Practical formulation of shrinkage and creep of concrete

Z. P. BAŽANT ([1]), E. OSMAN ([2]), W. THONGUTHAI ([2])

A set of algebraic formulas is proposed to describe the shrinkage and creep of concrete over the entire range of time durations of interest. The formulas cover: the effects of drying at various environmental relative humidities, the size and shape of cross section, aging (due to hydration), the effect of time lag of loading after the start of drying, creep of specimens predried to various humidities, the nonlinear dependence of stress, the increase of nonlinearity at simultaneous drying, and the decrease of strength for long-time loads. Simplification in the form of a linear dependence on stress is obtained as special case. The formulation is an extension of the double power creep law, which has been recently proposed for creep in absence of moisture exchange. The shape of the time curves of creep depends on the humidity. A rather close agreement with the extensive experimental data available in the literature is demonstrated.

OBJECTIVE

The widespread use of electronic computation has had two-fold effect on the analysis of creep and shrinkage effects in concrete structures: (1) More sophisticated formulations can be easily considered in design; and (2) more detailed analyses of the available test data can be carried out. A constitutive equation which is quite general and agrees well with a wide range of experimentally observed properties has been derived recently ([2], [7], [8]). However, for the analysis of ordinary concrete structures, this constitutive equation, which is inevitably in the form of differential or integral equations, is unnecessarily complicated, and simpler formulations consisting of algebraic formulas should be adequate. It is generally accepted that such formulations do not have to represent a valid constitutive equation satisfying all invariance requirements and principles of continuum mechanics; they may be concerned merely with the average behavior of a structural member in various situations and be restricted to uniaxial stress. Furthermore, algebraic formulas for creep and shrinkage are also urgently needed for statistical evaluation of test data and for the extrapolation of short-time tests into long times. A number of formulations of this type have recently been proposed ([1], [2], [5], [6], [9], [10], [11], [14], [17], [20], [25]) and a gradual improvement in the description of experimental data is being achieved.

The purpose of the present study is a further step in this direction. Creep and shrinkage of structural members of various sizes, ages at loading, and environmental conditions will be described by a set of algebraic formulas whose agreement with test data will be shown to be closer and broader than has been attained thus far. The formulation for creep will be conceived as a generalization of the double power law previously proposed ([2], [5], [6]) for the basic creep (creep without moisture exhange). This law has been previously shown ([5], [6]) to give a much better agreeme.t with test data than the creep formulation from reference 18, which approximates material behavior by a Maxwell solid of age-dependent viscosity and has been in 1976 incorporated into the C.E.B. International Recommendations.

PROPOSED SHRINKAGE FORMULAS AND THEIR VERIFICATION

After extensive studies of published experimental data, the following formula appears to be a reasonable compromise between simplicity and accuracy:

$$\varepsilon_{sh}(\hat{t}, t_0) = \varepsilon_{sh_\infty} k_h S(\hat{t}) \tag{1}$$

([1]) Professor of Civil Engineering, Northwestern University, Evanston, Illinois 60201; Active Member RILEM, U.S.A.

([2]) Graduate Research Assistant, Northwestern University, Evanston, Illinois, U.S.A.

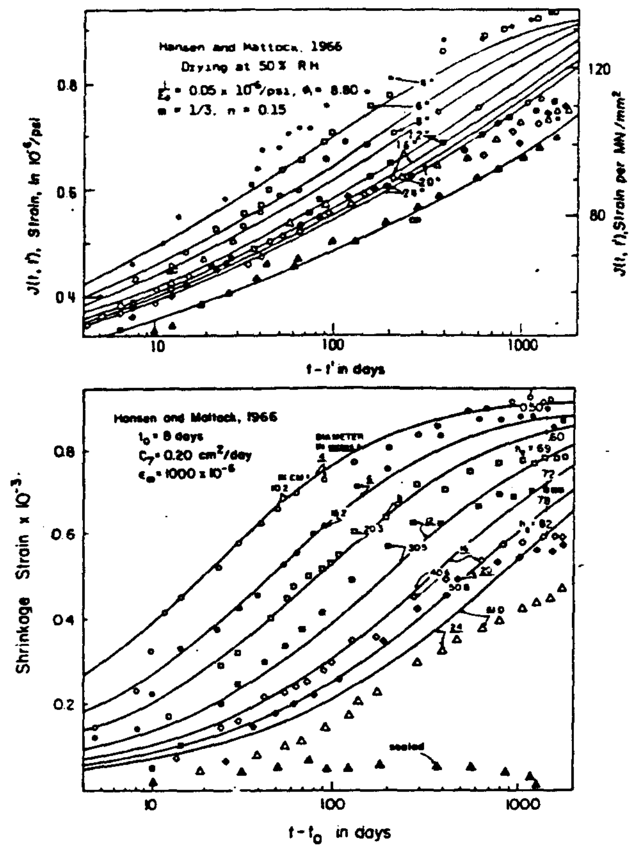

Fig. 1. — **Hansen and Mattock's data on the effect of size and shape on shrinkage and creep** (data constructed from reference [13]). Specimens drying at 50% relative humidity; 28-day cylinder strength; 6,000 psi; Elgin gravel (92% calcite, 8% quartz). Max. size of aggregate = 3/4"; 500 lbs. of ASTM type III cement per cubic yard of concrete; cured 2 days in mold, and 6 days in fog at 70°F; specimen loaded and exposed to drying at 8 days.

in which t_0 = curing time = age of concrete (in days) at the beginning of drying, $\hat{t}=t-t_0$ = duration of drying (in days), t being the current age of concrete; S = function of $\hat{t}$ giving the shape of shrinkage curve; k_h = humidity coefficient, and ε_{sh_∞} = ultimate shrinkage (at zero humidity). These quantities are determined as

$$S(\hat{t})=\left(\frac{\hat{t}}{\tau_{sh}+\hat{t}}\right)^{1/2}, \tag{2}$$

$$k_h=1-0.95h^3-0.25h^{200};$$

$$\varepsilon_{sh_\infty}=\varepsilon_{s_\infty}\frac{E(7+600)}{E(t_0+\tau_{sh})}. \tag{3}$$

Here $E=E(t)$ = elastic (Young's) modulus of concrete (which does not undergo drying) at age t (in days); h = environmental relative humidity (expressed as a decimal, $0 \leqq h \leqq 1$), and τ_{sh} = parameter which represents time at which $S^2=1/2$ and may be called shrinkage square half-time. It may be expressed as

$$\tau_{sh}=600\left(\frac{k_S}{150}D\right)^2\frac{C_1^{ref}}{C_1(t_0)}, \qquad D=2\frac{v}{s} \tag{4}$$

in which D = effective cross section thickness (given in millimetres); v/s, volume to surface ratio of concrete member; $C_1=C_1(t)$ = drying diffusivity of nearly-saturated concrete at age t (in days) and reference temperature T_0 (room temperature) [4]. By fitting shrinkage data for various ages t_0 at the start of drying (*fig.* 3) it has been found that

$$C_1(t)=C_7k'_T(0.05+\sqrt{6.3/t}), \qquad C_1(7)=C_7k'_T \tag{5}$$

$C_1^{ref}=10$ mm²/day, chosen reference value of C_1; k'_T = correction factor for the effect of temperature on drying (*see* reference [4]) affecting $C_1(t_0)$ in equation (4); k_S = shape factor of the cross section

$$\begin{aligned} k_S=\ &1.00 \text{ for an infinite slab;}\\ &1.15 \text{ for an infinite cylinder;}\\ &1.25 \text{ for an infinite square prism;}\\ &1.30 \text{ for a sphere;}\\ &1.55 \text{ for a cube.}\end{aligned} \tag{6}$$

The dependence of E upon t may be taken according to ACI Committee 209/II [1] as

$$E(t')=E(28)\left(\frac{t'}{4+0.85t'}\right)^{1/2}, \tag{6 a}$$

which has been used in all figures herein, or according to a more accurate expression in reference [6].

To express the effect of temperature T in evaluating $C_1(t)$ and $E(t)$, time t must be replaced by the equivalent hydration period t_e [4] and the dependence of C_1 upon T must be considered;

$$C_1\approx\frac{T}{T_0}\exp\left(\frac{5{,}000}{T_0}-\frac{5{,}000}{T}\right) \tag{6 b}$$

where T_0 = reference temperature = 298 K (25°C) and T must be given in Kelvin degrees [4]. Equations (2), (3) and (4) are applicable when the mean temperature roughly equals room temperature.

