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ON THE CHOICE OF CREEP FUNCTION FOR STANDARD RECOMMENDATIONS ON PRACTICAL ANALYSIS OF STRUCTURES

Zdeněk P. Bažant and ElMamoun Osman
Department of Civil Engineering
Northwestern University, Evanston, Illinois 60201

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ABSTRACT

In the proposed revision of the C.E.B. International Recommendations, the unit creep curves for various ages t' at loading are approximated in the form f(t-t')+g(t)-g(t') where t = time. Using optimization techniques to find functions f and g which give optimum fits of experimental data, it is shown that, by contrast with the present form, the proposed form of the C.E.B. creep function cannot reasonably approximate experimental creep curves over the full range of t' of interest. In addition, the simplification of creep structural analysis intended by this formulation of creep function can be achieved, even to a greater extent, by another recent method. Therefore, the creep function of form $\varphi(t')F(t-t')$, which is presently used by C.E.B., should be retained, although improvement in the definition of functions φ and F is in order.

Dans la révision proposée des recommendations internationales du C.E.B., les courbes du fluage unitaires sont représentées sous la forme f(t-t')+g(t)-g(t') où t = le temps et t' = l'âge au temps de chargement. Par des techniques d'optimization on trouve les formes optimales des fonctions f et g pour le données expérimentales et on montre que les courbes de fluage expérimentales ne peuvent pas être suffisament approximées pour tout le domaine des t' d'intérêt. De plus, la simplification du calcul des constructions que l'on recherche par cette représentation des courbes de fluage peut être obtenue par une autre méthode récente, qui rend le calcul encore plus simple. Par conséquent, la représentation actuelle des courbes de fluage du C.E.B., ayant la forme $\varphi(t')F(t-t')$, est préférable à la représentation proposée et devrait être retenue, bien que l'on puisse améliorer la forme des fonctions φ et F.

Introduction

Within the range of service stresses, the nonlinearity of concrete creep with respect to stress is not too pronounced. Therefore, in the interest of simplicity of structural creep analysis for service loads, it is generally assumed that the creep law is linear with respect to stress, which implies the validity of the principle of linear superposition of time histories. Under this assumption, creep is completely defined by the time curves of strain for time-constant unit stress applied at various ages t'. These curves are summarily characterized by the creep function, J(t,t'), which represents the strain, ε, (creep strain plus instantaneous strain) at time t caused by a unit stress acting since time t' (1), time being measured from setting of concrete.

The form of the creep function is embodied in codes or recommendations of various engineering societies (2,3), of which the widest international acceptance is enjoyed by CEB-FIP International Recommendations (3). According to this recommendation, as well as ACI Committee recommendation (2), the creep function is assumed to be of the form

$$J(t,t') = \frac{1}{E_0} + \phi(t')F(t - t') \tag{1}$$

in which E_0 = constant representing a typical value of the elastic modulus, and ϕ and F are positive monotonic continuous functions of one variable.

Recently, Rüsch et al. (4) proposed a number of revisions of the CEB-FIP International Recommendations, with a creep function of the form

$$J(t,t') = \frac{1}{E_0} + f(t - t') + g(t) - g(t') \tag{2}$$

in which E_0 = positive constant; f and g = certain positive increasing continuous functions, defined by a graph (4). The rationale behind this form, originally proposed in Refs. (5) and (6), is that it allows a simple method of creep structural analysis, known as the "rate-of-flow method" or, alternatively (4,7), "improved (or extended) Dischinger method" (cf. Ref. (16)).

However, although a number of revisions proposed by Rüsch et al. (4) represent important improvements, the introduction of Eq. 2 has not been sufficiently justified, as has been briefly pointed out in Ref. (8). In particular, Eq. 2 has not been compared in Ref. (4) with any set of experimental creep curves for various ages t' at loading. The purpose of this paper is to make such comparisons.

Optimization Technique for Fitting Test Data

The specific form of functions f and g which was proposed in Ref. (4) has been critically examined in Ref. (8) and it has been found that the agreement with experimental data is surprisingly poor. However, this fact per se does not justify abandonment of Eq. 2 because it is not known whether or not there exist some other functions f and g for which the agreement with experimental data would be substantially better. Hence, to give the ultimate proof of the usefulness of the general form of the creep function as given in Eq. 2 (or any other form, for that matter), it is necessary to find functions f and g which give the optimum fit of given test data. It is also necessary to fit each data set separately, because otherwise it would not be known how much of the misfit is due to the differences between various concretes and how much is due to the limitations of the chosen form of creep function.

According to the method of least squares, the condition of optimum fit may be expressed as

$$\Phi = \int_t \int_{t'} [J(t,t') - \tilde{J}(t,t')]^2 d\ln(t-t') d\ln t' = \text{Min.}, \tag{3}$$

where $\tilde{J}(t,t')$ is the measured creep function and $J(t,t')$ is the theoretical creep function. Note that, in order to give equal weight to short-time as well as long-time creep, the integration of the square deviation in Eq. 3 must be carried out in logarithmic scales of $t-t'$ and t'. For numerical implementation, the condition in Eq. 3 must be approximated in the discrete form

$$\Phi = \sum_p w_p \sum_q [J(t_q,t'_p) - \tilde{J}(t_q,t'_p)]^2 = \text{Min.}, \tag{4}$$

in which t'_p ($p = 1,2,\ldots,N_p$) are discrete values of the ages at load application; $t_q = t' + \bar{t}_q$ ($q = 1,\ldots,N_q$) where $\bar{t}_q$ are discrete values of the times elapsed from the instant of load application, t'; and w_p = chosen weights. Discrete elapsed times $t-t' = \bar{t}_q$ must be chosen with nearly uniform spacing in the logarithmic scale $\ln \bar{t}_q$ and for this purpose it is usually necessary to smooth the measured creep curves by hand. Weights w_p may be all chosen as unity if discrete ages t'_p are likewise nearly uniformly spaced in log t'-scale. Yet, in practice the test data usually do not satisfy this condition and then unequal weights w_p ought to be introduced; a suitable choice is

$$\begin{aligned} w_p &= \ln t'_{p+1} - \ln t'_{p-1} \text{ for } 1 < p < N_p, \\ w_p &= \bar{w}_1 + \ln t'_2 - \ln t'_1 \text{ for } p = 1, \\ w_p &= \bar{w}_1 + \ln t'_{N_p} - \ln t'_{N_p-1} \text{ for } p = N_p, \end{aligned} \tag{5}$$

where $\bar{w}_1 = (\ln t'_{N_p} - \ln t'_1)/N_p$.

To apply numerical optimization methods, functions f and g in Eq. 2 must be characterized by a set of discrete values $f_i = f(\bar{t}_i)$ ($i = 1,\ldots,m$) and $g_j = g(t_j)$ ($j = 1,\ldots,n$) where $\bar{t}_i$ and t_j are chosen discrete time values, spaced uniformly in the logarithmic scale of $\bar{t}_i$ or t_j. For arguments $\bar{t}$ or t lying between two discrete values, $\bar{t}_i$ or t_j, functions $f(\bar{t})$ and $g(t)$ are determined by linear interpolation in terms of $\ln \bar{t}$ or $\ln t$, and beyond the extreme discrete times linear extrapolation is used.

To assure that $f(\bar{t})$ and $g(t)$ are positive-valued monotonically increasing functions and that E_0 is positive, it is expedient to set

$$\begin{aligned} f_i &= x_1^2 + x_2^2 + \cdots + x_i^2 = \sum_{k=1}^{i} x_k^2 \\ g_j &= x_{m+1}^2 + \cdots + x_{m+j}^2 = \sum_{k=m+1}^{m+j} x_k^2 \\ 1/E_0 &= x_N^2 \quad (N = m+n+1). \end{aligned} \tag{6}$$

For a given set of test data, expression Φ in Eq. 4 may be regarded as a nonlinear function of unknowns $x_1,x_2,\ldots,x_N$ and thus the following optimization problem results:

$$\Phi = \Phi(x_1,x_2,\ldots,x_N) = \text{Min.} \tag{7}$$

The solution of the optimization problem is facilitated by the fact that Φ represents a sum of squares, so that Eq. 7 may be written as

$$\Phi = \sum_{r=1}^{M} [F_r(x_1,x_2,\ldots,x_N)]^2 = \text{Min.}, \tag{8}$$

in which

$$F_r = w_p[J(t_q,t'_p) - \tilde{J}(t_q,t'_p)], \quad r = (p-1)N_q + q, \quad M = N_q N_p. \tag{9}$$

In choosing the optimization method, it should be noted that it would be quite inconvenient to compute derivatives of functions F_r with respect to arguments x_k. For nonlinear sum-of-squares problems a very efficient nonlinear optimization method, which avoids computation of the derivatives, is the Marquardt algorithm (9) as modified in Ref. (10). This algorithm, for which standard library subroutines are available, has been used in the present study, in conjunction with a subroutine that has been written to compute functions F_r from any given values of $x_1,x_2,\ldots,x_N$.

Comparison with Experimental Data

To be able to pass judgment on the suitability of Eqs. 1 or 2, it is necessary to compare these equations with test data that cover a broad range of test durations, $t - t'$, as well as ages at loading, t'. Only a few such data are found in the literature, and the best ones satisfying this requirement are those of L'Hermite et al. (11), Figs. 1 and 2, of Pirtz (12), Fig. 3, and of Hanson and Harboe (13,14), Fig. 4. A few other creep data of nearly the same scope are summarized in Ref. (15).

To apply the optimization technique, the creep curves were first smoothed by hand, and representative data points were selected on such smoothed curves, with equal spacing in $\log(t - t')$ scale. The optimization algorithm described was run for widely different choices of starting values of f_i, g_j and E_0, and the same optimum values f_i, g_j and E_0 were always obtained. This confirmed the soundness of the optimization technique selected.

The optimum fits in terms of Eq. 2 are shown by the solid curves in Figs. 1 - 4 and the corresponding optimum functions f and g are graphically plotted in Figs. 1 - 4 in separate diagrams. It is apparent that the fits are very poor. The fits by Eq. 2 with functions f and g according to the proposed revision (4) would be even poorer because these functions are not the optimum ones. An example of such fit can be found in Ref. (8), although with a time-variable elastic modulus $E(t')$ instead of a constant modulus E_0 as in Eq. 2. Considering time-variable E somewhat improves the fits of test data but would be out of place in this study because introduction of time-variable E complicates the structural analysis based on Eq. 2 to such an extent that it defeats the purpose of Eq. 2.

For comparison, the test data were also fitted by Eq. 1. However, it has been found that one does not need to consider general functions ϕ and F because excellent fits can be obtained by limiting consideration to a certain special form of functions ϕ and F in Eq. 1, as indicated by the double power law

$$J(t,t') = \frac{1}{E_0} + \frac{\phi_1}{E_0} t'^{-m}(t - t')^n \tag{10}$$

proposed on p. 15 of Ref. (1); ϕ_1, m, n are non-dimensional constants and E_0 represents the typical value of the elastic modulus for very rapid loading (possibly a higher value than that obtained in the conventional, not too rapid, loading). Optimum fits obtained with Eq. 10 are shown by the dashed lines in Figs. 1 - 4. Obviously, a much better agreement with test data can be achieved by using Eq. 1. (If the forms of functions ϕ and F as given in the existing CEB-FIP International Recommendations (3) were used, the fits in Figs. 1 - 4 would not be so close. However, this discrepancy would not be inherent to the form of Eq. 1 as such.)

Discussion

The cause of the misfit in Figs. 1 - 4 may be traced to the fact that the term $g(t) - g(t')$ in Eq. 2 arbitrarily constrains the effect of age at loading to the shape of the creep curve. In previous studies, rather than using creep curves for various ages at loading, functions f and g in Eq. 2 were determined from one typical creep curve and one typical creep recovery curve. However, two objections to this approach can be raised: (a) The linear principle of superposition, which is the basic assumption underlying all current practical methods of creep analysis, is much more in error in case of large strain reversals (creep recovery) than it is in their absence, provided the stresses remain in the working stress range. (Note that no strain reversal occurs in stress relaxation.) (b) By fitting $J(t,t')$ to the behavior after complete unloading, a very rare case in practice, the fit of creep curves for constant or slowly relaxing stress, the regime close to that in most structures, is sacrificed.

It may appear striking that such a significant discrepancy from test data is revealed by Figs. 1 - 4, whereas in previous works on the "improved Dischinger method" or the "rate of flow method" the comparisons with experimental data looked acceptable. The reason is that only a very narrow range of ages t' at loading has been considered in previous works and that the comparisons were presented in actual (rather than logarithmic) time scale, in which only one order of magnitude of time can be shown and eventual misfits for longer as well as shorter creep durations are obscured.

The discrepancy apparent from Figs. 1 - 4 does not necessarily lead to significant errors in structural creep analysis. In many creep problems, the results are sensitive mainly to creep function values for a certain average creep duration and a certain average age, and these values can be given by Eq. 2 without gross error. In a detailed study of various approximate linear methods of creep analysis (16) many of the predictions based on the rate of flow method were found to be quite acceptable (and generally much better than the predictions of the old "rate-of-creep method" or "Dischinger method"). Nevertheless, in a number of other cases a significant error with regard to the theoretically exact solutions has been found (16).

Introduction of the "improved Dischinger method" has previously been justified by the need of keeping the creep calculations simple. However, this argument is unfounded. Although the "improved Dischinger method" does indeed simplify the creep analysis, yielding easily integrable differential equations in basic practical problems, it has been recently shown that with the help of another method utilizing a table of a certain auxiliary coefficient even simpler solutions, consisting in a single elastic analysis, are possible (16). Moreover, this method is applicable for any form of creep function $J(t,t')$, so that no distortions of the actual creep function to suit the method of analysis

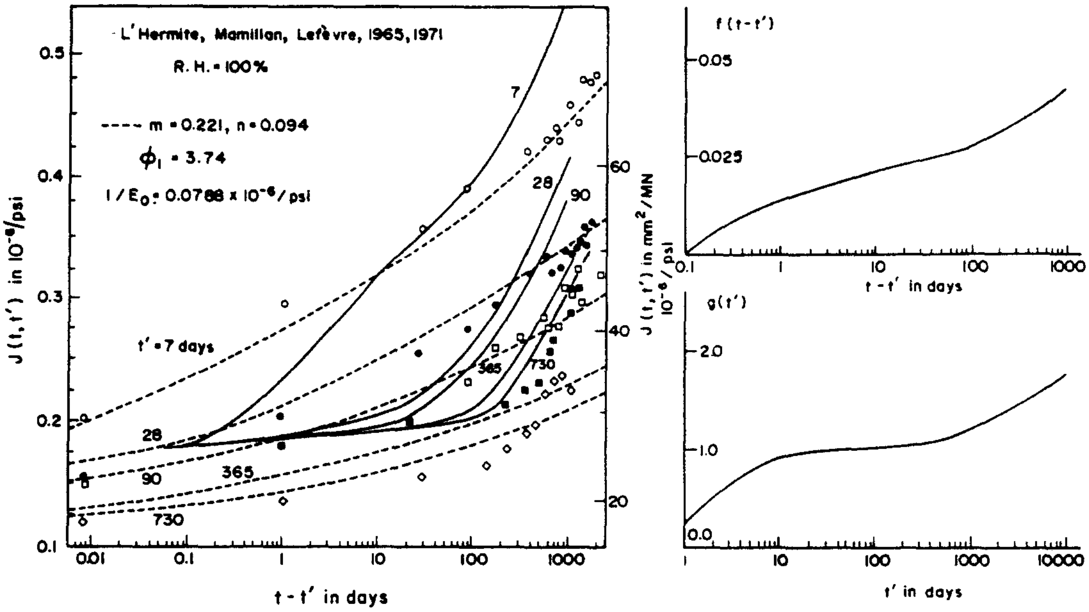

Fig. 1 L'Hermite and Mamillan's Creep Tests in Water. (Data constructed from Ref. 11; prisms 7 x 7 x 28 cm of 28-day strength 370 kgf/cm^2; in water; at room temperature; concrete French type 400/800; 350 kg of cement per m^3 of concrete; stress = ¼ strength; water-cement-sand-gravel ratio 0.49: 1:1.75:3.07; Seine gravel.)

