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CAN THE CREEP CURVES FOR DIFFERENT LOADING AGES DIVERGE?

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ABSTRACT

When creep recovery is predicted by the principle of superposition from the unit creep curves for various loading ages, both Branson's law (ACI 209/II) and the double power law give non-monotonic recovery curves. This has been criticized by some as thermodynamically inadmissible. Using an age-dependent Maxwell chain model, it is shown, however, that this phenomenon is thermodynamically admissible. Creep recovery, though, is not accurately predicted by the principle of superposition, and the result should rather be interpreted as the admissibility of a divergence of creep curves for different loading ages, which is indeed observed in many measurements. Such divergence can be modeled by Maxwell chains but not by Kelvin chains, so the latter imply an arbitrary, unjustified limitation. The same is characteristic of the "improved Dischinger formulation" (recently adopted from DIN for the new C.E.B. Model Code), which represents a special case of Kelvin chain. This is a further reason why the separation of reversible (delayed elastic) creep component is an arbitrary assumption lacking thermodynamic justification. For aging creep only Maxwell chain rheologic models should be used. Other related shortcomings of "improved Dischinger formulations" are also summarized.

Si le retour de fluage après le déchargement est calculé d'après le principe de superposition à partir des courbes unité de fluage pour chaque temps de déchargement, la recommendation de l'A.C.I., de même que la loi de double puissance, fournissent une courbe non-monotonique. Cette propriété a été critiquée du point de vue thermodynamique. Cependant, en utilisant la chaîne de Maxwell, on montre que ce phénomène est thermodynamiquement admissible. Du fait que le principe de superposition est assez inexact dans le cas de déchargements, cette conclusion peut être mieux interprétée comme l'admissibilité d'une divergence des courbes de fluage, que l'on rencontre souvent dans les mesures. On peut l'obtenir avec la chaîne de Maxwell, mais pas avec la chaîne de Kelvin. C'est ainsi qu'il faut éviter cette dernière pour les matériaux viellisants. La même observation est valable en ce qui concerne la méthode de Dischinger améliorée, récemment adoptée par le C.E.B., qui est un cas particulier de la chaîne de Kelvin et qui est basée sur l'hypothèse de la séparabilité de la déformation totale reversible (élasticité différée). On cite aussi quelques autres limitations de cette méthode.

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Statement of Problem

Within the service stress range, the creep law of concrete is approximately considered to be linear, i.e., to obey the principle of superposition. The uniaxial creep is then fully characterized by the creep function, $J(t,t')$, which represents the strain at time t caused by a unit sustained stress applied at time t' (1). The question of the optimum choice of the creep function for codes has been recently receiving vivid attention (2-5), especially since the old rate-of-creep (Dischinger) type formulation was revived in the form of the "improved Dischinger method" embodied in the DIN code which is now adopted for the new (1978) edition of C.E.B. Model Code (6).

Reacting to various criticisms of the "improved Dischinger" formulation, Rüsch et al. (4) argued that the theoretical creep recovery curves which are obtained by the principle of superposition from the creep function must be monotonic, and Nielsen (5) recently tried to support this argument by suggesting that it was thermodynamically inadmissible. Formulations of the "improved Dischinger" type always yield monotonic recovery curves. However, it seems to have been overlooked that application of the superposition principle to actually measured creep curves for various ages t' at loading usually leads to non-monotonic recovery curves. Those practical formulations which agree with test data more closely, also exhibit this phenomenon. This includes Branson's creep formula (7,8,9) embodied in the current ACI Committee 209 recommendation, as well as the double power law (10,11), as has been pointed out by Argyris et al. (12).

Non-monotonic recovery curves are obtained from the principle of superposition if the creep curves for various t', plotted versus age t, diverge. So, the claims of Rüsch et al. (4) and Nielsen (5) are equivalent to saying that the creep curves cannot diverge. This divergence is actually the essence of the question, since cases of decreasing strain, such as recovery, are not satisfactorily predicted by the principle of superposition in the first place. Thus, the question of recovery curves obtained by superposition is an academic one.

The purpose of this study is to decide whether it is admissible for the creep function to yield divergent creep curves, and whether or not the creep function should be capable of exhibiting this property. Some related questions of rate-type aging viscoelastic models, reversibility, and the choice of the creep function for codes will be also examined.

