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Size Effect in Brittle Failure of Unreinforced Pipes

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Unreinforced concrete pipes basically can fail in two modes: beam failure and ring failure. According to the existing test data, the nominal stress at beam failure is much less than that at ring failure. Furthermore, the nominal stress for either failure mode decreases as the pipe diameter or thickness increases. These size effects can be described in a unified manner by means of the recently established size-effect law for failures which are preceded by the formation of microcracking zones. A simple formula involving the size effect is proposed and is justified by comparisons with existing test data.

Keywords: concrete pipes; cracking (fracturing); dimensional analysis; **failure;** microcracking; regression analysis; **stresses;** structural analysis.

Fracture mechanics aspects of the failure of unreinforced as well as reinforced concrete structures are now coming to the center of attention. The most important consequence of fracture mechanics is the structural size effect in failure. According to limit analysis as well as allowable stress design, geometrically similar structures of different sizes fail at the same nominal stress. However, according to fracture mechanics, the nominal stress at failure decreases as the size of the structure increases. This effect has been demonstrated not only for notched fracture specimens, but also for diagonal shear failure of longitudinally reinforced beams, unprestressed as well as prestressed, and it probably is characteristic of all brittle failures of concrete structures. The present study demonstrates the effect for unreinforced concrete pipes.

Unreinforced concrete pipes exhibit basically two modes of failure: beam failure [Fig. 1(a)] and ring failure [Fig. 1(b)]. Test results show that the nominal stress at failure for the beam failure is much less than for the ring failure, and that for either case the nominal stress at failure decreases as the pipe diameter or thickness increases. Therefore, different strength values have been considered for various situations. Gustafsson[1] and Hillerborg,[2] however, have recently demonstrated that the existing test results are consistent with unique values of material strength characteristics provided that nonlinear fracture mechanics is applied. Gustafsson established this with the help of the finite element fictitious crack model of Hillerborg, and the same can be shown to be true for the crack band finite element model.[4-7] The objective of this study is not finite element analysis but development of an approximate simple formula for design.

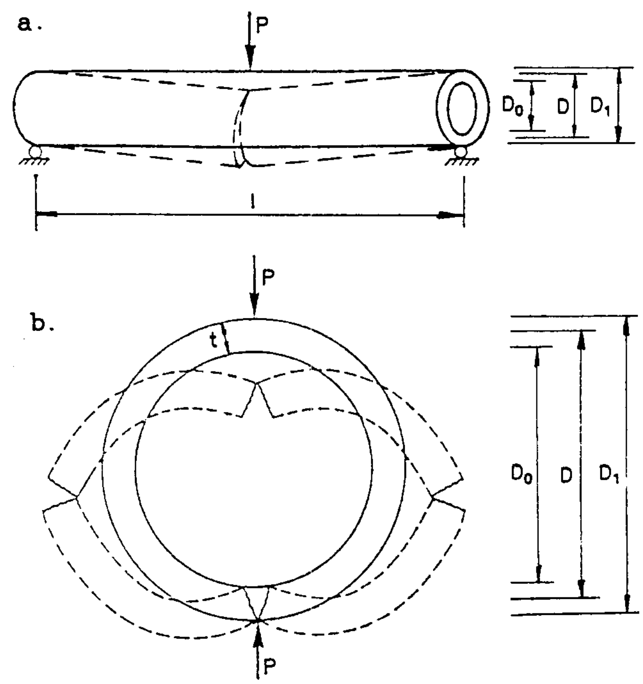

Fig. 1 — Beam and ring failures of pipe

SIZE EFFECT LAW

Due to its heterogeneity and brittleness, fracture propagation in concrete is preceded by dispersed mi-

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crocracking. As shown by dimensional analysis and similitude arguments,[4] the nominal stress at failure of geometrically similar structures of different sizes is approximately determined by the following size effect law

$$\sigma_N = Bf_t' \left(1 + \frac{d}{\lambda_0 d_a}\right)^{-1/2} \tag{1}$$

in which f_t' = direct tensile strength, d = characteristic dimension of the structure, d_a = maximum aggregate size, and B, λ_0 = empirical constants which characterize the structure geometry. For small structures, the second term in the parentheses is negligible compared to 1, which means that the nominal stress at failure σ_N is proportional to strength f_t', so that the limit analysis or the allowable stress design is applicable. For a very large structure, 1 is negligible compared to the second term in the parenthesis, which means that σ_N is proportional to $d^{-1/2}$; this is the size effect known from linear elastic fracture mechanics. Thus, Eq. (1) represents a gradual transition from limit analysis, for which there is no size effect, to linear elastic fracture mechanics.

