

## MATHEMATICAL MODEL FOR CREEP AND THERMAL SHRINKAGE OF CONCRETE AT HIGH TEMPERATURE

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Based on the existing limited test data, it is possible to set up an approximate constitutive model for creep and shrinkage at temperatures above 100°C, up to about 400°C. The model presented here describes the effect of various constant temperatures on the creep rate and the rate of aging, similar effects of the specific water content, the creep increase caused by simultaneous changes in moisture content, the thermal volume changes as well as the volume changes caused by changes in moisture content (drying shrinkage or thermal shrinkage), and the effect of pore pressure produced by heating. Generalizations to time-variable stresses and multiaxial stresses are also given. The model should allow more realistic analysis of nuclear reactor vessels and containments for accident situations, of concrete structures subjected to fire, of vessels for coal gassification or liquefaction, etc.

### 1. Introduction

Mathematical modeling of creep and hygral volume changes of concrete at temperatures over 100°C is needed for finite element analysis of hypothetical accident situations in nuclear reactor structures, as well as assessments of fire resistance. Another area of need is the design of concrete vessels for coal gasification and liquefaction. Although considerable relevant experimental information has been obtained in recent years [1], numerous gaps remain and much more research remains to be done. Yet, structural analyses have to be carried out. Therefore, it seems worthwhile to set up at this time an approximate and rather simplified model based on what is known now. This is chosen as the objective of this work, even though one should be aware that revisions are likely when further test data become available.

Analysis of deformations of concrete at high temperatures cannot be carried out without the knowledge of the changes in the specific content of water and its migration, as well as the pressure of pore water. We will not address here the question of calculating the pore pressure, not only because of necessary scope limitations but also because various approximate mathematical models have been developed in other works [2–10]. For our purpose, the pore pressure is assumed to be given.

The total, macroscopically measurable strain,  $\epsilon$ , can be subdivided into three parts:

$$\epsilon = \epsilon^M + \epsilon^T + \epsilon^H, \quad (1)$$

in which  $\epsilon^M$  = mechanical strain, i.e., the strain caused by stress,  $\epsilon^T$  = thermal dilatation, and  $\epsilon^H$  = hygral strain, i.e., the strain caused by humidity changes (shrinkage or swelling). We will analyze these three components separately. The analysis will be restricted to the state of uniaxial stress and possible multiaxial generalization will be indicated at the end.

### 2. Mechanical strain

For stresses less than about 0.5 of the strength limit, the strain caused by stress  $\epsilon^M$ , is approximately proportional to the stress,  $\sigma$ , which is considered for the time being to be constant in time. Thus,

$$\epsilon^M = \sigma J(t, t'), \quad (2)$$

in which  $J(t, t')$  = compliance function (also called the creep function), which represents the strain at age  $t$  caused by a unit stress that has been acting since age  $t'$  of concrete. At constant specific moisture content,  $w$ , and constant temperature  $T$ , the compliance function

may be reasonably well approximated by the double power law [11–13]. For an arbitrary time-constant specific water content  $w$  and an arbitrary time-constant temperature  $T$ , the double power law may be written in the form

$$J(t, t') = \frac{1}{E_0} + \frac{\phi_T f_w}{E_0} (t'_e^{-m} + \alpha)(t - t')^n, \quad (3)$$

in which  $E_0$  = asymptotic modulus = the elastic modulus extrapolated to an infinitely fast application of stress;  $n, m, \alpha$  = parameters for the given concrete, roughly independent of temperature (for their prediction, see refs. [12 and 13]);  $\phi_T$  = function of temperature,  $f_w$  = function of water content, and  $t'_e$  = equivalent hydration period (maturity) [11,14,15] = age of concrete at temperature  $T$  for which the creep is the same as age  $t'$  at reference temperature  $T_0 = 25^\circ\text{C}$ . The typical values of material parameters are  $n \approx 1/8$ ,  $m = 1/3$ ,  $\alpha = 0.3$ ; and, for reference temperature  $T_0 = 25^\circ\text{C}$ ,  $\phi_T f_w = \phi_1 = 3$  to 6 (the times must be given in days); approximately,  $E_0 = 1.5 E(t')$ . The conventional (static) elastic modulus is obtained from eq. (3) as  $E(t') = 1/J(t, t')$ , where  $t = t'$  plus 0.1 day.