Reversal of Creep Recovery

According to the principle of superposition, the curve of creep recovery is obtained by subtracting the creep curve for loading age t_1' from the creep curve for loading age t_0'; see Fig. 1. Using the double power law, this yields a recovery curve which is not monotonic, i.e., it first declines but after a certain period a recovery reversal occurs and the curve begins to rise. Referring to Fig. 1, we see that this happens when the slope of the curve for loading age t_1' at time t becomes less than the slope of the curve for loading age t_0' at the same time t; i.e., when these two curves begin to diverge. If this should never happen, we would have to require that

$$\frac{\partial J(t,t_1')}{\partial t} - \frac{\partial J(t,t_0')}{\partial t} \geq 0 \quad \text{for any } t_1' \geq t_0'. \tag{1}$$

Dividing this inequality by $t_1' - t_0'$ and taking the limit for $t_1' - t_0' \rightarrow 0$, we obtain

$$\frac{\partial^2 J(t,t')}{\partial t \partial t'} \geq 0. \tag{2}$$

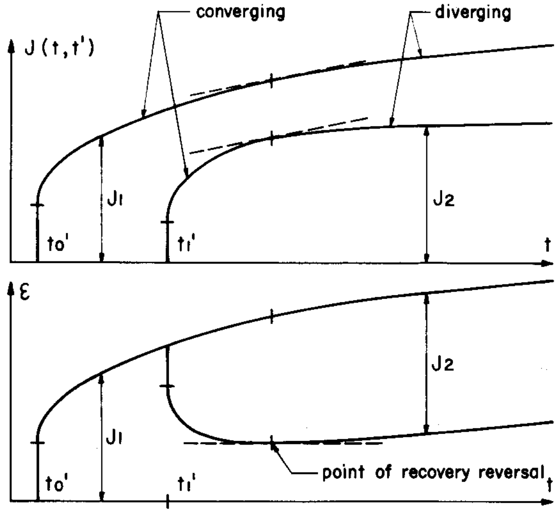

FIG. 1
Divergence of Creep Curves for Loading Ages t_0' and t_1' and Non-monotonic Theoretical Recovery Curve Obtained by Principle of Superposition

On the other hand, by integrating from $t' = t_0'$ to $t' = t_1'$, Eq. 1 may be obtained from Eq. 2. So, Eqs. 1 and 2 are equivalent.

It is generally accepted that the creep function must always satisfy the inequalities $[\partial J/\partial t]_{t'} \geq 0$, $[\partial^2 J/\partial t^2]_{t'} \leq 0$, $[\partial J/\partial t']_{t-t'} = 0$ and $[\partial^2 J/\partial t'^2]_{t-t'} \geq 0$, which follow from the second law of thermodynamics, the principle of fading memory, and the fact that concrete hardens in time and the hardening rate decays. Occasional contradictions of creep measurements with these conditions must be disregarded because they can be due only to experimental error or statistical scatter. In an early work on rate-type models (13), it was thought that, in addition to these inequalities, Eq. 2 must also hold, but in subsequent works of the writer this condition was dropped. We will show in this study that there is indeed no fundamental reason for imposing Eq. 2.

For the double power law, which reads (1,2,10,11):

$$J(t,t') = \frac{1}{E_0} + \frac{\phi_1}{E_0}(t'^{-m} + \alpha)(t - t')^n \tag{3}$$

we find that the condition in Eq. 1 is violated if

$$t - t' > \frac{1-n}{m} t'(1 + \alpha t'^m) = \tau_r \tag{4}$$

where τ_r denotes the creep duration beyond which the creep curves diverge and a recovery reversal is obtained. For typical values (11) $m = 1/3$, $n = 1/8$ and $\alpha = 0.05\ (\text{day})^{-1/3}$, we have

$$\begin{aligned} &\text{for } t' = 3,\quad 30,\quad 300,\quad 3000 \text{ days:} \\ &\tau_r = 8.4,\ 91.0,\ 1051,\ 13550 \text{ days.} \end{aligned} \tag{4a}$$

These are the recovery reversal times if the time of unloading t_1' approaches the time of loading, t_0'; τ_r increases beyond these values as the loading period $t_1' - t_0'$ increases. Anyway, though, an early recovery reversal is predicted if the age at loading is small.