The most extensive shrinkage data available in the literature are fitted in figures 1-4 and 9 by the preceding formulas [equations (1)-(6)] and a reasonable agreement is found. Closer fits of individual data could be obtained using more complicated functions; then, however, different types of functions would be required for different data sets.

The shrinkage formulas (1)-(6) contain only one basic material constant: the final shrinkage ε_{s_∞}. However, further material constants are indirectly involved by means of the auxiliary expressions for drying diffusivity C and for the time change of elastic modulus E, of which particularly C_7 affects shrinkage strongly. Additional, numerically given coefficients in equations (1)-(5) [e. g., 1/2 in equation (2); 0.95, 0.25 and 600 in equation (3); etc.] have also been considered as variables in optimizing the data fits; but it has been possible to fit all data with the same values of these parameters, as given in equations (1)-(5). Coefficient ε_{s_∞} and, in absence of drying test data, also C_7 have been considered as the characteristics of each particular concrete, and their values obtained by fitting test data are stated in captions of figures 1-4 and 9. When test data for the particular concrete to be used are lacking, $C_1(t_0)$ may be set equal to its reference value C_1^{ref} and ε_{s_∞} may be taken according to the current recommendations on shrinkage.

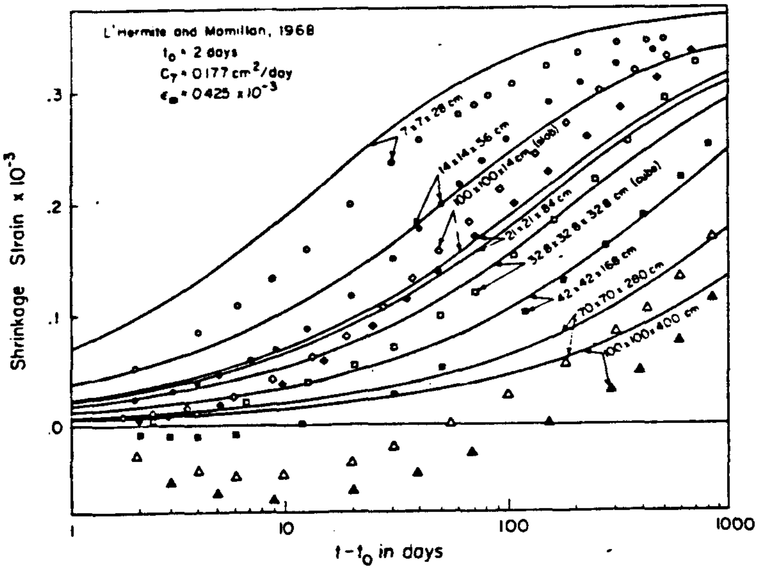

Fig. 2. — L'Hermite and Mamillan's tests on effect of size on shrinkage (data constructed from reference [15]). Specimens drying at 50% relative humidity; water-cement-sand-aggregate: 0.49 : 1 : 1.75 : 3.07. Max. size of aggregate 20 mm. Specimen cured in water then exposed to drying at 2 days, measurements of shrinkage were made along the longitudinal axis.

ANALYSIS OF PROPOSED SHRINKAGE FORMULAS

1. Shape of shrinkage curves in time

This is an aspect of particular importance for extrapolation of short-time shrinkage data. A number of different functions $S(\hat{t})$ have been tried. A single exponential curve does not give an acceptable fit because its slope decays in log-$\hat{t}$ scale too quickly both to the left and to the right of the shrinkage half-time. A sum of exponentials, $\sum c_i(1-e^{-\hat{t}/\tau_i})$, allows a good fit in the whole time range but at least 3 terms in the sum are needed to match the fit achieved with equation (2), which is simpler and has fewer constants. A more significant drawback is that the end portion of this curve is essentially independent of the first term, and this is disadvantageous when long term shrinkage is to be predicted from short-term data. Curve $\hat{t}/(\tau_{sh}+\hat{t})$ [1] is not as narrow in range as an exponential but still its slope ends too abruptly and too soon, as compared with the data in figures 1 and 4. However, the end portion of the curve is improved by exponent 1/2 in equation (2). Still more general functions of form $[\hat{t}^p/(\tau_{sh}^p+\hat{t}^p)]^q$ were also examined but best fits of all data on the average were obtained with $p\approx 1$, $q=1/2$ [equation (2)]; some of the data could be fitted better with different exponents (*fig.* 1) but others would be fitted poorer (*fig.* 2, 3). The function $a\,e^{-b/\hat{t}}$ [20], has also been considered, but has not been as good as equation (2). The infinite series expression of Becker and McInnis [9], representing the exact solution of the linear diffusion equation, has also been inferior, which is not surprising because drying of concrete is a highly nonlinear problem ([4], [2]).

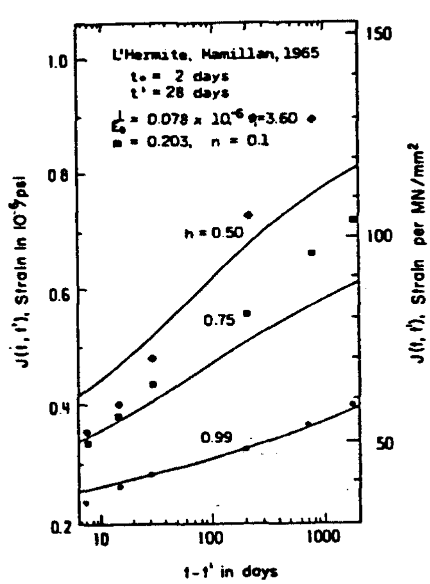

Fig. 3. — L'Hermite and Mamillan's tests on shrinkage and creep at various humidities and ages (data constructed from reference [15]). 28-days strength = 370 kgf/cm²; concrete of French type 400/800; 350 kg of cement per cubic meter of concrete; water-cement-sand-gravel ratio = 0.49 : 1 : 1.75 : 3.07; Seine gravel; specimens cured in water then exposed to drying.

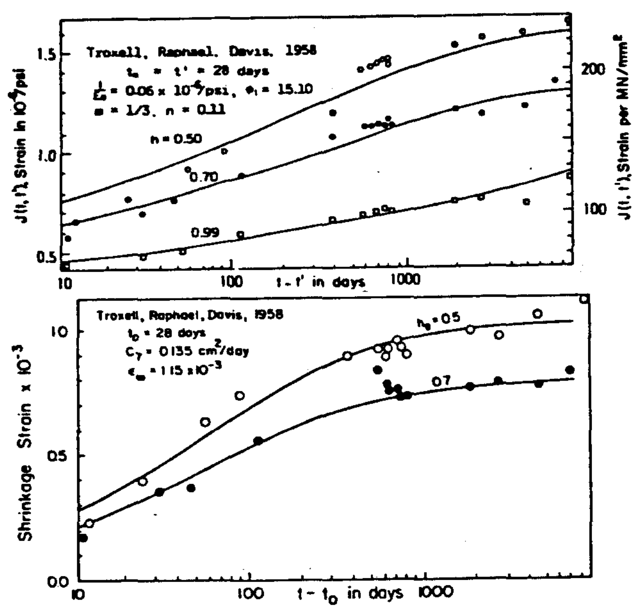

Fig. 4. — Troxell, Raphael, and Davis' data on shrinkage and creep at various humidities (data constructed from reference [20]). 28-days cylinder strength = 2,500 psi; cement type 1; water-cement-aggregate ratio = 0.59 : 1 : 5.67; granite aggregate: 1.5 in. max. size of aggregate; 4" x 14" cylinders; specimen exposed to drying at 28 days.

According to the linear diffusion theory, as well as the nonlinear diffusion theory for drying of concrete [4], the shrinkage curves should all be identical in non-dimensional time $t'=(t-t_0)C_1/D^2$. (This has been graphically demonstrated in [9].) Thus, the shape of shrinkage curves should be the same for all sizes. Moreover, equation (2) implies the same shape also for all h; but this is certainly a simplification because, for one reason, the diffusivity C depends upon h [4].

Another factor which is neglected although it must be affecting the shapes of shrinkage curves is hydration, especially when shrinkage for different ages at the start of drying is compared.

