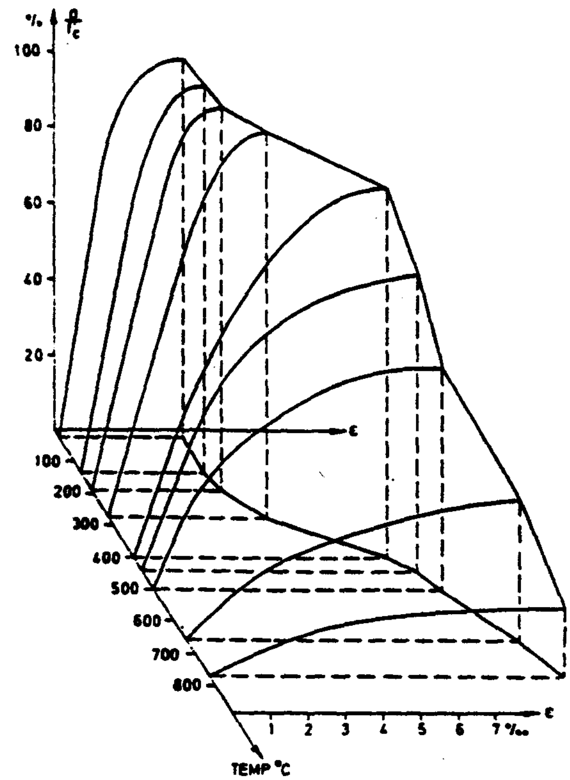

Fig. 13 Creep compliance surface as a function of temperature, as reported for high temperatures by Thelandersson (66).

time and temperature (Fig. 13), but also came with an important simple idea: The thermal strain rate is the sum of the thermal dilatation rate observed on a material element which is stress-free, and the stress-induced thermal strain. At the same time, the shrinkage rate is a sum of the basic shrinkage rate due to the rate of change of moisture content and of a stress-induced shrinkage rate. Later the stress-induced thermal strain rate and the stress-induced shrinkage rate were recognized to be identical to the so-called drying creep and transitional thermal creep and to represent a special limiting case of the thermodynamic theory for concrete creep by Bažant (42, 43). While Thelandersson (66) proposed a detailed model for the stress-induced thermal strain, Bažant and Chern formulated a detailed model for the stress-induced shrinkage; e.g. Chern et al. (67). The magnitude of these phenomena is well illustrated by Fig. 14 taken from Thelandersson (66), which compares the thermal compressive stress in a restrained specimen with the stress which is calculated in ignorance of the stress-induced thermal strain. This prediction was shown to closely agree with test results (Fig. 14). The aforementioned phenomena were mathematically formulated and implemented in the large TEMP-STRESS computer code at Argonne National Laboratory by Marchertas et al. (61).

The transient temperature effects on creep were the subjects of many other important studies, e.g. Roelfstra (68), Schneider and Diederichs (69), Schneider, Herbst and Diederichs (70), Schimmelpfenig and Altes (71), Wydra, Diederichs and Schneider (72), etc. Further important studies of thermal creep properties were presented by Schwesinger (73), Altes et al. (74), Kaplan and Roux (75), Seeberger, Belli and Hilsdorf (76), Kasami et al.(77), Schneider and Diederichs (78), Weber et al. (79, 80) and others.

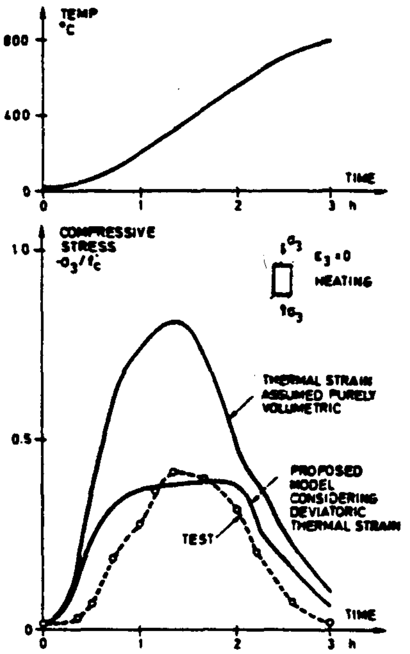

Fig. 14 **Thermal strain model with and without stress-induced deviatoric thermal strain compared with experiment; after Thelandersson (66).**

An important application aside from nuclear reactor structures is in the field of nuclear waste disposal - the design of a concrete plug for an underground depository; Pfeiffer and Kennedy (81).