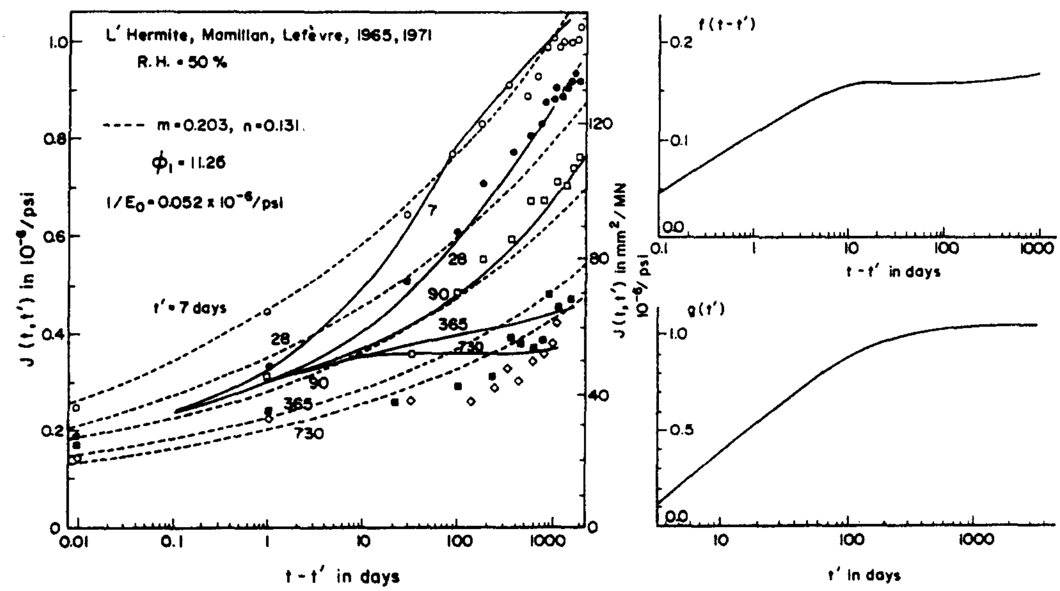

Fig. 2 L'Hermite and Mamillan's Creep Tests in Drying Environment. (Data constructed from Ref. 11; same test series as in Fig. 1, environmental relative humidity = 50%.)

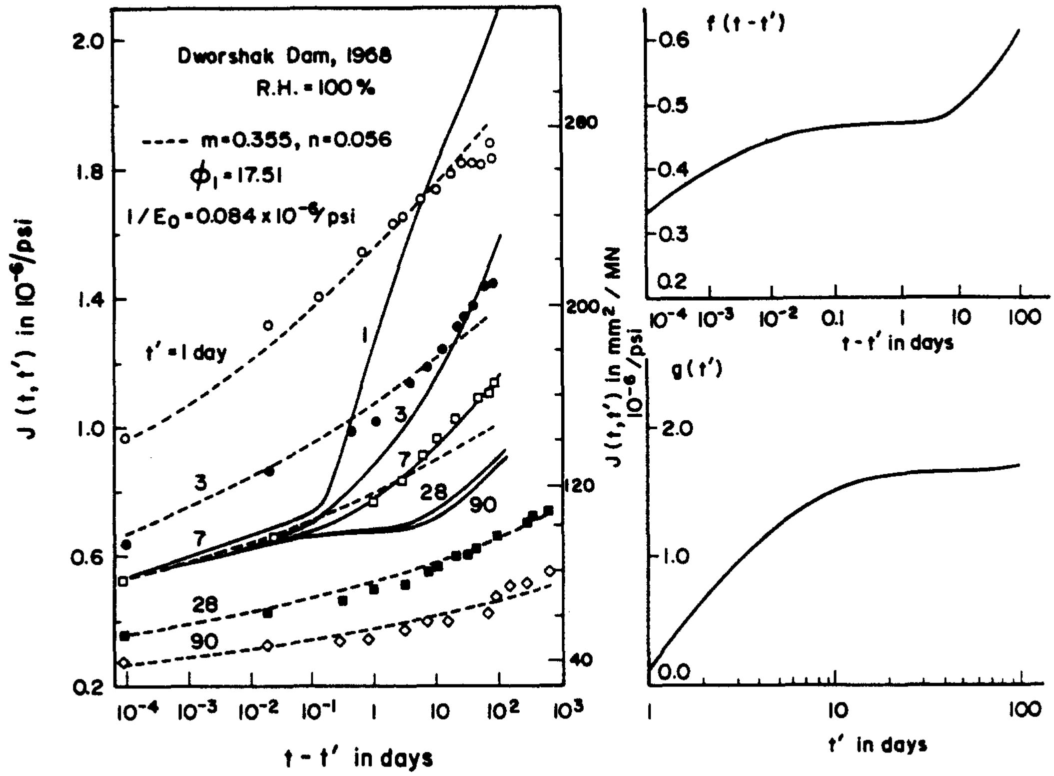

Fig. 3 Creep Tests for Dworshak Dam. (Data extracted from Ref. 12; cylinders 6 x 26 inch, sealed, at 70 F; 28-day cyl. strength = 3230 psi; stress ≤ 1/3 strength; water-cement ratio 0.58; cement type IV; max. aggregate size 1.5 in.)

are necessary. In addition, this method was also shown to yield theoretically more accurate results than the rate-of-flow method (16).

It has been argued that Eq. 2 follows from the possibility of decomposition of creep strain in reversible and irreversible components. This decomposition is indeed possible; but in a time-variable material it can be thermodynamically formulated only in a rate-type form which in general is not integrable in the form of an algebraic expression for total strains (1).

It has also been suggested that a good agreement of Eq. 2 with test data is obtained if the creep curves are arbitrarily shifted in the vertical direction (see Bild 2, reply following Ref. (8), p. 152). However, such shifting implies for elastic modulus E_0 in Eq. 2 a very strong time variation, which has a very substantial effect in structural analysis (16) and may not be neglected. This fact, of course, destroys the purpose of Eq. 2 because the use of the "improved Dischinger method" is impossible when E_0 is considered time-dependent. Therefore, arbitrary shifting of creep curves to make the fits look better is misleading and inadmissible.

Finally, it is also of interest to note that the optimum fits of creep data do not indicate that the creep curves should approach some final asymptotic value.

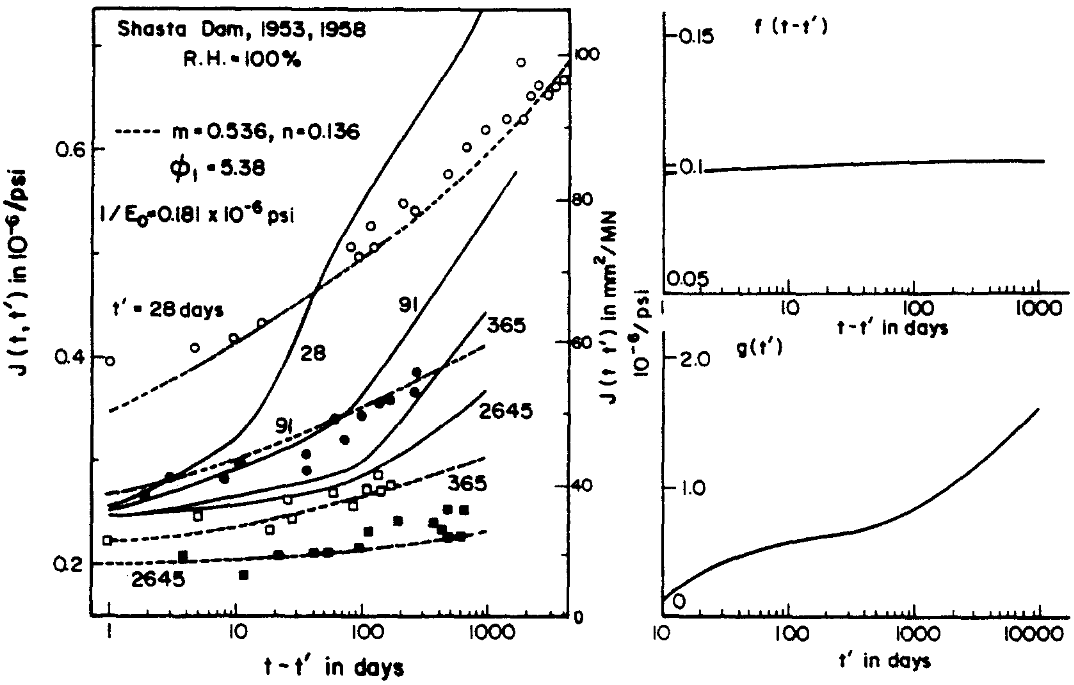

Fig. 4 Creep Tests for Shasta Dam. (Data extracted from Refs. 13 and 14; cylinders 6 x 26 inch, sealed, at 70 F; 28-day cyl. strength = 3230 psi, stress ≤ 1/3 strength; water-cement-sand-gravel ratio 0.58:1:2.5:7.1 by weight; cement type IV, max. size of aggregates 0.75 to 1.5 inch.)

Conclusions

The creep function of the type proposed for CEB International Recommendations (Eq. 2) is unable to give an acceptable approximation of experimental creep curves over the full range of ages at loading of interest. Therefore, introduction of this creep function would represent a step backward and the type of creep function (Eq. 1) embodied in the current recommendations (3) should be retained, although an improvement in defining its component functions is in order. In particular, the double power law (Eq. 10) appears to yield the desired improvement.

The total creep strain cannot be decomposed into reversible and irreversible components, even though infinitesimal creep increments can.

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DISCUSSIONS

A DISCUSSION OF THE PAPER "ON THE CHOICE OF CREEP FUNCTION FOR STANDARD RECOMMENDATIONS ON PRACTICAL ANALYSIS OF STRUCTURES"*

by Z.P. Bazant and E.M. Osman

H. Rüsch, D. Jungwirth and H. K. Hilsdorf
Institut für Beton und Stahlbeton Universität Karlsruhe
Karlsruhe, Germany

In this paper, as in previous discussions by Mr. Bazant e.a. the authors stated that the prediction method for creep coefficients and the analytical procedure to estimate the time dependent behavior of concrete structures as proposed by the writers (1) are not suitable for this purpose since the proposed creep function cannot reasonably approximate experimental creep data. Since many of the readers of "Cement and Concrete Research" are with all likelihood not familiar with the method described in (1) some additional information shall be given in the following: In the CEB-FIP recommendations 197o (2) a method to predict shrinkage and creep of structural concrete was given. There, creep was expressed in terms of the product of five coefficients taking into account age at loading, mix proportions, specimen size, relative humidity and duration of loading. A strict application of the law of superposition was suggested to evaluate creep under variable stress.

After this method had been used for some time it became apparent that it had a number of shortcomings:

1. It does not show clearly that parts of the creep strains are reversible.
2. The estimate of creep coefficients from the product of five independent variables is unsatisfactory since experimental data show some interdependence between these variables. This is particularly true for the effect of specimensize which influences both the magnitude of creep as well as the development of creep with time and is different for different values of concrete age at the time of load application.
3. An estimate of stress relaxation and creep under variable loads using the CEB-FIP methods was cumbersome and the results were for some cases unsatisfactory.

Together with the formulation of an improved prediction method for creep coefficients also a new method of analysis was developed ("extended Dischinger

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method"). The new methods avoid some of the shortcomings of the previous methods, and the new method of analysis is particularly suitable for the proposed prediction method. This, however, does not imply the new prediction method for creep coefficients to be applicable only to the proposed method of analysis.

In the proposed prediction method total concrete strain is subdevided into instantaneous strain, reversible and irreversible strains:

$$\varepsilon_t = \varepsilon_e + \varepsilon_v + \varepsilon_f + \varepsilon_s \qquad (1)$$

where

ε_t = concrete strain at time t

ε_e = instantaneous strain

ε_v = delayed elastic strain (reversible creep)

ε_f = plastic flow (irreversible creep)

ε_s = shrinkage strain

The prediction method is phenomenological and depicts the observed behavior of concrete under sustained loads. It makes the various components of creep and their effect on structural behavior particularly clear to the designer. In (1) it was shown that the delayed elastic strain can be expressed as a constant ratio of the elastic strain. Plastic flow can be estimated from tables and diagrams taking into account the effect of concrete age at loading, mix proportions, specimen size, relative humidity and duration of loading. Particularly the effect of specimen size has been developed clearly and the law of superposition now may be used in a simplyfied manner compared to the previous CEB-FIP recommendations (2).

The coefficients and relationships for the prediction of creep coefficients given in (1) have been deduced from a continued survey of various experimental creep data including experiments on the effect of age at time of load application. It is not true that they were deduced from one typical creep curve and one typical creep recovery curve. In Fig. 1 of this discussion which had already been used in a previous discussion between Mr. Bazant and the writers (3) the experimental data given on the left hand side of Fig. 1 of the original paper are compared with the creep functions as determined on the basis of the creep prediction method suggested in (1) over the range of practical interest. In this figure the first term of equation 1, the instantaneous strain, was determined using an age dependent modulus of elasticity on the basis of the respective relationship given in the CEB-FIP recommendations (2).

In most instances the instantaneous strain ε_e in equation 1 may be assumed to be age independent and constant since in many cases of engineering practice concrete is subjected to sustained loads at an age of at least 28 days. Subsequent change in modulus of elasticity may - as an approximation - be neglected. The method of analysis described in (1) does indeed assume a constant modulus of elasticity. However, in cases where concrete is subjected to sustained loads at an earlier age the variation of modulus of elasticity with time can be taken into account using a method of stepwise integration described in (4).

Comparison of experimental creep data for specimens loaded at an age ranging from 7 days to 73o days with the prediction method for creep coefficients is meaningless if the obvious differences in the instantaneous strain ε_e for young concrete are neglected. It is difficult to understand that the writers label such justified adjustment "arbitrary shifting". Accepting the age dependent variation of the instantaeous strain the predicted creep curves agree as well or better with experimental data as any other comparatively simple prediction method currently in use and known to the writers. Even if a constant value

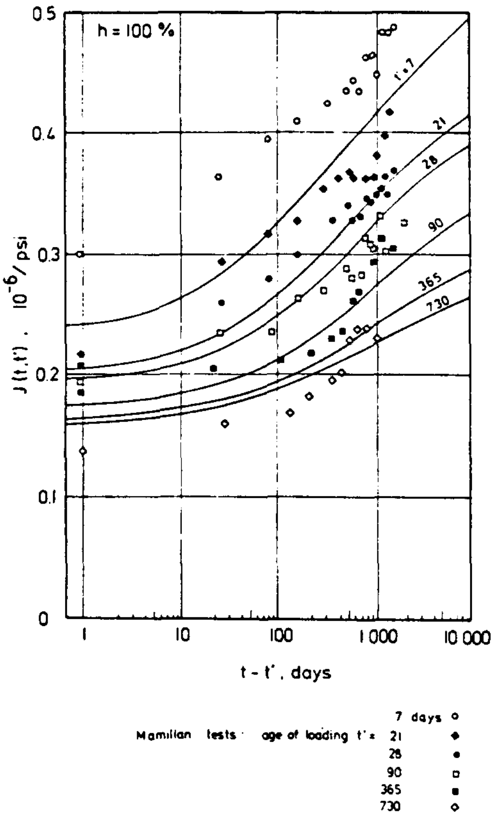

FIG. 1

(From Discussion to "Kritische Sichtung der Verfahren zur Berücksichtigung der Einflüsse von Kriechen," by H. Rüsch, D. Jungwirth, H. K. Hilsdorf, Beton-und Stahlbetonbau, Heft 6, 150-152 (1974).

of ε_e were assumed for an age at loading equal to or larger than 28 days the agreement is still satisfactory. The agreement may be also acceptable for a duration of loading of less than 1 day. We leave it up to the authors to check. It is of no value for engineering practice. There is no doubt that better agreement can be obtained by numerical optimization of a particular function with a large number of variable coefficients for a particular creep curve. However, this does not lead to a prediction method.

It should be pointed out again that the prediction method for creep coefficients described in (1) may be used for other analytical procedures as well. In (1) it has been compared with the previous CEB-FIP method (2) and it was shown that both methods agree well except for those cases for which the old CEB-FIP method needed improvement.

The suggested method of analysis is an approximation as are all other methods of practical significance. The writers have no particular reason to condemn Mr. Bazant's approach. For most structural problems various methods exist for their solution. It has been shown in (1) that the results of creep analyses obtained by different methods do not differ greatly from each other particularly if the still existing uncertainties in predicting the actual creep behavior of structural concrete are taken into account; however, they do differ in the amount of work involved. Let the users judge.

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REPLY TO RÜSCH, JUNGWIRTH, AND HILSDORF'S DISCUSSION OF THE PAPER "ON THE CHOICE OF CREEP FUNCTION FOR STANDARD RECOMMENDATIONS ON PRACTICAL ANALYSIS OF STRUCTURES" *

Zdeněk P. Bažant and ElMamoun Osman
Department of Civil Engineering
Northwestern University, Evanston, Illinois 60201

The authors welcome the Discussion by H. Rüsch et al., for it raises several important questions on which, unfortunately, no agreement has yet been reached by specialists in the field.

Comparison of the Proposed C.E.B. Creep Function with Test Data

Effect of Vertical Shifting

The figure of the discussers does not correspond to their proposed C.E.B. creep function (Ref. 4). The correct plot is shown in Fig. 6 and it is seen that the deviations from test data are unacceptably large. They are also greater than those in Fig. 1 of the paper, which pertains to the best possible fit by a function of the type proposed for C.E.B. recommendations.