In figures 1 and 2, the slope of the proposed curves seems to be too high at the end. By contrast, the slopes at end are too small in figure 2, and especially in figure 3. Thus, the proposed formula is a compromise between the data in these figures.

2. Size and shape effect

The source of shrinkage is the drying process and, consequently, the size effect in shrinkage should be essentially the same as that for drying. Considering a linear diffusion equation, the effect of changing the size should consist in a horizontal shift of the plot of ε_{sh} versus $\log \hat{t}$ by a distance proportional to $\log D^2$. The same appears to be true when the very strong nonlinearity of drying of concrete [4] is considered, and this suggests that the shrinkage square half-time τ_{sh} should be proportional to D^2, as is the case in equation (4). Test data agree with this reasonably well (*figs.* 1, 2). However, the effect of hydration after the start of drying exposure (occurring only in the core, until the core begins to lose water) distorts the D^2-dependence of τ_{sh}. This is in part accounted for by the ratio of E-moduli in equation (3) which vertically scales the curve according to the final value.

Some data in the literature indicate the change in τ_{sh} with size (or horizontal shift) to be much smaller than that corresponding to D^2. However, these data (e. g. [22]) pertain to surface shrinkage of short specimens which is sufficiently influenced by deformation of the core and is, therefore, little dependent on specimen size. But this is not representative of the behavior of long structural members.

The volume-to-surface ratio v/s has been suggested long ago as a good indicator of the size effect. There are, however, some deviations from proportionality between D and v/s. As one possible remedy, attaching an exponent to (v/s) has been suggested in the past. However, this is dimensionally unacceptable because both v/s and D have a dimension of length. Therefore, the corrections must be accomplished by a multiplicative non-dimensional factor, which can describe only shape but not size (because of the dimensionality of size). This is the shape factor k_S in equation (4).

The effective thickness, D, for a body of any shape may thus be defined in terms of v/s. The factor 2 in equation (4) is inserted to achieve that, for an infinitely extending slab, D would be equal to the thickness of slab.

The insufficiency of $D \sim v/s$ as the single factor is most blatant when comparing a long cylinder of diameter a and a long square prism of side a. Both have the same v/s ratio, yet the cylinder is known to dry and shrink distinctly faster. The same is true when a sphere and a cube are compared. Furthermore, the v/s ratios of a slab and a cylinder differ by a factor of 2, but the drying and shrinking times are in a ratio less than 2^2. These discrepancies are approximately corrected by the factor k_S. The values in equation (6) have been derived by adjusting the time to reach half final shrinkage to the time to reach half pore humidity drop, as determined from the charts of drying in figure 16 of reference [4]. (These charts were constructed on the basis of extensive test data on drying.) It has been subsequently checked that a reasonable agreement is obtained with the test data, not only those in figures 1 and 2 (which include cylinders, prisms, slabs, and cubes), but also other data available in the literature ([13], [21]).

3. Shrinkage differences within cross section

The formulas presented describe shrinkage of the whole member only in a certain average sense. In an infinitely long test cylinder, though, the shrinkage strains in the axis and on the surface are forced to be equal (due to the condition of translational symmetry of deformation). However, in typical, relatively short test specimens, the two strains substantially differ. The shrinkage strain in the axis is more representative of long members and, consequently, the data pertaining to surface shrinkage (e. g. [22]) have been excluded from this study, except for the data in [13] which give shrinkage on the surface of specimens with sealed ends in which the difference between axial and surface shrinkage is not too large.

4. Effect of environmental humidity

The dependence of shrinkage upon environmental relative humidity h is expressed by coefficient k_h. The choice of a cubic function for k_h follows the results of previous investigators (e. g. [15]) and agrees well with the data shown here (*fig.* 3). Term $-0.25\,h^{200}$ in equation (3) is negligible except for $0.98 \leq h \leq 1$ and becomes -0.25 at $h=1$; this gives $k_h=-0.20$ at $h=1$, which approximately gives the value of swelling of concrete under water ($h=1$). The case of swelling may be distinguished from the case of autogeneous shrinkage of sealed concrete if one takes into account that sealed concrete exhibits self-desiccation, i. e., a drop in pore humidity to about 0.98. For $h<0.98$ the term $-0.25\,h^{200}$ is negligible and, by virtue of coefficient 0.95 (rather than 1.0), equation (3) gives $k_h \approx 0.05$ for $h=0.98$, in agreement with the fact that shrinkage of sealed concrete, called autogeneous shrinkage, represents about 5 % of the total drying shrinkage. (The autogeneous shrinkage should not depend on the size, D, but it seems unworthy complicating the formulas to model this small effect accurately.)

5. Effect of permeability and diffusivity of concrete

A dense concrete of low moisture permeability and diffusivity dries slower, and it also shrinks slower because drying shrinkage is chiefly determined by moisture loss. According to the theory of drying, the drying time, and thus also the shrinkage square half-time τ_{sh} [equation (4)], should be inversely proportional to diffusivity, as is the case in equation (4). This is true in spite of the strong dependence of diffusivity upon pore humidity [4], but it is necessary to use diffusivity C at specified h; e. g., C_1 corresponding to $h=1$. The reference value C_1^{ref} is introduced to achieve that the ratio C_1^{ref}/C_1 (7) be equal to unity for a typical structural concrete exposed to drying at $t_0=7$ days. When measurements of drying were available (e. g., *fig*. 1), the value of C_1 (7) was determined directly from the drying data. This corroborates the relationship of drying and shrinkage.

6. Effect of hydration (aging)

Extending the period of wet curing, the shrinkage on subsequent drying exposure gets diminished (which is apparent from figure 3). This is obviously due to the progress of hydration in concrete that is kept wet. The effect of hydration occurs simultaneously in two different ways: elastic modulus $E(t)$ increases, and moisture diffusivity $C_1(t)$ decreases as long as concrete remains wet. The source of shrinkage is largely the compressive stress in solid microstructure of cement paste produced by capillary tension and surface tension. Assuming that this tension for a given moisture loss is constant (although actually it changes with hydration due to a change in porosity and mean pore size), the shrinkage strain should be proportional to the value of $1/E$ at a time when moisture content begins to drop appreciably (because hydration greatly slows down as pore humidity drops below 95 %). This time does not coincide with time t_0, the beginning of drying exposure, because the loss of water in the core begins with a certain delay after t_0. This delay was guessed to be τ_{sh}, which explains why τ_{sh} is added to t_0 (and 600 to 7) in equation (3).

Note that τ_{sh} depends on the diffusivity of concrete and on the size of specimen, in accord with the fact that in thicker members the core concrete remains wet for a longer time. The numerator $E(7+600)$ is introduced so as to make the constant ε_{s_∞} represent the final shrinkage of a 150 mm thick slab.

The decrease of moisture diffusivity with hydration is approximately accounted for by taking C_1 as the value of C at age t_0 [equation (4)], the beginning of drying exposure. It might be again more reasonable to consider the value of C at t_0 plus a certain delay, but this did not appear to improve data fits appreciably. The dependence of C_1 upon age, as given by equation (5), has been justified by test data in reference [8].

In thick specimens, hydration heat causes a significant temperature rise in the core of specimens and this in turn produces thermal dilatation. This effect, which explains the negative shrinkage at early times as exhibited by thick specimens in figure 2, is not included in equations (1)-(6).

7. Effect of creep and other phenomena

Creep causes a relaxation of shrinkage stress due to nonuniform drying within the cross section. Because a thicker specimen takes longer to shrink, a role of creep in the size effect might be suspected. However, as is shown in Appendix I, for a linear aging creep law the creep has no effect on shrinkage strain. Deviations from this creep law, which are of lesser significance, do affect shrinkage and their effect depends on specimen size. These deviations include the nonlinearities of creep due to drying (which are higher near the surface than in the interior), and the nonlinearities due to the stress level, to the cracking caused by tensile shrinkage stress, and to the nonuniformity of creep properties caused by drying and by the hydration heat. These effects are included in the preceding formulas, but only in an overall and empirical manner.

The effect of carbonation is not included herein; but for specimens which are not very thin the carbonation effect is unimportant.

PROPOSED CREEP FORMULAS AND THEIR VERIFICATION

In current practical formulations, the shape of the curve of creep versus time is tacitly considered to be independent of humidity conditions. It appears, however, that this assumption has to be abandoned if formulas of better accuracy should be introduced. Thus, it will be necessary to describe the basic creep and the drying creep by different functions of time.