In view of certain postulated core-disruptive accidents, thermal analyses were also conducted for various possible designs of prestressed concrete reactor vessels made of refractory concretes; e.g. Bažant, Chern, Seidensticker and Marchertas (82).

7. HEAT AND MOISTURE TRANSFER IN CONCRETE

The hygrothermal effects on concrete creep which we just discussed cannot be analyzed without calculating the distributions of temperature, moisture content and pore pressure at various times throughout the structure. This is also a rather complex task, which occupied the researchers at SMiRT since the early days. At the time of SMiRT 1, the diffusion of moisture through the pores of concrete at constant temperature was generally considered to be adequately described by a linear diffusion equation, as proposed in the 1930's by Carlsson. However, extensive fitting exercises of the measurements of humidity distributions and weight loss curves measured by Monfore at Portland Cement Association, Skokie, and others revealed, in an early study at Northwestern University by Bažant and Najjar, that the moisture diffusion problem is strongly nonlinear (24). Surprisingly, the moisture permeability and diffussivity decrease approximately 20-times as the pore relative humidity decreases from 90% to 60%. This produces a very strong nonlinearity of the initial-boundary value problem, which must be taken into account in finite element codes. Bažant and Najjar's (24) nonlinear diffusion model has been generally adopted by programmers, e.g.

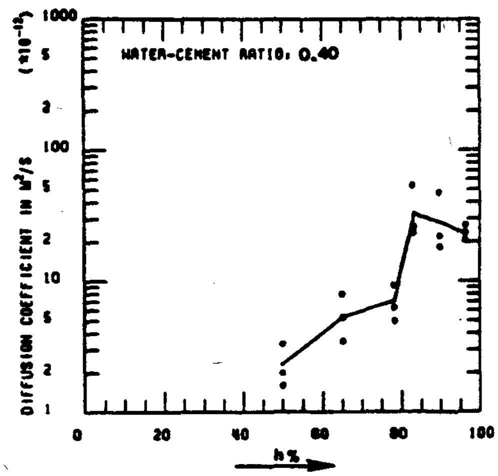

Fig. 15 Measurements of water diffusion coefficient in concrete, confirming its approximately 20-fold drop due to water content, previously deduced indirectly by fitting transient measurements; after Wittmann, Roelfsta and Kamp (88).

Argyris et al. (83), Marchertas et al. (84), and was further verified by experimentalists (Fig. 15) and given more detailed explanations. Wittmann, Roelfstra and Kamp (85) presented results of sophisticated diffusion studies in concrete in which the aforementioned nonlinearity of the diffusion equation was taken into account and the microstructure was treated at three levels: micro, meso, and macro (Fig. 16). Roelfstra (17) simulated the shrinkage strains using finite element analysis of randomly generated microstructure and taking into account cracking, including the phenomenon of instability of parallel cracks which enforces crack arrest and increases opening of the leading cracks, as previously established by Bazant and Ohtsubo; see Fig. 16. This study included a relative estimate of the ratio of shrinkage strains with and without cracking (Fig. 16e).

