

# Temperature Effect on Concrete Creep Modeled by Microprestress-Solidification Theory

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**Abstract:** The previously developed microprestress-solidification theory for concrete creep and shrinkage is generalized for the effect of temperature (not exceeding 100°C). The solidification model separates the viscoelasticity of the solid constituent, the cement gel, from the chemical aging of material caused by solidification of cement and characterized by the growth of volume fraction of hydration products. This permits considering the viscoelastic constituent as non-aging. The temperature dependence of the rates of creep and of volume growth is characterized by two transformed time variables based on the activation energies of hydration and creep. The concept of microprestress achieves a grand unification of theory in which the long-term aging and all transient hygrothermal effects simply become different consequences of one and the same physical phenomenon. The microprestress, which is independent of the applied load, is initially produced by incompatible volume changes in the microstructure during hydration, and later builds up when changes of moisture content and temperature create a thermodynamic imbalance between the chemical potentials of vapor and adsorbed water in the nanopores of cement gel. As recently shown, this simultaneously captures two basic effects: First, the creep decreases with increasing age at loading after the growth of the volume fraction of hydrated cement has ceased; and, second, the drying creep, i.e., the transient creep increases due to drying (Pickett effect) which overpowers the effect of steady-state moisture content (i.e., less moisture—less creep). Now it is demonstrated that the microprestress buildup and relaxation also captures a third effect: The transitional thermal creep, i.e., the transient creep increase due to temperature change. For computations, an efficient (exponential-type) integration algorithm is developed. Finite element simulations, in which the apparent creep due to microcracking is taken into account separately, are used to identify the constitutive parameters and a satisfactory agreement with typical test data is achieved.

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## Introduction and Overview of Physical Mechanisms

The arduous protracted quest for a realistic physically based creep and shrinkage model for Portland cement concrete has been confounded during the last several decades by three complex phenomena:

1. The *aging of concrete*, which is manifested by a significant decrease of creep with the age at loading and is of two types:
  - a. Shorter-term chemical aging, which ceases at room temperature after about a year and is caused by the fact that new solids are produced by the slowly advancing chemical reactions of cement hydration and deposited (in an

essentially stress-free state) on the walls of capillary pores; and

- b. Long-term non-chemical aging, manifested by the fact that the decrease of creep with the age at loading continues unabated even for many years after the degree of hydration of cement ceased to grow. Recently, this phenomenon was explained by the relaxation of microprestress (Bažant et al. 1997a), although a long-term increase of bonding due to “polymerization” in calcium silicate hydrates might also play a role (Bažant and Prasanna 1989a).
2. The *drying creep effect*, also called the stress-induced shrinkage (or Pickett effect), which is a transient effect consisting in the fact that the apparent creep during drying is much larger than the basic creep (i.e., creep at moisture saturation) while the creep after drying (i.e., after reaching thermodynamic equilibrium with a reduced environmental humidity) is much smaller than the basic creep. The physical source of drying creep is now known (Bažant and Xi 1994) to involve two different mechanisms:
    - a. The apparent mechanism consisting of an apparent additional creep due to microcracking (Wittmann 1974, 1980, 1982; Wittmann and Roelfstra 1980) or strain-softening damage (Bažant and Wu 1974a,b; Bažant 1975; Bažant and Chern 1985a,b; Bažant and Xi 1994; Granger et al. 1994), which are equivalent processes from the viewpoint of constitutive modeling. The creep at variable humidity is conventionally defined by the difference in deformation

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between a loaded creep specimen and its companion load-free shrinkage specimen. This difference is increased not because of creep amplification in the loaded specimen (in which the microcracking is suppressed by compressive load, considered as compressive), but because microcracking due to nonuniform local shrinkage tends to decrease the deformation of the load-free companion specimen, causing the overall deformation measured on a drying load-free specimen to be less than the “true” material shrinkage. The reason for calling this mechanism “apparent” is twofold: (1) It does not reside in the constitutive material properties, and (2) it is not happening in the creep specimen itself, but in its companion. Obviously, this mechanism has little effect in bending creep or torsional creep, and works in the opposite sense for tensile creep (Bažant and Moschovidis 1973; Bažant and Xi 1994).

