

COMPUTER MODELLING OF CONCRETE STRUCTURES

Edited by

H. MANG

Technical University, Vienna, Austria

N. BIĆANIĆ

University of Wales, Swansea, U. K.

R. DE BORST

Delft University of Technology, Netherlands

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Creep and Thermal Effects in Concrete Structures: A Conspectus of Some New Developments

By ZDENĚK P. BAŽANT¹

ABSTRACT. — The lecture reviews some recent developments in the theory of creep and thermal effects in concrete structures. The formulation of the solidification theory for the effect of aging due to cement hydration on concrete creep is summarized, a new operator-type formulation for the theory is given and a new general statement of the computational algorithm is presented. Also, a reformulation of the solidification theory on the basis of a continuous retardation spectrum is outlined. The dichotomy of the two-fold influence of temperature, consisting in the acceleration of creep rate and the acceleration of hardening due to hydration, is discussed and its formulation given. Attention is then focused on the interaction of creep with fracture mechanics, and a time-dependent generalization of the cohesive (fictitious) crack model is described, along with the compliance method of structural analysis for time-dependent cohesive fracture. Finally, some recent novel experiments that separate the mechanisms of the creep increase due to drying are reviewed and the effects of drying on a structure such as a box girder bridge segment is explored. The lecture concludes with a brief discussion of currently important research directions.

1 Introduction

The long-time creep strains of concrete under service stresses are typically 2 to-6 times larger than the short-time elastic strains, and even short-time deformations of about 10 minute duration comprise 25 to 50% of creep. Thus, consideration of creep is obviously important for a realistic analysis of concrete structures.

Major advances have been made in the understanding of creep during the last two decades, principally driven by the needs of design of nuclear power

¹Walter P. Murphy Professor of Civil Engineering, Northwestern University, Evanston, IL 60208, USA

plants. Although this subject has recently become politically unpopular, many nuclear plants exist around the world. Their state of stress and deformation is changing over the years. Insuring their continued safety requires paying attention to creep. The main need for creep analysis now arises from the ubiquitous problems with concrete infrastructure of most nations. Misunderstanding and miscalculations of creep, thermal effects and moisture effects is no doubt a strong contributing factor in these problems. Equally important is creep for the analysis, evaluations and long-time fracture predictions of dams—a technology that has so far caused many more fatalities than nuclear power.

The theory of creep and of the related effects of moisture and temperature is greatly complicated by several phenomena: (1) the aging, caused by long-time chemical changes; (2) long-time cracking and fracture; (3) complex and multifaceted thermal effects, which affect hardening and creep rate in mutually competing ways; (4) coupling to the diffusion problems of moisture and heat; and (5) strong stochastic variability engendered by high sensitivity to environmental conditions and to the variability of concrete mixes and ingredients. No wonder, therefore, that the theory of creep remains one of the most important and challenging aspects of concrete structural design.

The present lecture attempts to give a conspectus of some new developments, focusing chiefly on those attained at the writer's home institution (with which he is most intimate). One recent development in the constitutive modeling of creep is the solidification theory, whose latest improvements will be discussed first. Subsequently, attention will be focused on the interaction of creep with the damage due to cracking and with the growth of fracture. Furthermore, some recent developments in the modeling of thermal effects will be highlighted and an improved analysis of the drying creep mechanism will be briefly mentioned, along with some finite element applications. The lecture will conclude with comments on promising research directions.

2 Solidification Theory

The aging, which complicates the formulation of linear viscoelasticity of concrete, cannot be regarded as an actual change of the mechanical properties of the matter constituting concrete. Chemical substances generally have fixed properties in time. The change of mechanical properties in time must be regarded as a change in the concentrations of various substances in concrete. This is the same as in chemical thermodynamics where the properties of a system change not because of a change of the properties of its constituents but because of a change in the concentrations of time-invariable constituents. To describe a system of time-variable properties in the spirit of chemical thermodynamics, the system must be broken down to its time-invariant constituents and the changes in their quantities must be considered as the basic variables. Without this approach, the thermodynamic theory for concrete viscoelasticity cannot be properly formulated.

In the case of concrete, the non-aging constituent is the hardened cement gel (roughly the tobermorite gel, consisting mainly of tri- and bi-calcium sili-

cate hydrates). From a more precise viewpoint, the hardened cement gel might further be broken into certain subconstituents because its structure also undergoes changes. There is a certain process of polymerization representing a gradual increase of the internal binding of the hardened cement gel. Such a process may explain why the creep decreases with the age at loading even long after the volume growth of the hydration products has ceased.

To sum up, consideration of the change of volume of some nonaging constituent is the basic requirement for a sound thermodynamic description of the effect of aging on the creep of concrete. Such a description was introduced in [2], in which the model visualized in Fig. 1 was proposed, based on a general concept promulgated in [3]. It was assumed that a nonaging constituent, such as the non-polymerized hardened cement gel, is growing by additions of layers coupled in parallel, as shown in Fig. 1. Now the basic fact to take into account is that at the moment a layer of the material solidifies and gets attached to the existing solid, it is stress free. This simple fact dictates a certain form of the compliance function, which is used in the solidification theory, and also a certain special form of the equations for the aging springs in the rheologic model (some of the previous formulations in the literature violated this form, which must be regarded as physically unacceptable).