As is well known, the temperature effect on creep is two-fold. First, at a higher temperature the hydration process, manifested in aging, progresses faster, and since for loads applied at the higher age the creep is smaller, this effect tends to decrease creep. Second, at the higher temperature the rate of bond ruptures which are responsible for creep and are manifested in viscosity parameters (e.g., those for the Maxwell chain model), is larger. This effect tends to increase creep and usually prevails over the first one [11,14,15].

The effect of temperature on the rate of hydration (aging) may be described [15,11,14] by expressing the equivalent hydration period (maturity) as

$$t'_e = \int_0^t \beta_T [T(t'')] dt'' \quad (4)$$

For a two-step temperature history in which the temperature is equal to the reference temperature  $T_0$  until age  $t_1$  at which the temperature is raised to a new constant temperature,  $T_1$ , eq. (4) yields  $t'_e = t_0 + \beta_{T_1}(t' - t_1)$ ; see fig. 1c. Also  $t'_e = t_0$  if  $T_1 = T_0$ . Coefficient  $\beta_T$  is a function of temperature, and since the chemical reaction of hydration is a thermally activated process, this coefficient may be expressed with the help of the rate-process theory [16,15,11,14] and:

$$\beta_T = \exp \left[ \frac{U_h}{R} \left( \frac{1}{T_0} - \frac{1}{T} \right) \right], \quad (5)$$

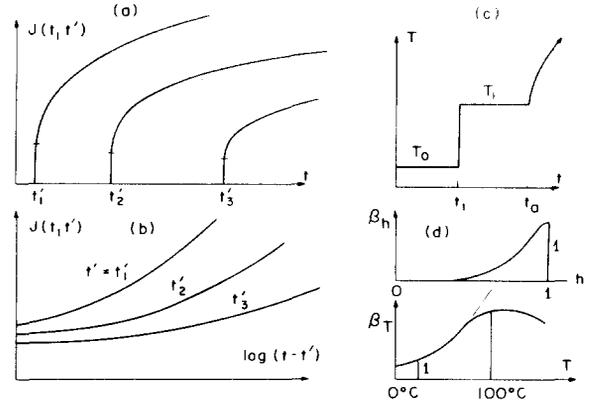


Fig. 1.

in which  $R$  = gas constant,  $U_h$  = the activation energy of hydration; and  $T, T_0$  must be given as the absolute temperatures. By definition,  $\beta_T = 1$  when  $T = T_0$ . The value of  $U_h$  depends on the type of cement and concrete, and actually eq. (5) is itself a simplification since hydration involves a number of different chemical processes with different activation energies. Nevertheless, as a good approximation up to about  $100^\circ\text{C}$  one can use  $U_h/R = 4000$  K. Above  $100^\circ\text{C}$ , the hydration process does not proceed, and above approximately  $200^\circ\text{C}$ , the reverse process, dehydration, takes place. Dehydration, however, is not too significant below  $400^\circ\text{C}$  and in this range may be neglected, as an approximation [1].

For nuclear accident analysis, one needs to determine the value of  $t'_e$  just before the postulated accident happens, i.e., for operating temperature conditions. The variations of  $t'_e$  during a short-time accident may probably be neglected, and the same is true for calculations of response to fire.

The change of viscosity aspect of the temperature effect on creep is logically also modeled according to the rate-process theory since the mechanism of creep no doubt consists in thermally activated ruptures of bonds within the solid microstructure. Therefore,

$$\phi_T = \phi_0 \exp \left[ \frac{U}{R} \left( \frac{1}{T_0} - \frac{1}{T} \right) \right], \quad (6)$$

in which  $\phi_0$  = constant parameter for the given type of concrete [12,13], and  $U$  = activation energy for the creep rate (i.e., for the bond ruptures). While below  $100^\circ\text{C}$  the dependence expressed by eq. (6) is obscured by the simultaneous effect of temperature on the hydration rate, from  $100^\circ\text{C}$  to about  $400^\circ\text{C}$  this dependence can be directly measured, as the hydration effects are negli-

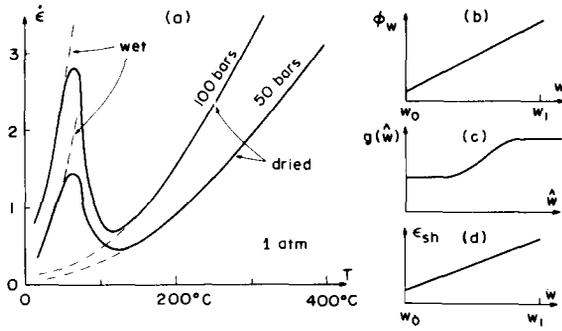


Fig. 2.