Branson's formula (7,8) used in the ACI Committee 209 recommendation (9) gives the creep function:

$$J(t,t') = \frac{1}{E(t')} + \frac{\phi_1}{E_0} t'^{-m} \frac{(t - t')^n}{a + (t-t')^n} \tag{5}$$

where m = 0.118, n = 0.6, a = 10 days. It is easy to check that this function violates Eq. 2 if

$$t' < m\hat{t}\left(1 - n + \frac{2n\hat{t}^n}{a+\hat{t}^n}\right)^{-1}, \quad \hat{t} = t - t' \tag{6}$$

i.e., for any chosen $t-t'$ we can find t' such that Eq. 2 is not true. As will be seen, the common property which causes both Branson's formula (ACI 209) and the double power law to violate Eq. 1 is that they do not separate a total reversible creep component (delayed elasticity) and a total irreversible creep component (flow).

Divergence of creep curves for different ages at loading is also observed experimentally. This is demonstrated in Fig. 2 where, for several well-known creep test data, the measured creep curve J_2 for loading age t_2' has been subtracted from the measured creep curve J_1 for loading age t_1'; it is seen that the predicted hypothetical recovery curves exhibit blatant reversals. (For more information on the test data used, see Ref. 11.)

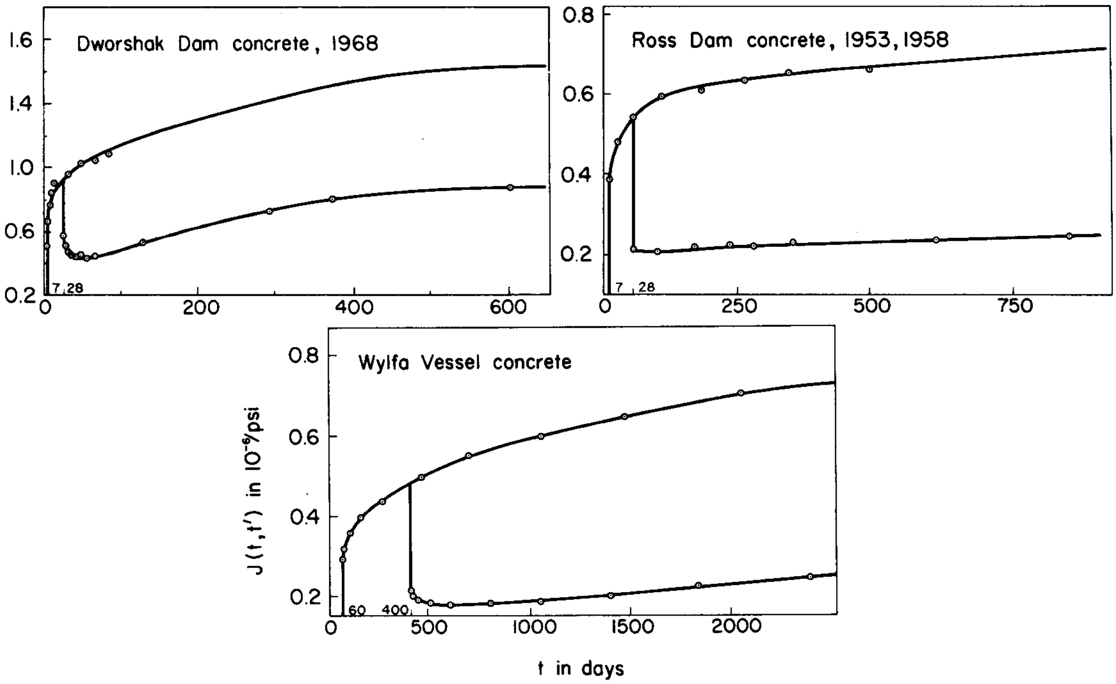

FIG. 2

Difference between Measured Creep Curves J_1 and J_2 for Two Ages at Loading, Demonstrating that Divergence Exists and Superposition Predicts Recovery Unreasonably (after Ref. 11; for references on test data used, see Ref. 11 or Ref. 10). (These are not measured recovery curves.)

In measurements of creep recovery such strong reversals are never observed, although mild reversals are occasionally observed; see Fig. 11 of (2). However, the principle of superposition is known to give especially poor results when strain (not stress) decreases, and so the creep recovery should be properly treated by means of a nonlinear stress-strain relation. So, in the light of experimental data the existence of non-monotonic recovery for the double power law is not disturbing. However, the question remains whether perhaps some laws of thermodynamics are not violated, since in non-aging materials the recovery curves are always monotonic. We will now look at this question more closely.