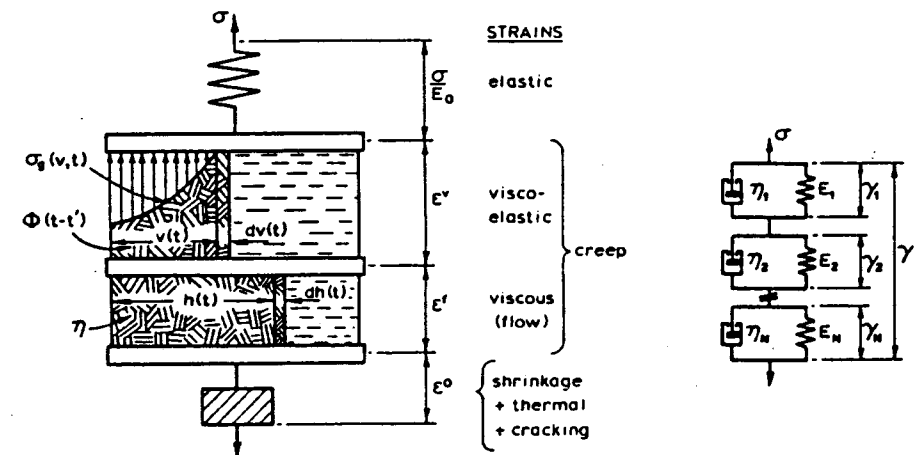


Figure 1: Visualization of solidification theory in conjunction with the Kelvin chain model; after [2-3].

According to the solidification theory [2], the constitutive law for linear aging creep of concrete is written as follows:

$$\dot{\epsilon} = \frac{\dot{\sigma}(t)}{E_0} + \dot{\epsilon}^v + \dot{\epsilon}^f \quad (1)$$

in which

$$\dot{\epsilon}^v(t) = \frac{1}{v(t)} \dot{\gamma}(t), \quad \dot{\epsilon}^f(t) = \frac{\sigma(t)}{\eta(t)} \quad (2)$$

$$\gamma(t) = \int_{t_0}^t \Phi(t-t') d\sigma(t') \quad (3)$$

Here σ, ϵ = uniaxial stress and strain in concrete, γ = effective strain in the solidifying constituent on the microscale, ϵ^v = viscoelastic strain, ϵ^f = viscous strain (or flow), $\eta(t)$ = associated viscosity which depends on the age t of concrete, $v(t)$ = volume (or volume concentration) of the solidified constituent at age t , E_0 = asymptotic elastic modulus representing the material stiffness for extremely fast deformations (extrapolated to zero load duration), and $\Phi(t-t')$ = nonaging linear viscoelastic micro-compliance function for the solidifying constituent (t' = integration variable). Note that the solidification theory prohibits replacing Eq. (2) by $\epsilon^v(t) = \gamma(t)/v(t)$, which would violate the second law of thermodynamics (however, some formulations which imply such a relation can be found in the literature). The aging is described by two functions, $v(t)$ and $\eta(t)$. Note that one of these functions would not suffice, except as an approximation when long-time creep of concrete loaded at a young age is not of interest. Note also that the compliance function Φ should include even the very rapid short-time creep that causes the difference between the dynamic elastic modulus and the static elastic modulus; otherwise the elastic modulus E would have to be considered as age-dependent and the formulation would actually become more complex.

Based on the essential properties of creep curves observed in experiment, the following expressions were proposed and verified by test data [2]:

$$\Phi(t-t') = q_2 \ln(1 + \xi^n), \quad \text{with } \xi = (t-t_0)/\lambda_0, \quad \lambda_0 = 1 \text{ day} \quad (4)$$

$$\frac{1}{\eta(t)} = \frac{q_4}{t}, \quad \frac{1}{v(t)} = \left(\frac{\lambda_0}{t}\right)^{-m} + \alpha \quad (5)$$

in which t_0 is the time of applying a constant unit uniaxial stress; m, n and λ_0 are constants that can be fixed once for all, for all the concretes; $m = 0.5, n = 0.1$; and q_2, q_4 are empirical parameters to be identified by fitting of data on each particular concrete or predicted from some approximate creep prediction model based on the composition and the strength of concrete. The macroscopic uniaxial compliance function which ensues from Eqs. (4) and (5) is then obtained as

$$J(t, t_0) = q_1 + \int_{t_0}^t j(\tau, t_0) d\tau \quad \left(q_1 = \frac{1}{E_0}\right) \quad (6)$$

The compliance rate is

$$j(t, t_0) = \frac{n(q_2 t^{-m} + q_3)}{(t-t_0) + (t-t_0)^{1-n}} + \frac{q_4}{t} \quad (q_3 = \alpha q_2) \quad (7)$$

in which q_1 and q_3 are additional empirical constants to be identified from test data for each particular concrete or predicted from the composition and strength. An important advantage of this formulation is that the parameters q_1, \dots, q_4 are all involved linearly, which means that they can be obtained from

test data by linear regression (which is always convergent, unlike nonlinear regression).

The compliance function is given according to Eq. (6) by an integral which, for the expression in (7), is a binomial integral and is not integrable in a closed form. This is no problem for computer programs solving the creep problem in time steps, because only the compliance rate j , is needed for that purpose. For simple code-type calculations, the values of J can be obtained by numerical integration, or can be calculated from a simple explicit approximate formula given in [2]. Tables and graphs for the values of J have also been prepared [2].

Aside from the physical and thermodynamic justification, the solidification theory has also two important practical advantages: First, it guarantees avoidance of the so-called divergence of the creep curves for different ages at loading, which also avoids problems with nonmonotonic creep recovery and nonmonotonic relaxation recovery. Second, it lends itself to a much simpler rate-type formulation suitable for finite element programs, which we review next.

3 Operator-Algorithmic Method with Internal Variables

The integral type description of linear viscoelastic behavior is a strictly phenomenological concept lacking any physical basis. It implies the notion of memory, yet the material has no memory *per se*. The memory property arises indirectly. It is embedded in the current values of many variables characterizing the state of the microstructure. These variables are approximately reflected in continuum theories of materials by means of the so-called internal variables (or hidden variables). Their current values take into account the previous stress or strain history of the material.

The internal variables can be regarded, in linear viscoelasticity, as the deformations or stresses in the elements of spring-dashpot rheological models. Although there are infinitely many possible geometric configurations of such models, it was proven by Roscoe [1], for nonaging viscoelasticity, that the most general possible viscoelastic behavior can be approximated with any degree of accuracy by either the Kelvin chain model or the Maxwell chain model. This makes it unnecessary to consider any more complicated models. Each unit of such a model is described by a first-order differential equation in time, and thus the integral-type creep law is converted to a system of first-order differential equations in time. These differential equations, unlike the integral equation, can be correlated to various physical concepts for the processes in the microstructure, for example the activation energy controlled rate process for the breakages of bonds that are the source of creep.