It is most interesting that the transitional creep phenomena due to changes of temperature and humidity are exhibited also in shear or deviatoric deformations, as proposed by Thelandersson (66) as well as Bažant and Chern (Fig. 14). This was confirmed in torsional-axial creep tests at various confining pressures, conducted at Northwestern University in a novel high-temperature creep testing machine by Bažant and Prasannan (86). These tests reveal intricate effects of various moisture regimes; see e.g. Fig. 17, which shows the response curves at 120°C measured on pressurized specimens subjected to torque, which were: (1) tested after being sealed in the wet state (test no. 1), (2) tested after being sealed in a dried state (test no. 2), and (3) tested in an unsealed state, being initially wet (test no. 3). While at room temperature, the largest creep in such tests is obtained in the specimen which is unsealed and is drying simultaneously with creep, the opposite is surprisingly found to happen at high temperatures. The highest creep is exhibited by the specimen sealed in the wet state, while the creep of the unsealed and drying specimen is substantially lower. So is the creep of a specimen tested in water, which imbibes water during the creep. The explanation, at present still hypothetical, is that the moisture leaves the unsealed drying specimen so fast (due to the aforementioned increase in diffussivity) that the specimen becomes essentially dry already at the beginning of the test (which agrees with the fact that dried concrete has negligible creep). The reason that a specimen submerged in highly pressurized water at high temperatures creeps less than a sealed specimen may be due to some chemical reactions similar to autoclaving, which may