- b. A true mechanism that resides in the nanostructure and consists in the fact that the rate of shear (slip) due to breakages and restorations of bonds in the calcium silicate hydrates is reduced (or amplified) by a decrease (or increase) in the magnitude of compressive microprestress that is acting across the slip planes, the stress change being produced by a change in the chemical potential (i.e., the Gibbs free energy per unit mass) of pore water due to drying (Bažant 1972a,b, 1975).
3. The *transitional thermal creep*, which represents a transient increase of creep after a temperature change, both heating and cooling. In the case of cooling, the transient increase is of the opposite sign than the final change in creep rate after a steady-state lower temperature has been regained. Like the drying creep effect, this effect has two analogous mechanisms:
    - a. An apparent macroscopic mechanism, due to thermally induced microcracking and similar to drying creep; and
    - b. A nanoscale mechanism due to a change in the level of microprestress caused by a change of chemical potential of nanopore water with a temperature change.

While the apparent mechanisms 2a and 3a operate on the macroscopic level of the whole specimen or structure (on the scale of centimeters and meters), mechanism 1a operates on the level of capillary pores (on the scale of micrometers), and mechanisms 1b, 2b and 3b operate on the level of nanopores in calcium silicate hydrates (on the scale of nanometers).

Development of the solidification theory (Bažant and Prasannan 1989a,b) showed that the chemical aging (mechanism 1a) can be separated from the viscoelastic constitutive model if that model is formulated not for concrete but for the solidifying constituent—the hardened cement gel (i.e., the solid, consisting mostly of calcium silicate hydrates, which forms the skeleton of hardened cement paste), and if the chemical aging is interpreted as a volume growth of the solidifying constituent (per unit volume of concrete). Since epitaxial growth must be ruled out because crystal structure is lacking, the model is formulated under the restriction that the newly solidified material is stress free at the moment it solidifies (certain earlier models purportedly based on thermodynamics violated this restriction; Bažant 1977). The solidification theory greatly reduced the number of unknowns in the modeling of aging creep and allowed removing the ambiguity (nonuniqueness) that previously plagued the identification of age-dependent moduli of the Kelvin or Maxwell chain model from the test data. The fact that, in the solidification theory, these moduli are constant allows describing the viscoelastic properties of the

solidifying constituent with a continuous relaxation (or retardation) spectrum, the benefit of which is a smooth spectrum that is unambiguous, i.e., uniquely identifiable from the test data (Bažant and Xi 1995; Zi and Bažant 2001, 2002).

Mechanisms 1a and 1b were initially modeled separately (Bažant and Prasannan 1989a). In a later study (Bažant et al. 1997a,b), both 1b and 2b were explained by one and the same physical theory resting on the idea of relaxation of microprestress that is created in the solid nanostructure of cement gel either by microscopic chemical volume changes of various chemical species during hydration or by an imbalance of chemical potentials (Gibbs free energy density per unit mass) among the four phases of pore water (vapor, capillary, adsorbed, and hindered-adsorbed phases).

This paper shows how the microprestress-solidification theory, proposed and verified in Bažant et al. (1997a,b) for mechanism 2b (and suggested only in general terms, without details and verification, for mechanism 3b) can be extended for mechanism 3b in detail, and verifies that the experimental evidence can be matched. This extension (broadly hinted in Bažant et al. 1997a,b) rests on recognizing that an imbalance among the chemical potentials of various phases of pore water is created not only by a change in the pore vapor pressure but also a change of temperature. The reason is that the chemical potential is a function of both.

The present study finally provides the last building block needed to achieve a grand unity of physical modeling of all the creep-influencing phenomena on the nanoscale. The fact that phenomena, as diverse as the long-term aging, drying creep, and transitional thermal creep, can all be explained by one and the same model lends credence to the validity of the microprestress solidification model. The new unified model, which can be easily handled numerically with the powerful computers that exist today, may be expected to provide a more realistic common basis for the analysis many practically important problems—fire resistance of concrete structures, response to various hypothetical extreme nuclear reactor accidents, long-term safety of radioactive waste disposal, behavior of chemical technology vessels, effects of hydration heat in massive structures, and effects of environmental variations on structures.

Interpretation of the available test data on the effect of temperature changes is complicated by two counteracting phenomena, namely the fact that, due to an acceleration of bond breakages causing shear slips in the nanostructure, (1) a temperature rise amplifies the creep rate, while (2) it also accelerates the aging due to hydration which is responsible for deceleration of creep. This dichotomy of the thermal effect, modeled in a more limited context long ago (Bažant 1970a,b; Bažant and Wu 1974a,b; Bažant 1975), will be automatically embedded in the present formulation. The detailed physical arguments justifying the microprestress-solidification theory are beyond the scope of this paper but can be found in an earlier paper (Bažant et al. 1997a,b).