The rate-type form of the creep law can be obtained by expanding the micro-compliance function Φ into Dirichlet series (or Dirichlet-Prony series):

$$\Phi(\xi) = \sum_{\mu=1}^N \frac{1}{E_{\mu}} (1 - e^{-\xi/\tau_{\mu}}) \quad (\xi = t - t') \quad (8)$$

in which ξ is the elapsed time from the moment of loading, t' ; τ_{μ} are constants, called the retardation times, and E_{μ} are the associated elastic moduli. It can

be shown that these moduli correspond to the spring constants of the Kelvin chain model (Fig. 1) and $\tau_\mu = \eta_\mu/E_\mu$ where $\eta_\mu =$ viscosity of the dashpot of the μ -th Kelvin unit. The expansion in Eq. (8) makes it possible to replace the integral-type constitutive relation based on principle of superposition by the following system of first-order linear differential equations:

$$E_\mu \dot{\gamma}_\mu + \eta_\mu \dot{\gamma}_\mu = \sigma \quad (\mu = 1, \dots, N) \quad \text{with} \quad \gamma = \frac{\sigma}{E_0} + \sum_{\mu} \gamma_\mu \quad (9)$$

in which a set of internal variables γ_μ , representing the strains of the individual units of the Kelvin chain, is introduced, and $\eta_\mu = \tau_\mu E_\mu =$ viscosities of dashpots.

The most efficient method of integration of these differential equations, which leads to an algorithm that is stable for every value of the time step, is obtained by assuming that $\sigma, \dot{\gamma}, \dot{\gamma}_\mu, E_\mu$ and η_μ are constant within each time step and change only by jumps between the time steps. Then the differential equations (9) can be integrated exactly. This yields the so-called exponential algorithm, which was formulated for nonaging materials in [4, 5] and for aging materials in [6, 7]. To the solidification theory this algorithm was adapted in [2]. In [2], it was formulated for the relationship between stress and strain, which leads a certain incremental quasielastic stress-strain relation.

We will now indicate a new, more general statement of this algorithm in the form of a difference-algorithmic operator \underline{E}^{-1} . This operator can be substituted for E^{-1} in any elastic relation of the type $E^{-1}Cf = u$ in which $u =$ any type of strain variable or generalized displacement variable, $f =$ the associated stress or generalized force variable, and $C =$ geometric compliance, representing the compliance of the structure for $f = 1$ and for a unit value of the elastic modulus E . According to the exponential algorithm described by Eqs. (5) - (14) in part II of [2], the required generalization is obtained by replacing E^{-1} with the aforementioned operator as follows:

$$\underline{E}^{-1}Cf = \frac{1}{\Delta} \left(\frac{C}{E''} \Delta f + \Delta u'' \right) \quad \text{or} \quad \frac{C}{E''} \Delta f + \Delta u'' = \Delta u \quad (10)$$

Here Δ is the difference operator for the increment of a variable during the current time step Δt from time t_i to time t_{i+1} , and

$$\Delta u'' = \frac{1}{v_{i+\frac{1}{2}}} \Delta q'' + \frac{C \Delta t}{\eta} f_{i+\frac{1}{2}}, \quad \frac{1}{\eta} = \frac{q_4}{t_{i+\frac{1}{2}}} \quad (11)$$

$$\Delta q'' = \sum_{\mu=1}^N \left(\frac{C}{E_\mu} f_i - q_{\mu i} \right) (1 - e^{-\Delta y_\mu}) \quad (\Delta y_\mu = \Delta t / \tau_\mu) \quad (12)$$

$$q_{\mu i+1} = q_{\mu i} e^{-\Delta y_\mu} + \frac{C f_i}{E_\mu} (1 - e^{-\Delta y_\mu}) + (1 - \lambda_\mu) \frac{C \Delta f}{E_\mu} \quad (\mu = 1, \dots, N) \quad (13)$$

Subscripts i and $i+1$ refer to the values at times t_i and t_{i+1} and

$$\frac{1}{E''} = q_1 + \frac{1}{v_{i+\frac{1}{2}} D}, \quad \frac{1}{D} = \sum_{\mu=1}^N \frac{1 - \lambda_\mu}{E_\mu}, \quad \Delta y_\mu = \frac{\Delta f}{\tau_\mu}, \quad \lambda_\mu = \frac{1 - e^{-\Delta y_\mu}}{\Delta y_\mu} \quad (14)$$

The meaning of the difference algorithmic operator \underline{E}^{-1} includes create variables $\Delta u'', \Delta q''$ and q_μ associated with u , and (2) following in the time step (t_i, t_{i+1}) , in which the values $f_i, u_i, q_i, q_{\mu i}$ are already known, the following procedure:

1. Estimate the value $f_{i+\frac{1}{2}} = f_i + (\Delta f/2)$ using the value of Δf from the previous iteration or, for the first iteration, the value $\Delta f = 0$.
2. Calculate the values $D, E'', \Delta q''$ and $\Delta u''$ using equations (14), (12) and (11) for this time step and this iteration.
3. Solve Eq. (10) based on either prescribed Δf or prescribed Δu .
4. Check the convergence criterion, expressed as a norm on the change of Δf or Δu from the previous iteration to this iteration. If it is violated, go to step 1 and carry out another iteration of this time step.
5. Calculate from (13) the values of internal variables q_μ for the end of the time step, and update the values of u and f for the beginning of the next time step. Return to 1 to begin the next step.

In the foregoing formulation we used a short writing device in which the elastic deformation with the modulus E_0 is considered as a deformation of an additional Kelvin unit with a very short retardation time, for example $\gamma_\mu = 10^{-9}$ s. This elastic deformation is considered as the first unit in the Kelvin chain ($\mu = 1$).