In the figure of the Discussion the creep curves have been vertically shifted, which gives the appearance of a better agreement with test data, but implies a very strong age-dependence of the associated (not the actual) elastic modulus E. By deleting the time range from 0.01 day to 1 day, the associated values of 1/E have been obscured. In Fig. 7 the curves of the discussers are extended to 0.01 day and the 1/E-values obtained by taking the strain at 0.01 day are also plotted.

It is claimed in the Discussion that the disagreement for loadings of duration of less than 1 day "is of no value to the engineering practice". However, this is not true. To be sure, for long-time structural creep effects the detailed shape of creep curves up to 1 day (Fig. 8), as well as the strain increment from 0.01 day to 1 day, is indeed unimportant when the concrete is more than 7 days old at loading. For long-time predictions it does not matter much when only the short-time strain, 1/E, is arbitrarily distorted (see shifts a or b in Fig. 8, yielding curves 423 or 723). However, the total strain due to load at 1 day and beyond is very important. When 1/E is changed by shifting the whole creep curve (shifts c or d in Fig.

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* CCR 5, 129 (1975)

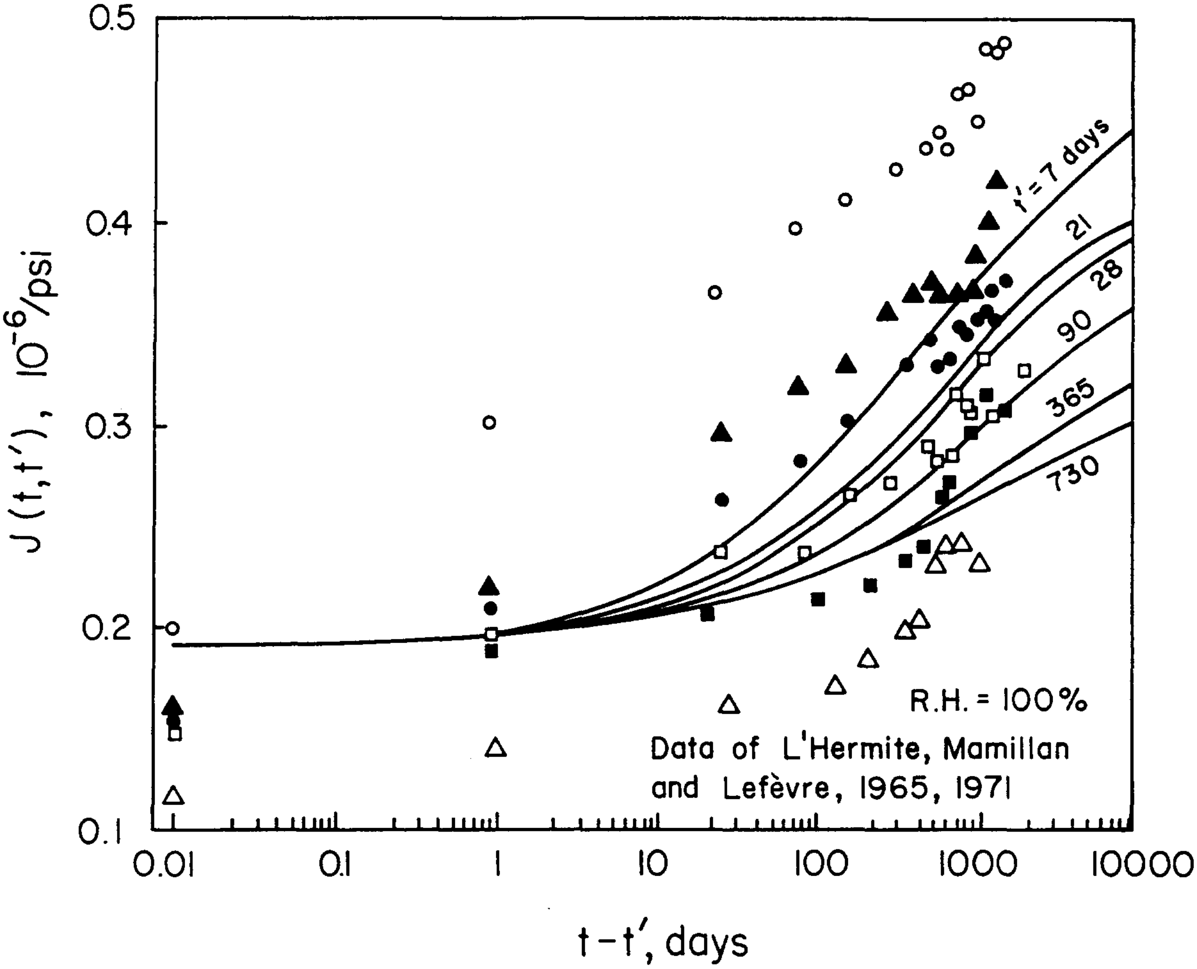

Fig. 6 The Plot of Creep Curves Exactly as Proposed for the C.E.B. Recommendations, Compared with Test Data

8, yielding curves 456 or 789), rather different total long-time creep strain may be obtained, which may result in a gross error in the predictions of long-time creep effects. It is the latter type of distorsion that was done in the figure of the Discussion.

By shifting the creep curves, the discussers transfer the age-dependence into E and assume that in E the age-dependence does not matter. But this is only true when the change in E is small (up to roughly 7%). It has been demonstrated by computer calculations (Ref. 16) that often the time-variation of E does have considerable effect on the theoretical predictions of creep effects. Nevertheless, since the discussers say that they "leave it up to the authors to check", it will be useful to do so by means of a simple example. According to the principle of superposition, strain $\epsilon(t)$ caused at age t by stress history $\sigma(t)$ that has begun at age t_0 is

$$\epsilon(t) = \int_{t_0}^{t} \left[\frac{1}{E(t')} + C(t,t')\right] d\sigma(t') \tag{11}$$

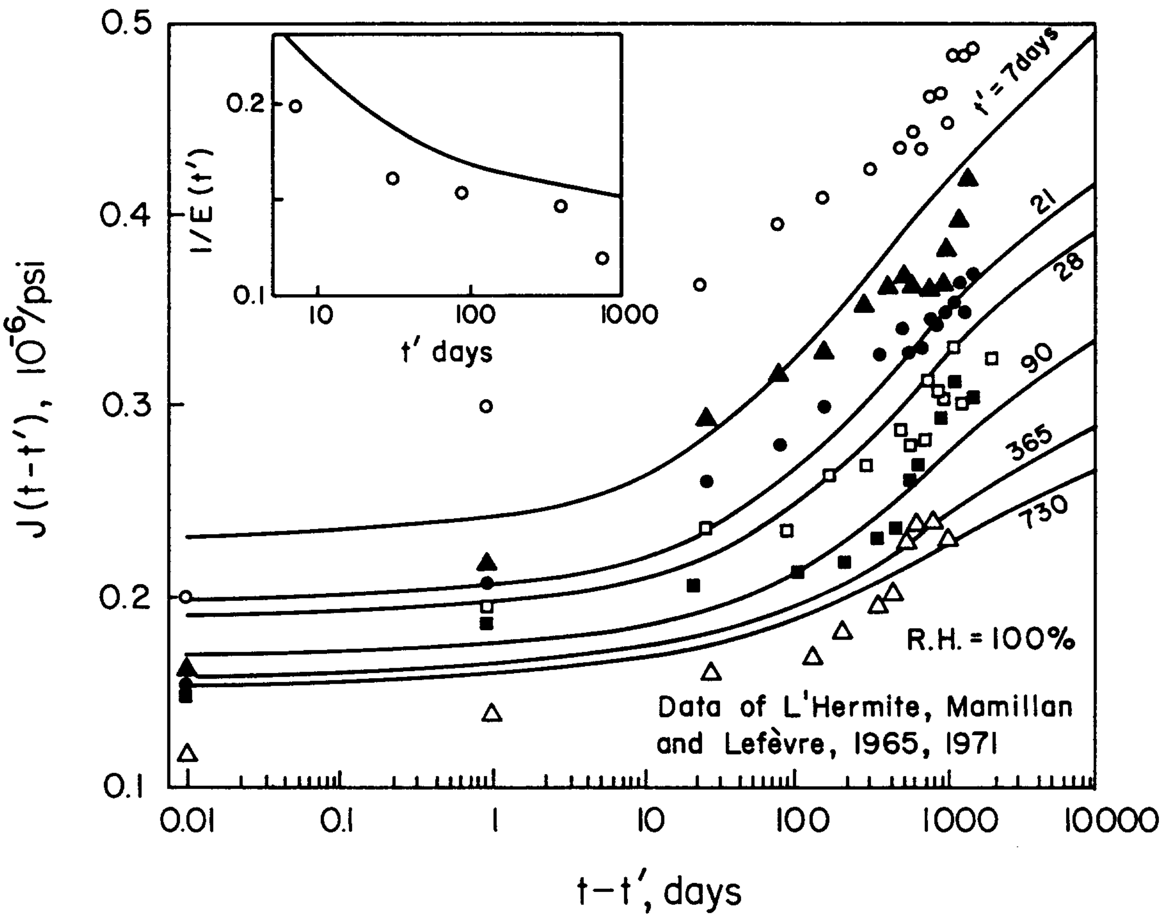

Fig. 7 Extension of the Vertically Shifted Creep Curves from the Discussion into Short Times, Intended to Show the Associated Variation of Elastic Modulus E

in which C(t,t') = specific creep = creep strain (total strain minus instantaneous strain) at time t caused by a unit stress acting since time t'. If the actual function E(t') is replaced by some arbitrary function $E_a(t')$ without changing C(t,t') (see the vertical shift c or d in Fig. 8), the error committed in the final strain is

$$\text{Error } (\epsilon) = \left[\frac{1}{E_a(t_0)} - \frac{1}{E(t_0)}\right] \sigma(t_0) + \int_{t_0^+}^{\infty} \left[\frac{1}{E_a(t')} - \frac{1}{E(t')}\right] d\sigma(t') \qquad (12)$$

To allow easy integration, one may quite realistically assume (18) that $1/E_a(t') = [1+\alpha(28/t')^{1/3}]/E_0$ where E_0 and α are constants and t' is in days. According to the proposed C.E.B. creep function, E(t') is taken as a constant, $E(t') = E_{28}$. As an example of stress variation, one may consider that stress $\sigma(t_0) = \sigma_0$ induced at age t_0 = 7 days gradually relaxes to a final value $\sigma_\infty = 0.25\ \sigma_0$ and that the relaxation curve is roughly similar to $t^{-1/3}$; this yields $\sigma(t') = \sigma_\infty\ [1+3(7/t')^{1/3}]$. Substitution of the fore-

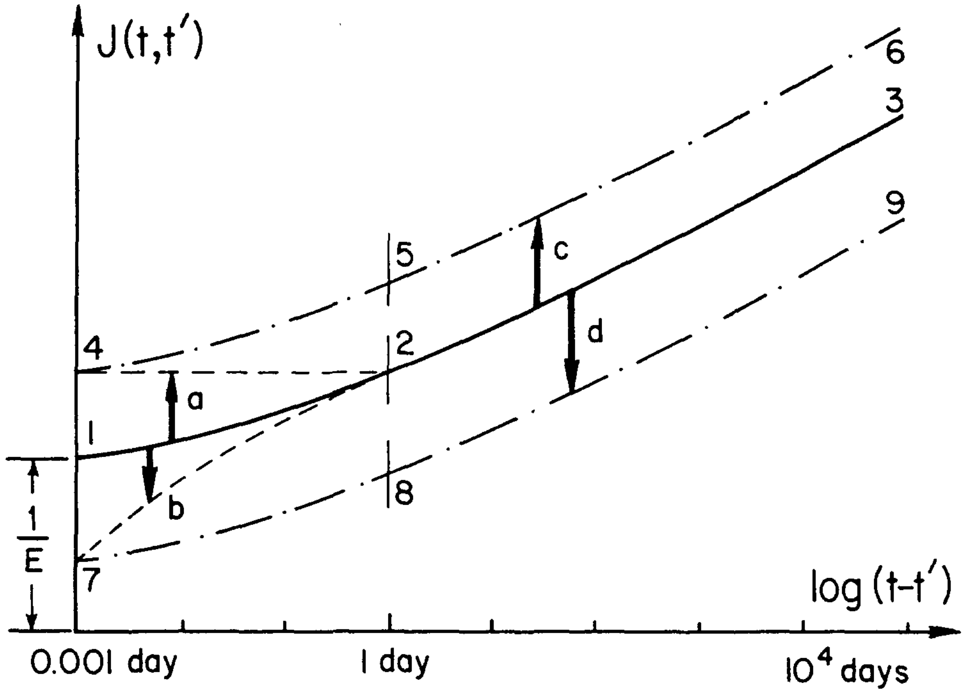

Fig. 8 Admissible (a,b) and Inadmissible (c,d) Distorsions of the Correct Creep Curve (123) with Regard to the Prediction of Long-Time Effects for Concrete More than a Few Days Old

going expressions into Eq. 12 and integration provides 0.74 $\alpha\ \sigma_0/E_0$. The variation of E in Fig. 7 is well described by function $E_a(t')$ with $\alpha = 0.43$ and this provides

$$\text{Error } (\epsilon) \doteq 0.32\ (\sigma_0/E_0) = 32\% \text{ of total strain} \qquad (13)$$

i.e., the error that would be committed in strain by vertical shifting of creep curves in the discussers' figure is in this example about 32% of the total strain causing stress relaxation.

Thus, it is obvious that the creep curves must accurately describe the total strain due to stress. Although the apportionment of the total strain in the elastic and creep parts is insignificant, arbitrary vertical shifting of creep curves leads to a serious error because it alters the total strain.

The variation of the associated elastic modulus such as that in Fig. 7 can, of course, be taken into account using step-wise numerical integration, as the discussers suggest. However, then the "improved Dischinger's method" which they proposed in Ref. 4 cannot be applied and the calculation is much more complex. Aside from that, the proposed C.E.B. recommendation does not tell the designer how to determine the variation of 1/E.

Other Aspects

It is wondered why the range of ages at loading from 7 days to 730 days is labeled "meaningless". Structures are designed for a life of about 40 years and when any long-time creep effect on stress distribution occurs, the stress varies gradually up to 40 years. According to the principle of superposition, the creep caused by all stress increments, even those after 730 days, must be included to reach correct long-time predictions. It is not the question of whether or not the loads applied on the structure will change after 730 days.

It is not understood how the discussers could attribute the better agreement of the creep curves obtained by optimization to a "large number of variable coefficients" and say that this "does not lead to a prediction method". The double power law underlying these curves (Eq. 10 of the paper) involves only four constants, namely E_0, φ_1, m and n, of which one (E_0) defines the basic value of elastic modulus E and two other (φ_1, m) define creep while at the same time defining the age-dependence of E (E represents 1/J for $t-t' \approx 0.01$ day). This is the least number of constants one could possibly desire. On the other hand, in the proposed C.E.B. creep function (Eq. 2), the functions f and g are not characterized by any law and to define them at least one discrete value (f_i and g_j) is needed in every decade of log(t-t')– and log t'– scales, which amounts to at least 10 unknown parameters. Moreover, since the adjacent values f_i and g_j are not tied mutually by any law, Eq. 2 cannot be used for extrapolating short-time creep data into long-time creep data, whereas the double power law (Eq. 10) can be used for this purpose very effectively and does lead, therefore, to a prediction method.

The discussers deny that their proposed creep function has been "deduced from one typical creep curve and one typical recovery curve", disregarding data on the age effect. But then it is not clear how the age effect could have been taken into account because a single creep curve and a single recovery curve is sufficient to define the creep function in Eq. 2 uniquely, unless the recovery curve is disregarded even though its use is implied by introducing the notion of reversible creep. (A formulation using as the basic information the recovery curve instead of the creep curves at various ages at loading is disadvantageous for reasons which were stated on page 133 in the second paragraph, which was not commented upon in the Discussion.)

The fact that insufficient agreement of the existing (1970) C.E.B. Recommendation with relaxation data is sometimes found (item 3 in the Disdussion) is due, in the writer's opinion, mainly to the effect of drying. This is a nonlinear effect that cannot be described by any linear creep law based on principle of superposition (19). However, for massive structural members, in which the rate of moisture loss is small or nonexistent, the linear creep law predicts relaxation very accurately, provided that it fits also the creep data (Ref. 15). The writers concur that the effect of specimen size on creep does not agree very well with the existing (1970) C.E.B. Recommendation; but, according to their own studies, improvement of this shortcoming does not necessitate abandoning creep functions of the form of Eq. 1 in the paper.