$$\frac{U}{R} \approx \left(1 + \frac{w - w_0}{w_1 - w_0}\right) 2000 \text{ K}, \tag{8}$$

where a linear dependence on  $w$  is assumed due to lack of more precise information;  $w_0$  = water content of dried concrete (i.e., at 105°C and 1 atm), and  $w_1$  = water content of saturated concrete at 25°C and 1 atm (1 atm = 101 325 Pa).

**3. Creep at various constant water contents**

Aside from the direct effect of water content on activation energy  $U$ , as just explained, there exists, no doubt, an additional direct effect of water content  $w$  on the creep rate. This is because eq. (8) does not give a sufficiently large difference between the creep rates of wet and dried concretes, which is an order of magnitude difference. Apparently, the presence of water not only reduces the activation energy but also generally increases the mobility of the solid particles that are displaced during creep. Again, from Maréchal's data [17, his figs. 4 and 5], the ratio of strain rate  $\dot{\epsilon}$  values for wet and dried concretes is about 2.8/0.3  $\approx$  9 (from his fig. 5), or 560/70 = 8 (from his fig. 4). As another experimental evidence, Bažant, et al. [4] find in their fig. 6 for 200°C the ratio of  $\dot{\epsilon}$  values at two hours after loading for predried and drying concrete to be about 1/5, and that for drying concrete and sealed concrete to be about 1/2. This gives for the ratio of  $\dot{\epsilon}$  values for predried concrete and sealed concrete approximately 1/10, which is about the same result. Other data, e.g., those by Wittmann [21,22,23], give also about the same result. Thus, assuming again a linear dependence due to lack of better evidence, we may propose that (see fig. 2b):

$$f_w = 1 - k_w \frac{w_1 - w}{w_1 - w_0}, \text{ with } k_w \approx \frac{7}{8}. \tag{9}$$

Obviously, the effect of  $w$  is a very important one and can never be neglected.

One should also realize that the pore pressures in concrete can far exceed the saturation pressure [2,3,10,4,5,6], and concrete then becomes significantly oversaturated, i.e.,  $w > w_1(t)$ . Eq. (9) would certainly give too much variation for oversaturated concrete, and is not applicable for  $w > w_1$ . Actually, according to the test results of Bažant, Kim and Meiri [24], it appears that oversaturation significantly decreases rather than increases the creep rate, and roughly  $f_w \approx 0.5$  which ensues from measurements on cement paste exposed to liquid pressurized water at 200°C. However, more extensive testing is required.

gible, in comparison, and anyhow small. Pertinent test data were reported, e.g., by Maréchal [17–20]. To determine  $U$  from Maréchal's data, we note that  $\dot{\epsilon}_T = \sigma dJ/dt = \epsilon_{T_1} \exp[T_1^{-1} - T^{-1}]U/R$  or  $U/R = [\ln(\dot{\epsilon}_T/\dot{\epsilon}_{T_1})]/(T_1^{-1} - T^{-1})^{-1}$ . Using Maréchal's [17] figs. 5 and 4 (p. 113), we find from his curve for  $\sigma = 100$  bars and points at 100°C and 300°C that  $U/R = [\ln(3.3/0.5)](373^{-1} - 573^{-1})^{-1} = 2017 \text{ K}$ , and from his curve for  $\sigma = 50$  bars, and points at 100°C and 400°C, that  $U/R = [\ln(3.2/0.28)](373^{-1} - 673^{-1})^{-1} = 2038 \text{ K}$ . Thus, it seems one may consider (fig. 2):

$$U/R \approx 2000 \text{ K}. \tag{7}$$

This applies for dried portland cement concrete, same as Maréchal's data.

Maréchal's points for 20°C and 50°C give  $U/R = [\ln(2.8/1.07)](323^{-1} - 293^{-1})^{-1} = 3030 \text{ K}$ , a higher value. In an analysis of many test data in the literature [15, p. 275], the values of  $U/R$  in the range 0°C to 80°C are found to range from 3000 K to 10000 K, with a median roughly 4000 K. One must remember, however, that for temperatures below 100°C, which are not of direct interest here, the value of  $U$  can be different.