Thermodynamic Restrictions on Recovery of Aging Creep

With the aim of determining thermodynamic restrictions, Nielsen (5) considered isothermal energy changes during the recovery. The balance of energy (first law of thermodynamics) requires that $\dot{W} = \dot{U} + \dot{D}$, where $\dot{U}$ is the rate of supply of elastic strain energy (free energy per unit mass), $\dot{D}$ is the rate of energy dissipation, and $\dot{W}$ is the rate of work of the applied loads (per unit mass of the specimen); superimposed dots refer to time derivative d/dt. During creep recovery, $\dot{W} = 0$, and so $\dot{U} = -\dot{D}$. The second law of thermodynamics requires that $\dot{D} \geq 0$, which indicates that the elastic strain energy is being released:

$$\dot{U} \leq 0 \quad \text{(during recovery).} \tag{7}$$

From this observation, Nielsen (5) inferred that (for $\varepsilon > 0$) the strain rate must be negative, i.e., $\dot{\varepsilon} < 0$. However, for an aging material this last logical step is unjustified, and we will now demonstrate it by considering an aging standard solid (Fig. 3) as an example.

The spring constants E_1 and E_2 and viscosity η_1 of the standard solid are considered to depend on age t. If the material solidifies (i.e., hydrates rather than dehydrates), we must have

$$\dot{E}_1(t) \geq 0, \quad \dot{E}_2(t) \geq 0. \tag{8}$$

The deformations of the aging springs and the damper are described by the relations:

$$\dot{\sigma}_1 = E_1(t)\dot{\varepsilon}_1^{e\ell}, \quad \dot{\sigma}_2 = E_2(t)\dot{\varepsilon}, \quad \sigma_1 = \eta_1(t)\dot{\varepsilon}_1 \tag{9}$$

where σ_1 and σ_2 are the forces in the two springs, while $\varepsilon_1^{e\ell}$ and ε are their extensions, and ε_1 is the extension of the damper. Note that the forms $\sigma_1 = E_1(t)\varepsilon_1^{e\ell}$ and $\sigma_2 = E_2(t)\varepsilon$, which have sometimes been erroneously used, would be

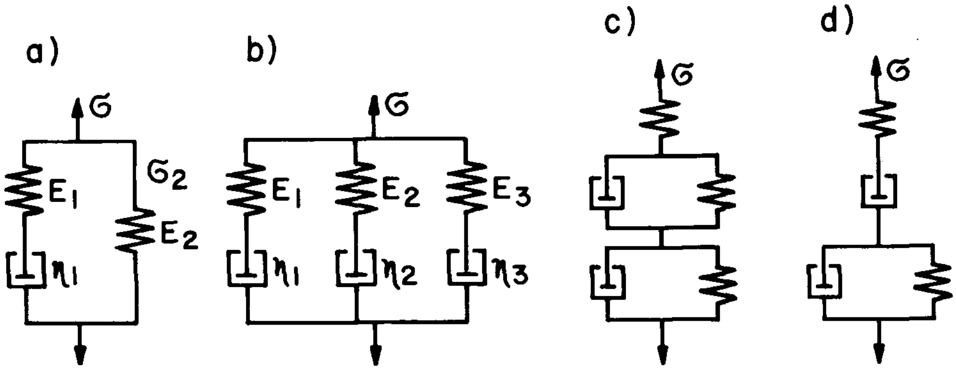

FIG. 3

Standard Solid Model (a), Maxwell Chain (b), Kelvin Chain (c), and Burgers Model (d).

incorrect for a solidifying material (13,14), whether the solidification is due to volume growth of cement gel or to gradual increase in the number of bonds in the solid microstructure (polymerization). This is because the material that is being added to the load-carrying part of the microstructure must be in the unstressed state at the moment it is deposited.

The fact that the relation $\sigma = E(t)\varepsilon$ is impossible can be shown from thermodynamics. For $\sigma = E(t)\varepsilon$ the total energy expression is $W = \sigma^2/2E(t)$, which yields $\dot{W} = \sigma\dot{\sigma}/E(t) + \dot{D}_e$ where $\dot{D}_e = -\dot{E}(t)\sigma^2/2E(t)^2$ = rate of dissipation of strain energy by dissolution of stressed material (chemical dissipation). The second law of thermodynamics requires that $\dot{D}_e \geq 0$ or $\dot{E}(t) \leq 0$. So, the relation $\sigma = E(t)\varepsilon(t)$ is possible only if $\dot{E} \leq 0$ (which is true, e.g., of dehydration above 100°C).