BEAM AND RING FAILURES OF PIPE

For the beam failure sketched in Fig. 1(a), the nominal stress at failure σ_N may be set equal to the maximum longitudinal bending stress σ_b

$$\sigma_b = \frac{M_b}{I}\frac{D_1}{2}, \; I = \frac{\pi}{64} D_1^3\left[1 - \left(\frac{D_0}{D_1}\right)^4\right] \tag{2}$$

in which M_b = bending moment in the cross section of pipe, D_0, D_1 = interior and exterior diameters of the pipe, and I = centroidal moment of inertia of the cross section of pipe. For the ring failure depicted in Fig. 1(b), elastic statically indeterminate analysis of the ring subjected to a pair of concentrated forces yields the following value of the maximum normal stress, taken as a nominal stress at failure

$$\sigma_r = \frac{3PD}{\pi t^2} \tag{3}$$

in which $t = (D_1 - D_0)/2$ = thickness of the pipe wall, and $D = (D_0 + D_1)/2$ = diameter of the middle surface of wall.

To take into account the size effect of fracture mechanics, the foregoing expressions for σ_N must be equated not to the strength but to the expression from Eq. (1). Thus, the failure conditions are, for the beam failure

$$\frac{M_b D_1}{2\,I} = Bf_t' \left(1 + \frac{d}{\lambda_0 d_a}\right)^{-1/2} = \sigma_N \tag{4}$$

and for the ring failure

$$\frac{3PD}{\pi t^2} = Bf_t' \left(1 + \frac{d}{\lambda_0 d_a}\right)^{-1/2} = \sigma_N \tag{5}$$

from which the failure load P or the failure bending moment M_b can be solved, provided that the meaning of characteristic size d is specified. We may choose, for beam failure

$$d = \alpha D \tag{6}$$

and for ring failure

$$d = t \tag{7}$$

α is an empirical coefficient, probably close to 1, which is introduced in order to correlate ring failures and beam failures.

The size-effect law has the advantage that its coefficients can be identified from test data simply by linear regression. For this purpose, Eq. (1) may be algebraically rearranged as a straight line equation

$$Y = AX + C \tag{8}$$

in which

$$X = d, \quad Y = \left(\frac{f_t'}{\sigma_N}\right)^2, \quad A = \frac{1}{B^2}, \quad C = \frac{A}{\lambda_0} \tag{9}$$

Specifically, for the beam failure

$$Y = \left(\frac{2\,If_t'}{M_b D_1}\right)^2, \quad X = \alpha D \tag{10}$$

and for the ring failure

$$Y = \left(\frac{\pi t^2 f_t'}{3PD}\right)^2, \quad X = D \tag{11}$$

Thus, if the test data are plotted as Y versus X, A is obtained as the slope of the regression line (i.e., the straight line minimizing the sum of squared deviations from data), and C as the Y-intercept of the regression line.

First we conduct separate regression analyses of the ring failure data and the beam failure data, for which the available data sets are those reported by Gustafsson[8] in 1982 and Brennan[9] in 1978, which are