Separation of Creep in Reversible and Irreversible Components

To make a definition of the reversible component of concrete creep meaningful, the strain which is ultimately recovered after a stress cycle, such as a pulse of constant stress beginning at age t_0 and ending at age t_1, would have to be essentially independent of ages t_0 and t_1. However, there is no test data indicating that for various ages t_0 and t_1 the ultimate recovery strains do not significantly differ. Therefore, reversible creep cannot be uniquely defined. Although it has been suggested that at least after longer creep periods the ultimate creep recovery is almost constant and equal to 0.4 of the instantaneous strain, no data on recovery of long (many year) duration are available, and when the available recovery curves are plotted versus the logarithm of the time elapsed since unloading, no approach to an asymptotic final value is usually apparent (even though it may appear so in the actual time scale).

It is illuminating to consider a rate-type stress-strain law for an aging viscoelastic material. Such a law has been shown to be capable of approximating a given creep function with any desired accuracy (1). The components of the reversible strain increments are in the rate-type law expressed as $d\sigma_\mu/E_\mu(t)$, where σ_μ are the hidden stresses (e.g., the stresses in the springs of the Kelvin chain model) and E_μ are the associated elastic moduli. For a definition of the total reversible creep strain to be admissible, it would have to be possible to integrate $d\sigma_\mu/E_\mu(t)$ as $\sigma_\mu/E_\mu(t)$; but this is impossible because, as a result of aging, E_μ is strongly time-variable. In fact, the E_μ-variation is much stronger than that of the instantaneous modulus E. Consequently, there is no physical and mathematical justification for the separation of the reversible component of total creep strain, as introduced in Eq. 1 of the Discussion.

The foregoing arguments do not imply, of course, that a separation of reversible creep could not be a useful practical expedient. Nevertheless, the fact that the creep function in Eq. 2 of the paper compares with the test data on creep (at various t') much poorer than other equally simple creep functions does prove that the separation of the total reversible creep strain is practically useless.

Method of Analysis of Structural Creep Effects

In view of the preceding analysis, it is hard to understand that the proposed C.E.B. creep function could have any other purpose but to tailor the creep description to the "improved Dischinger method" of structural creep analysis. It is true that the proposed C.E.B. creep function can be applied with other methods of analysis; but the "improved Dischinger method" cannot be applied for creep functions of other forms.

Mention has been made of the age-adjusted effective modulus method (Refs. 15 and 16), which represents a refinement of the method originally discovered by H. Trost. Here, one has a method which allows predicting creep effects in structures by a simple elastic analysis using the age-adjusted effective modulus E" in place of the actual elastic modulus. By contrast, in the "improved Dischinger method" one needs formulas based on integration of differential equations, and this is obviously more involved. It is unclear why the discussers claim the opposite. It has also been shown that, aside from greater simplicity, the age-adjusted effective modulus method is much more accurate in comparison with the exact solutions based on principle of superposition (Ref. 16). The preceding facts have recently been independently confirmed at the University of Toronto in an extensive study by Bruegger (20), who compared various methods of analysis in a vast number of carefully documented examples involving essentially all practical creep problems.

The objection has been previously raised that in Trost's approach a table of a certain coefficient is needed for determining E", so that an engineer on an isolated island would be unable to use the method. However, he could not use the "improved Dischinger method" either because he would need a table or graph of the creep function. A table or graph of the coefficient needed does not take more space than the graphs for the creep function itself and could be published simultaneously with it.

Conclusion

From the foregoing analysis it becomes even clearer that the general form of the creep function in the existing (1970) C.E.B. Recommendations is better than the proposed one and should be retained until a truly improved form is found.

"Let the users judge", the concluding call of the discussers, certainly sounds logical. However, the vast majority of engineers in the design offices do not have time to make their own comparisons with test data and with other methods of analysis. They need standard recommendations which they can take for granted. Let the creep specialists in committees judge first.

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CEMENT and CONCRETE RESEARCH. Vol. 6, pp. 155-158, 1976. Pergamon Press, Inc.
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A DISCUSSION OF THE PAPER "ON THE CHOICE OF CREEP FUNCTION FOR STANDARD RECOMMENDATIONS ON PRACTICAL ANALYSIS OF STRUCTURES" by Z. P. Bažant and E. M. Osman*

W. Haas
Rechen- und Entwicklungsinstitut
für EDV im Bauwesen - RIB e.V.
Schulze-Delitzsch-Str. 28
7000 Stuttgart 80 (Vaih.), Germany

The authors show quite clearly that the equation for the creep-function proposed for CEB International Recommendations (see, e.g., Ref. 2) is not suitable to approximate experimental creep curves satisfactorily over the full range of ages at loading of interest.

Similar results can also be obtained with test data from Ref. 1. A comparison of these measured creep curves with the creep curves calculated on the basis of the equation for the creep function proposed for CEB International Recommendations can be done easily if we further assume the delayed elastic strain according to the proposed CEB International Recommendations. Then the specific flow strain corresponding to the test data of Ref. 1 (dashed line in Fig. 1) can be calculated as follows (t' = 3 days)

$$\underbrace{\bar{\epsilon}_f(t) - \bar{\epsilon}_f(t')}_{\text{specific flow}} = \underbrace{\bar{\epsilon}_{el} + \bar{\epsilon}_c}_{\substack{\text{specific elastic}\\ \text{and creep strain}\\ \text{(from test data)}}} - \underbrace{\frac{1}{E(t')}}_{\substack{\text{modulus of}\\ \text{elasticity}\\ \text{(from test data)}}} - \underbrace{\frac{0.4}{E(28)}\beta_d(t-t')}_{\substack{\text{delayed elastic}\\ \text{strain}\\ \text{(from Ref. 2)}}} \qquad (1b)$$

With this flow curve, one can easily calculate the creep curves for t' = 28, 90 days, i.e.

$$\bar{\epsilon}_{el} + \bar{\epsilon}_c = \frac{1}{E(t')} + \bar{\epsilon}_f(t) - \bar{\epsilon}_f(t') + \frac{0.4}{E(28)}\beta_d(t-t') \qquad (1a)$$

These curves are shown and compared with the corresponding measured curves in Figs. 2, 3. The agreement of measured and calculated curves is not satisfactory.

Similar results can be obtained if other test data from Ref. 1 are studied or if the flow-curves are calculated for other ages at loading (t' in Eq. 1b).

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* CCR 5, 129 (1975)

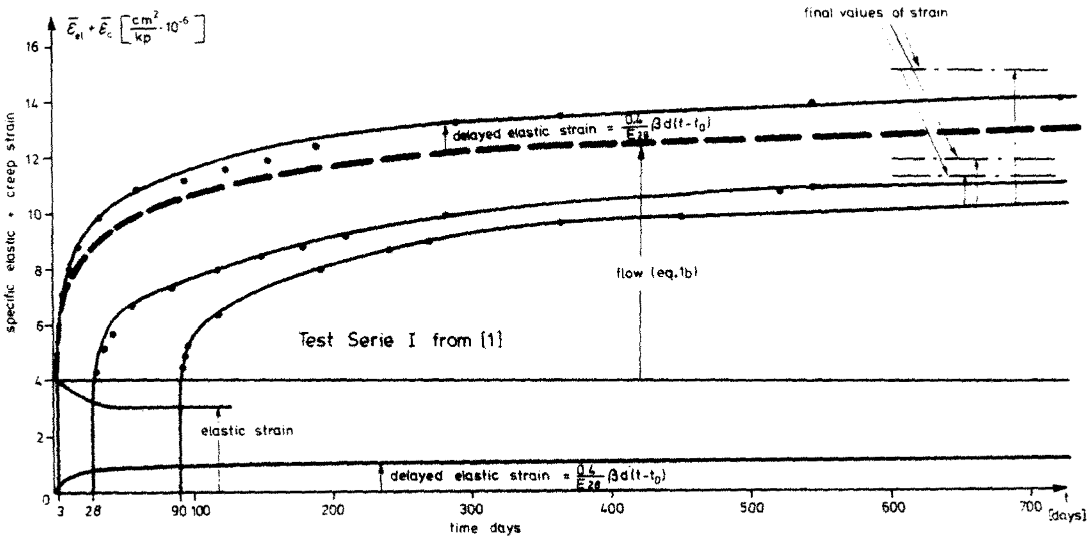

FIG. 1

Calculation of the flow-curve from test data [1], using the delayed elasticity-curve, given in [2]

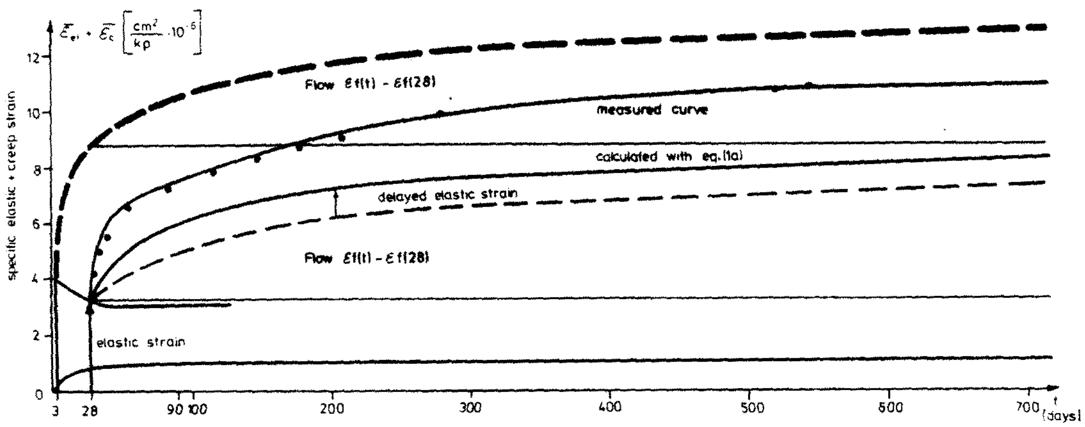

FIG. 2

Comparison of a measured creep curve t = 28 days and a creep curve, calculated with eq. (b) and the flow-curve from Fig. 1.

Although this analysis is not as refined as the optimization technique used in the paper, it shows that either the curve for the delayed elastic strain or the equation for the creep function proposed for CEB International Recommendations are not in a satisfactory agreement also with the test data from Ref. 1.

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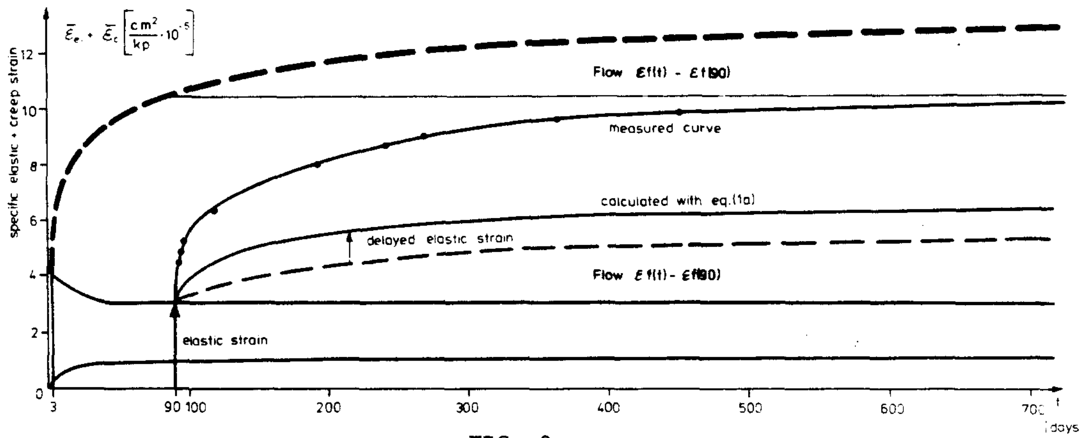

FIG. 3

Comparison of a measured creep curve t_0 = 90 cays and a creep curve, calculated with eq. (1b) and the flow-curve from Fig. 1.

2. H. Rüsch, D. Jungwirth and H. Hilsdorf: "Kritische Sichtung der Verfahren zur Berücksichtigung der Einflüsse von Kriechen," Beton- und Stahlbetonbau 68, 49-60, 76-86, 152-158 (1973).

Authors' Reply

We are in full agreement with the Discussion. It is a valuable addition to the paper and reinforces its conclusions.

Z. P. Bažant and E. M. Osman

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Printed in the United States.

DISCUSSIONS

DISCUSSION OF THE PAPER "ON THE CHOICE OF CREEP FUNCTION FOR STANDARD RECOMMENDATIONS ON PRACTICAL ANALYSIS OF STRUCTURES"*

by Zdenek P. Bazant and ElMamoun Osman

I. J. Jordaan and G. L. England
University of Calgary, Canada and King's College,
University of London, England

The writers have studied practical problems of creep in concrete for some years and have come to the conclusion that the form of creep law proposed for the CEB-FIP (reference 4 of the paper) represents the best compromise between simplicity and accuracy for purposes of structural analysis. Therefore a different conclusion has been reached from that of Messrs. Bazant and Osman and there are several points in which it is felt that the interpretation presented in the paper is in accord with only part of the published experimental results in the field of creep of concrete. Accordingly, the writers would value the response of the authors to the following points.

(1) It has been shown repeatedly (1, for example) that creep prediction based on superposition of strains from virgin creep specimens loaded at successive ages overestimates the response to <u>both</u> stress increases and decreases (not necessarily total removal of stress). The close agreement between experimental strains found in reference 6 of the paper, and elsewhere, is due to the fact that the rate of flow (and also the improved Dischinger) method takes into account, albeit approximately, these stress history effects. Hence, from this standpoint, the fits of creep strains for virgin specimens loaded at large ages are largely academic. Under conditions of changing stress (such as occurs in stress relaxation also) the superposition of virgin creep curves causes greater errors than the rate of flow or improved Dischinger methods, as shown in reference 6 of the paper.

However, it is easy to exaggerate the differences between the methods, which are not great if the data in the analysis is obtained directly from experiment. A more serious disadvantage of the form of equation 1 of the paper is that under raised temperatures or decreased humidity, experimental evidence shows that the irreversible component of creep is substantially increased while the creep recovery is sensibly independent of the temperature

* CCR 5, 129 (1975)

or humidity change. The usual manner of specifying creep in codes of practice is to provide factors which enlarge the entire creep expression for, say, a given humidity. If applied to equation 1, this would have the effect of increasing the creep recovery in the same ratio as the total creep and could well result in a recovery that is many times the correct value, thus making the prediction of creep under decreasing stress inadequate. In equation 2, only the term accounting for irreversible creep need be amplified, thus giving a more accurate response.

(2) The term "theoretically exact" appears to apply to analysis based on the superposition method that is derived from strains of virgin specimens. As described in (1) above, these solutions are not exact and are therefore a spurious basis for comparison. The only valid basis for comparing methods is to compare them with appropriate experimental results.

(3) Notwithstanding the comments in (1) above, some aspects of the fitting of the creep curves are disturbing. The writers do not agree that elastic strains should be included since most change in stress or strain occurs within a comparatively short period after loads are applied and the effect of the increase in modulus decreases in importance. Also, the same weight in the least squares analysis seems to have been placed on the strains at very small durations as on those at large durations; it is felt that a close fit in the first few days after load application is, from a practical point of view, of secondary importance.

(4) The statement that "the total creep strain cannot be decomposed into reversible and irreversible components, even though infinitesmal creep increments can" needs some amplification. It seems to the writers to suggest that creep occurs in infinitesmal increments and cannot be summed! The "infinitesmal increments", when summed will give the values of the components for various loading ages and durations. All representations of creep include the two components; in some cases they are implied by the form of the equation and not given explicitly.

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CEMENT and CONCRETE RESEARCH. Vol. 6, pp. 151-154, 1976. Pergamon Press, Inc.
Printed in the United States.

REPLY TO JORDAAN AND ENGLAND'S DISCUSSION OF THE PAPER "ON THE CHOICE OF CREEP FUNCTION FOR STANDARD RECOMMENDATIONS ON PRACTICAL ANALYSIS OF STRUCTURES" *

Zdeněk P. Bažant and ElMamoun Osman
Department of Civil Engineering
Northwestern University, Evanston, Illinois 60201

The authors appreciate the discussion by Jordaan and England but cannot agree with their four objections for the following reasons.

(1) Validity of the principle of superposition for concrete creep is, of course, limited. However, all practical methods in use today, including the improved Dischinger's method and the rate-of-flow method, are described by linear relationships and this automatically implies the principle of superposition as the underlying assumption, whether or not the creep function has been set up by considering the creep curves at various ages at loading, t'. The deviations from the principle of superposition, as mentioned in the discussion, are nonlinear effects and in the authors' opinion it is a misconception when one is trying to correct them by any creep law which is linear. The fit of the test data for unloading is improved by the afore-mentioned methods only at the expense of sacrificing something else, i.e., the fit of unit creep curves at various t'. (This fact is, however, obscured when the creep curves are plotted in the actual rather than the logarithmic time scale.) The only possible remedy is a nonlinear creep law.