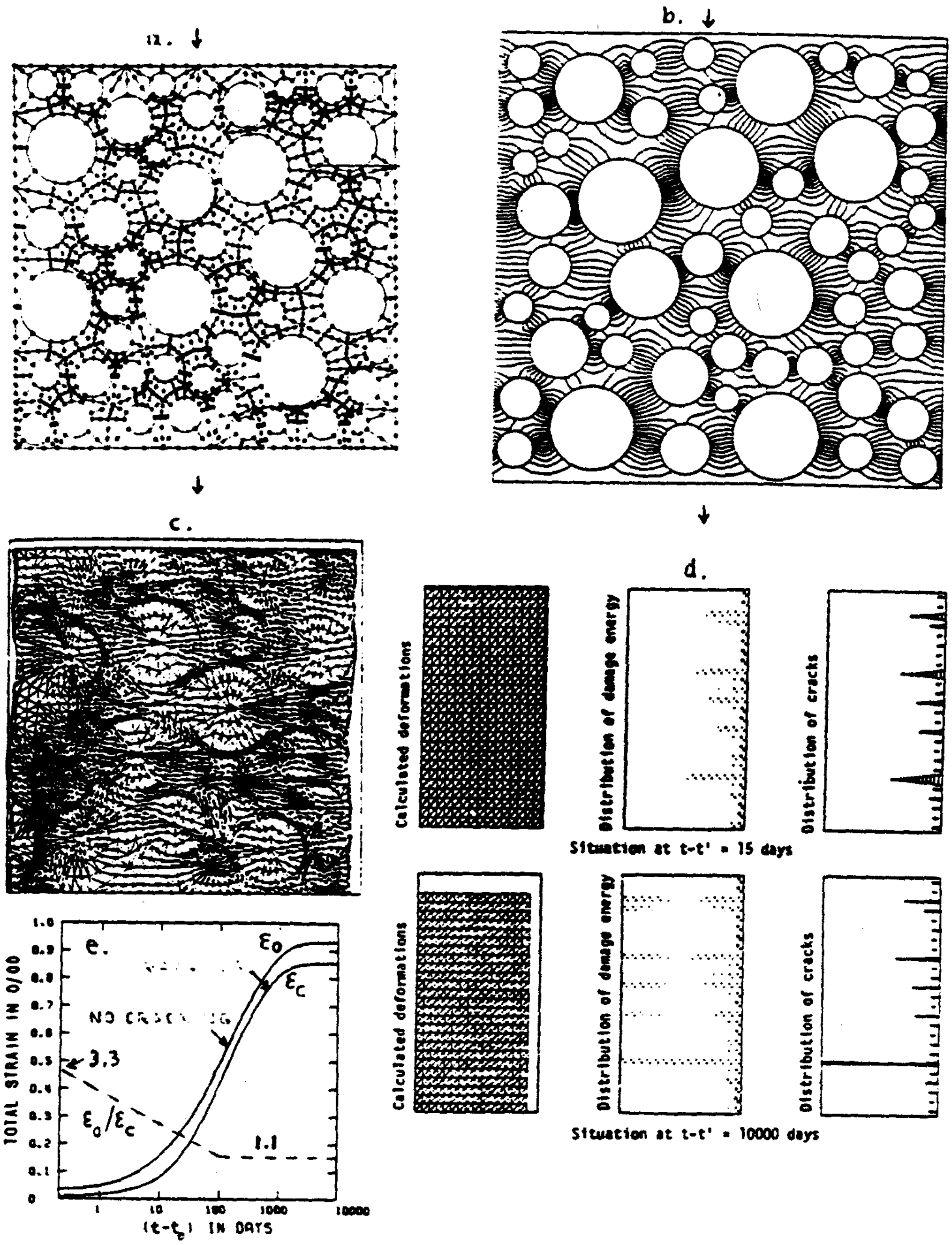

Fig. 16 Micromodeling of water diffusion through the mortar in concrete (a, b), with calculated isohygres (constant R. H. lines) (b); micromodeling of shrinkage (c, d, e), with the evolution of cracking (d); and comparison of overall strains with and without cracking; after Wittmann, Roelfstra and Kamp (88), and Roelfstra (17).

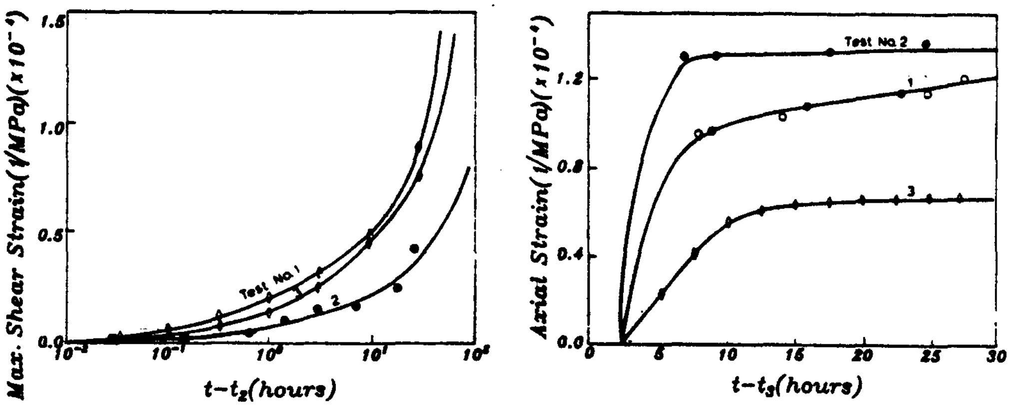

Fig. 17 Measured effects of moisture regimes on shear and normal creep strain of cylindrical specimens (of 6 in. diameters) at 120°C, due to torque (left) and to axial compression (right) (1 - sealed wet, 2 - sealed dry, 3 - unsealed, initially wet); after Bažant and Prasannan (88).

strengthen concrete. The tests shown at the left of Fig. 16 were conducted under torque, so as to eliminate the hygrothermal volume changes from their evaluation.

The thermal creep phenomena are of considerable interest for the analysis of accidents, e.g. sodium spills or molten core, in which the surface of a massive concrete wall protected by a steel liner is very rapidly heated to high temperatures. If the liner remains intact, moisture cannot escape at the surface but is forced deep into the concrete wall. This creates a layer of dried hot concrete right under the liner, and behind it a layer of supersaturated concrete with an increased water content, which is also very hot. Much effort has been devoted, especially at Argonne National Laboratory, to the calculation of the response of a concrete vessel to such an accident.