A differential algorithmic operator \underline{E}^{-1} can also be defined to transform an elastic relation of the type $E^{-1}Cf = u$ to the differential equations of the solidification theory. The meaning of the operator is in that case as follows:

1. Create new variables q_μ associated with u and replace the foregoing elastic relation with the differential equation

$$\dot{u} = \frac{C}{E_0} \dot{f} + \frac{C}{\eta(t)} f + \frac{1}{v(t)} \sum_{\mu} \dot{q}_\mu \quad (15)$$

in which superimposed dots denote time derivatives.

2. Introduce a differential equation for q_μ , with a differential operator defined as follows:

$$Cf = \underline{E} q_\mu, \quad \underline{E} = E_\mu \left(1 + \tau_\mu \frac{d}{dt} \right) \quad (16)$$

The foregoing relations of course imply that the structure exhibits homogeneous creep properties. If not, a generalization would be required, in which the foregoing relations are written separately for each homogeneous part of the structure.

4 Continuous Retardation Spectrum

The diagram of the values of $1/E_\mu$ versus $\log \tau_\mu$ (Eq. 8) is called the retardation spectrum. For a Kelvin chain model with a finite number of Kelvin units, this spectrum is discrete, consisting of a set of points. However, it is advantageous to consider a generalization of Eq. (8) in which the spectrum is continuous, that is, there are infinitely many Kelvin chain units with retardation times τ distributed infinitely closely. According to this generalization, well known from classical viscoelasticity [8], one has, as the limit case of Eq. (8),

$$\Phi(\xi) = \int_{-\infty}^{\infty} L(\tau) (1 - e^{-\xi/\tau}) d(\ln \tau) \quad (17)$$

in which the function $L(\tau)$ characterizes the continuous spectrum and ξ is the duration of unit uniaxial stress. The solidification theory has the major advantage that this spectrum is age-independent.

The continuous spectrum has one important advantage over the discrete spectrum: it eliminates the aforementioned ill-posedness and ambiguity in the determination of the retardation spectrum from the given compliance data. For the discrete spectrum corresponding to Eq. (8), the problem of determining the unknown material parameters τ_μ and E_μ from the given test data is ill-posed [22], that is, very different sets of E_μ -values can give nearly equally good fits of the given creep data on $\Phi(\xi)$. If both τ_μ and E_μ are being determined from test data, the resulting system of equations is ill-conditioned. Now, for the continuous retardation spectrum, the values of τ need not be determined as τ is a continuous variable, and the problem of determining the function $L(\tau)$ is not ill-conditioned. This means, that Eq. (17) can be unambiguously fitted to the given creep compliance data. The discrete spectrum for Eq. (8) can then be obtained by a numerical quadrature of $L(\tau)$.

Eq. (18) can be rearranged by introducing a new variable ζ by setting $\tau = 1/\zeta$. This yields [10]:

$$\begin{aligned} \Phi(\xi) &= \int_{-\infty}^{\infty} L\left(\frac{1}{\zeta}\right) (1 - e^{-\zeta\xi}) d(\ln \zeta) \\ &= - \int_0^{\infty} \frac{1}{\zeta} L\left(\frac{1}{\zeta}\right) e^{-\zeta\xi} d\zeta + \int_0^{\infty} \frac{1}{\zeta} L\left(\frac{1}{\zeta}\right) d\zeta = -f(\xi) + f(0) \end{aligned} \quad (18)$$

where $f(0)$ is a constant. We now recognize that function $f(\xi)$ represents the Laplace transform of the function $\zeta^{-1}L(\zeta^{-1})$. So the continuous spectrum can be obtained from the measured compliance function by means of an inversion of Laplace transform. Such an inversion is provided by the following Widder's formula known from linear viscoelasticity [8, 9]:

$$L(\tau) = \lim_{k \rightarrow \infty} L_k(\tau), \quad L_k(\tau) = \frac{(-k\tau)^k}{(k-1)!} f^{(k)}(k\tau) = -\frac{(-k\tau)^k}{(k-1)!} \Phi^{(k)}(k\tau) \quad (19)$$

(for $k \geq 1$). This inversion formula can be applied only if the compliance function $\Phi(\xi)$ is sufficiently smooth, i.e., has derivatives of a sufficiently high

order and no jumps. Therefore, experimental data must be fitted by a smooth function before Eq. (9) is applied. In [10], where the foregoing formulation of a continuous spectrum is described in more detail, $\Phi(\xi)$ is approximated by the log-power law, which is defined as

$$\Phi(\xi) = q_2 \ln [1 + (\xi/\lambda_0)^n] \quad \text{with} \quad \lambda_0 = 1 \text{ day} \quad (20)$$

in which λ_0 , η and q_2 are three empirical constants. Applying Widder's formula (19) of order $k = 3$, we obtain

$$\begin{aligned} L(\tau) &= \left[\frac{-2n^2(3\tau)^{2n-3} [n-1 - (3\tau)^n]}{[1 + (3\tau)^n]^3} \right. \\ &\quad \left. + \frac{n(n-2)(3\tau)^{n-3} [n-1 - (3\tau)^n] - n^2(3\tau)^{2n-3}}{[1 + (3\tau)^n]^2} \right] \frac{(3\tau)^3}{2} q_2 \end{aligned} \quad (21)$$

Because the value of n is quite small ($n = 0.1$), the foregoing expression may be approximated by [10]:

$$L(\tau) \simeq n(1-n) \frac{(3\tau)^n}{1 + (3\tau)^n} q_2 \quad (22)$$

For computational purposes, the continuous spectrum may be approximated by a discrete spectrum corresponding to a Kelvin chain with a finite number of units and $\tau_\mu = \tau_1 10^{\mu-1}$. According to the trapezoidal integration rule,

$$\frac{1}{E_\mu} = L(\tau_\mu) \ln 10 \log \tau_\mu \quad (23)$$

The reason for first determining the continuous spectrum is two-fold: (1) a smooth continuous spectrum has no ambiguity, no ill-posedness in the identification of its parameters; and (2) it can be obtained by explicit formulas.