The authors also disagree with the statement that the principle of superposition overestimates stress relaxation. Within the working stress range this is found only for relatively small and rapidly drying specimens, the cause being the nonlinearity of the effect of drying on creep; see (21) and Ref. 1 of the paper. This error cannot be corrected by means of a linear creep law.

Using a more accurate computer algorithm, the authors have recalculated the stress relaxation curves from the creep curves for the data of Ross (Ref. 1 of the Discussion) as well as the data of Bureau of Reclamation; see Figs. 10 and 15 in Ref. 15 of the paper. It appeared that the predictions agree as closely as one might desire.

* CCR 5, 129 (1975)

The error of the principle of superposition of creep curves of virgin concrete in the working stress range is not serious unless not merely the stress but also the strain decreases, as at sudden unloading; but compared to the relaxation regimes this is a case of lesser practical interest for structures.

For these reasons, the authors dispute the claim that "the fits of virgin creep strains for virgin specimens loaded at large ages are largely academic". It should be also noted that the close agreement of Eq. 2 with the experimental data mentioned by the discussers is found only when creep curves are plotted in the actual time scale, which permits only one order of magnitude of the time delays (say, from 10 to 100 days) to be graphically represented. When replotted in the logarithm of creep duration, the same comparisons look unfavorable. There is no reason why the stress redistributions due to creep between 10 and 100 days should be more important than those between 100 or 1000 days, 1000 and 10,000 days, or 1 and 10 days, provided that the creep properties change substantially (due to aging) in each of these spans.

The increase of irrecoverable creep at transient temperature or humidity conditions can be modeled by Eq. 2, as mentioned by the discussers, only to a limited extent, especially when both short and long delays are considered and the opposite effects of humidity during and after its change are taken into account. According to authors' recent (as yet unpublished) analysis of available test data, a better model can be attained when the creep rate derived from Eq. 1 of the paper is multiplied by a factor which grows with the rates of drying shrinkage (or swelling) and thermal shrinkage and decreases with decreasing humidity or temperature. This formulation can reflect the increase of irreversible creep which occurred during or shortly after drying or temperature change and, at the same time, it can correctly model the fact that at a decreased humidity or raised temperature the reversibility of creep is about the same as that for saturated concrete at room temperature, provided that sufficient time needed to achieve internal moisture equilibrium has been allowed.

(2) The term "theoretically exact" does indeed apply here to analysis based on the superposition method. This is justified by the fact that all formulations under consideration are linear and, therefore, imply the principle of superposition as the basic assumption. The only difference is that the method which the discussers call "the superposition method" applies the superposition to the actual creep curves as measured, while other methods (e.g., Eq. 2) are equivalent to applying it to distorted creep curves. Direct comparisons of structural creep calculations with measurements on structures are important; but if they were used as the only basis for validation the method could not be regarded as a general one and could not be applied with confidence to structures other than those measured. To obtain a general method it is essential to base it on a certain well defined creep law and validate this creep law directly by comparisons with appropriate measurements of creep specimens. If a disagreement with measurements on structures is subsequently detected, one must decide whether the error is in creep law or in the method of calculation.

(3) The reply to the question of including the elastic strains and the age-dependence of elastic modulus coincides with that in the reply to a preceding discussion; see Ref. 22, Fig. 8, Eq. 13, and the associated comments.

(4) The discussers objection to the impossibility of decomposing the

total creep in reversible and irreversible components is also answered in Ref. 22. To be sure, summing the infinitesimal reversible increments is always possible but it is of no advantage if the result is not independent of stress history. For a constant load followed by a zero load the recovered strain component can be, of course identified, but it cannot be applied to the cases of other load durations and ages, and of time-varying stress.

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Printed in the United States.

SECOND DISCUSSION OF THE PAPER

"ON THE CHOICE OF CREEP FUNCTION FOR STANDARD RECOMMENDATIONS ON PRACTICAL ANALYSIS OF STRUCTURES"+

by Z.P. Bazant and E.M. Osman

H. Rüsch, Institut für Massivbau, TU München

D. Jungwirth, Dyckerhoff und Widmann, München

H.K. Hilsdorf, Institut für Baustofftechnologie
University of Karlsruhe, Germany

The reply to our first discussion++ has been formulated in such a way that it cannot be left unanswered. We sternly object against the manner in which this discussion is conducted. By now at least three papers are known to us in which Mr. Bazant attempts to condemn the CEB-FIP method for the prediction of creep coefficients. In particular we object against the modes in which experimental and predicted creep values are compared. In this context we refer to a recent paper by Argyris et al where the product form of a creep function as proposed by Bazant, the summation form as proposed by the discussers as well as other creep functions are compared objectively with experimental creep data (Ref. 1). For this also a least square optimization applying a modified Marquardt algorithm has been used. The age at loading for the experimentel data ranged from 7 days to 4560 days. The result of this study was that the summation form of the creep function was superior to all other functions investigated though also the product form gave good results.

In the following we will discuss some of the points which have been raised in the authors reply to our first discussion.

1. The "Inadmissible Shifting" of Creep Curves

In the new CEB-FIP method the stress induced and time dependent strain of concrete under a constant stress σ_{co} is defined as follows:

$$\underbrace{\varepsilon_{c\ tot}(t, t_o)}_{\text{total strain}} = \sigma_{co}\left[\underbrace{\frac{1}{E_c(t_o)}}_{\text{elastic strain}} + \underbrace{\frac{\phi_c\ (t, t_o)}{E_{c28}}}_{\text{creep strain}}\right] \tag{1}$$

Eq. 1 gives the strain of concrete at time t, loaded at time t_o. This formulation is by no means new, but has been common practice since many years. Thus, by definition, creep under constant stress is the difference between total strain and the elastic strain at time of load application, t_o, expressed by the modulus of elasticity at time of load application $E_c(t_o)$. A change in modulus

+ CCR, 5, 129 (1975)
++ CCR, 5, 631 (1975)

of elasticity during the period under load is not taken into account. The same definition has been used in the previous CEB-FIP method 197o.

The coefficient $\phi(t, t_o)$ from eq. 1 of this discussion is defined as follows:

$$\phi_c(t, t_o) = \underbrace{\phi_d \beta_d(t - t_o)}_{\text{delayed elastic strain}} + \underbrace{\phi_f \left[\beta_f(t) - \beta_f(t_o)\right]}_{\text{plastic flow}} \qquad (2)$$

Thus creep is subdivided into delayed elastic strain and plastic flow. In eq. 1, the creep coefficient ϕ_c is related to the modulus of elasticity of concrete at an age of 28 days. However, E_{c28} could be replaced by some other value, if the coefficients β_d and β_f in eq. 2 would be altered accordingly.

In Fig. 1 of our first discussion the creep functions for a constant unit stress were given using eqs. 1 and 2 and the appropriate coefficients as given in the CEB-FIP method. These coefficients were average values obtained in an evaluation of numerous experimental creep studies. The definitions given in eqs. 1 and 2 have been used in presenting the data in Fig. 1 of our first discussion. Thus it is completely incomprehensible to us how the authors can state that Fig. 1 does not correspond to the CEB creep function and repeatedly refer to "inadmissible shifting" of creep curves when we correctly assume $E_c(t_o) \neq E_{c28}$.

2. Effect of the Time Dependence of Elastic Strain on Relaxation

In a numerical example the authors in their reply to our first discussion seem to prove that assuming a constant modulus of elasticity during a relaxation process may lead to an error of 32 percent of the elastic strain. This error may take place if a stress relaxes from an initial value σ_o to a final value of o.25 σ_o assuming the modulus of elasticity for an age t_o = 28 days to be valid for the entire period from t = 7 days to t = ∞.

Eqs. 1 and 2 of this discussion correctly describe the a v e r a g e elastic and creep strain of concrete. Thus, through a stepwise analysis the problem delt with by the authors can be solved. If the time dependence of E is expressed in an analytical form also closed analytical solutions can be deduced.

The time dependence of E chosen by the authors

$$1/E_a(t') = \left[1 + 0.43(28/t')^{1/3}\right]/E_o \qquad (3)$$

indicates that E for t' = 7 and t' = 28 days is 59 percent and 7o percent of E for t' = ∞. According to our own evaluation of experimental data (Ref. 2) these percentages would be for a common German cement (Type I - Type III) concrete 85 percent and 92 percent respectively. Obviously the more extreme age dependence of E chosen by the authors will lead to a pronounced error.

Nevertheless, the same error can be deduced assuming the authors time dependence of E if a simple analysis in three steps as described in (Ref. 2) is applied. In this analysis E is assumed to be constant within each of the following periods: 0 < t' < 7; 7 < t' < 28 and 28 < t' < ∞.

The stress intervals corresponding to these time intervals are shown in Fig. 1 based on the authors time-stress relationship. Fig. 2 shows the age dependence of the modulus of elasticity according to the authors relationship and according to Ref. 2 for a concrete made with a 35o F cement (approx. Type I - Type III cement).

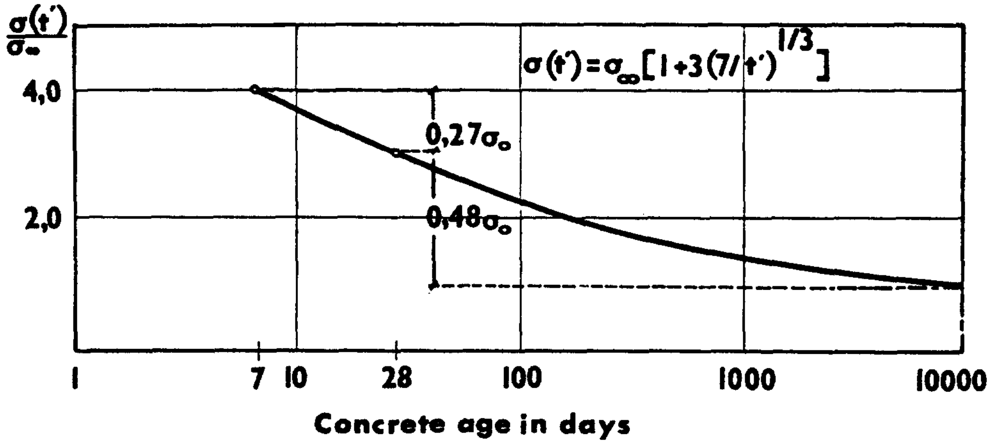

FIG. 1

Stress relaxation as assumed by the authors

(a) For E_o/E_{28} = const. = 1.43 the change in elastic strain during the relaxation process is

$$\varepsilon = 1.43\ (\sigma_o - 0.27\ \sigma_o - 0.48\ \sigma_o)/E_o = 0.36\ \frac{\sigma_o}{E_o}$$

(b) For $E_o/E_a(t')$ according to eq. 3 and as proposed by the authors:

$$\varepsilon = (1.68\ \sigma_o - 0.27\ \sigma_o \cdot \frac{1.68 + 1.43}{2} - 0.48\ \sigma_o \cdot \frac{1.43 + 1.0}{2})/E_o$$

$$= 0.68\ \frac{\sigma_o}{E_o}$$

The difference between (a) and (b) = $0.32\ \frac{\sigma_o}{E_o}$. This is exactly the value as deduced by the authors.

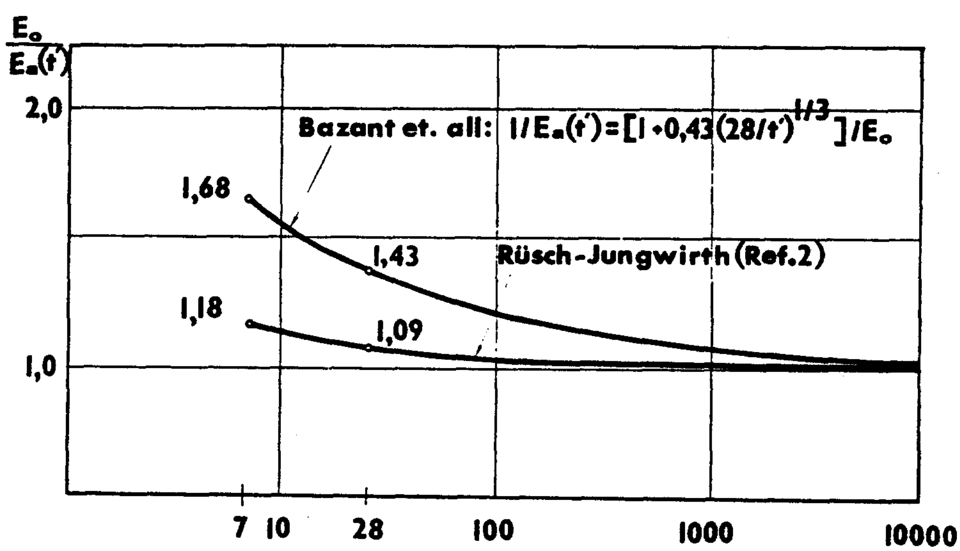

FIG. 2

Time dependence of modulus of elasticity of concrete

(c) For E_o/E_{28} = const. = 1.09 as proposed in (Ref. 2)

$$\varepsilon = 1.09\ (\sigma_o - 0.27\ \sigma_o - 0.48\ \sigma_o)/E_o = 0.28\ \frac{\sigma_o}{E_o}$$

(d) For $E_o/E_7 = 1.18$; $E_o/E_{28} = 1.09$ and $E_o/E_\infty = 1.00$ according to (Ref. 2)

$$\varepsilon = (1.18\ \sigma_o - 0.27\ \sigma_o \cdot \frac{1.18 - 1.09}{2} - 0.48\ \sigma_o \cdot \frac{1.09 + 1.00}{2})/E_o$$

$$= 0.37\ \frac{\sigma_o}{E_o}$$

The difference between (c) and (d) is 0.09 σ_o/E_o. Thus, if realistic values of E(t') are assumed the error in strain is only 9 percent. Furthermore, it should be pointed out that an error in strain is not equal to an error in stress, and that elastic strain is only part of the total strain. Creep strains which may be twice the elastic strain have been neglected. Thus the error due to the time dependence of E becomes even of less significance and may be neglected.

3 . Fitting of Creep Functions to Experimental Data

In Figs. 1, 2 and 3 of the paper by Bazant and Osman it is shown that the creep function eq. 10 agrees well with experimental data. The agreement was obtained by optimization of four coefficients E_o; ϕ; m and n. For each test series investigated four different coefficients have been obtained in order to obtain an optimum fit for each case. However, a prediction method is only of significance if it has general validity enabling the user to predict the creep properties of various types of concrete. Therefore coefficients of general validity have to be developed. Unfortunately such coefficients have not been proposed by the authors.

At this point the difference in opinion between the authors and the discussers become particularly clear: We attempt to interpret the creep behavior of an actual concrete specimen in a way which is lucid to the practical engineer: Concrete initially exhibits an instantenous "elastic" strain which may be expressed by stress and modulus of elasticity at the time of load application. It is followed by the time dependent creep strain which may be obtained from the total deformation of a concrete specimen after subtracting the shrinkage strain observed on companion specimens not subjected to load and the instantenous strain at the time of load application. Upon unloading the instantenous strain is recovered. Due to aging the instantenous strain upon unloading is smaller than the instantenous strain at load application. However, in many cases this difference may be neglected. (See section 2). Following unloading delayed elastic strain recovery takes place. This behavior can be taken into account by subdividing creep into plastic flow and into delayed elastic strain. This general behavior can be described by a few coefficients which have been deduced from a multitude of experimental data and which depend on parameters known to the designer.