Table 1 — Beam failure data

Pipe No.	D_0, in.	t, in.	D, in.	d_a, in.	D/d_a	M_u kip-in.	f'_t, psi
Gustafsson's 1982 data[8]							
1	3.94	1.38	6.70	0.472	14.19	30.980	710.7
2	3.94	1.38	6.70	0.472	14.19	28.770	710.7
3	3.94	1.38	6.70	0.472	14.19	30.980	710.7
4	3.94	1.38	6.70	0.472	14.19	30.980	710.7
5	5.91	1.22	8.35	0.472	17.69	53.110	710.7
6	5.91	1.22	8.35	0.472	17.69	48.680	710.7
7	5.91	1.22	8.35	0.472	17.69	46.470	710.7
8	5.91	1.22	8.35	0.472	17.69	53.110	710.7
9	8.86	1.34	11.54	0.472	24.45	117.280	739.7
Brennan's 1978 data[9]							
1	8.86	1.17	11.20	0.394	28.43	80.550	594.6
2	8.86	1.17	11.20	0.394	28.43	56.650	594.6
3	8.86	1.18	11.22	0.394	28.48	80.810	594.6
4	8.86	1.19	11.24	0.394	28.53	47.180	594.6
5	8.85	1.45	11.76	0.394	29.85	70.860	594.6
6	11.81	1.70	15.21	0.394	38.60	169.410	623.6
7	11.81	1.70	15.21	0.394	38.60	164.010	623.6
8	11.81	1.65	15.11	0.394	38.35	165.780	623.6
9	11.81	1.65	15.11	0.394	38.35	168.350	623.6
10	11.81	1.98	15.77	0.394	40.03	167.380	623.6
11	11.81	2.12	16.05	0.394	40.74	145.870	623.6
12	11.81	1.90	15.61	0.394	39.62	250.220	681.7
13	11.81	2.03	15.87	0.394	40.28	190.390	681.7
14	11.81	1.87	15.55	0.394	39.47	249.600	681.7
15	11.81	1.84	15.49	0.394	39.31	176.300	681.7
16	11.81	2.08	15.97	0.394	40.53	205.350	681.7
17	11.81	2.06	15.93	0.394	40.43	250.670	681.7
18	11.81	2.04	15.89	0.394	40.33	279.790	681.7
19	11.81	1.33	14.47	0.394	36.73	172.070	681.7
20	11.81	1.33	14.47	0.394	36.73	215.700	681.7
21	14.76	2.26	19.28	0.394	48.93	441.140	710.7
22	14.76	2.33	19.42	0.394	49.29	413.350	710.7

1 in. = 25.4 mm; 1 kip-in. = 6.895 MPa; 1 psi = 6.895 kPa.

Table 2 — Ring failure data

Pipe No.	D_0, in.	t, in.	D, in.	d_a, in.	t/d_a	p_u kip-in.	f'_t, psi
Gustafsson's 1982 data[8]							
1	8.86	1.38	11.62	0.472	2.92	0.388	710.7
2	8.86	1.38	11.62	0.472	2.92	0.388	710.7
3	8.86	1.38	11.62	0.472	2.92	0.383	710.7
4	8.86	1.38	11.62	0.472	2.92	0.371	710.7
5	15.75	2.17	20.09	0.472	4.60	0.500	710.7
6	15.75	2.17	20.09	0.472	4.60	0.518	710.7
7	15.75	2.17	20.09	0.472	4.60	0.488	710.7
8	15.75	2.17	20.09	0.472	4.60	0.506	710.7
9	15.75	2.70	21.15	0.472	5.72	0.691	710.7
10	15.75	2.70	21.15	0.472	5.72	0.654	710.7
11	15.75	2.70	21.15	0.472	5.72	0.667	710.7
12	15.75	2.70	21.15	0.472	5.72	0.648	710.7
Brennan's 1978 data[9]							
1	8.86	1.17	11.20	0.394	2.97	0.238	594.6
2	8.86	1.18	11.22	0.394	2.99	0.229	594.6
3	8.86	1.19	11.24	0.394	3.02	0.257	594.6
4	8.86	1.45	11.76	0.394	3.68	0.375	594.6
5	11.81	1.70	15.21	0.394	4.31	0.279	623.6
6	11.81	1.70	15.21	0.394	4.31	0.279	623.6
7	11.81	1.65	15.11	0.394	4.19	0.286	623.6
8	11.81	1.65	15.11	0.394	4.19	0.294	623.6
9	11.81	1.90	15.61	0.394	4.82	0.498	681.7
10	11.81	2.03	15.87	0.394	5.15	0.475	681.7
11	11.81	1.87	15.55	0.394	4.75	0.508	681.7
12	11.81	1.84	15.49	0.394	4.67	0.415	681.7
13	11.81	2.08	15.97	0.394	5.28	0.424	681.7
14	11.81	2.06	15.93	0.394	5.23	0.528	681.7
15	11.81	2.04	15.89	0.394	5.18	0.481	681.7
16	11.81	1.33	14.47	0.394	3.38	0.281	681.7
17	11.81	1.33	14.47	0.394	3.38	0.270	681.7
18	14.76	2.26	19.28	0.394	5.74	0.416	710.7
19	14.76	2.33	19.42	0.394	5.91	0.461	710.7