The rate of supply of energy to the elastic springs and the rate of viscous energy dissipation in the damper are

$$\dot{U} = \frac{(\sigma_1^2)^\bullet}{2E_1(t)} + \frac{(\sigma_2^2)^\bullet}{2E_2(t)}, \qquad \dot{D} = \frac{\sigma_1^2}{\eta_1(t)} \tag{10}$$

while the energy that can be recovered upon sudden unloading is

$$U_0 = \frac{\sigma^2}{2[E_1(t) + E_2(t)]} . \tag{11}$$

The second law of thermodynamics requires that $\dot{U} \geq 0$ for any σ_1, σ_2 such that $(\sigma_1^2)^\bullet \geq 0$ and $(\sigma_2^2)^\bullet \geq 0$ and that $\dot{D} \geq 0$ and $U_0 \geq 0$. These conditions dictate that

$$E_1(t) \geq 0, \quad E_2(t) \geq 0, \quad \eta_1(t) \geq 0. \tag{12}$$

No other thermodynamic restriction exists.

Setting $\sigma_1 = \sigma - \sigma_2$, $\varepsilon_1^{e\ell} = \varepsilon - \varepsilon_1$, and eliminating ε from Eqs. 9, we obtain (13) the differential equation

$$\dot{\sigma}_2 + f(t)\sigma_2 = g(t) \tag{13}$$

in which $f(t) = \dfrac{E_1(t)E_2(t)}{E(t)\eta_1(t)}$, $E(t) = E_1(t) + E_2(t)$ (14)

$$g(t) = \frac{E_2(t)}{E(t)} \left[\dot{\sigma} + \frac{E_1(t)}{\eta_1(t)}\sigma\right]. \tag{15}$$

The solution of Eq. 13 is

$$\sigma_2 = e^{-F(t,t')}\left[\sigma_2(t') + \int_{t'}^{t} g(\tau)e^{F(\tau,t')}d\tau\right]; \quad F(t,t') = \int_{t'}^{t} f(\xi)d\xi \tag{16}$$

where $\sigma_2(t')$ is the initial value at $t = t'$.

To calculate $J(t,t')$ for the standard solid, let $\sigma = 0$ for $t < t'$ and $\sigma = 1$ for $t \geq t'$. Then $\sigma_2(t') = E_2(t')/E(t')$ and $g(t) = f(t)$, which yields

$$\sigma_2 = 1 - \frac{E_1(t')}{E(t')} e^{-F(t,t')}. \tag{17}$$

Noting that $\dot{\varepsilon} = \dot{\sigma}_2/E_2(t)$ and $J(t,t') = \int \dot{\varepsilon}dt$, we get the creep function for the aging standard solid:

$$J(t,t') = \frac{1}{E(t')} + \frac{E_1(t')}{E(t')} \int_{t'}^{t} \frac{E_1(\tau)}{E(\tau)\eta_1(\tau)} e^{-F(\tau,t')}d\tau. \tag{18}$$

By differentiating, we finally acquire:

$$\frac{\partial^2 J(t,t')}{\partial t \partial t'} = \frac{E_1(t)}{E(t)\eta_1(t)} \frac{1}{E(t')^2} \Big(\frac{E_1(t')^2 E_2(t')}{\eta_1(t')} + \dot{E}_1(t')E(t') - E_1(t')\dot{E}(t')\Big) e^{-F(t,t')} . \tag{19}$$

In case of a non-aging material, $\dot{E}_1 = \dot{E} = 0$, and so Eq. 2 is always satisfied. However, for an aging material, the expression in Eq. 19 can be negative if $\dot{E}$ is sufficiently large and $\dot{E}_1 = 0$. So, Eq. 2 can be violated without contradicting any law of thermodynamics.

An intuitive insight is helped by realizing that if a creeping non-aging material is unloaded only partially, the strain would continue to increase. In an aging material, the increase of $E_2(t)$ reduces the instantaneously recovered strain, which is similar to partial unloading of a non-aging material.

The rate of supply of elastic strain energy during recovery ($\sigma = 0$) may be calculated from Eq. 10 by substituting $\sigma_1 = -\sigma_2$ and using Eq. 17. This provides

$$\dot{U} = \Big(1 - \frac{E_1(t')}{E(t')} e^{-F(t,t')}\Big) \frac{E_1(t')}{E(t')} e^{-F(t,t')} . \tag{20}$$

Obviously, $\dot{U} \leq 0$ during recovery, as Nielsen (5) has stated, yet $\dot{\varepsilon}$ can be positive. So, the fallacy of the argument based on Eq. 7 consists in the fact that $\dot{U} \leq 0$ does not imply $\dot{\varepsilon} \leq 0$, in case of an aging material.