1. Basic creep in linear range

The creep without moisture exchange, called basic creep, can be closely described for various ages t' at loading by the double power law:

$$J(t,t')=\frac{1}{E_0}+C_0(t,t'),\qquad C_0(t,t')=\frac{\varphi_1}{E_0}(t')^{-m}(t-t')^n \tag{7}$$

proposed in reference [2] and verified by test data in reference [6]; $J(t,t')$ = strain at time t caused by a unit stress sustained since time $t'=C_0(t,t')$, unit basic creep strain; E_0, φ_1, m, n = constants; E_0 is a constant which indicates the left side asymptote of the creep curve in the log-time plot. This asymptote is reached only at ultrashort times which are much below the range of validity of equation (7). Thus, E_0 does not represent the elastic modulus, not even the dynamic modulus, and may be called the asymptotic modulus. The conventional (static) elastic modulus E corresponds to loading of about 0.001 day (1.44 minutes) duration, and equation (7) gives a correct value for $1/E$ at any age t' by substituting $t-t'\approx 0.001$ day, i. e., [6]:

$$\frac{1}{E(t')}=\frac{1}{E_{stat}(t')}=\frac{1}{E_0}+\frac{\varphi_1}{E_0}10^{-3n}(t')^{-m}. \tag{8}$$

The dynamic modulus, E_{dyn}, corresponds to loading of about 10^{-7} day (0.0086 seconds) duration, and it has been found that equation (7) gives also roughly correct values for $1/E_{dyn}$ at various ages t', i. e.,

$$\frac{1}{E_{dyn}(t')}=\frac{1}{E_0}+\frac{\varphi_1}{E_0}10^{-7n}(t')^{-m}. \qquad (9)$$

The age dependence of elastic modulus is described by three constants (E_0, φ_1, and m), and it is most remarkable that creep over the entire range, as well as the relationship between the dynamic modulus and the static modulus, is characterized merely by one additional constant, n.

2. Creep under general conditions

Drying has a twofold effect upon creep [2]:

(1) Movements of water induced by drying (as well as those induced by wetting) increase creep, and (2) the reduction of moisture content caused by drying reduces creep. This suggests that

$$J(t,t')=\frac{1}{E_0}+C_0(t,t')+C_d(t,t',t_0)-C_p(t,t',t_0), \qquad (10)$$

where C_d represents the increase of creep during drying and C_p represents the decrease of creep after drying, i. e., in predried specimens (subscript d refers to "drying", and p to "predried". These components of creep are depicted in figure 10.

To deduce reasonable expressions for C_d and C_p, the following twelve basic facts on creep at variable humidity must be considered.

1) In saturated condition ($h=1.0$), equation (7) for basic creep must be obtained.

2) Up to a certain time, the creep of drying concrete is higher than the basic creep.

3) At the beginning of drying, the creep curve has the shape of power function (or a sum of power functions) with an exponent higher than n. This fact has been established by analysis of test data in reference [6].

4) For specimens loaded after the termination of drying ($t'-t_0 \gg \tau_{sh}$), i. e., at reduced and constant moisture content, the creep curves have the shape of power function [equation (7)] with a reduced value of φ_1 and roughly the same exponent ([25], [24], [2]).

5) Creep after drying ($t-t' \gg \tau_{sh}$) is independent of the thickness of specimen.

6) The longer is the period $t'-t_0$ from the start of drying to the instant of loading, the smaller is the increase of creep due to drying. When $t'-t_0 \gg \tau_{sh}$, the increase of creep due to drying vanishes.

7) The thicker is the specimen (greater τ_{sh}), the smaller must be the increase of creep due to drying.

8) For a sufficiently thick specimen ($\tau_{sh}\to\infty$), equation (7) for basic creep must be obtained not only for $h=1$ but for any h.

9) At higher age t', the increase of creep due to drying must be less, similarly to basic creep.

10) Concrete which exhibits higher shrinkage (higher ε_{sh_∞}), exhibits also a greater increase of creep during drying.

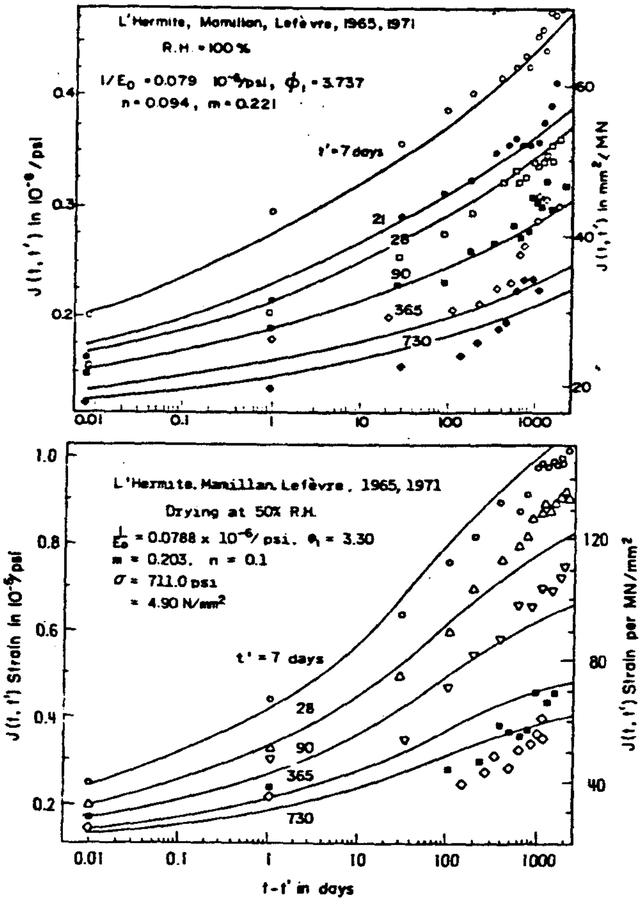

Fig. 5. — L'Hermite and Mamillan's tests of basic and drying creep for various ages at loading (data constructed from reference [15]). Specimens same as figure 3 cured in water then exposed to drying at 2 days and loaded at ages 7, 28, 90, 365 and 730 days.

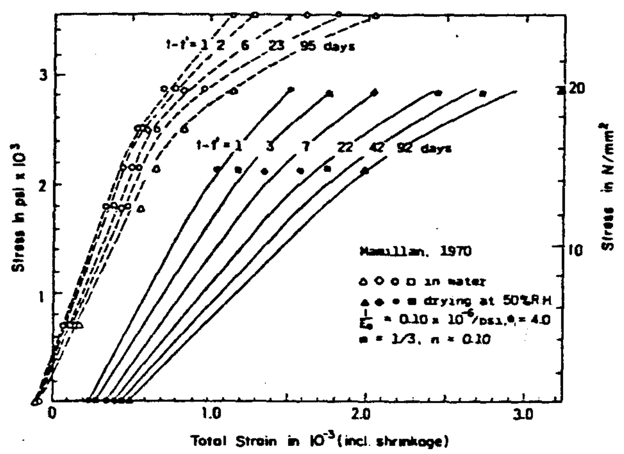

Fig. 6. — Mamillan's tests of saturated and drying specimens of various stress levels (data constructed from reference [15]). Prisms 7 x 7 x 28 cm; short-time failure stress 250-300 kgf/cm². 28 days old when loaded; cured 2 days in mold; 5 days in water then drying in air of $h = 0.5$ at 20°C; other factor probably same as figure 3.

11) The decrease of creep due to the lower moisture content after drying is not proportional to ε_{sh_∞} when various concretes are considered, and is approximately independent of ε_{sh_∞}.

12) Unlike shrinkage, creep of saturated specimens and creep of sealed specimens (in which the pore humidity drops to about 0.98) are about the same.