5 Thermal Effect on Creep

There have been repeated but futile attempts to modify the integral-type stress-strain relation based on the principle of superposition, in order to take into account variable temperature and variable humidity. Various generalizations of superposition have been tried, for example

$$\epsilon(t) = \int_0^t J[\theta(t), \theta(t')] d\sigma(t') \quad (24)$$

$$\dot{\epsilon}(t) = \frac{\dot{\sigma}(t)}{E(t)} + \int_0^t j[\theta(t), \theta(t')] d\sigma(t') \quad (25)$$

$$\epsilon(t) = \int_0^t J[t, t', T(t), T(t')] d\sigma(t') \quad (26)$$

in which θ is a certain increasing function of time, dependent on temperature T . But such efforts are doomed to failure. It is easier and physically more

realistic to consider a Kelvin chain, and assume that each unit of the chain corresponds to a thermally activated rate process which controls the rate of creep. Furthermore, the acceleration of hydration due to an increase of temperature may be taken into account by replacing the actual time t by an equivalent time t_e in the calculation of functions $v(t)$ and $\eta(t)$, that is

$$\begin{aligned} \eta_\mu(t) &= \tau_\mu E_\mu f_\mu [T(t), h(t)] \\ v &= v(t_e), \quad \eta = \eta(t_e), \quad t_e = \int \beta_T(t) \beta_h(t) dt \end{aligned} \quad (27)$$

in which

$$f_\mu [T(t)] = f_\mu^0 \exp \left(\frac{Q_c}{RT(t)} - \frac{Q_c}{RT_0} \right), \quad \beta_T(t) = \exp \left(\frac{Q_h}{RT(t)} - \frac{Q_h}{RT_0} \right) \quad (28)$$

[7] in which f_μ are functions of the current temperature T at time t , and also of the current pore relative humidity h , β_T is a function of temperature, f_μ^h is a function of pore relative humidity h , T is the absolute temperature, T_0 is the chosen reference temperature, R is the gas constant, Q_c is the activation energy of creep, governing the ruptures of bonds that cause creep, and Q_h is the activation energy of the chemical reaction of hydration.

Note that temperature has two opposite and mutually competing influences: its increase magnifies the rate of creep, which increases the creep strain, but it also magnifies the rate of hydration and thus of hardening, which decreases the creep strain. It seems virtually impossible to capture these two competing influences in an integral-type formulation. In more detail see [7, 11].

6 Interaction of Fracture with Creep

This is an important subject which has only recently come to the forefront of attention. Although the classical fracture mechanics is a time and rate independent theory, fracture is of course a time-dependent process. There is no instantaneous fracture. Fracture results from the breakage of bonds, and breakage of bonds is a thermally activated process promoted by stress. The thermal vibration energies of molecules or atoms have random values following the Maxwell distribution. Applying stress, the probability of jumps of the atoms or molecules over their activation energy barriers Q is altered in proportion to the stress. From this one can theoretically calculate the dependence of the bond breakage rate, and thus the rate of displacement v due to fracture, or the temperature. This calculation has been done in [12], with the following result

$$\dot{w} = C_T \sinh \left\{ \frac{T_0}{T} k_0 [\sigma - \phi(w)] \right\}, \quad C_T = \exp \left(\frac{Q}{RT_0} - \frac{Q}{RT} \right) \quad (29)$$

in which w is a crack opening displacement, used in the cohesive (fictitious) crack model, σ is the crack bridging stress (cohesive stress), Q = activation energy of bond breakage, and k_0 = a constant (depending on the activation volume). For practical calculations, the sinh-function can approximately be

replaced by a power function. The function $\phi(w)$ is similar to the dependence of the crack bridging stress on the crack opening displacement used in the time-independent cohesive (fictitious) crack model, but is scaled down by a ratio of approximately 0.8 to 0.9 and is shifted up by approximately 0.1 f_c' ; see Fig. 2.

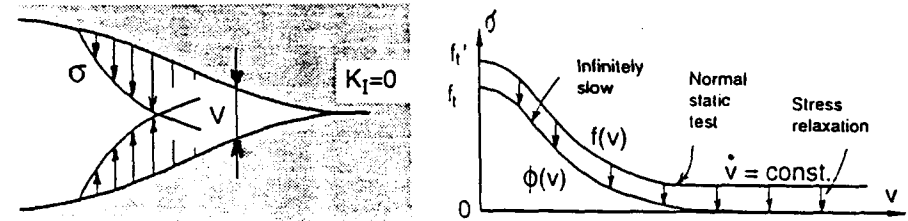


Figure 2: The stress-displacement relation for a cohesive crack at finite and infinitely slow opening rates (upper and lower curves); after [12-13].

Eq. (29) needs to be complemented by the compliance relation between the crack stresses and the displacements due to the loads applied on the structure. The elastic form of this relation can be written in the form [14, 15]

$$w(x) = \int_{a_0}^a \bar{C}(x, \xi) E^{-1} \sigma(\xi) d\xi + \bar{C}_P(x) E^{-1} P \quad (30)$$

$$u = \int_{a_0}^a \bar{C}_P(\xi) E^{-1} \sigma(\xi) d\xi + \bar{C}_{PP} E^{-1} P \quad (31)$$

in which P = applied load, u = load-point displacement, a = length of the cohesive crack, a_0 = length of notch or initial traction-free crack, and \bar{C} , \bar{C}_P and \bar{C}_{PP} are the geometric compliances (for $E = 1$), defined by the position in these equations. These compliances can approximately be determined by the finite element method. In that case the integrals are replaced by discrete sums, which leads to a formulation essentially equivalent to that used by Petersson [16].