A curious fact, difficult to model mathematically, is the observation (made from various tests of large heated blocks) that the pore pressures measured inside the concrete are surprisingly low. They are far less than the pressures which would be calculated assuming a saturated pore to behave as a rigid container completely filled by water at room temperature and subsequently heated. Certain, not implausible, hypotheses about extra pore space, as well as creation of further pore space by pressurized pore water, made it possible to obtain reasonable agreements of theory with various types of test results. However, extensive further research will be needed to achieve full understanding. Calculation of the pore pressure is very important, since it causes cracking and macroscopic fracture and may even lead to the so-called explosive spalling of concrete (observed e.g. by Sullivan and Zamann (87), in which buckling of the surface layer separated by a fracture no doubt plays an important role.

Calculations of this type are also a necessary part of the analysis of the chemical interaction of liquid sodium, molten steel or molten reactor core with concrete. Many tests have been conducted at various laboratories; see references in Bažant (24), as well as Fritzke and Schultheiss (88), Schneider, Ehm and Diederichs (89), Fritzke and Schultheiss (90), Massa et al. (91), etc.

If gradients of both specific moisture content and temperature are present, one needs to consider the so-called coupled diffusion. The general theory was expounded long ago by Luikov and others. In their formulations, the fluxes of moisture and temperature depend on both gradients, i.e. coupled fluxes are present. However, a more careful study for concrete indicated that this formulation is for the most part, an unnecessary complication. If the driving force of moisture transfer through the pores of concrete is considered to be the pore pressure, which amounts to Darcy's diffusion law, then the moisture transfer can be considered to be a consequence of a single driving force and the separate effects of temperature gradient and pore water content gradient come merely as different manifestations.

Compared to the early picture at the first SMiRT, a surprising phenomenon was discovered with regard to the dependence of the permeability and diffussivity of moisture in concrete on temperature. As shown in 1977 at the Brazilian nuclear conference and later at SMiRT by Bažant (92, 93) (also Bažant, Chern and Thonguthai (94)), and independently by Chapman and England (95), the diffussivity of moisture increases about 200-times as the temperature passes from 90 to 110°C. This creates a strong nonlinearity in the diffusion equation for moisture transfer and causes problems in numerical calculations. The physical explanation of this interesting phenomenon is still only conjectured; probably the increase of temperature removes very narrow, molecular-size necks on the passages of water through the pores of concrete, such that the pore volume change caused by the flattening of these necks is negligible, since no appreciable change in porosity is observed. This phenomenon was modeled in the TEMP-STRESS code at Argonne National Laboratory by Marchertas and Kennedy. Some partially similar phenomena were also observed by Dayan and Glueckler (96), and various discussions of physical mechanisms and further insights were provided by Kamp, Roelfstra and Wittmann (97), Schneider and Herbst (98), Jonasson (99) and others.

CONCLUDING THOUGHTS

Having completed this apercu of research highlights, one cannot miss to observe that advances over the nearly two decades that elasped since the first SMiRT have probably been greater than over any two previous decades in the history of materials research in concrete. This has of course come due to the fact that the availability of computational tools makes sophisticated material models worthwhile and at the time makes their development, identification and evaluation possible. The advances in viscoelastic behavior, hygrothermal behavior, constitutive models, and fracture mechanics created a scene that fundamentally differs from that at the time of the first conference.

The progress in fracture is perhaps most conspicuous. Only about a decade ago, fracture mechanics of concrete was considered to be a curiosity for the researchers and was generally deemed inapplicable to concrete structures. This was of course not due to any ignorance on the part of concrete designers. Linear elastic fracture mechanics had been tried and found invalid for concrete, at least on the structure scales explored at those times. Then, after the strain-softening models ran into a quagmire, research on fracture started, first perhaps from desperation; but soon it led to lively dialogue and disagreements, and recently crystalized into a broad consensus on most basic issues. At the moment, the fracture mechanics theory of concrete seems to be ready for practical application, not only in the finite element analysis of concrete structures, which should be particularly important for nuclear vessels and containments, but also for implementation in design codes. We might well be standing today at the threshold of a revolution in concrete design and structural analysis.

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