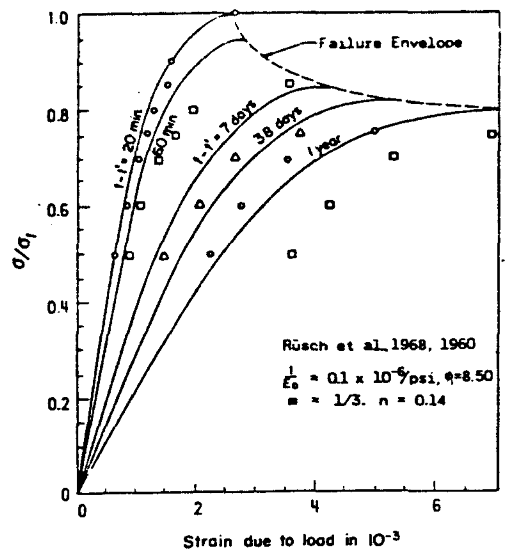

Fig. 7. — **Rüsch's tests of creep and long-time strength at high stress** (data adapted from reference [15]). Prisms 10 × 15 × 60 cm with widened ends; water-cement-aggregate ratio 0.55 : 1 : 4.9. Rhine gravel (mostly quartz); 28 day cube strength 350 kgf/cm²; moist cured for 7 days at 20°C, then drying at $h = 0.65$ and 20°C. Load applied at 56 days at a strain rate 0.003/20 min.; σ_1 is failure stress of specimen. Data points are interpolated and smoothed.

The preceding conditions can be satisfied by the expressions:

$$C_d(t,t',t_0)=\frac{\varphi'_d}{E_0}(t')^{-m}k'_h\,\varepsilon_{sh\infty}\,S_d(t,t'),$$

$$\varphi'_d=\left(1+\frac{t'-t_0}{k_a\tau_{sh}}\right)^{-1}\varphi_d, \tag{11}$$

$$C_p(t,t',t_0)=c_p k''_h S_p(t,t_0)\,C_0(t,t'), \tag{12}$$

$$S_d(t,t')=\left(1+\frac{10\,\tau_{sh}}{t-t'}\right)^{-c_d n},$$

$$S_p(t,t_0)=\left(1+\frac{100\,\tau_{sh}}{t-t_0}\right)^{-n}, \tag{13}$$

$$k'_h=|h_0^{1.5}-h^{1.5}|,\qquad k''_h=h_0^2-h^2. \tag{14}$$

Here t' = age at loading, t_0 = age at which drying begins (both in days), h_0 = initial relative humidity (prior to loading) at which the specimen was in equilibrium just before the drying started (usually $h_0 = 1$). Attention is here restricted to $t_0 \leq t'$, i. e., drying which begins after loading is not considered.

In addition to shrinkage square half-time τ_{sh_∞}, shrinkage constant ε_{sh_∞}, and the four material constants for basic creep (m, n, φ_1, E_0), the foregoing formulas contain four more constants, k_a, c_p, c_d, φ_d. Fitting of test data indicated that $k_a = 42.0$ and $c_p = 0.83$ for all concretes considered, while c_d and φ_d appeared

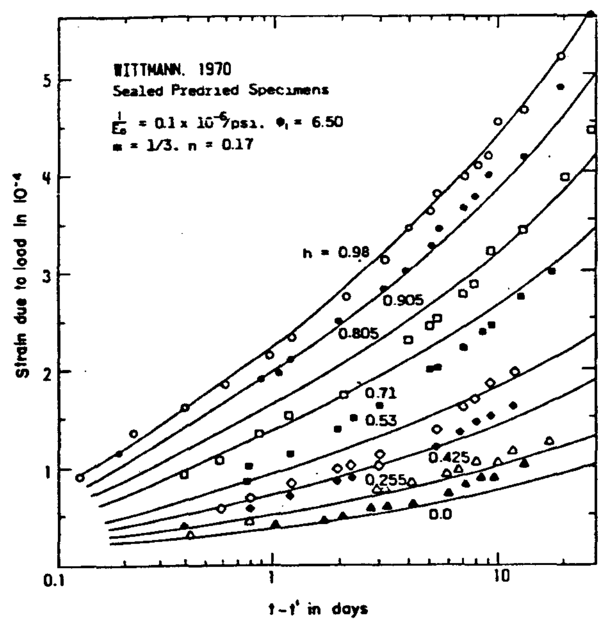

Fig. 8. — **Wittmann's tests of creep at various constant water content** (data constructed from reference [23]). Solid cement paste cylinders 18 × 60 mm; water cement ratio 0.4; cured sealed for 28 days at 20°C; then dried in oven at 105°C for 2 days; resaturated for 3 months at various constant humidities h shown at 20°C. Then tested for creep in the same environment under stress 150 kgf/cm² equal 0.2 of failure stress before test; E = 210,000 kgf/cm² for 1 minute loading; strain at 20 minutes under load is subtracted from values shown.

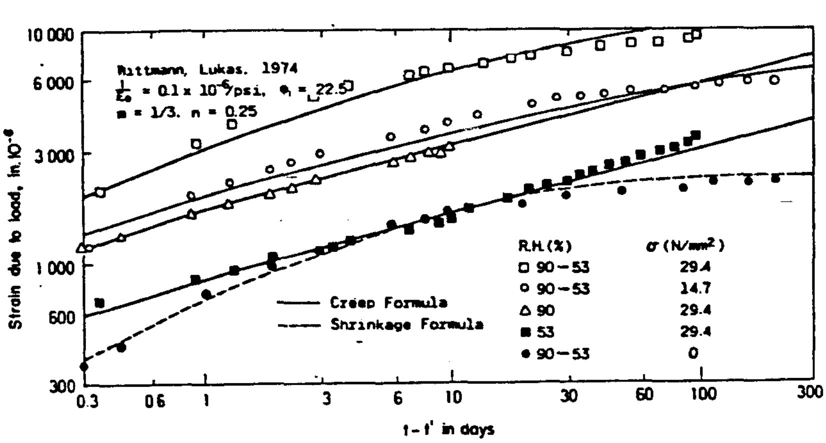

Fig. 9. — **Wittman and Lukas' tests for creep and shrinkage** (data from reference [24]). Cylinders of hardened cement paste 18 × 60 mm, w/c ratio of 0.4. All specimens are cured sealed at 20°C for 28 days. Then some are placed into desiccators at 90% R.H. and some at 53% R.H. After a 3-months storage they were sealed and tested, and some of those stored were tested sealed at 90% R.H. and were also tested unsealed at 53% R.H. Shrinkage of unloaded specimens has been determined at the same time. Since the time which corresponds to the subtracted short-time strain was not precisely controlled, it has been assumed so as to get the best fits: 15 minutes for the top creep curve, 0.5 minute for the other creep curves. (If same values were assumed for all curves, then f_a would be smaller for the higher stress, which is impossible.)

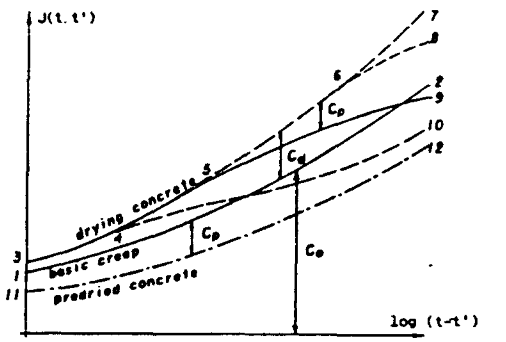

Fig. 10. — Sketch of basic creep, drying creep and creep after predrying.

to vary from concrete to concrete and it was found that approximately

$$c_d = 2.6 - 6n \qquad \text{and} \qquad \varphi_d = 116{,}400 - 13.8 f'_{c_{28}}$$

in which $f'_{c_{28}}$, cylindrical compressive strength at age 28 days. These formulas appeared to hold only for concretes (*figs.* 1-7), but not for mortars or cement pastes (*figs.* 8, 9).