If Eq. 18 gives a negative value for some t, it does so for all $t \geq t'$, i.e., the recovery is always monotonic, although it could represent not only a decrease but also an increase of deformation. To obtain a non-monotonic recovery curve, it is necessary to consider Maxwell chain with at least three Maxwell units (Fig. 3b). If these three units have substantially different relaxation times, the model acts at early times approximately as a standard solid with Maxwell unit E_1, η_1 and parallel spring ($E_2 + E_3$). Later, after the stress in damper η_1 has almost dissipated, the model acts approximately as a standard solid with Maxwell unit E_2, η_2 and parallel spring E_3. According to the preceding analysis, it is possible that one of these Maxwell units gives a recovery of deformation while the other one gives continued deformation after unloading, with the result that the combined behavior exhibits a reversal. Here is an example which gives a non-monotonic recovery:

$$\left.\begin{array}{lll} E_1 = 1, & E_2 = 2 - e^{-0.1\,t}, & E_3 = 5 - 4e^{-0.3\,t} \\ \eta_1 = 1, & \eta_2 = 1 + t, & \eta_3 = 1 + 0.1\,t^2 \end{array}\right\} . \tag{21}$$

When constant load is applied at t = 0 and is removed at t = 5, recovery reversal is obtained at t = 15.5, as the reader can easily check by calculating the response. This means that the creep curves for t' = 0 and t' = 5 diverge at t > 15.5.

Limitations of Kelvin Chain

A Kelvin unit (spring and damper in parallel) may be considered as a special case of standard solid for $E_1 \to \infty$. In this case we have $E_1/E \to 1$ and $(\dot{E}_1 E - E_1\dot{E})/E^2 \to 0$, and so we see from Eq. 18 that Eq. 2 is always satisfied. By induction, it follows that for a Kelvin chain model (Fig. 3c) the creep recovery is always monotonic, and the creep curves for various ages t' at loading are never divergent.

The proper conclusion from this result is that the aging Kelvin chain model, which has so far been favored over the Maxwell chain in concrete creep literature, is arbitrarily restrictive and its use is therefore incorrect (unless one allows the spring moduli to be negative). This conclusion has previously been made on the basis of numerical experience (1). The Kelvin chain is incapable of describing diverging creep curves, which are often observed experimentally, and are characteristic of the double power law, as well as the recommendation of ACI Committee 209.

It is instructive to observe that the impossibility of $\partial^2 J/\partial t \partial t' < 0$ for a Kelvin chain is due to the fact that Kelvin units have no instantaneous deformation and at the moment of loading the initial stresses in the springs of all Kelvin units are zero, thus being independent of t'. In a Maxwell chain the initial stresses σ_μ in all springs are non-zero and depend on t'. Therefore, if E_μ changes in time, only part of the initial strain in the spring is recoverable at a later time.

The Maxwell chain model does not suffer from the aforementioned unjustified limitations, as has been shown. This also proves that our conclusions apply to the creep function in general, because any creep function of concrete can be approximated by Maxwell chain with any desired accuracy (15).

It may be checked in a similar manner that both Maxwell and Kelvin chain models allow non-monotonic recovery of stress if constant strain is enforced.

It would further be of interest to determine for aging creep the thermodynamic potentials with internal variables. This fact, which has not yet been accomplished, requires the system to be decomposed in time-invariable material components, similarly as in thermodynamics of chemical reactions. This calls for a more extensive analysis and must be relegated to a subsequent paper.

Implications for the New C.E.B. Creep Function and Related Questions

The "improved Dischinger formulation," which has just been incorporated in the midst of criticism into the new C.E.B. Model Code (1978) (6) and will remain in it for some period of time, is based on the assumption that a so-called "reversible" (or "delayed elastic") strain can be separated from the total creep strain. This formulation is equivalent to adopting Burgers' rheologic model (Fig. 3d), consisting of a non-aging Kelvin unit which is supposed to represent the "delayed elastic" strain and is coupled in series to an aging spring and an aging damper. This model is a special case of the Kelvin chain model (in which the viscosity of the damper parallel to one spring is zero), and according to the preceding analysis this implies an arbitrary restriction which lacks thermodynamic justification. This is a point that has not been made before. It explains why this model cannot be made to fit measured creep curves at later times, which typically diverge.