The foregoing compliance relations must be converted to creep. The most direct way is to implement the principle of superposition based on the compliance function of the material, $J(t, \tau)$, in which case Eqs. (30) and (31) are replaced by:

$$w(x, t) = \int_{a_0}^a \bar{C}(x, \xi) \int_{t_0}^t J(t, \tau) \sigma(\xi, d\tau) d\xi + \bar{C}_P(x) \int_{t_0}^t J(t, \tau) P(d\tau) \quad (32)$$

$$u(t) = \int_{a_0}^a \bar{C}_P(\xi) \int_{t_0}^t J(t, \tau) \sigma(\xi, d\tau) d\xi + \bar{C}_{PP} \int_{t_0}^t J(t, \tau) P(d\tau) \quad (33)$$

Here, for continuous variation of P and σ , $\sigma(\xi, d\tau) = [\partial\sigma(\xi)/\partial\tau]d\tau$, $P(d\tau) = dP(\tau) = [dP(\tau)/d\tau]d\tau$.

Computationally, however, Eqs. (32) and (33) are inefficient because long sums must be evaluated in each time step of a numerical integration in time. It is much more efficient to use the rate-type form of the creep law, which leads to the following system of integro-differential equations

$$\dot{w}(x) = \int_{a_0}^a \bar{C}(x, \xi) \left(\frac{\dot{\sigma}(\xi)}{E_0} + \frac{\sigma(\xi)}{\eta(t)} \right) d\xi + \frac{1}{v(t)} \sum_{\mu} \dot{w}_{\mu}(x) \quad (34)$$

$$\dot{u} = \int_{a_0}^a \bar{C}_P(\xi) \left(\frac{\dot{\sigma}(\xi)}{E_0} + \frac{\sigma(\xi)}{\eta(t)} \right) d\xi + \frac{1}{v(t)} \sum_{\mu} \dot{u}_{\mu} \quad (35)$$

$$w_{\mu}(x) + \tau_{\mu} \dot{w}_{\mu}(x) = \frac{1}{E_{\mu}} \left[\int_{a_0}^a \bar{C}(x, \xi) \sigma(\xi) d\xi + \bar{C}_P(x) P \right] \quad (36)$$

$$\mu_{\mu} + \tau_{\mu} \dot{\mu}_{\mu} = \frac{1}{E_{\mu}} \left[\int_{a_0}^a \bar{C}_P(\xi) \sigma(\xi) d\xi + \bar{C}_{PP} P \right] \quad (37)$$

Here w_{μ} and u_{μ} are internal variables associated with w and u , and $v(t)$ is the volume growth function of the solidification theory.

Furthermore, the algorithm described in general by Eqs. (10) – (14) can similarly be applied to Eqs. (30) – (31) to obtain the incremental equations of the exponential algorithm. Alternatively, these incremental equations can also be deduced in a clearer but more lengthy fashion by expanding the compliance function in (32) – (33) into the Dirichlet series and integrating under the assumption that σ, P, \dot{w} and \dot{u} , as well as all the material characteristics are constant during each time step [15]. Under than assumption, the integrals can be expressed exactly.

From the foregoing relations, it is clear that fracture and creep interact closely. For concrete structures it is important to make a proper distinction between a short time loading of the duration of a couple of hours, dynamic loading of a fraction of a second, and long-time loading over many years. For example, in concrete dams, cracks may grow slowly over a period of 50 years and then propagate suddenly under seismic excitation. The fracture properties for these two regimes of fracture are very different, and the response is greatly affected by differences in creep.

The interaction of creep and fracture, for example, is manifested by the effect of loading rate on the peak load, exemplified in Fig. 3, or by the dependence of the nominal strength of a fracture specimen on the specimen size at various loading rates (Fig. 3). A sudden change of the loading causes an immediate change of stiffness, and can even change a softening response to hardening response [18], as illustrated in Fig. 3 (which actually represents the results from a rate-dependent generalization of the R-curve model with creep, from [19]).

Aside from the rate and creep effects on fracture, there is of course the effect of fracturing on creep. Creep at drying or creep at high deformations is typically accompanied by cracking, and may cause distinct fractures. The distributed cracking is normally characterized by the stress-strain relations for creep, but a physically correct model would call for a material model that is based on the rate of growth of the distributed cracks.