It has been observed by L'Hermite (*cf.* [2], [14]) that under drying conditions the nonlinear dependence of the increased creep upon stress becomes much more pronounced, and for this reason it is appropriate that the term $C_d(t, t', t_0)$ in equation (10) be multiplied by a function of stress. Then, however, the dependence of the remaining terms in equation (10) upon σ may be included as well, extending thus the validity of the expression into the high stress range. Thus, equation (10) may be generalized as

$$J(t, t') = \left[\frac{1}{E_0} + C_0(t, t') + C_d(t, t', t_0)\, g_\sigma - C_p(t, t', t_0)\right] f_\sigma, \tag{15}$$

where f_σ accounts for the nonlinearity of the basic creep with the instantaneous deformation, and g_σ accounts for the stronger nonlinearity of the drying creep. It is convenient to introduce f_σ and g_σ in such a manner that f_σ and $g_\sigma = 1$ under typical permanent loads of structures. For such loads, the sustained stress equals roughly 0.3 of the strength. The following expressions have been found to give good predictions

$$f_\sigma = \frac{R(r)}{R(0.3)}, \qquad g_\sigma = \frac{R_d(r)}{R_d(0.3)}, \qquad r = \frac{\sigma}{f'}, \tag{16}$$

$$\left.\begin{aligned} R(r) &= 1 + a\left(1 - \sqrt{1 - r^6}\right), \\ R_d(r) &= 1 + a_d r^2 (3 - r), \end{aligned}\right\} \tag{17}$$

$$\left.\begin{aligned} f' &= f'_c\left[0.8 + 0.2\left(\frac{t}{\tau_f}\right)^{-0.25}\right], \\ f'_c &= f'_{c_{28}} \frac{t'}{4 + 0.85\, t'}, \end{aligned}\right\} \tag{18}$$

in which f'_c = cylindrical strength of (virgin) concrete at age t', $f'_{c_{28}} = f'_c$ at 28 days of age; f' = cylindrical strength for sustained load of duration $t - t'$ (long-time strength); $f'_{c_{28}}$, a, a_d, τ_f are constants for a given concrete. When test data are lacking, one may use $a = 1.1$, $a_d = 0.05$, $\tau_f = 0.0135$; these values apply for all figures shown herein.

The most extensive and representative data sets available in the literature have been fitted using equation (7)-(18). The fits are shown by solid lines in figures 1-9 and a good agreement is apparent. The values of material parameters are stated in the figure captions.

Coefficients $1/E_0$, m, n, and φ_1 have been the only parameters which differed from concrete to concrete. Lacking test data, one may consider the values: $1/E_0 = 0.10 \times 10^{-6}$ per psi, $m = 1/3$, $n = 1/8$.

ANALYSIS OF PROPOSED CREEP FORMULAS

1. Limiting cases

Short times of drying creep are those which are small compared to shrinkage square half-time τ_{sh}.

For $t - t' \ll \tau_{sh}$ one has

$$t - t_0 \approx t' - t_0$$

and

$$S_d(t, t') \approx \left(\frac{t - t'}{10\,\tau_{sh}}\right)^{c_d n},$$

and so equations (11)-(15) reduce to the form

$$J(t, t') = \frac{1}{E_0} + \frac{\varphi'_1}{E_0}(t')^{-m}\left[(t - t')^n + b_1 (t - t')^{c_d n}\right] \tag{19}$$

where φ'_1 and b_1 are coefficients independent of t and t', although they depend on $t' - t_0$. Thus, the creep curve is a sum of power curves.

When the load is applied long after the loss of moisture has ended, i. e., $t' - t_0 \gg \tau_{sh}$, then $S_p \approx 1$, $\varphi'_d \approx 0$, $C_d \approx 0$, and so

$$J(t, t') = \frac{1}{E_0} + (1 - c_p k''_h)\frac{\varphi_1}{E_0}(t')^{-m}(t - t')^n. \tag{20}$$

Thus, power law of the same exponent is preserved. This is in agreement with the findings of Wittmann *et al.* [24]. However, the function $(t')^{-m}$ in equation (20) is inappropriate for aging because no hydration is going on in a dried concrete. Properly, t' should be evaluated [6] as the equivalent hydration period, t'_e, which is not increasing with time when h is low.

2. Delay of the effect of drying on creep

The fact that the increase of creep due to drying is delayed with regard to drying and persists for some time after all moisture loss has ended is expressed by multiplying τ_{sh} by 10 in equation (13) for S_d, which is confirmed by test data. This corresponds to the assumption that this increase in creep develops about 10-times slower than shrinkage.

The decrease of creep due to moisture loss is observed only very long after the moisture loss has finished. In equation (13) for S_p, this is interpreted by multiplying τ_{sh} by 100, which is again confirmed by test data.

The curves S_d and S_p have a form which differs from the shrinkage curve $S(\hat{t})$ [equation (2)] by the low values of their exponents, n and $c_d n$. These small exponents again model the fact that the drying effect on creep is a slower and more gradual process than shrinkage.

Because of the delay of drying effect in creep, the region $\overline{59}$ in figure 10, in which the slope of the creep curve in log $(t-t')$-scale begins to decline, can only be reached in relatively thin specimens of rather permeable concrete. This has been the case of the data of Troxell *et al.* [21]; but in most other test data the declining region has not been reached within the duration of the test. In such cases the last term C_p of the creep expression does not enter the picture and may be dropped; this includes most practical situations.

The slope of the creep curve may never be negative, and to assure it, coefficient c_p may not be too large. The values given herein satisfy this condition.

Dependence of φ'_d upon $t'-t_0$ is needed to express the fact that the increase of creep due to drying is smaller, the later is the load applied after the start of drying. For higher τ_{sh} (e. g., for a thicker specimen), the decrease of φ'_d with $t'-t_0$ must get slower because the drying process unfolds slower. Therefore, φ'_d should be a function of the nondimensional ratio $(t'-t_0)/\tau_{sh}$. Equation (11), with empirical constant k_a, represents the simplest possible choice for this function.

3. Effect of humidity

The effect of different environmental relative humidities h upon the increase of creep due to drying is expressed by coefficient k'_h, equation (14). The lower is h, the greater increase of creep is obtained, but at lower h the change of h has a lesser effect. Exponent 1.5 was obtained chiefly from the data in figures 3 and 4 ([2], [4]). Usually $h_0=1$, but the formulas also apply for a specimen which has reached hygral equilibrium at some humidity $h_0<1$ before the environmental humidity is changed. The absolute value is used in equation (14) so as to achieve that not only a humidity decrease $(h<h_0)$ but also a humidity increase $(h>h_0)$ intensifies creep until a new hygral equilibrium is reached. The humidity dependence of creep after drying is given by coefficient k''_h, equation (14); here, the lower is h, the smaller is creep, by contrast to the previous case. Function k''_h results chiefly from Wittmann's data in figure 8.

4. Effects of size, shape, diffusivity, and hydration

The effect of size and shape upon drying creep is introduced by means of coefficient τ_{sh} which enters equation (11) and (13) and is given by equation (4). The data in figure 1 confirm that this formulation is acceptable. When the specimen is very massive ($\tau_{sh}=\infty$), equation (11)-(15) yield $S_d(t,t')=S_p(t,t_0)=0$, and the basic creep law, equation (7), is recovered.

The effect of permeability and diffusivity C_1 is similar to that on shrinkage and is introduced also by means of τ_{sh}.

The effect of aging due to hydration upon basic creep is given by $(t')^{-m}$. The effect on drying creep includes function $(t')^{-m}$ as well, and through coefficients ε_{sh_∞} and C_1 it also includes the aging effects of shrinkage. It should be noted, however, that after drying (term C_p), the equivalent hydration period t'_e $(t'_e<t')$ should be substituted for t'.

5. Nonlinear dependence on stress

The nonlinearity of basic creep is described by function f_σ, equation (16). This function should be introduced in a form which at low stress reduces to a constant, so as to conform to the fact that the basic creep is almost linear with regard to stress σ up to 0.3 of the strength. Function f_σ should also exhibit a horizontal tangent when strength $\sigma=f'$ is reached. Function R in equation (17) satisfies both of these conditions. (However, this function is not intended to describe the declining branch after reaching the peak.) The degree of nonlinearity is given by parameter a in equation (17), characterizing the offset of the peak point from the initial tangent.

Function g_σ introduces the additional nonlinearity due to drying, whose existence has been discovered by L'Hermite [14]. While without drying the concrete creep is almost linear with stress up to 0.5 of the strength, in presence of drying it is markedly nonlinear even in the low stress range (see *figs* 6 and 7). (This effect may be explained by a nonlinear coupling between simultaneous diffusions of water molecules and molecules (or ions) of the solid microstructure [2], [8]). The drying rate does not seem to have much effect on strength, and theoretically it indeed should not have much effect because the nonlinear dependence on σ at high stress range is due almost entirely to microcracking and not to molecular diffusion processes. Therefore, function g_σ should intensify the nonlinearity only in the low stress range, but not in the high stress range. Function R_d in equation (17) satisfies this requirement; R_d was derived as the polynomial whose curvature decreases linearly with σ and reaches 0 at $\sigma=f'$. Figure 6 indicates a very good agreement with test data. This figure, however, is contradicted by the data in figure 7, in which the dependence of creep on stress seems to be atypically strong. Values $R(0.3)$ and $R_d(0.3)$ were introduced in equation (16) in order to make f_σ and g_σ equal unity when the stress has the typical value of the sustained stress component in structures.