1 in. = 25.4 mm; 1 kip-in. = 175 kNm; 1 psi = 6.895 kPa.

summarized in Tables 1 and 2. The regression plots and size effects plots obtained (with $\alpha = 1$) for the beam and ring failures are shown in Fig. 2 and 3. Although we clearly see the presence of size effect (according to plastic limit analysis these plots would have to be horizontal lines), we also notice large scatter of the data, as indicated in the figures by the coefficients of variation $\omega_{y|x}$ of the vertical deviations from the straight regression line. Thus, the resulting regression parameters A and B have a large uncertainty, especially for the ring failures. This is due primarily to the relatively narrow range of sizes involved in each of these test series.

To obtain less uncertain results, we need to broaden the range of sizes. One way to achieve this without carrying costly tests of very large pipes in the laboratory is to analyze the data for ring and beam failures collec-

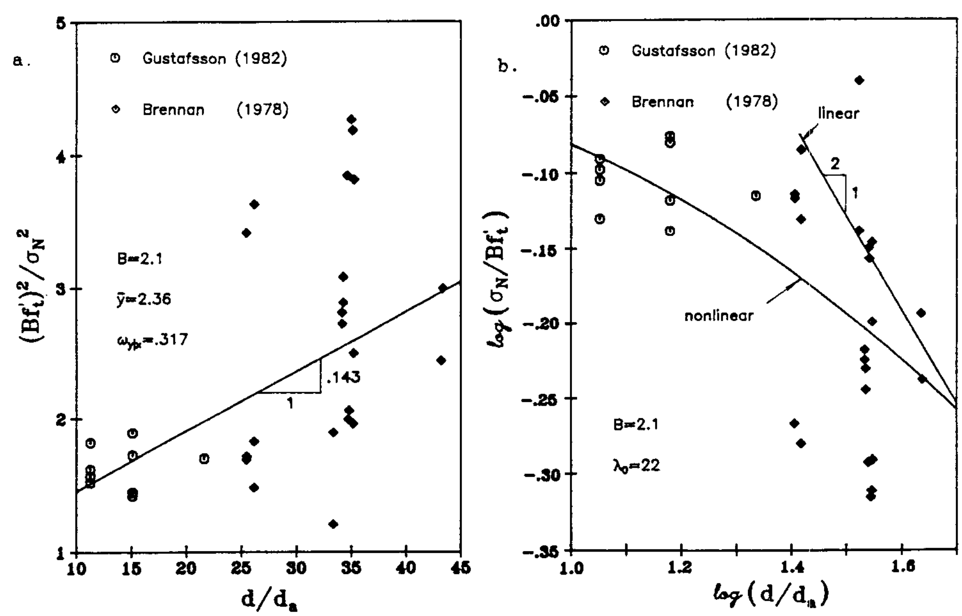

Fig. 2 — Statistical analysis of beam failures of pipes

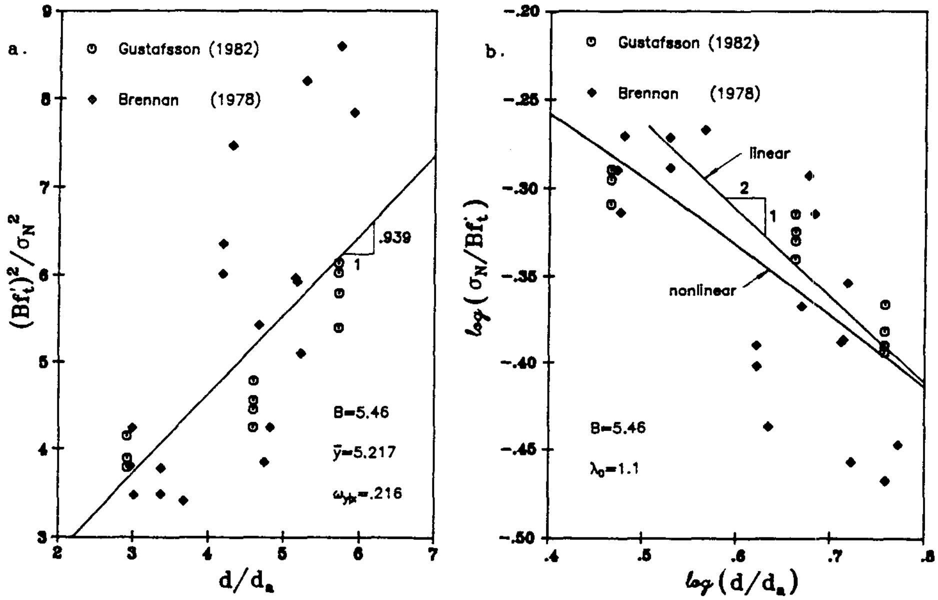

Fig. 3 — Statistical analysis of ring failures of pipes

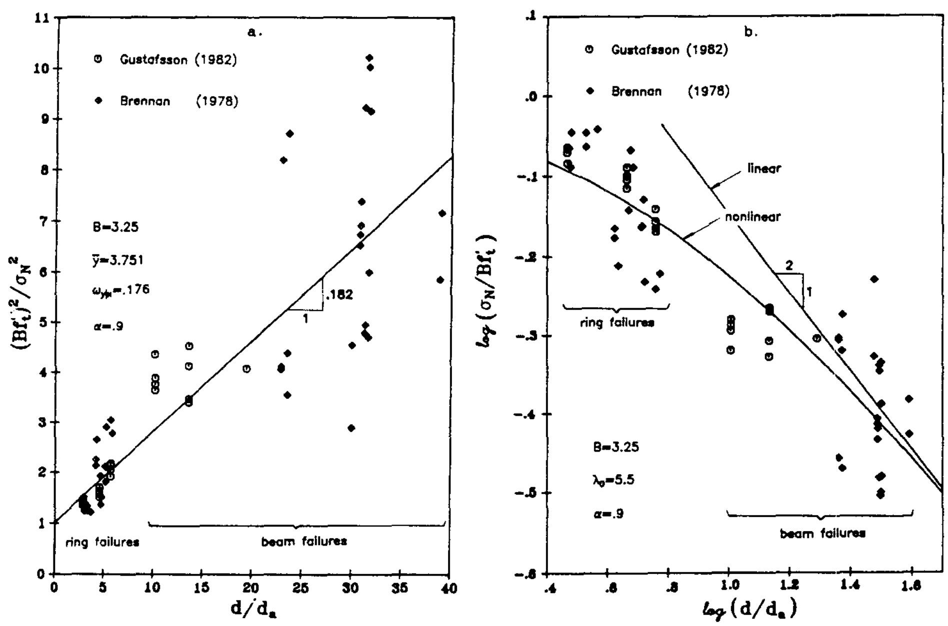

Fig. 4 — Combined statistical analysis of ring and beam failures of pipes

tively, as one statistical set. Combining the data would be unjustified if the ring and beam failures had very different failure mechanisms. However, both are essentially bending failures with tensile (Mode I) final fracture, and with a zero shear force and a zero (or almost zero) normal force in the failure cross section. Thus, combining ring and beam data should be possible at least as an approximation. The fact that the failure cross sections are not similar — a rectangle for the case of ring failure and an annulus for the case of beam failure — is taken into account by coefficient α, introduced in Eq. (6). The value of this coefficient may be determined from the minimizing condition for the sum of the squared deviations from the common regression line for beam and ring data plotted as one data set. This can be done either by choosing various α-values, running the linear regression for each one of them and picking among them the optimal case, or, more directly, by using a computer library nonlinear optimization subroutine, such as the Marquardt-Levenberg algorithm, to determine B, λ_0, and α simultaneously. The nonlinear optimization may be based on the deviations from data in the size effect plot [Fig. 4(b)], which seems more realistic than using the deviations in the regression plot in Fig. 4(a).