Since the solids forming hardened cement paste are strongly hydrophylic, and an ingress of water into the pores reduces the surface energy and thus weakens the bonds (as the water molecules are trying to enter and split the bonds), the value of  $U$  no doubt depends on the specific water content,  $w$  (kg of water per m<sup>3</sup> of concrete). Comparing the value 2000 K, which applies for dried concrete above 100°C, with the value roughly 4000 K found in ref. [15] for temperatures below 100°C, and assuming the same value above 100°C, we may extrapolate eq. (7) as

#### 4. Creep at variable moisture content and temperature

As clearly demonstrated first by Pickett [25] and studied by many others [11,14], the creep of concrete in a transient moisture state can be much higher than at constant moisture content. This appears to be true both for decreasing and increasing moisture contents, as well as for elevated temperatures, most likely also above 100°C. The phenomenon may be explained by the hypothesis that movement of water molecules may facilitate changes in the bonding of the solid microstructure [11].

Above 100°C the measurements on specimens exposed to dry environments show much less creep than those which are sealed [26,24], contrary to what is seen at room temperature. This is probably due to the fact that the rate of moisture loss above 100°C is much faster (about 200-times) than at room temperature [2,3], so that the existing measurements at high temperature pertain to specimens that have already dried and achieved equilibrium before the measurement. To model this effect, we may generalize the double power law, eq. (3), as follows

$$J(t, t') = \frac{1}{E_0} + g(\hat{w}) \frac{f_w \phi_T}{E_0} (t'_e^{-m} + \alpha)(t - t')^n, \quad (10)$$

in which  $g$  is a function of  $\hat{w}$ , and  $\hat{w}$  represents some suitable proper measure of the rate of the moisture content. It would be, however, unrealistic to assume that  $g = g(\hat{w})$ , since the relative increase of the creep rate seems to be of the same order of magnitude for both the short-time creep (say, several hours after load application) and the long-time creep (say, many years after load application, at room temperature), even though the strain rates  $\dot{\epsilon}$  differ by orders of magnitude. Therefore, we must assume that  $g$  depends on the ratio  $\dot{w}/\dot{\epsilon}$  rather than just  $\dot{w}$ . Furthermore, we should note that the effect seems to be relatively the same for weak concretes of high moisture contents (high  $w_1$ ) and for dense concretes of small water contents. Also, the relative effect seems to be the same for concretes which creep much (large  $\epsilon$ ) and those which creep little (small  $\epsilon$ ). Therefore, instead of  $\dot{w}$  one should use  $\dot{w}/w_1$ , and instead of  $\dot{\epsilon}$  one should use  $\dot{\epsilon}/\epsilon$ . This leads us to postulate that  $\hat{w}$  may be defined as

$$\hat{w} = \dot{w}\epsilon/\dot{\epsilon}w_1, \quad (11)$$

in which  $w_1$  represents the total saturation water content at room temperature. We may call  $\hat{w}$  the relative drying rate.

Now we need to specify functions  $g(\hat{w})$ . We should

realize that for extremely high drying rates, such as those experienced by the thin surface layer of a drying specimen at the start of drying, or by cylinders at high temperature, the increase of creep caused by simultaneous drying is not much larger than it is for moderate drying rates, which are orders of magnitude smaller [27]. A similar comparison could be made between moderate and very low drying rates. Therefore, it seems that the function  $g(\hat{w})$  should have the form sketched in fig. 2c, consisting of a smooth transition from an asymptotic value for very low drying rates to an asymptotic value for very high drying rates. The simplest function to describe such a transition is

$$g(\hat{w}) = 1 + \hat{k}_w [1 + \tanh \hat{c}_w (\hat{w} - \hat{b}_w)], \quad (12)$$

in which one may set  $\hat{k}_w = \frac{1}{2}$ ,  $\hat{c}_w = 1$ , and  $\hat{b}_w$  is the value  $\hat{w}$  for which the slope of curve  $g(\hat{w})$  is highest. Approximately take  $\hat{b}_w \approx 1$ , which means that the drying creep effect becomes significant when the value of  $\dot{w}/w$  becomes comparable to  $\dot{\epsilon}/\epsilon$ . This should of course be checked carefully against test data after detailed studies are completed.