On the other hand the authors show that it is possible to describe the creep behavior of one particular type of concrete by a single function (eq. 10)

$$J(t, t') = \frac{1}{E_o} + \frac{\phi_1}{E_o} \cdot t'^{-m}(t - t')^n$$

more accurately than the discussers can with their generalized formulation. However, eq. 10 expresses an instantenous strain $1/E_o$ which is independent of the age at the time of load application. However, the total strain J(t, t')

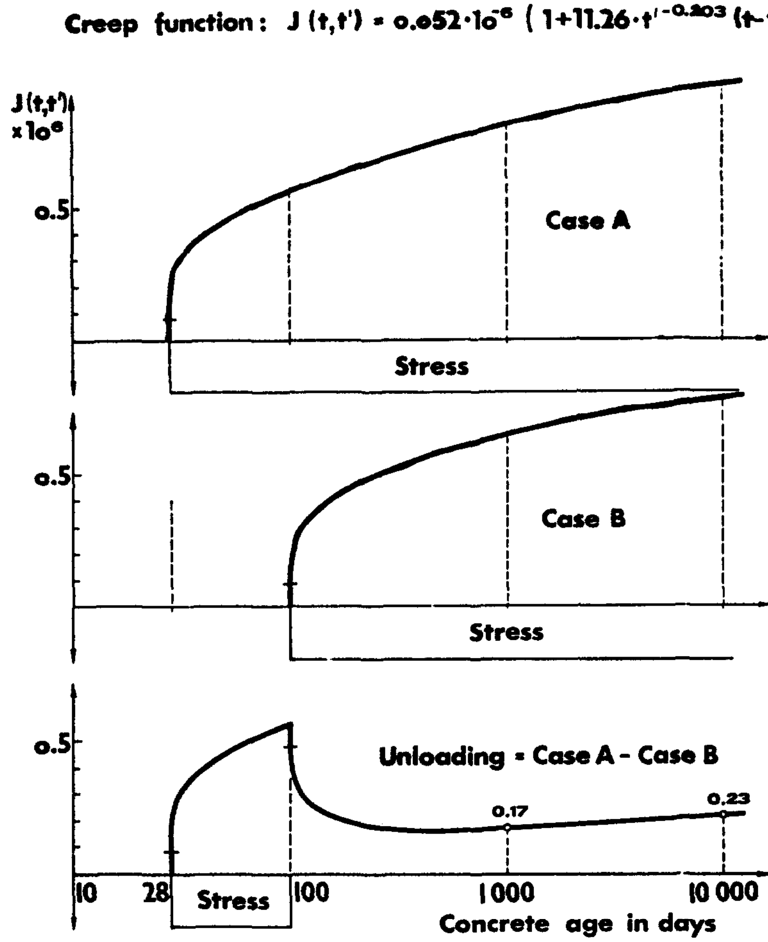

FIG. 3

Creep and creep recovery as predicted by the creep function proposed by the authors. (Based on tests by L'Hermite, Mamillan, Lefèure.) rel. H = 50%.

gives correct values already for very short durations of loading e.g. (t - t') = o.o1 days, due to the type of function which had been chosen. Nevertheless, the limiting value of J for t - t' = 0 is erroneous. Furthermore, eq. 1o cannot correctly describe the strain behavior of concrete after unloading or stress reversals. In Fig. 3 it is shown that after unloading the predicted total strain initially decreases, however, eventually it increases again. On the other hand the CEB-FIP method correctly describes the continued decrease of an average concrete upon unloading. In contrast to the authors opinion stress reversals and unloading of a particular concrete fiber are e.g. for prestressed concrete members by far more frequent, particularly at early ages, than the case of pure relaxation.

4. Deduction of Creep Functions from Experimental Data

As has been stated repeatedly, for the new CEB-FIP method the relations for the delayed elastic strain and for plastic flow have been deduced from a multitude of experimental data published in the literature. Evaluation of these data showed, that with an acceptable accuracy the creep behavior of various types of concrete can be described, if a single function for plastic flow and a single function for delayed elastic strain are chosen. However, these unique functions have not been deduced from one set of experimental data. They are average values or a compromise between the differences observed in different test series. Deviations of a single test series from the average are shown in (Ref.3). Thus, the accuracy of the proposed method differs from test series to test series. At no time it has been stated that this simplified method of presenting creep data is the only valid procedure. However, it is the result of numerous numerical evaluations and in most instances it works acceptable well.

5. The Magnitude of Delayed Elasticity

As stated above data published in the literature were evaluated in order to determine the effect of various parameters such as age at loading, relative humidity, specimen size etc. on this particular propoerty. This evaluation showed that it is at this stage impossible to give different relations for the delayed elastic strain which are significantly different and which result in meaningful tendencies. On the basis of the available data it was sufficient to express delayed elastic strain as a fraction of the modulus of elasticity at an age of 28 days and to neglect other possible parameters. Of course it is possible, as it is with all creep parameters and creep data to show that individual experiments deviate from the average functions given by the CEB-FIP method. (See also section 7).

6. Advantages and Disadvantages of Both Methods

In section 3 it was shown that the CEB-FIP method is a clear phenomenological presentation of the development of time dependent strains of concrete. It is of a general form and thus can be applied to a variety of analytical methods such as the Dischinger method, the improved Dischinger method, as well as the methods proposed by Trost and by Bazant. The CEB-FIP method is by no means deduced from one series of tests and is not of value only to one analytical method such as the Dischinger method. However, it describes only the behavior of average types of concrete and it cannot be adjusted as easily to describe exactly the creep behavior of one particular type of concrete.

On the other hand the creep function proposed by Bazant et al is sufficient to describe the creep behavior of a particular type of concrete with great accuracy with the exception of the instantenous strain for $t - t' = 0$. It gives incorrect values for the strains after unloading or for stress reversals. Because of its analytical form it is particularly adaptable to computer applications.

7. Contribution by Other Discussers

In CCR 6, 149 (1976) two other discussions of the paper by Bazant and Osman have been presented. W. Haas checks the validity of the new CEB-FIP method on the basis of one test series in which a particular type of concrete was subjected to sustained loads at an age of 3; 28 and 9o days, respectively. This test series, well known to the discussers always causes some interpretation problems, since there is hardly any difference in creep for concrete loaded at ages of 28 days and 9o days, respectively. This observation is in contrast to most other test series which describe the effect of loading on concrete creep. In his discussion W. Haas determines plastic flow of concrete from the creep curve, age at loading 3 days, by subtracting delayed elasticity as proposed in the new CEB-FIP method from total creep. On the basis of this flow curve he calculates creep for an age of loading of 28 and 9o days and obtains poor agreement with the experimentel data. At first sight this is a valid approach and the authors reply was so brief and remarkable that we will quote it: "We are in full agreement with the discussion. It is a valuable addition to the paper and reinforces its conclusions."

Creep of concrete depends on at least 2o different external and internal parameters. Only a small fraction of these parameters can be taken into account in a prediction method of practical significance or is known even for controlled laboratory experiments. Consequently, it is already difficult to give a unique relation for a parameter as common as the age of loading if all test series available and pertinent to this particular parameter are taken into account. The new CEB-FIP method is based on the evaluation of numerous experimental data. Naturally it describes only average behavior. An attempt to prove its faults on the basis of one other set of experiments is as meaningless as to prove its validity on the basis of one other set of experiments. Whoever does so is either inexperienced or unobjective.

In the following table creep of concrete after 6oo days of sustained loading as measured in the test series evaluated by W. Haas and estimated using the new CEB-FIP method are compared:

age at loading t_o, days	$\sigma_{c\ tot}$ x $1o^{-6}/kp/cm^2$ measured	predicted	error in percent
3	13.6	14.2	+ 4.4
28	1o.8	1o.1	- 6.5
9o	1o.o	7.7	- 23

For a general prediction method which takes into account only a limited number of significant parameters the error committed is acceptable to someone who has ever attempted to formulate a prediction method for creep coefficients of general validity.

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2. H. Rüsch, D. Jungwirth "Berücksichtigung der Einflüsse von Kriechen und Schwinden auf das Verhalten der Tragwerke" Werner-Verlag Düsseldorf, 1975

3. H. Rüsch, D. Jungwirth, H.K. Hilsdorf "Kritische Sichtung der Verfahren zur Berücksichtigung der Einflüsse von Kriechen" Beton- und Stahlbetonbau, Heft 3, 49-6o; Heft 4, 7o-86; Heft 6, 152-158, 1973

CEMENT and CONCRETE RESEARCH. Vol. 7, pp. 119-130, 1977. Pergamon Press, Inc.
Printed in the United States.

REPLY TO RUSCH, JUNGWIRTH, AND HILSDORF'S SECOND DISCUSSION OF THE PAPER[1] "ON THE CHOICE OF CREEP FUNCTION FOR STANDARD RECOMMENDATIONS ON PRACTICAL ANALYSIS OF STRUCTURES"

Zdeněk P. Bažant and ElMamoun Osman[2]
Department of Civil Engineering
Northwestern University, Evanston, Illinois 60201

It is rather unusual to receive a discussion of authors' reply[3], and it is very welcomed as an indication of the interest in this topic and its importance. Since the second discussion makes it clear that some of the questions still persist, the authors are pleased to provide clarification.

On Wylfa Vessel Concrete (First Paragraph of Discussion)

Argyris et al. (21) compared various creep functions with one set of experiments for one particular concrete, namely the Wylfa Vessel concrete (22,23). It is unclear why the discussers try to prove the faults of the product form by refering to a study of this particular set of experiments, for the discussers themselves state in their next to the last paragraph that "an attempt to prove...faults on the basis of one other set of experiments is ...meaningless" and "whoever does so is either inexperienced or unobjective."

The data used by Argyris et al. (21) represent smoothed experimental results (design curves). It should be noted that, from among the data of Browne et al. (22), Argyris et al. tacitly excluded the creep curves for some ages at loading (60 and 180 days, see Fig. 9c). Although this might be statistically questionable, exclusion of some curves in smoothing the data set seems to be justified by the fact that the three curves for t'=28, 60 and 180 days show an increase of creep with age (Fig. 9c) rather than a decrease, which can only be a random feature. It so happens that it is least favorable for the product form if the excluded data curves are those for t'=60 and 180 days.

Nevertheless, let it be assumed for the present that this exclusion is proper (Fig. 9a,b). Then, after studying such data it appears that the curves

[1] CCR 5, 129-138 (1975)

[2] Presently Instructor in Civil Engineering, University of Petroleum and Minerals, Dhahran, Saudi Arabia

[3] CCR 6, 635-641 (1976)

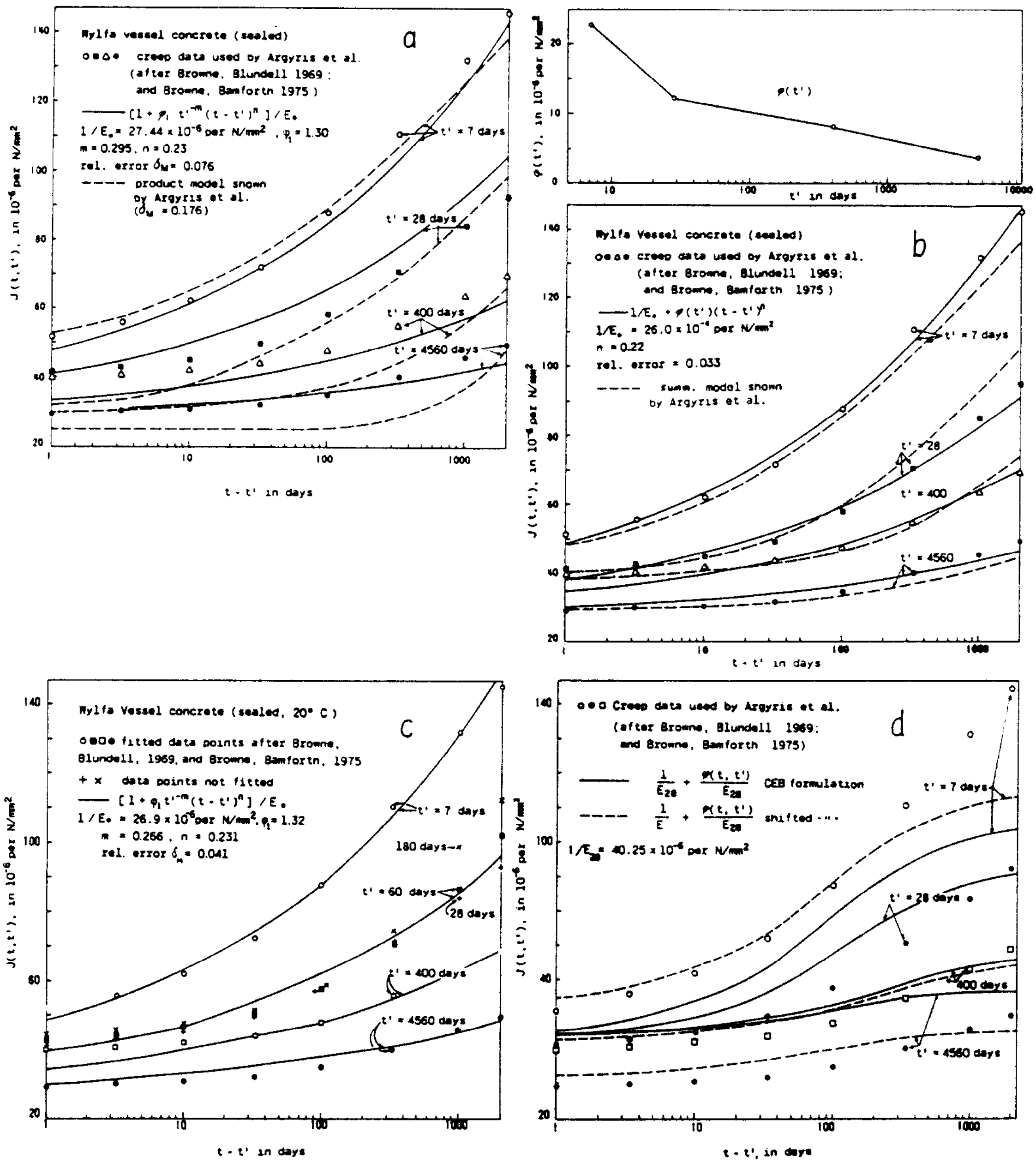

Fig. 9 Comparisons of Various Creep Functions with Test Data of Browne et al. (22,23).

for the product form, as indicated by Argyris et al. (21), are far from optimum fits. Using Marquardt optimization algorithm, the creep data have been fitted (26) by the product form $1/E_0 + \varphi(t')\ F(t-t')$ (Eq. 1 of the paper) of two special types: (a) the double power law (Eq. 10 of the paper), and (b) a more general form of the type $J(t,t') = 1/E_0 + \varphi(t')(t-t')^n$ where $\varphi(t')$ is an arbitrary function. The fits are drawn as solid lines in Fig. 9a, b and it is seen that the more general form gives a better fit. However, each of these fits is much closer than that indicated by Argyris et al. (21); their relative errors δ_M (root mean square error in J divided by root mean square of J) are as small as 0.076 and 0.033, respectively, while the fits shown by Argyris et al. (21) have $\delta_M = 0.176$ for the product form and $\delta_M = 0.147$ for the summation form. The errors in both cases are in fact so small that any effort for further improvement is meaningless in view of the random scatter which is apparent from the reversed sequence of the creep data for t'=28, 60 and 180 days (Fig. 9c). Furthermore, in addition to the scatter with regard to t', the measurements exhibited also considerable scatter with regard to t-t'. This is not apparent from the smoothed data (22) used by Argyris et al. (21), but it is clear from Ref. 24, in which the same data were published in greater detail and averages of measured values were indicated. These averages differ appreciably from the data (design curves) (22) used by Argyris et al. (Fig. 9a,b); but they are less smooth, which lends some degree of justification for preferring the data from Ref. 22.

Comparison of these data with the prediction by the discussers' formulation now adopted by C.E.B. (European Concrete Committee) (25) is shown in Fig. 9d (29). The comparison is made both for constant E, which is the case to be considered in accord with C.E.B. recommendations during the period under load, and for arbitrarily variable E. In the latter case a large vertical shift of creep curves is necessary to achieve an acceptable fit, just like that in Fig. 1 of the first discussion[4]; the fallacies in such vertical shift are discussed in the first reply and also later in this reply.

The same data (22,23) have also been fitted excluding the curves for t'=28 and 180 days, instead of those for t'=60 and 180 days (Fig. 9c). In this case the double power law fitted extremely well (see Fig. 9c), giving a relative error of only 0.041, while the fits for the summation form and for the new C.E.B. formulation became even worse than those in Fig. 9a,b. Since the power-type dependence on age t' agrees with most other data, it seems to be more appropriate to exclude the curves for t'=28 and 180 days (rather than those for 60 and 180 days) if any such data smoothing is carried out.

Consequently, the authors are afraid that the discussers' interpretation of the example given by Argyris et al. (21) might not be complete. Indeed, the data on Wylfa Vessel concrete (22,23) show again the product form to be vastly superior.

On Section 1.- Inadmissible Shifting of Creep Curves

In their last paragraph of Sec. 1, the discussers state that "Eqs. 1 and 2 have been used in presenting the data in Fig. 1" of their first discussion. However, the fact that Eq. 1 is written with a variable elastic modulus $E_c(t_0)$ does not mean that $E_c(t_0)$ can be given an arbitrary value, and especially not such unreasonable values (Eq. 3 of second discussion) as those implied by Fig. 1 of first discussion (E_c=E). Furthermore, even if realistic values of $E_c(t_0)$ were considered (Fig. 2 of second discussion), new C.E.B. recommendations (1976) do not indicate how these values should be determined. Anyhow, consideration of the E-variation during the period under load is not intended in C.E.B. recommendations, as the discussers admit below their Eq. 1. Undoubtedly, the reason is that the "improved Dischinger method", whose applicability is contingent upon the use of the summation form for the creep function, would become too complicated in case of variable E.