The decrease of strength with the duration of sustained load is expressed by f' in equation (18) and figure 7 demonstrates a favorable comparison with the test data. The increase of strength with age is introduced in equation (18) according to the formula recommended by A.C.I. Committee 209/II [1].

FURTHER EXTENSIONS

1. Various temperatures

The effect of temperature may be approximately considered by replacing the actual creep duration, $t-t'$, by a reduced creep duration $k_T(t-t')$, and also replacing age at loading t' by the value $t_e-(t-t')$, where t_e is the equivalent hydration period at temperature T, as given in [4] and [7], or [2];

$$k_T=\exp\left[\left(\frac{1}{T_0}-\frac{1}{T}\right)\frac{Q}{R}\right], \tag{20 a}$$

where $Q/R-5{,}000$ K (Q = activation energy of creep; R = gas constant). Note that temperature also enters shrinkage parameter τ_{sh} which appears in creep formulas.

In case of a sudden change of temperature during creep, a significant temporary increase of the creep rate, called transitional thermal creep, is produced. A further correction coefficient would have to be introduced to model this phenomenon.

2. Cyclic creep, fatigue, strength

The acceleration of creep due to superimposing a cyclic stress component $\Delta\sigma \sin \omega t$ upon constant stress σ can be easily modeled by replacing f_σ in equation (16) with the expression $f_\sigma[1+(\Delta\sigma/c')^p]$, which ensues from recent experimental observations of Neville and Whaley [23]. The effect of fatigue due to load repetitions could be included in f' [equation (18)]. The effect of drying on strength and the increase of strength produced by a low sustained compression could be also incorporated through f'.

TABLE I

SUMMARY OF NUMERICAL PARAMETERS WHICH CHARACTERIZE THE FITS

Figure	Data	ε_{s_∞} (10^{-6})	C_7 (mm²/day)	$1/E_0$ (10^{-6})	φ_1	m	n	φ_d (10^6)
1......	Hansen, Mattock, 1966 (various sizes and shapes)	1,000	20.2	0.05	8.80	1/3	0.15	.034
2......	L'Hermite, Mamillan, 1968, 1970 (various sizes and shapes)	425	17.7	–	–	–	–	–
3......	L'Hermite, Mamillan, 1965 (various humidities)	575	3.3	0.078	3.60	0.203	0.1	.044
4......	Troxell, Raphael, Davis, 1958 (various humidities)	1,150	13.5	0.06	15.10	1/3	0.11	.082
5......	L'Hermite, Mamillan, 1965 (various ages at loading)	575	3.3	0.078	3.30	0.203	0.1	.044
6......	Mamillan, 1970 (various stress levels)	425	17.7	0.1	4.00	1/3	0.1	.063
7......	Rüsch, 1960 (various stress levels)	1,000	10.0	0.1	8.50	1/3	0.14	.044
8......	Wittmann, 1970 (various humidities)	3,950	10.0	0.1	6.50	1/3	0.17	–
9......	Wittmann, Lukas, 1974	3,950	10.0	0.1	22.5	1/3	0.25	.247

CONCLUSION

The creep and shrinkage behavior at various humidity regimes, specimen sizes and shapes, ages of concrete, and stress levels can be satisfactorily described by algebraic formulas which represent a generalization of the double power law presented previously.

ACKNOWLEDGMENT

Sponsorship by the U.S. National Science Foundation under Grants GK-26030 and ENG75-14848 is gratefully acknowledged.

APPENDIX I

The question of size effect in shrinkage due to creep

For simplicity, assume that the creep law is linear and the effect of lateral stresses can be neglected. Consider an axially shrinking prismatic specimen which is sufficiently long for the axial strain ε to be the same within the whole cross section. Then, for any point of the cross section one may write

$$\varepsilon(t)=\varepsilon_{sh}(x,y,t)+\int_{t_0}^{t} J(t,t')\frac{\partial\sigma(x,y\ t')}{\partial t'}\,dt', \tag{21}$$

where ε_{sh} = theoretical unrestrained shrinkage strain at individual points; σ = normal stress in axial direction z; x, y = rectangular coordinates in the cross section. Integrating this equation over the area A of cross section, one obtains

$$A\,\varepsilon(t)=\iint_A \varepsilon_{sh}(x,y,t)\,dx\,dy + \int_{t_0}^{t} J(t,t')\frac{\partial}{\partial t'} \times\left\{\iint_A \sigma(x,y,t')\,dx\,dy\right\}dt'=0. \tag{22}$$

Here, because of the condition of equilibrium of stresses σ, the second area integral must vanish, and so

$$\varepsilon(t)=\frac{1}{A}\iint_A \varepsilon_{sh}(x,y,t)\,dx\,dy. \tag{23}$$

It is seen that the shrinkage strain of a long specimen is the average of the free shrinkage strains in all points, and that linear aging creep has no effect on shrinkage strains, even though it greatly affects shrinkage stresses. (In three dimensions, this result also follows from McHenry's analogy [2].)

APPENDIX II

Other formulations examined

A similar expression for drying creep, of the type

$$C(t,t')=[1-c_0 S(t,t_0)]\,C_0(t,t')+c_1 S(t,t_0), \tag{24}$$

with $S(t,t_0)=S(\hat{t})$ from equation (2) and

$$C(t,t')=J(t,t')-\frac{1}{E_0},$$

has been previously proposed by Wittmann [25]. The present expressions represent an extension of Wittmann's formulation. Equation (24) does not satisfy several of the twelve conditions listed before, as the reader can check. Many other expressions have been also explored in the course of this study, but neither of them satisfied fully the twelve conditions nor provided acceptable fits of test data. These include:

$$C(t,t')=C_0(t,t') + c_1\bar{S}(t,t',t_0)-c_2 S(t,t_0)\,C_0(t,t') \tag{25}$$

in which $\bar{S}(t,t',t_0)=S(t,t_0)-S(t',t_0)$; and

$$C(t,t')=f_h(\varphi_1/E_0)(t')^{-m}(t-t')^n+f_d(t,t') \tag{26}$$

in which $f_d(t,t')$=increase of creep due to drying function of $\bar{S}(t,t',t_0)$, and f_h= function of h accounting for the decrease of creep with h in predried specimens. In particular detail, a formulation which is obtained by integrating the rate relation

$$\frac{\partial}{\partial\xi}\ln C(t,t')=n\left[1+c_1\frac{dS_d(t,t')}{d\xi}-c_2 S_p(t,t')\right], \tag{27}$$

$$\xi=\ln(t-t')$$

and is of the form $C(t,t')=C_0(t,t')f_d f_p f_\sigma$ in which f_d and f_p are functions similar to $\exp C_d(t,t')$ and $\exp C_p(t,t')$, has been extensively investigated; but the results were worse than those shown in figures.

APPENDIX III

Possibilities of refining the double power law

Since the double power law contains only one parameter in addition to those which define the elastic modulus, the formulation may seem to be oversimplified. It is then natural to ask whether an extension with a greater number of parameters could yield a better description of basic creep data. The following formula with three additional constants p, q, and φ_2 has been examined in detail:

$$J(t,t')=\frac{1}{E_0}+\frac{\varphi_1}{E_0}t'^{-m}(t^p-t'^p)+\frac{\varphi_2}{E_0}(t^q-t'^q), \tag{28}$$

where the last term represents what has been called the "flow term" by others and the replacement of t by t^p corresponds to the concept of "reduced time", also proposed previously. A similar formula in which $t^q-t'^q$ is replaced by $\log t-\log t'$ (which is equivalent to $q\to 0$) has also been investigated. Optimum fits have been again obtained by minimizing a sum of the squares of the deviations from test data. When individual data sets for various particular concretes have been analyzed independently, improvements of data fits, as compared to those in reference [6], have been achieved. However, p was always close to 1, and the last term in equation (28) was always small. Moreover, the values of φ_2 varied entirely inconsistently from concrete to concrete, and when data fits for various concretes were optimized simultaneously under the condition of the same values of p, q and φ_2/φ_1 (or φ_2), the optimum fits corresponded to $\varphi_2\approx 0$ and $p\approx 1$. For some data (e. g., figures 3 and 4 of reference [6]), equation (28) conforms to the need of making the creep curves for high t' steeper at the end, but for other data (e. g., figure 1 of reference [6]) such a change makes the fit worse. Thus, as far as the formulas which describe concrete creep in general and are suitable for prediction, the double power law appears to be the best choice.