#### 5. Generalization to multiaxial stress

While important nonlinear effects certainly exist above 100°C, we may probably use a linear formulation for many purposes, just like at room temperature. Then the multiaxial generalization is completely specified by the values of the Poisson's ratio  $\nu$  for instantaneous deformations and for creep. Both are approximately equal below 100°C ( $\nu \approx 0.18$ ).

There is a question, however, as to the value of  $\nu$  above 100°C, since no direct measurements seem to exist at the present for concrete. We will therefore try to make a logical deduction from the measurements on small cement paste specimens [24], which showed that the value  $\nu = 0.25$  applicable for cement pastes at 25°C increases to about 0.46 at 200°C. This means that a large increase of temperature intensifies the deviatoric creep much more than the volumetric creep, for cement pastes. For concretes, however, the relative magnitude of this effect should be much weaker, since the Poisson's ratio of concrete is dominated by the aggregates rather than the binder. At 25°C,  $\nu \approx 0.18$  for both short-time and long-time deformations.

For approximate inferences, one may use the simple laws of mixtures known from the theory of composites, which were shown to be applicable to concrete as a satisfactory approximation; see Dougill [28]. There are

two simple laws of mixture, corresponding to the so-called series and parallel couplings of the deformable elements representing the one and the other component of the composite solid—cement paste (c) and aggregate (a). The reality lies between these two laws, which represent bounds, and is much closer to the mixture law for the parallel coupling [28]. Based on this, the Poisson's ratio of the composite solid may be approximated as

$$\nu = \frac{E_c \nu_c \nu_a + E_a \nu_a \nu_c}{E_c \nu_c + E_a \nu_a}, \quad (13)$$

in which  $E_c$ ,  $E_a$ ,  $\nu_c$ , and  $\nu_a$  = the elastic moduli and Poisson's ratios for cement paste and for aggregate, and  $\nu_c$ ,  $\nu_a$  = volume fractions of hardened cement paste and aggregate in concrete. For the aggregate skeleton without cement we may assume  $\nu_a = 0.175$ . Then, for a typical concrete with  $E_a \nu_a / E_c \nu_c \approx 20$ , we get  $\nu = (0.25 + 20 \times 0.175) / (1 + 20) = 0.18$ . This agrees with many observations, and thus supports our way of argument. Now, for concrete at 200°C, we may estimate in the same manner  $\nu = (0.46 + 20 \times 0.175) / (1 + 20) = 0.19$ , which is only a little different from the previous value 0.18. Our theory is rather crude anyway, and so it appears that we may use for concrete at all temperatures

$$\nu \approx 0.18. \quad (14)$$

Using the approximate effective modulus approach, the same value of  $\nu$  is obtained for creep of concrete at all temperatures. Clearly, it is the dominating influence of the aggregate skeleton which causes the Poisson's ratio of concrete to be insensitive to the value for cement paste.

The value of  $\nu$  being known, the volumetric and deviatoric compliance functions may be expressed as [11,14]:

$$\begin{aligned} J^V(t, t') &= 6\left(\frac{1}{2} - \nu\right)J(t, t'), \\ J^D(t, t') &= 2(1 + \nu)J(t, t'). \end{aligned} \quad (15)$$

## 6. Effect of pore pressure

At high temperatures, the pressure of pore water,  $p$ , can attain significant values and must be taken into account in evaluating tensile fracture. A similar phenomenon has been studied in depth in soil mechanics, and for concrete the effect of pore pressure was examined in many works on the problem of uplift pressure in dams (for review see ref. [29]). From these works it transpires that a distinction must be made between

the total stress, representing a resultant (over a unit area of concrete) of the microstresses in solid as well as water, and the effective stress, representing the resultant of the microstresses from the solid part of the microstructure only. Further, there is a fine point whether the unit area should be perfectly plane or microscopically tortuous so as to pass through the weakest connections in the solid microstructure. The second possibility is more relevant for the assessments of fracture, and in that case the porosity  $n_p$  to be used with pressure  $p$  should not be the actual porosity  $n$  (i.e., the volume fraction of pores) but the so-called boundary porosity, as used in soil mechanics, which represents the area fraction of water over the macroscopically plane but microscopically tortuous weakest section through the material [29]. The value of  $n_p$  is always  $\leq 1$  and always  $\geq$  the actual porosity. From studies of the uplift pressure in dams [29], it seems that  $n_p = 0.9$  is a reasonable estimate (some investigators recommend for dams  $n_p \approx 1$ , but for our purposes the difference is not significant). The effective volumetric stress may be calculated as

$$\sigma^V = \sigma^V + n_p p \quad (n_p \approx 0.9), \quad (16)$$

in which  $\sigma^V$  = volumetric total stress = mean total stress. As a simplification, one may use this expression both for the stress-strain relation and the fracture criterion.