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[4]CCR 5, 631-634 (1975)

However, the point is not whether one "correctly assumes $E_c(t_0) \neq E_{c_{28}}$," as the discussers state in conclusion on Sec. 1. Rather, the point is whether the total strain J(t,t') produced by unit stress (sum of elastic and creep strains) is predicted correctly by the formula used for the creep function (for $t-t' \geq 1$ day). How the total strain is subdivided into instantaneous (elastic) strain and creep strain is of little importance in most structural calculations, for it is well known that if the age at loading is t_0, then the values of creep function at stress durations $t-t' < 0.1\ t_0$ are irrelevant for long-time response (provided the loading is steady). This means that the stress relaxation predicted on the basis of creep curve 123 from Fig. 8 of the first reply is the same as that predicted on the basis of creep curve 423 or 723. Thus, part of strain called creep strain can in fact be adjusted at will by a vertical shift of the whole creep curve (for $t-t' > 1$ day; see Fig. 8 of first discussion), but only if the shift is compensated for by a fictitious variation of E to be used in calculations so as to keep the total strain unchanged. Without such a shift-compensating variation of E (Eq. 3 of second discussion), there is no way to cancel the 32% error found in the example of stress relaxation in the first reply. As long as the user does not intend to complicate his creep calculations by taking into account during the period under load the shift-compensating variation of E according to Eq. 3 of second discussion, the vertical shifting of creep curves which was used in Fig. 1 of the first discussion to obtain a better fit is i n a d m i s s i b l e because it would imply altered values of total strain or J(t,t').

The fictitious variation of E which would have to be used in conjunction with the discusser's shifted curves in order to preserve the same J(t,t')-values was figured out by the leftward extensions of the shifted creep curves, as shown in Fig. 7 of first reply. The discussers apparently thought that these E(t)-values were proposed in the first reply, although this was not the case. Thus, the curve which is labeled "Bažant et al." in Fig. 2 of the second discussion and is reproduced here as Fig. 10 should actually be labeled as is shown in Fig. 10.

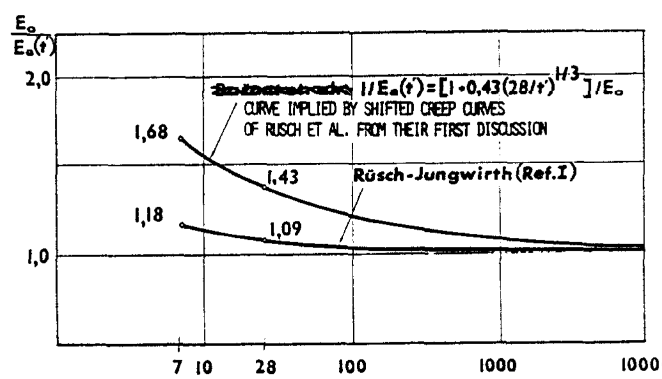

Fig. 10 Reproduction of Fig. 2 of the Discussion with Corrected Text.

<u>On Section 2.- Effect of Time Dependence of Elastic Strain on Relaxation</u>

The shifting of creep curves in Fig. 1 of the first discussion would be of no practical consequence if it had little effect in practical calculations. The example of stress relaxation in the first reply was intended to show that there exist some practically important cases where this is not so.

Strictly speaking, one ought to compare relaxation predictions based on (I) actual creep curves (Fig. 6 of first reply), and on (II) shifted creep curves (Fig. 1 of first discussion and Fig. 7 of first reply). However, calculations may be simplified by noting that the difference between these two cases results solely from the differences in elastic modulus E and can be evaluated from Eq. 12 of first reply. To make the calculations in a simple way which the reader can check without a computer, the shift-compensating fictitious variation of E(t) was approximated by a formula (as quoted in Fig. 2 of second discussion). Then an example was solved to show what is the difference in the prediction of stress relaxation when this shift-compensating E(t)-variation is considered, as it ought to be, and when it is neglected, which would be dictated by practicality of design office calculations and would not be disallowed by the C.E.B. recommendations. Again, to keep the calculations simple, it was chosen to compare the strains corresponding to a typical chosen relaxation curve rather than the stresses corresponding to constant strain. (This is possible because the percentage error in both cases is in fact about the same; see the sequel.) For convenience, a typical relaxation curve was described by a formula, quoted in Eq. 3 and Fig. 1 of the second discussion. The discussers may have overlooked the intended purpose of calculating the effect of E(t)-variation, as stated on pages 636-637 of the first reply.

Nevertheless, it is reassuring to see that the discussers obtain the same value ($0.32\ \sigma_0/E_0$) when they calculate in their own way the effect of E(t)-variation (cases (a) and (b) below Eq. 3 of the second discussion). However, it should be noted again that this is not how much the new C.E.B. formulation differs from some method "proposed by the authors" (case (b)), but how much it differs from Eq. 3 implied by the shifted creep curves in Fig. 1 of the first discussion. The error of 32% is the error caused when the creep curves are arbitrarily shifted without compensating for it by means of a change in E(t). Thus, the authors are afraid that the argument below Eq. 3 of the second discussion does not address the point.

Discussers' calculation of cases (c) and (d) demonstrates that the effect of the actual variation of E(t) compared to the assumption of constant E is relatively small ($0.09\sigma_0/E_0$), which the authors have not disputed.

The discussers state at the bottom of the page below Eq. 3 that "the error due to the time dependence of E becomes of even less signigicance", referring to the fact that the "creep strains which may be twice the elastic strains have been neglected" (in the example calculated). As a matter of fact, however, they have not been neglected. Rather, creep functions of the same long-time creep component and different elastic components have been compared. Thus, there is no reason to expect an error of lesser significance.

The discussers also add in this respect that "an error in strain is not equal to an error in stress". However, this is not true, as far as linearity of the creep law is assumed. For a linear creep law, the histories of stress and strain are related as $\epsilon(t)=\tilde{E}^{-1}\sigma(t)$ where $\tilde{E}^{-1}$ is the Volterra integral operator of creep (Ref. 1 of the paper). Consider an error in stress $\delta\sigma(t)=k\sigma(t)$ where k is a small number. Then owing to the linearity of operator $\tilde{E}^{-1}$, the relative error in $\epsilon(t)$ is $\tilde{E}^{-1}[k\sigma(t)]/\tilde{E}^{-1}\sigma(t) = k\tilde{E}^{-1}\sigma(t)/\epsilon(t)=k\epsilon(t)\epsilon(t)= k$. Hence, the relative error is the same. (For a time-dependent error $\delta\sigma(t)$, the comparison is more complicated and requires defining a suitable norm of the error; but the same result is obtained for the norm of the error.)

On Sections 3 and 4.- Fitting of Creep Functions to Experimental Data and Deduction of Creep Functions from Experimental Data

The discussers state repeatedly that their formulation, now adopted by C.E.B. (25), has been deduced from a "multitude of experimental data" and that it "describes the behavior of average types of concrete" (Sec. 3, end of 2nd par.; Sec. 4, lines 3 and 12; Sec. 6, line 7; Sec. 7, 2nd par.). However, the writers are aware of no publication showing h o w . In the writers' opinion, this would require showing comparisons of the new C.E.B. formulation with the relevant sets of data available in the literature. The only data comparisons shown in Ref. 3 of the second discussion with respect to both time and age at loading are Fig. 3, which is a creep recovery curve (not indicating, incidentally, any approach to some "final" value); and the creep curves for various t' in Figs. 4 and 5 and for a single t' in Figs. 9-11, in which no comparison of data with creep function is shown. The new C.E.B. creep function has been compared there only with the old C.E.B. creep function and with one measured deflection curve of a certain bridge, but not with any test data on both age and load duration effects. No more comparisons are given in the book quoted by discussers as Ref. 2. Although one of the writers raised the questions now discussed while serving as ACI representative on a C.E.B. Working Group on creep (since 1971), he was unable to receive any more comparisons with test data. Thus, it seems as if the new C.E.B. creep function has in fact not been compared with any extensive data set on the effect of both time and age at loading. Yet, the time curves of $J(t,t')$ for various t' are the most fundamental characteristic of creep because every structure which is suffering stress changes due to creep is aging in the process.

By contrast, the following data sets, involving broad ranges of both t-t' and t', have been fitted (27) by the product form: 1) L'Hermite and Mamillan's data, 2) Dworshak Dam, 3) Shasta Dam, 4) Ross Dam, 5) Canyon Ferry Dam, 6) Gable and Thomass' data, 7) A. D. Ross' data, 8) Wylfa Vessel data by Browne et al. (26). (For various humidities many further comparisons are made for an extension of double power law in Ref. 28.)

Justification of any creep function should involve two steps: (a) Show that the mathematical form selected is capable of individually representing well any of the relevant test data available in the literature; and (b) determine the dependence of the coefficients in this creep function upon the type of concrete, and estimate the random differences from various test data within each particular type of concrete. It appears that the first step, which is essential for chosing the right mathematical form, has been omitted in deriving the new C.E.B. creep function. The fact that the creep parameters of double power law found by fitting differ from concrete to concrete is not surprising, and the dependence of these parameters on the type of concrete can be established. The main point is that the product form is capable of representing well various test data, while the summation form, now adopted by C.E.B., is not.

It has been also objected that only the general form of the creep function has been analyzed in the paper. Suppose, however, that the actual creep curves as proposed at that time for C.E.B. recommendations were considered, being found to disagree with test data. From experience, a possible response would then be to merely modify the creep curves keeping the same basic form, again without full-scope comparisons. Then another paper would have to be written, comparing these modified curves to full-scope test data, etc. It was for saving years of delay and the labor of doing this that the general form of creep function was considered in the paper. The purpose was to show that no matter how the creep curves are modified, better fits than the optimum ones shown in the paper cannot be obtained, unless the summation form itself, along with the "improved Dischinger method", is abandoned.

At the end of Section 3, the discussers point out that in double power law "the limiting value of J for $t-t'=0$ is erroneous". This seems to be a misinterpretation. The value of J for $t-t'=0$ is beyond the range of validity; it merely represents the left-hand asymptote of the creep curve plotted in $\log(t-t')$-scale. What only matters is that the elastic modulus E, obtained as 1/J for $t-t'\approx 10^{-3}$ day, and even the dynamic modulus E_{dyn}, obtained as 1/J for $t-t'\approx 10^{-7}$ day, is represented by the double power law quite well.

The writers have been aware that superposition of the creep curves for double power law sometimes yields unrealistic shapes of recovery curves (Fig. 3 of second discussion). However, creep recovery is beyond the range of applicability of any linear creep law. Furthermore, the C.E.B. formulation itself represents recovery inadequately; see discussion of Fig. 11 in the next section.

It should be also noted that a reversal of recovery curve (Fig. 3 of second discussion) is not theoretically impossible. Indeed, there exist some recovery experiments which show just that (see Fig. 11c,i; and also Ref. 42), although majority of recovery tests follows a different trend (see Fig. 11).

On Section 5.- The Magnitude of Delayed Elasticity

The discussers refer again in Section 5 as well as 4 to creep recovery data which have been used to characterize the delayed elastic part of strain. As has been already mentioned in the second paragraph on p. 133 of the paper, it is inappropriate to use recovery data for determining $J(t,t')$ because the principle of superposition, assumed in C.E.B. recommendations, does not apply when strain decreases (as in creep recovery), although it does apply when only stress decreases (which covers most practical situations). To fit creep recovery data, a nonlinear creep law would be necessary.

However, let us for a while disregard with the discussers this fact. The C.E.B. formulation is based on these two hypotheses: (I) The creep recovery curves are bounded; and (II) the ultimate recovered strain is independent of age at loading, t', and age at unloading, t_1. These hypotheses appear to be true when the creep recovery curves are plotted in actual time scale for $t-t^1$, as has usually been done in the past. The trouble is, however, that if the scale on the paper ends by 1 day, then the plot looks as if an asymptote were to be reached at 2 days; if the scale ends on the paper by 100 days, then the plot looks as if an asymptote were to be reached at 200 days, and so on. So, the "asympotic" value can be manipulated largely at will. Thus, the plots in actual $t-t_1$ scale obscure the creep recovery curve for times $t-t_1$ which fall out of a chosen limited time range. This would not matter if only $t-t_1$, say, from 0.1 to 1 day or from 10 to 100 days, were of interest; but if a creep function covering the full time range from 1 minute to 40 years is of interest, then plots in log $(t-t_1)$-scale must be considered. Such plots have been constructed (29) from nearly all relevant recovery data available in the literature (30-44), and they are shown in Fig. 11 (30-41). It is evident from these plots that in most cases the creep recovery curves are essentially straight in $\log(t-t_1)$-scale and normally do not approach any asymptote, especially not within a 100-day or 1-year recovery period as has been previously assumed. (Actually, creep recovery data were plotted on p. 55 of Ref. 3 of

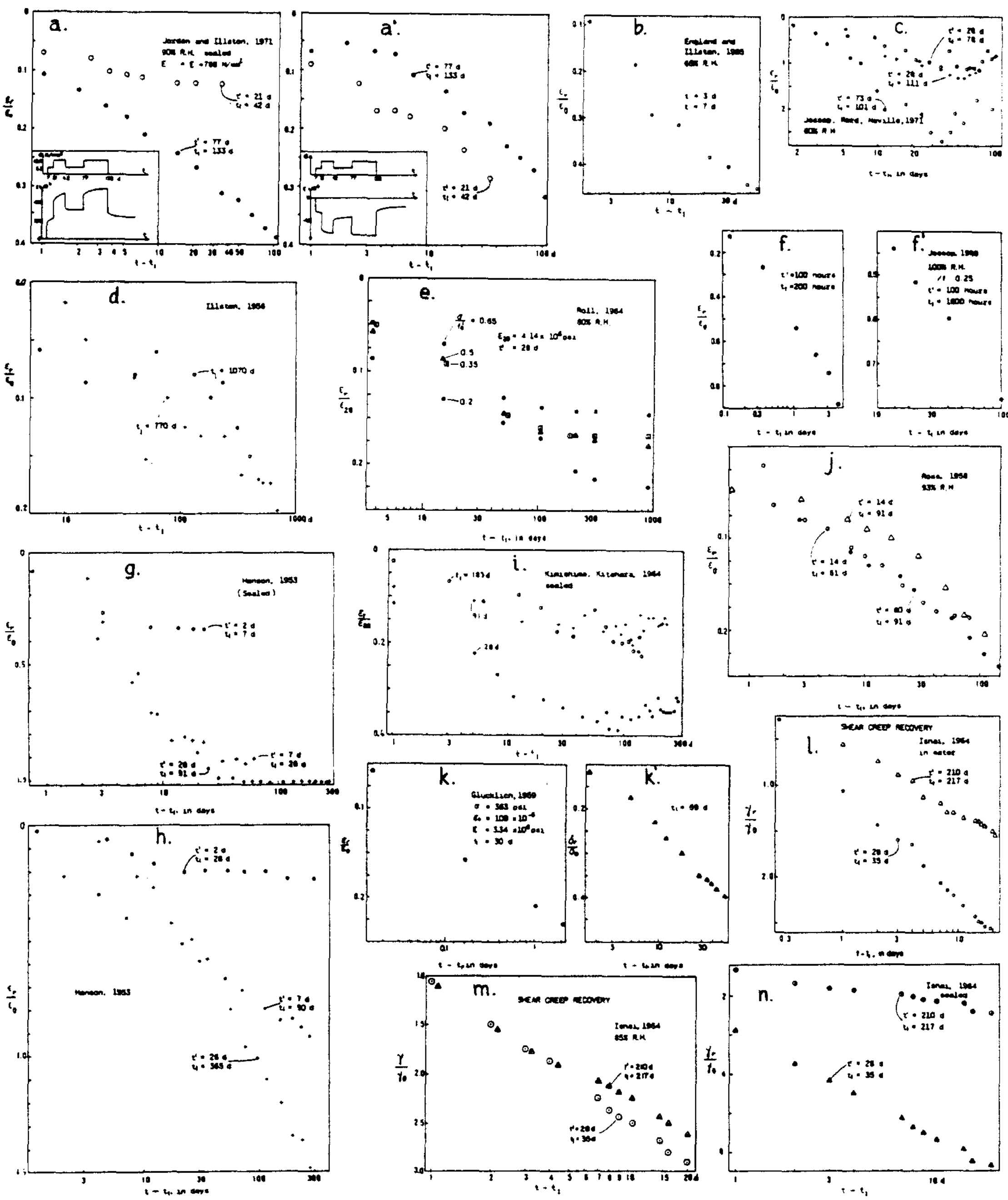

Fig. 11 Summary of Basic Experimental Data on Creep Recovery Available in the Literature (30-41)

the second discussion in log-time; the data point band was also steadily inclined up to the last point, and it is unclear why a horizontal asymptote was drawn there right behind the last point.) Furthermore, it is seen from Fig. 11 that creep recovery within a one-year period may range from 0.14 to 0.9 of the elastic strain.

Thus, hypotheses (I) and (II) on which the new C.E.B. formulation rests are tenable only for a rather limited time range, and for the full time range of interest they appear to be quite illusory.