Since a lively polemics has been underway, it may be of interest to summarize the various arguments against resurrecting the old Dischinger-type formulation and adopting it for the C.E.B. Model Code:

1) The long-time trend of the creep curves has not been justified by experimental data. The comparisons with test data reveal huge discrepancies (16) and are clearly inferior to other available formulations.

2) Contrary to the situation prior to the development of Trost's method (1967) and its refinement as age-adjusted effective modulus method, this formulation is no longer necessary for making structural creep calculations feasible and simple.

3) The implied assumption that the shape of creep curves and the effect of age at loading are both described by one and the same function is unjustified and contradicted by tests.

4) The preceding assumption is related to the assumption of "parallelness" of the creep curves for different loading ages at longer creep durations. This parallelness is, however, an optical illusion caused by plotting the creep curves in a linear rather than logarithmic time scale, and is acceptable only for a rather limited time range.

5) Creep recovery (delayed elastic component) does not reach a final value after a relatively short period (as seen in log-time plots), (Ref. 2, 7, 126).

6) The apparent final value seen in actual time plots is not independent of age, not even approximately, (Ref 2, 7, 126).

7) Taking the recovery curves instead of the creep curves as the basis is improper because attaining a good fit of the creep curves themselves is more important (and one cannot fit both).

8) The principle of superposition (linearity) is not applicable to cases of decreasing strain as in recovery.

9) Divergence of creep curves for various ages at loading is observed in tests.

10) The separation of reversible (delayed elastic) component is an arbitrary limitation which is thermodynamically unjustified in case of an aging material.

11) The activation energy concept for creep leads logically to a power law in time, as shown by Wittmann (17). The measured creep curves are smooth and regular, and give no impression of being a collection of terms representing supposedly different physical components.

12) A power law also results from a stochastic process model for creep (19), based on physical mechanism of creep.

13) A realistic model of the solidification process (both as volume growth of gel or as polymerization) admits the double power law and other product forms (Branson, ACI 209), but not the laws of Dischinger type (14).

14) Certain structural predictions are dangerously in error. In addition to the examples given before (2,19) this includes the long-time critical creep buckling load, which is incorrectly predicted by improved Dischinger methods as independent of age (as criticized by Distéfano) (cf. 1).

The argument number one is, of course, most important. If only some conveniently selected, isolated test data are used and if the comparisons are plotted in the linear time scale, good fits can be easily achieved with any of the numerous models for creep advanced in the past. Such comparisons mean nothing. Time has come to agree on not making general conclusions from such comparisons. Computer methods now make data fitting far less tedious than it used to be. Moreover, the relevant creep data from the literature (several dozen sets) have recently been collected and organized in a recent report (11) and are available to anyone who wishes to check the creep formulation.

In view of the foregoing arguments, it is advisable for the engineers who would use the 1978 C.E.B. Model Code to substitute, until a revision is made, some other formulation (e.g., C.E.B. 1970) for the long-time creep curves.

Regarding creep recovery predictions, there has been recently suggested to use a so-called "non-virgin superposition," in which a different creep function, supposedly describing the creep of previously loaded concrete, would be used in case of variable stress. However, in spite of much verbal gymnastics the creep law for "non-virgin" superposition has not yet been clearly formulated and no systematic comparisons with test data were made.

Moreover, it is apparently overlooked that one linear differential equation has only one Green's function associated with it. Thus, using "virgin" creep curves for constant stress, and "non-virgin" creep curves for variable stress does not correspond to a linear creep law, and could be workable only with a non-linear creep law.

Conclusions

1. Non-monotonic creep recovery in an aging (solidifying) material does not violate the laws of thermodynamics.

2. Divergence of creep curves for different ages at loading is thermodynamically admissible.

3. Creep curves of concrete for different ages of loading always first converge but after a longer creep duration they usually start to diverge.

4. The use of the Kelvin chain model with positive elastic moduli implies an arbitrary limitation that is thermodynamically unjustified. Hence, for aging creep the Kelvin chain should be avoided, unless negative elastic moduli are admitted. Kelvin chain would be admissible if test data indicated non-divergent creep curves, which is not the case.