## 7. Hygral and thermal volume changes

The thermal and hygral strains, representing the volumetric (mean) strains caused by changes of temperature  $T$  and water content  $w$ , may be calculated as

$$\epsilon^V = \int_{T_0}^T \alpha_T dT, \quad \epsilon^H = \int_{w_0}^w k_S dw, \quad (17)$$

in which the thermal dilatation coefficient  $\alpha_T$  and the shrinkage coefficient  $k_S$  depend, strictly viewed, on  $T$  and  $w$ , and  $T_0$ ,  $w_0$  = initial values of  $T$  and  $w$ .

For hardened cement paste,  $\alpha$  depends significantly on  $w$ , and the average value is about  $\alpha_T \approx 25 \times 10^{-6}/^\circ\text{C}$ . For siliceous or calcareous aggregates,  $\alpha_T \approx 7$  to  $8 \times 10^{-6}/^\circ\text{C}$ . For concrete, the  $\alpha_T$  values are generally around  $\alpha_T \approx 10 \times 10^{-6}/^\circ\text{C}$ . From this it is clear that again the aggregates dominate the value for concrete. Thus, the variations of  $\alpha_T$  due to changes in  $w$  should be manifested only very weakly for concrete. Therefore, one may normally assume for concrete, as an approximate constant value,  $\alpha_T \approx 10 \times 10^{-6}/^\circ\text{C}$ . The scatter is however large. For the range 100°C–430°C, the secant value for Harmarthy's data [30] for calcareous concrete



above the observed curves for concrete. Note that the dashed-line extrapolations represent roughly straight-line extrapolations of the initial trend of the measured curve, which seems reasonable from the physical point of view.

Further deductions can be made with regard to the value of shrinkage coefficient  $k_S$ . Assume that the water-cement ratio (by weight) is 0.50 and that the final water loss due to drying at zero humidity is about  $0.2c$  where  $c$  = weight of cement in unit volume of concrete. If  $v_c/v_a \approx 0.1$  and the concrete weights  $2300 \text{ kg/m}^3$ , we then get the total water loss  $\Delta w = 0.2 \times 0.1 \times 2300 = 46 \text{ kg/m}^3$ . Then, if the corresponding total shrinkage is  $\Delta \epsilon^S = 300 \times 10^{-6}$ , we obtain  $k_S \approx 10^{-5} \text{ m}^3/\text{kg}$ . On the other hand, for cement paste taken alone, we have  $\Delta w = 0.2 \times 2000 \text{ kg/m}^3 = 400 \text{ kg/m}^3$ , and then, if the total shrinkage is  $\Delta \epsilon^S = 4000 \times 10^{-6}$ , we obtain  $k_S = 10^{-5} \text{ m}^3/\text{kg}$ .

Let us now estimate the values for high temperature, say  $500^\circ\text{C}$ . The water loss is about 0.8 of  $0.5c$  or  $0.4c$ , and the total drying shrinkage is roughly 0.01. This yields  $k_S = 10^{-5} \text{ m}^3/\text{kg}$  for cement paste. For concrete  $\Delta w$  is about the double of the previous value  $46 \text{ kg/m}^3$ , and if  $\Delta \epsilon^S$  is then about double the value at room temperature, then we obtain again about the same value of  $k_S$ . In consequence of all these arguments, it seems reasonable to assume

$$\Delta \epsilon^S = k_S \Delta w, \quad \text{with } k_S = 10^{-5} \text{ m}^3/\text{kg}. \quad (20)$$

However, further measurements should be carried out to ascertain that  $k_S$  is indeed approximately constant.