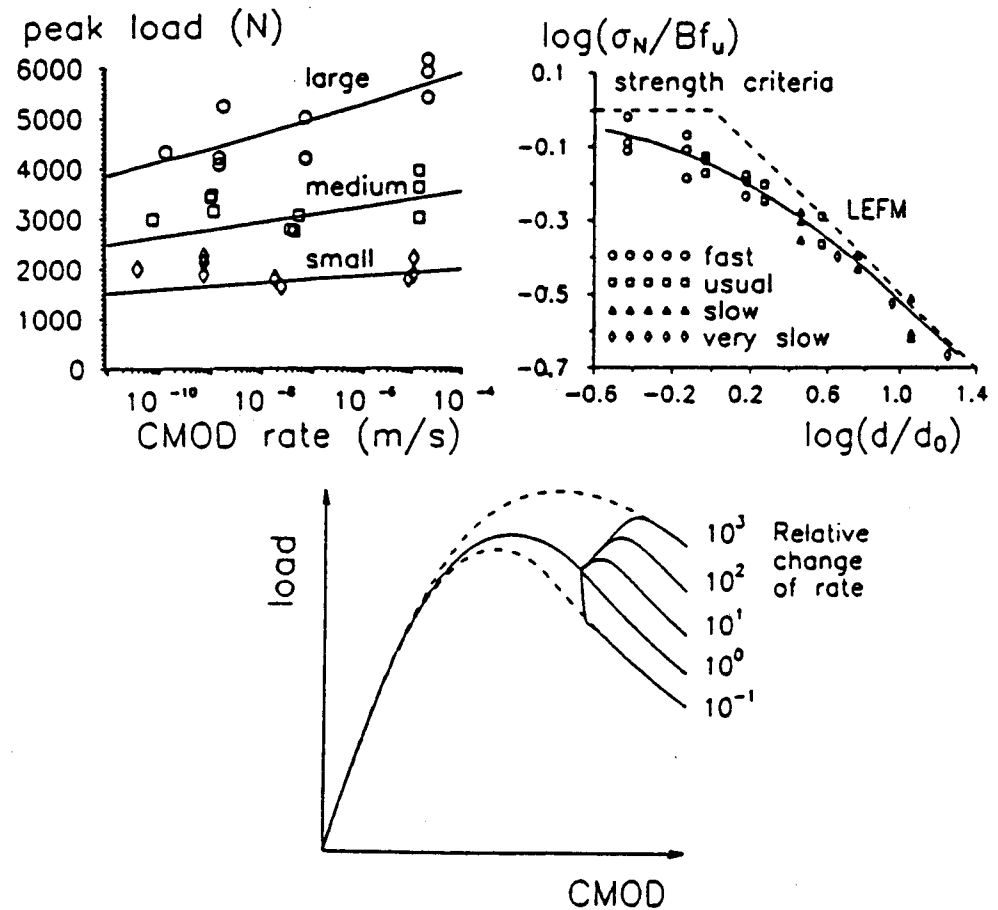


Figure 3: Typical effects of loading rate and its change on fracture response; after [13, 17–19].

7 Interaction of Creep, Fracturing and Humidity

This is a complex subject in which a gradual but significant progress has taken place over the last two decades [11]. The most important phenomenon is the increase of creep caused by drying, called the drying creep or the Pickett effect. After the rejection of several hypotheses, the physical explanation that is now considered viable is two-fold. The increase is caused by: (1) The so-called stress-induced shrinkage, the mechanism of which takes place on the molecular level and consists in the fact that microdiffusion of water between gel pores and capillary pores increases the probability of bond breakages within the solid microstructure of gel; and (2) an apparent effect, consisting in the fact that microcracking caused by drying shrinkage reduces the strain observed in load-free shrinkage specimens. This means that the difference of deformation between the compressed and load-free specimens, commonly considered as creep, actually includes the strain due to suppression of cracking in the compressed specimens. From the classical type of tests, these two mechanisms could not be separated except by an indirect analytical approach.

Recently, a new type of test which separates these two mechanisms directly and clearly has been carried out [21]. The idea is to compare curvature creep rather than axial creep, and test two specimens subjected to the same bending moment but different axial forces. For a small axial compression, there is cracking, and for a high axial compression, the cracking is suppressed. From such tests, it transpired [21] that more than half of the additional creep due to drying is caused by the stress induced shrinkage, as measured on the specimens under high axial compression. The remaining part is due to microcracking.

For illustration, Fig. 4 shows the measured effective compliance as a function of time for sealed and drying specimens at small eccentricity of the axial load resultant (the case of high compression) and at large eccentricity (the case of small axial compression). These results have been used to calibrate the cracking model coupled with the solidification theory, based on finite element analysis of creep, shrinkage and moisture diffusion in the specimen.

8 Analysis of Humidity Effects in Structures

Exchange of moisture with the environment has a major effect on long-time deformations of concrete and often causes severe cracking. This effect is complicated and is described in the existing standard code-type recommendations for predicting concrete creep and shrinkage only in a crude way. If these recommendations are used, it normally makes no sense to engage in finite element analysis. Compared to simplistic hand calculations, the gain in accuracy achieved by finite element analysis is much less than the typical error of the creep and shrinkage material model in the code-type recommendations. The recommendations cannot be made more accurate as long as they deal with the average properties of the cross section at variable humidity. To get accurate results, it is inevitable to analyze the evolutions of moisture and stress distributions throughout the cross section. A realistic and accurate material model, suited for finite element analysis, ought to have the following ingredients [11]:

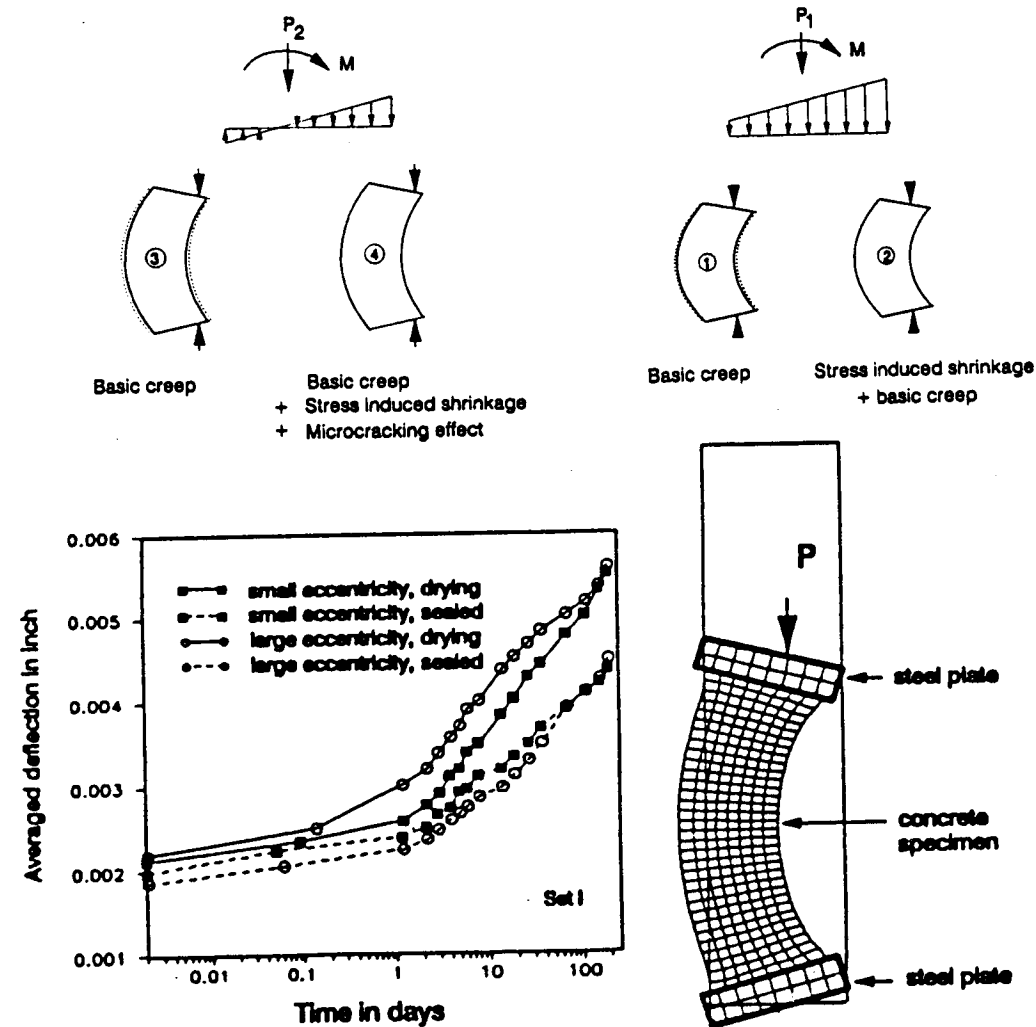


Figure 4: New tests of curvature creep at equal bending moments and different axial forces, and their consequence for the separation of stress induced shrinkage from the effect of microcracking on drying creep; after [21].

1. The profiles of the pore relative humidity and their evolution in time must be calculated by solving with finite elements the moisture diffusion problem, which is nonlinear.
2. The rate of local free shrinkage strains at various points must be determined from the space-time solution of pore humidity.
3. The stress-induced shrinkage—a large strain whose rate depends on the stress and the rate of pore humidity—must be taken into account. If temperature varies, this term also depends on the rate of temperature (see, e.g., the test data in [24]). (These phenomena are the sources of the Pickett effect [20] and the stress-induced thermal strains [24].)
4. The decrease of the viscosities of the Kelvin (or Maxwell) chain units caused by a decrease of pore humidity (e.g., the test data in [25]) must be considered. If temperature varies, the temperature dependence of these viscosities, governed by activation energy, must be taken into consideration as well.
5. The effect of aging on the creep, shrinkage and thermal properties must be calculated on the basis of the degree of hydration characterized for each point x and time t by its equivalent hydration period $t_e(x, t)$ (or maturity), whose rate of increase depends on the local pore humidity as well as (according to the activation energy of hydration) on the local temperature.
6. The strain-softening damage due to distributed cracking must be included in the analysis, and in the case of thicker cross sections it is also necessary to consider the localization of damage and propagation of distinct cracks.

To give an example of such analysis, Fig. 5 shows the profile of the normal stresses throughout the thickness of a slab of a typical precast segment of a box girder bridge (the Kishwaukee River Bridge in Illinois). This profile represents the stresses caused solely by a two-month exposure to the drying environment. The point to note is how large these stresses are—they reach the tensile strength limit, and in the surface layers there is significant tensile strain-softening (which was assumed to be stabilized against localization by the adjacent hardening layers). Such a stress and strain state, which develops at the beginning, has some significant effects on the response to the dead load and the traffic loads introduced later, particularly on the cracking strains, which play a major role in the long-time serviceability of the bridge.

8 Concluding Remarks

The present lecture has attempted a review of several recent developments which make possible a more realistic analysis of creep and thermal effects in concrete structures. The basic formulation of the solidification theory, which describes the effect of aging due to hydration of cement in a physically sound

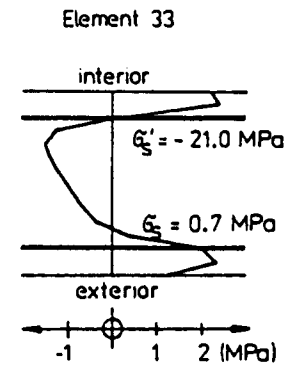
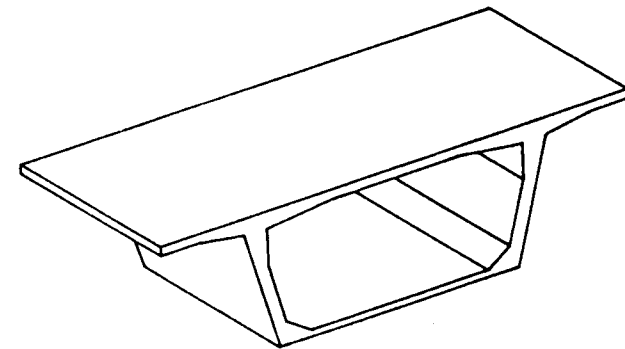


Figure 5: Stress distribution with tensile damage effect in a box girder bridge segment (after [23]).

manner, has been presented. A new, more general statement of the operator approach to applying this theory in structural analysis has been given. Furthermore, a new formulation which permits the identification and use of a continuous retardation spectrum for the solidification theory has been shown. Generalization of the cohesive crack model to rate dependent crack opening and creep in the bulk of the structure has been formulated in the form of integro-differential equations as well as in a purely rate-type form. Some new developments clarifying the modeling of the additional creep caused by drying and illustrating the drying effect on a box girder bridge have been also reviewed.

However, the theory of creep and thermal effects in concrete structures is far from complete. The most promising research directions may be identified as follows:

1. Interface of creep theory with fracture and damage mechanics, including localization.
2. Interface of creep with thermal and humidity effects.
3. Stochastic phenomena and probabilistic models, including their implementation in the context of finite element analysis.
4. Micromechanics of creep, including direct modeling of the physical phenomena in the microstructure and the composite aspects of concrete and hardened cement paste.
5. Development of effective numerical algorithms for problems involving interaction of creep with fracture, humidity and thermal behavior.

Important advances have recently also been made in the aforementioned subjects 3, 4, and 5. However their discussion is beyond the scope of a single lecture.

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