In terms of rheological models, the new C.E.B. formulation with the "improved Dischinger method" corresponds to the well-known Maxwell model, in which the viscosity coefficient grows with age and the spring represents the effective modulus for the sum of the elastic strain and the final value of "delayed elastic strain". However, it is known from viscoelasticity that for a broader range of creep durations this model is an oversimplification for all real materials whose creep law is linear in stress.

In Ref. 28, optimization of data fits with the function $\varphi(t')F(t-t') + g(t)-g(t')$ has been reported. This function involves both the product form and the summation form (Eqs. 1 and 2 of the paper) as special cases. The optimum fits have not been appreciably better than those for $\varphi(t')F(t-t')$ alone and the flow term, $g(t)-g(t')$, came out to be negligible for the optimum fits. Thus, the flow term, which is basic to the "improved Dischinger method", appears to be generally a concept of dubious usefulness.

This conclusion agrees with the fact that in microstructure of cemment paste and concrete no viscous (or inelastic) strain can occur without producing elastic microstresses at the same time, while in a Maxwell model the viscous deformation of the dashpot representing the flow term does occur freely, without producing stress in any spring.

From the point of view of mathematical analysis and approximation theory, it is known that if a function of two variables, such as $J(t,t')$, is to be approximated by means of functions of one variable, it is normally much better to assume a product rather than a sum of functions of one variable. In fact, the product form corresponds to the well-known technique of separation of variables and represents the first term of the widely used expansion in a series of products, such as $\Sigma_\mu f_\mu(t')g_\mu(t-t')$ with $\mu = 1,2,\ldots.$

On Section 6.- Advantages and Disadvantages of Both Methods

The discussers state that the new C.E.B. creep function "is not of value only to one analytical method such as Dischinger method" (Sec. 6). The point to note, however, is that the "improved Dischinger method" is inapplicable for other formulations of creep, and the writers question whether this motivation was involved in selecting the new C.E.B. formulation. The Dischinger method, while hardly ever used in English and French speaking countries, has taken deep root in Central European countries. From this point of view, of course, the Trost method or its refinement, the age-adjusted effective modulus method, has the disadvantage of being new (although it is formally equivalent to effective modulus method which has been prevalent in English speaking countries). Yet, this method is applicable to any creep function, is simpler to use, and is more accurate. The only disadvantage, in the eyes of some, is that this method requires a table or graph of a certain coefficient; but such a graph does not take more space than a graph of the creep function (25).

The argument for the product form and against the summation form has so far been based on: (a) comparisons with experimental creep curves, (b) lack of thermodynamic justification of separating reversible and irreversible creep strain for an aging material, and (c) the preceeding critique of the concept of "delayed elastic strain". Recently, two further arguments have appeared: (d) a stochastic process model has been developed as an extension of double power law, and it yielded quite realistic empirical distributions of extrapolated long-term creep values (4); (e) the form of the creep function has been deduced from the fact that the viscoelastic properties of cement gel are essentially constant and the age dependence is due to the growth of the volume fraction of cement gel (4). This led to a certain power-type law for creep rate ("triple power law"), which seems to be reasonably approximated, for not-too-young concrete, by the double power law. It is also noteworthy that the triple power law (46) has time-dependent E_0 and that the theory (46) indicates that, for the double power law approximation, E_0 ought to reduce to a constant, as data fitting has already shown (27).

On Section 7.- Contribution by Other Discussers

It is unclear why the discussers question the fact that in the data of Hummel et al. the creep curves for concrete loaded at 28 and 90 days of age are rather close. This property is not "in contrast to most other test series"; see the multitude of test data plotted in Ref. 15 or Ref. 1 of the paper. The reason for the small difference is that log 28 and log 90 differ little compared with the full range of log t'.

In the penultimate paragraph of the second discussion, the question of being "inexperienced and unobjective" is raised. While the experience of the discussers is certainly above question, the important point is that of objectivity. It has been the pervading concern of the writers to rely on objective methods of evaluation; i.e., to use quantitative methods such as optimization techniques, to plot creep curves in log-time scales which do not obscure disagreement for short and long times, to show comparisons with all relevant data sets available in the literature, to avoid the temptation of presenting the fits in a manner which seems to indicate better agreement than there actually is, etc. The reader must make the final judgment on whether an objective approach has been taken.

Reply to the discussers' comments on the data comparison from Haas' discussion is left to that discusser. The writers are disappointed that the discussers "sternly object against the manner" in which the C.E.B. formulation has been discussed, and if the writers have done anything other than raise objective criticism, they offer their most sincere apologies. They would also be most happy to reply to further discussions.

Conclusion

It has now become still more firmly established that the newly adopted C.E.B. formulation (25) of the effects of load duration and age at loading needs to be revised. Until this is done, designers are well advised to use with regard to these effects the previous (1970) C.E.B. formulation.

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REPLY TO RÜSCH, JUNGWIRTH AND HILSDORF'S SECOND DISCUSSION OF THE PAPER "ON THE CHOICE OF CREEP FUNCTION FOR STANDARD RECOMMENDATIONS ON PRACTICAL ANALYSIS OF STRUCTURES"[2] by Z. P. Bažant and E. M. Osman

W. Haas
Rechen- und Entwicklungsinstitut
für EDV im Bauwesen - RIB e.V.
Schulze-Delitzsch-Str. 28
7000 Stuttgart 80 (Vaih.), Germany

In their second discussion, Rüsch, Jungwirth and Hilsdorf make comments on my discussion[2] which cannot remain unanswered.

They state that I intend to prove the faults of the new CEB-FIP method using a single additional set of experiments. Such an insufficient proof was, of course, never intended. Obviously they misinterpreted the last section of my discussion, which may be repeated for convenience:

"Although this analysis is not as refined as the optimization technique used in the paper, it shows that either the curve for the delayed elastic strain or the equation for the creep function proposed for CEB international recommendations are not in a satisfactory agreement also with the test data from Ref. 1."

These results, however, additionally confirm the conclusions of the discussed paper that the creep function of the type proposed for CEB international recommendations is unable to give an acceptable approximation of experimental creep curves over the full interesting range of ages at loading.

In their second discussion, Rüsch, Jungwirth and Hilsdorf also compare the test data from Ref. 1 with the creep strains calculated according to the new CEB-FIP method. In contradiction to my discussion they obtain an acceptable agreement of measured and predicted creep strains after t-t' = 600 days of sustained loading. As the authors of the discussion did not specify how they calculated these creep strains, comparative calculations must be carried out. These calculations have shown that the modulus of elasticity has not been calculated by the discussers according to the new CEB-FIP recommendations, which provide $E_{(28)} = 9450 \sqrt[3]{f_{cj}} = 300\ N/mm^2$. They rather used the elastic strain from the test data of Ref. 1 to calculate $E_{(28)} = 1/\varepsilon_{el} = 244\ N/mm^2$.

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[1] CCR 5, 129 (1975)
[2] CCR 6, 155 (1976)

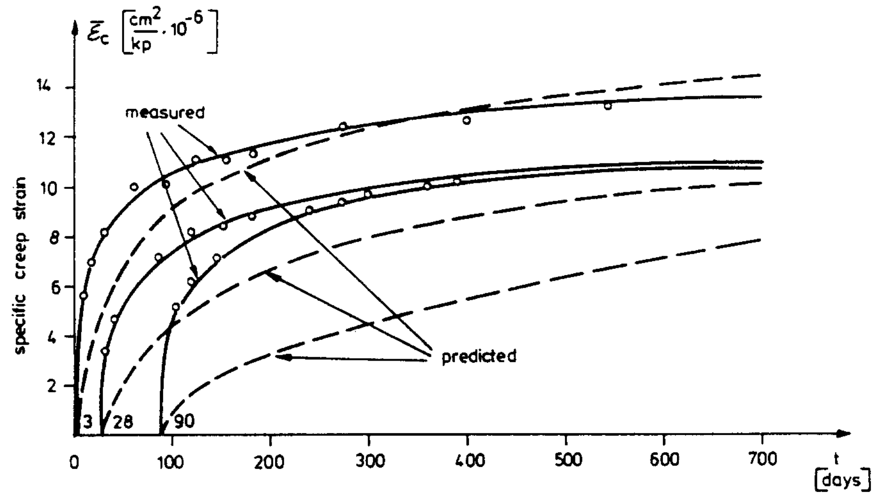

Fig. 1 Comparison of measured and predicted creep strain

—— measured (Ref. 1)

– – predicted according to the new CEB-FIP recommendations, elastic modulus $E_{(28)}$ = 244 kp/mm^2 from test data (Ref. 1)

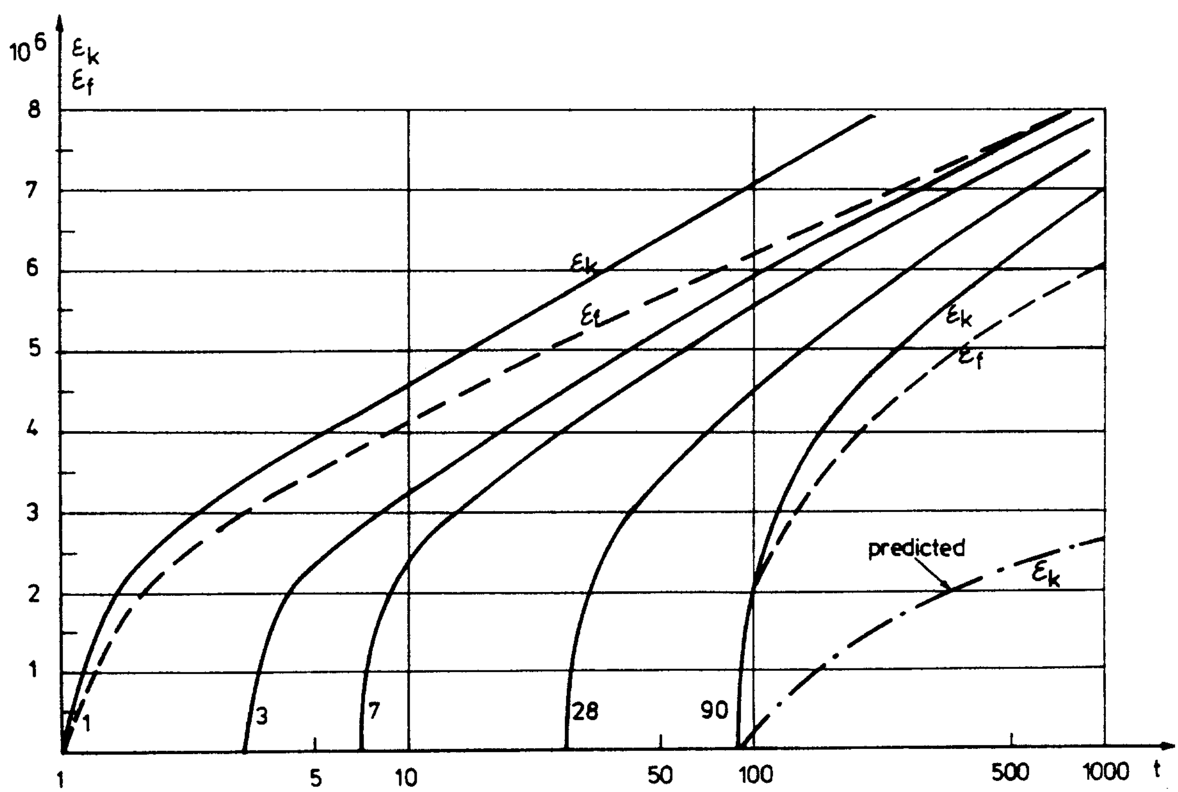

Fig. 2: Influence of the age at loading on creep strain ε_k (solid lines) and flow ε_f (dashed lines) Fig. 4 in Ref. 2

—·—creep strain calculated with the flow curve ε_f for t' = 1 day (added to the original Fig. 4 in Ref. 2)

If the authors of the discussion used the modulus of elasticity $E_{(28)}$ according to the new CEB-FIP recommendations, they would have obtained the values:

age at loading t' days	$\bar{\varepsilon}_c \times 10^6$ $\frac{cm^2}{kp}$ measured	predicted	error in percent
3	13.6	11.5	-15
28	10.8	8.2	-24
90	10.0	6.3	-37

which do not represent a satisfactory agreement. Even if the discussers' approach is accepted, the agreement of measured and predicted creep strain is not satisfactory if the whole period of t-t' = 600 days from the beginning of the sustained loading is considered, as shown in Fig. 1.

The authors further claim that the test data from Ref. 1 do not represent typical creep behaviour. This is inexplicable to me since they show in Ref. 2 their own test data with similar behaviour, i.e. small differences in creep for concrete loaded at ages of 28 days and 90 days, respectively.

If the flow curve ε_f of Fig. 2 for t' = 1 day is used to calculate the creep curve ε_k for t' = 90 days (the method is shown in my first discussion), a satisfactory agreement of measured and calculated creep strain cannot be achieved (see Fig. 2). Obviously the authors of the discussion have been aware of this fact since they show in Fig. 2 a second flow curve ε_f for t' = 90 days which is, obviously, in contradiction to the new CEB-FIP recommendations.

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ADDENDUM TO REPLY[1] TO RUSCH, JUNGWIRTH AND HILSDORF'S SECOND DISCUSSION OF THE PAPER "ON THE CHOICE OF CREEP FUNCTION FOR STANDARD RECOMMENDATIONS ON PRACTICAL ANALYSIS OF STRUCTURES"

Zdeněk P. Bažant and ElMamoun Osman[2]
Department of Civil Engineering
Northwestern University, Evanston, Illinois 60201

This addendum is published in response to a suggestion by Dr. K. Willam, Stuttgart, to whom the writers are obliged for pointing out that the reference to Argyris, Pister, and Willam's report[3] in the first section of preceding reply[1] was incomplete and could have been misinterpreted. In that report the term "product model" for creep did not have the same meaning as previously used in the literature. It actually referred to a degenerate form of the creep memory function, i.e., to the exponential series representation of aging material response to a unit stress impulse (see Eqs. 2.39 and 3.36 of that report), and not to a creep function chosen at the outset in the form of a product $\varphi(t')F(t-t')$ (although this form can be obtained from Argyris et al.'s exponential series by integration). This takes, however, nothing away from the conclusion[1] that the product form in the form of the double power law agrees with Wylfa vessel test data distinctly better than does the summation form, although the differences between the two fits are not significant in view of experimental scatter.

Furthermore, in Fig. 9a (p. 120), $\delta_M = 0.176$ should read $\delta_M^c = 0.176$ because in Argyris et al.'s report[3] the definition of the relative root-mean square error was not the same. There, δ_M^c referred to the "time-dependent part" of strain, while in the writers' reply[1] to Rusch et al.'s second discussion, δ_M was based on the total strain caused by stress. Thus, the δ_M-values and δ_M^c -values are not comparable.

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[1]CCR 7, 119-130 (1977)

[2]Presently Instructor in Civil Engineering, University of Petroleum and Minerals, Dhahran, Saudi Arabia

[3]Ref. 21 in the reply to second discussion, p. 128, CCR 7 (1977); see also J. H. Argyris, K. S. Pister, J. Szimmat, K. J. Willam, "Unified Concepts of Constitutive Modelling and Numerical Solution Methods for Concrete Creep Problems", Comp. Meth. Appl. Mech. Eng., Vol. 10, 199-246 (1977).

According to a private communication by Argyris et al., their δ_M-value based on total strain is 0.109 for what they call product model (the afore-mentioned exponential series, dashed line fits in Fig. 9a), and 0.093 for their summation model (not shown in Fig. 9; see Argyris et al.'s report[3]). It is noteworthy that the value 0.093 (of which 0.084 corresponds to the creep part and 0.009 to the elastic part) is not much worse that the value δ_M = 0.076 for the product model in Fig. 9a. Argyris et al.[3] obtained this δ_M-value by adding the initial values from their fit of the elastic curve (Fig. 3.6) to their fits for the "time-dependent part" of strain from their Fig. 3.11. However, although this does provide identically defined δ_M-values, the results are not directly comparable because the data were not fitted in the same manner.

The afore-mentioned degenerate forms of creep function, which are equivalent to a rate-type creep formulation, greatly reduce time and storage requirements in computer analysis for creep. The product form (e.g., the double power law) is not of this form. However, this is no disadvantage because a simple subroutine converting any creep function into a degenerate form (based on Maxwell of Kelvin chain models) is available and the degenerate form obtained is so close that it is graphically undistinguishable from the double power law. This subroutine forms an internal part of a program for creep analysis of concrete structures and automatically converts the input function J(t,t') into a degenerate form. This enables one to deal on input with functions given by only a few parameters, as in double power law. Alternatively, conversion of double power law into a degenerate creep function can also be accomplished by an explicit formula (see Ref. 18 of first reply).