## 8. Generalization to time-variable stress

In most practical applications, the stresses are not constant but vary in time; not just because of a possible variation of applied loads but also as a consequence of the creep itself. To generalize the preceding formulation to time-variable stresses, one can proceed, under the assumption of linearity stated at the outset, basically in two different ways. Either one can apply directly the principle of superposition, which leads to integral equations in time, or one can determine an equivalent rate-type formulation. The latter is much more convenient for finite element computations, particularly since no previous history needs to be stored. The determination of a rate-type formulation is exactly the same as for moderate temperatures, as described before in detail [11,14,15]. Therefore, we give here only a brief outline

and refer the interested reader to the preceding works.

Assuming, as in previous works, the Maxwell chain representation, we may express the stress, or more precisely the effective stress, for the porous material as a sum of partial stresses corresponding to Maxwell units of various relaxation times, i.e.,

$$\sigma^V + n_p p = \sum_{\mu} \sigma_{\mu}^V, \quad \sigma_{ij}^D = \sum_{\mu} \sigma_{ij\mu}^D, \quad (21)$$

in which  $i, j$  = subscripts referring to cartesian coordinates  $x_i$  ( $i = 1, 2, 3$ ),  $\sigma^V, \sigma_{ij}^D$  = volumetric and deviatoric component of total stress tensor  $\sigma_{ij}$ , and  $\mu$  refers to the number of the Maxwell unit in the chain model ( $\mu = 1, 2, \dots, N$ ). The volumetric and deviatoric stress-strain relation then has the form

$$\epsilon^V = \frac{\dot{\sigma}_{\mu}^V}{3K_{\mu}(t_e)} + \frac{\sigma_{\mu}^V}{3\eta_{\mu}^V(t_e)} + \alpha_T \dot{T} + k_S \dot{w}, \quad (22)$$

$$\epsilon_{ij}^D = \frac{\dot{\sigma}_{ij}^D}{2G_{\mu}(t_e)} + \frac{\sigma_{ij}^D}{3\eta_{\mu}^D(t_e)}, \quad (23)$$

in which  $\epsilon^V, \epsilon_{ij}^D$  = volumetric and deviatoric components of the strain tensor  $\epsilon_{ij}$ ;  $K_{\mu}, G_{\mu}$  = bulk and shear moduli for the individual units of the Maxwell chain; and  $\eta_{\mu}^V, \eta_{\mu}^D$  = age-dependent viscosities. Due to the condition of isotropy,

$$K_{\mu}(t) = \frac{E_{\mu}(t)}{6(\frac{1}{2} - \nu)}, \quad G_{\mu}(t) = \frac{E_{\mu}(t)}{2(1 + \nu)}, \quad (24)$$

in which  $E_{\mu}$  = Young's elastic moduli for the units of the Maxwell chain model for uniaxial stress. Furthermore, the viscosities may be expressed as

$$\frac{1}{\eta_{\mu}^V(t_e)} = g_w f_w \phi_T \frac{3(1 - 2\nu)}{\tau_{\mu} E_{\mu}(t_e)}, \quad (25)$$

$$\eta_{\mu}^D(t_e) = \frac{6(\frac{1}{2} - \nu)}{2(1 + \nu)} \eta_{\mu}^V(t_e) = \frac{G_{\mu}}{K_{\mu}} \eta_{\mu}^V(t_e), \quad (26)$$

in which  $\tau_{\mu}$  represent the relaxation times of the individual Maxwell units at reference temperature, determined as described before [11,14,15]. Together with the equations defining  $t_e, \phi_T, f_w$ , and  $g_w$  ( $t_e = \int \beta_T dt$ , eq. (4)), eqs. (20–26) define the constitutive relation. Efficient numerical algorithms for a step-by-step solution of these equations have been presented before [36,11,14,15].

As a possible generalization, which is easy in the case of the rate-type formulation, function  $g_w$  could depend on  $\mu$ , i.e., have different values for various units of the Maxwell chain model (with different coefficients in eq.

(12)). Test data would be needed, however, to decide that.

## 9. Conclusions

Based on the existing limited test data, an approximate constitutive model for creep and hygral volume changes of concrete at temperatures over 100°C can be set up and its material parameters estimated. The model covers constant as well as variable temperature and specific water content, defines the effect of these variables on the creep rate and the rate of aging, and describes the volume changes due to changes of temperature and of moisture content. The effect of pore pressure is taken into account through the effective stress concept. Generalizations to multiaxial stresses and time variable stresses can be carried out. The model should allow more realistic analysis of nuclear reactor structures under hypothetical accident conditions, concrete structures subjected to fire, concrete vessels for coal gassification or liquefaction, etc.

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