This result also confirms that a decomposition of the total creep strain into a "flow term" and a "delayed elastic term", which has been unfortunately adopted for the current C.E.B. International Recommendations on the basis of reference 18, is not appropriate for concrete. This is in addition to other reasons enunciated in reference [5] and in No. 50 (Vol. 9, pp. 91-98) of this Journal.

Note added in proof: It appears, however, that improvement can be obtained with the so-called "triple power law" for creep rate, $\dot{\varepsilon}\sim(t')^{-m}(t-t')^{n-1}t^q$ which reduces to the double power law when $q=0$ (*see* Z. P. Bazant, *Viscoelasticity of solidifying porous material*, Swedish Cement and Concrete Institute-C.B.I., Report, Stockholm, August 1976).

REFERENCES

[1] A.C.I. Committee 209/II (chaired by D. E. Branson), *Prediction of creep, shrinkage and temperature effects in concrete structures, ACI-SP27.* Designing for Effects of Creep, Shrinkage and Temperature, Detroit, 1971, pp. 51-93.

[2] Bažant Z. P. — *Theory of creep and shrinkage in concrete structures: A précis of recent developments.* Mechanics Today, Vol. 2, 1975, pp. 1-93, Pergamon Press.

[3] BAŽANT Z. P., ASGHARI A. A., SCHMIDT J. — *Experimental study of creep of hardened Portland cement paste at variable water content.* Materials and Structures. Vol. 9, No. 52, July-August 1976, pp. 279-290.

[4] BAŽANT Z. P., NAJJAR L. J. — *Non-linear water diffusion in non-saturated concrete.* Materials and Structures (RILEM, Paris), Vol. 5, 1972, pp. 3-20.

[5] BAŽANT Z. P., OSMAN E. — *On the choice of creep function for standard recommendations on practical analysis of structures.* Cement and Concrete Research, Vol. 5, 1975, pp. 129-138, 631-641; Vol. 6, pp. 149-153; and disc in press, Vol. 7, No. 1.

[6] BAŽANT Z. P., OSMAN E. — *Double power law for basic creep of concrete.* Materials and Structures (RILEM, Paris), Vol. 9, No. 49, 1976, pp. 3-11.

[7] BAŽANT Z. P., WU S. T. — *Thermoviscoelasticity of aging concrete.* J. Eng. Mech. Div., Proc. A.S.C.E., Vol. 100, 1974, pp. 1183-1209.

[8] BAŽANT Z. P., WU S. T. — *Creep and shrinkage law for concrete at variable humidity.* J. Eng. Mech. Div., Proc. A.S.C.E., Vol. 100, 1974, pp. 1183-1209.

[9] BECKER N. K., MACINNIS C. — *A theoretical method for predicting the shrinkage of concrete.* Amer. Concrete Inst. J., Vol. 70, 1973, pp. 652-657.

[10] CAMPBELL-ALLEN M. A., BROOKER J. R. — *Size effects in drying and shrinkage of concrete.* Materials and Structures (RILEM, Paris), Vol. 6, 1973, pp. 151-152.

[11] CARLSON R. W. — *Drying shrinkage of large concrete members.* Amer. Concrete Inst. J., Proc., Vol. 33, 1937, pp. 327-336.

[12] C.E.B.-F.I.P. (Comité Européen de Béton, Fédération Internationale de la Précontrainte). *International Recommendations for Analysis and Construction of Concrete Structures,* 2nd Ed., Cement & Concrete Assoc., London, 1970.

[13] HANSEN T. C., MATTOCK A. H. — *Influence of size and shape of member on the shrinkage and creep of concrete.* Amer. Concrete Inst. J., Vol. 63, 1966, pp. 267-290.

[14] L'HERMITE R. — *Volume changes of concrete.* U. S. National Bureau of Standards Monograph 43, 4th Int. Symp. on Chemistry of Cements (held in Washington, D. C.), October 1960, pp. 659-694.

[15] L'HERMITE R., MAMILLAN M. — *Retrait et fluage des bétons.* Annales de l'Inst. Techn. du Bâtiment et des Travaux Publics (Supplément), Vol. 21, No. 249, 1968, p. 1334; *Nouveaux résultats et récentes études sur le fluage du béton.* Materials and Structures, Vol. 2, 1969, pp. 35-41; MAMILLAN M., BOUINEAU A. — *Influence de la dimension des éprouvettes sur le retrait.* Annales Inst. Techn. du Bâtiment et des Travaux Publics (Supplément), Vol. 23, No. 270, 1970, pp. 5-6.

[16] L'HERMITE R. G., MAMILLAN M., LEFÈVRE C. — *Nouveaux résultats de recherches sur la déformation et la rupture du béton.* Annales de l'Institut Techn. du Bâtiment et des Travaux Publics, Vol. 18, No. 207-208, 1965, pp. 323-360 (*see* also International Conference on the Structure of Concrete, Cement and Concrete Association, London, 1968, pp. 423-433).

[17] ROSS A. D. — *Shape, size and shrinkage.* Concrete and Constructional Engineering, August 1944, pp. 193-199.

[18] RÜSCH H., JUNGWIRTH D., HILSDORF H. — *Kritische Sichtung der Verfahren zur Berücksichtigung der Einflüsse von Kriechen.* Beton- und Stahlbetonbau, Vol. 68, 1973, pp. 40-60, 76-86 and 152-158.

[19] RÜSCH H. *et al.* — *Festigkeit und Verformung von unbewehrten Beton unter konstanter Dauerlast.* Deutcher Ausschuss für Stahlbeton Heft 198, W. Ernst & Sohn, Berlin, 1968; *see* also Amer. Concrete Inst. J., Vol. 57, 1968, pp. 1-58.

[20] STOCKETT A. L., SCHNEIDER A. M., MARDULIER F. J. — *An analysis of drying shrinkage data for Portland cement mortar and concrete.* A.S.T.M. Jour. of Materials, Vol. 2, 1967, pp. 829-842.

[21] TROXELL G. E., RAPHAEL J. M., DAVIS R. W. — *Long-time creep and shrinkage tests of plain and reinforced concrete.* Proc. A.S.T.M., Vol. 58, 1958, pp. 1101-1120.

[22] WEIL G. — *Influence des dimensions et des tensions sur le retrait et le fluage du béton,* RILEM Bull., No. 3, 1959, pp. 4-14.

[23] WHALEY C. P., NEVILLE A. M. — *Non-elastic deformation of concrete under cyclic compression.* Magazine of Concrete Research, Vol. 25, No. 84, September 1973, pp. 145-154.

[24] WITTMANN F. — *Kriechverformung des Betons unter statischer und unter dynamischer Belastung.* Rheol. Acta, Vol. 10, 1971, pp. 422-428.

[25] WITTMANN F. H., LUKAS J. — *The application of rate theory to time-dependent deformation of concrete.* Magazine of Concrete Research, Vol. 26, 1974, pp. 191-197.

RÉSUMÉ

Méthodes pratiques de formulation du retrait et du fluage du béton. — *On propose une série de formules algébriques qui traduisent le retrait et le fluage du béton considéré pour toutes les durées intéressantes. Ces formules prennent en compte : les effets du séchage à diverses humidités relatives ambiantes, dimensions et formes de la section, le vieillissement (dû à l'hydratation), l'effet retardé du chargement après le début du séchage, le fluage d'éprouvettes préséchées à diverses humidités, la fonction non linéaire de contrainte, l'accroissement de la non-linéarité durant le séchage, et la diminution de la résistance sous charge de longue durée. On a obtenu comme un cas particulier une simplification sous forme de fonction linéaire de la contrainte. La formulation est une extension de la loi de fluage dite « à double puissance » qui a été récemment proposée pour l'étude du fluage en l'absence d'échange d'humidité. La forme des courbes de fluage en fonction du temps dépend des conditions d'humidité. On montre qu'on obtient une assez bonne concordance avec les nombreuses données expérimentales disponibles.*