5. The separation of total reversible and irreversible creep strains, serving as the basis for the traditional Dischinger-type (rate-of-flow type) formulations (including the formulation from the DIN Code that is currently adopted for the new C.E.B. Model Code) is an arbitrary assumption that lacks thermodynamic justification, as well as experimental support.

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References

1. Z. P. Bažant, "Theory of Creep and Shrinkage in Concrete Structures: A Précis of Recent Developments," Mechanics Today, Vol. 2, Pergamon Press, pp. 1-93, New York (1975).

2. Z. P. Bažant and E. Osman, "On the Choice of Creep Function for Standard Recommendations on Practical Analysis of Structures," Cement and Concrete Research 5, 129-138 (1975); discussions 5, 629-644 (1975), 6, 149-154 (1976), 7, 111-130 (1977), 8, 129-130 (1978).

3. Z. P. Bažant and W. Thonguthai, "Optimization Check of Certain Recent Practical Formulations for Concrete Creep," Materials and Structures (RILEM, Paris) 9, 91-98 (1976); disc. 10, 55-58 (1977).

4. H. Rüsch, D. Jungwirth and H. Hilsdorf, First and Second Discussions of Ref. 2, 5, 629 (1975), 7, 111 (1977).

5. L. F. Nielsen, Reply to Discussions of Nielsen's Paper "On the Applicability of Modified Dischinger Equations," Cement and Concrete Research 8, 123-128 (1978).

6. C.E.B. (Comité Européen de Béton)-F.I.P. Model Code, Congress of F.I.P. (Fédération Internationale de la Précontrainte), London, May 1978.

7. D. E. Branson, B. L. Meyers and K. M. Kripanarayanan, "Time-Dependent Deformation of Non-composite and Composite Prestressed Concrete Struc-

tures," Highway Research Record No. 324, 1970, pp. 15-43.

8. D. E. Branson, M. L. Christiason, "Time-Dependent Concrete Properties Related to Design-Strength and Elastic Properties, Creep and Shrinkage," Am. Concrete Inst. Special Publ. SP-27, "Designing for Creep, Shrinkage and Temperature," Detroit 1971, pp. 257-277.

9. ACI Committee 209/II, "Prediction of Creep, Shrinkage and Temperature Effects in Concrete Structures," ACI Spec. Publ. No. SP-27, "Designing for Effects of Creep, Shrinkage and Temperature," Detroit, 51-93 (1971).

10. Z. P. Bažant and E. Osman, "Double Power Law for Basic Creep of Concrete," Materials and Structures (RILEM, Paris), 9, 3-11 (1976).

11. Z. P. Bažant and L. Panula, "Practical Prediction of Creep and Shrinkage of Concrete," Structural Engineering Report No. 78-3/640, Northwestern University (March 1978); to appear (in five parts) in Materials and Structures (RILEM, Paris).

12. J. H. Argyris, K. S. Pister and K. J. Willam, "Thermomechanical Creep of Aging Concrete," Trans., 3rd Int. Conf. on Structural Mechanics in Reactor Technology, ed. by T. A. Jaeger, Paper H2/1, London (1975).

13. Z. P. Bažant, "Phenomenological Theories for Creep of Concrete Based on Rheological Models," Acta Technica ČSAV (Prague) 11, 82-109 (1966).

14. Z. P. Bažant, "Viscoelasticity of Solidifying Porous Material - Concrete," J. of the Engng. Mech. Div., Proc. ASCE 103, 1049-1067 (1977).

15. Z. P. Bažant and S. T. Wu, "Rate-Type Creep Law of Aging Concrete Based on Maxwell Chain," Materials and Structures (RILEM, Paris) 7, 45-60 (1974).

16. Z. P. Bažant and L. Panula, "A Note on Amelioration of the Creep Function for 'Improved Dischinger Method'," Cement and Concrete Research 8, 381-385 (1978).

17. F. H. Wittmann, "Bestimmung physikalisher Eigenschaften des Zementsteins," Deutscher Ausschuss für Stahlbeton, Heft 232, W. Ernst und Sohn, Berlin (1974).

18. E. Çinlar and Z. P. Bažant, "Stochastic Process for Extrapolating Concrete Creep," J. of the Engng. Mech. Div., Proc. ASCE 103, 1069-1088 (1977).

19. Z. P. Bažant, L. J. Najjar, "Comparison of Approximate Linear Methods for Concrete Creep," J. of the Struct. Div., Proc. ASCE 99, 1851-1874 (1973).