The optimum regression of the combined ring and beam data, along with the parameter values obtained from the regression, is shown in Fig. 4(a), and the corresponding size effect plot in Fig. 4(b). In this combined regression the coefficient of variation $\omega_{y|x}$ is considerably lower than that for regressions of either the ring data or the beam data taken separately. The parameters of the size effect law and of α are

$$B = 3.25, \lambda_0 = 5.5, \alpha = 0.9 \qquad (12)$$

Even in the combined regression plot the scatter is not small enough for being able to say that the data in Fig. 4 validate the size-effect law. This law has been validated by theoretical arguments (dimensional analysis and similitude) and has been verified by comparison with test results for other types of failure for which the scatter is much smaller. On the other hand, note that the straight line in Fig. 4(a), as well as the size effect plot of the same data in Fig. 4(b), describes the mean trend of the data as well as can be expected in view of the large experimental scatter seen. Especially note that the limit analysis (i.e., absence of the size effect), for which the straight regression lines in Fig 2 through 4 would have to be horizontal, would not agree at all with the basic trend of the data.

The regression line in Fig. 4(a) may be used for the mean prediction. For the purpose of design, the prediction must be scaled down using an appropriate safety factor (the strength reduction factor or understrength factor).

CONCLUSIONS

1. The existing test data on brittle beam and ring failures of unreinforced concrete pipes are not consistent with plastic limit analysis or allowable strength design in that they exhibit a significant size effect of fracture mechanics type.

2. The observed size effect is consistent with the recently proposed size-effect law. However, it cannot be said to validate this law in view of the very large scatter and limited range of the existing data.

3. Using Eq. (4) and (5), both beam and ring failures can be approximately predicted on the basis of the same material properties.

REFERENCES

1. Gustafsson, P. J., "Oarmerade betongrörs böjrottlast och ringbrottlast," *Report* No. TVBM-3012, Division of Building Materials, Lund Institute of Technology, 1983.

2. Hillerborg, A., "Examples of Practical Results Achieved by Means of the Fictitious Crack Model," *Preprints*, W. Prager IUTAM Symposium on Mechanics of Geomaterials; Rocks, Concretes, Soils, Northwestern University, Evanston, Sept. 1983, pp. 611-614.

3. Bažant, Zdeněk P., "Crack Band Model for Fracture of Geomaterials," *Proceedings*, 4th International Conference on Numerical Methods in Geomechanics, Edmonton, 1982, V. 3, pp. 1137-1152.

4. Bažant, Zdeněk P., "Size Effect in Blunt Fracture: Concrete, Rock, Metal," *Journal of Engineering Mechanics*, ASCE, V. 110, No. 4, Apr. 1984, pp. 518-535.

5. Bažant, Zdeněk P., and Cao, Zhiping, "Size Effect in Shear Failure of Prestressed Concrete Beams," *Report* No. 84-10/428s, Center for Concrete and Geomaterials, Northwestern University, Evanston, Oct. 1984. Also, ACI JOURNAL, *Proceedings* V. 83, No. 2, Mar.-Apr. 1986, pp. 260-268.

6. Bažant, Zdeněk P., and Kim, Jin-Keun, "Size Effect in Shear Failure of Longitudinally Reinforced Concrete Beams," ACI JOURNAL, *Proceedings* V. 81, No. 5, Sept.-Oct. 1984, pp. 456-468. Also, Discussion, V. 82, No. 4, July-Aug. 1985, pp. 579-583.

7. Bažant, Zdeněk P., and Oh, B. H., "Crack Band Theory for Fracture of Concrete," *Materials and Structures, Research and Testing* (RILEM, Paris), V. 16, No. 93, May-June 1983, pp. 155-177.

8. Gustafsson, P. J., "Hallfasthetsprovning av Oarmerade Betongrör," *Report*, Division of Building Materials, Lund Institute of Technology, July 1982 and Nov. 1982.

9. Brennan, Guy, "A Test to Determine the Bending Moment Resistance of Rigid Pipes," *TRRL Supplementary Report* No. SR 348, Transport and Road Research Laboratory, Crowthorne, Berkshire, 1978.

10. Gustafsson, P. J., and Hillerborg, A., "Improvements in Concrete Design Achieved through the Application of Fracture Mechanics," *Preprints*, NATO Advanced Research Workshop on Application of Fracture Mechanics to Cementitious Composites, Northwestern University, Evanston, Sept. 1984, pp. 487-500.