Note that the present constitutive law is not usable as a relation between the average stress and average strain in a cross section of beam or plate. Such relations (e.g., model B3, Bažant 2000; Bažant and Baweja 2000), popular in preliminary design, take into account the nonuniformity of stress over the cross section caused by drying and cracking, but only in a very crude manner.

## Microprestress-Solidification Theory

Under uniaxial stress  $\sigma$  (and in the absence of significant plastic strains that may arise at high confining pressures), the normal strain  $\varepsilon$  of concrete can be decomposed as follows (Fig. 1)

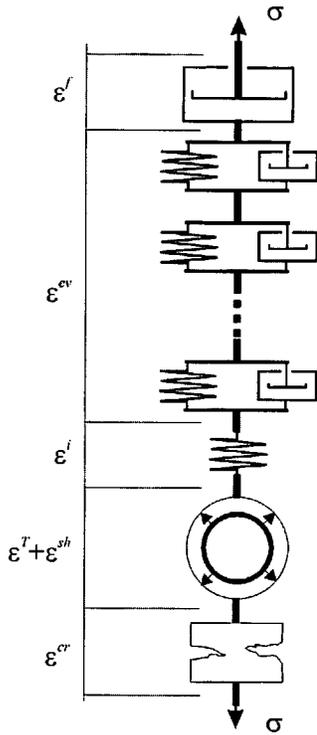


Fig. 1. Rheological model

$$\varepsilon = \varepsilon^i + \varepsilon^v + \varepsilon^f + \varepsilon^{cr} + \varepsilon^{sh} + \varepsilon^T \quad (1)$$

where  $\varepsilon^i$  = instantaneous strain;  $\varepsilon^v$  = viscoelastic strain;  $\varepsilon^f$  = purely viscous strain;  $\varepsilon^{cr}$  = inelastic strain due to cracking; and  $\varepsilon^{sh}$  and  $\varepsilon^T$  = shrinkage and thermal strains caused by variations of humidity and temperature, respectively. The triaxial generalization of all the strain components except  $\varepsilon^{cr}$  can be based on the restrictions of material isotropy, and need not be discussed because it is well known (e.g., Bažant 1975, 1982; RILEM 1988). The generalization of cracking strain  $\varepsilon^{cr}$  to triaxial stress conditions breaks isotropy but it, too, need not be discussed as it is the subject of the works on fracture and damage (e.g., Bažant and Planas 1998).

The instantaneous strain, i.e., the strain appearing immediately after applying uniaxial stress  $\sigma$ , may be written as  $\varepsilon^i = q_1 \sigma$ . At room temperature  $T = T_0 = 296$  K (23°C) and for saturation condition  $h = h_0 = 1$ , the coefficient  $q_1$  is age independent. Bažant and Osman (1976) and Bažant and Baweja (1995, 2000) demonstrated this property by considering the compliances for load durations  $t$  ranging from 0.001 s to 10 h. They fitted the compliances with a smooth formula of the type  $J = q_1 + ct^n$ . Then, they obtained  $q_1$  by optimizing the fit of data for various ages  $t'$  at loading. The  $q_1$  values for various  $t'$  were nearly the same, and the coefficient of variation of errors of the fit did not increase significantly when  $q_1$  was forced to be exactly the same for all  $t'$ .

The viscoelastic strain  $\varepsilon^v$ , originating in the solid gel of calcium silicate hydrates, will be described according to the solidification theory (Bažant and Prasanna 1989a,b), which achieves separation of  $\varepsilon^v$  from the solidification process that causes long-time aging of concrete

$$\dot{\varepsilon}^v(t) = \frac{\dot{\gamma}(t)}{v(t)} \quad (2)$$

$$\gamma(t) = \int_0^t \Phi(t-\tau) \dot{\sigma}(\tau) d\tau \quad (3)$$

where  $\Phi(t-t') = q_2 \ln[1 + \xi^n]$ ;  $\xi = (t-t')/\lambda_0$ ; and  $v(t)^{-1} = (\lambda_0/t)^m + \alpha$ . The function  $v(t)$  approximately describes the volume fraction of solidified material during the hydration processes. The free parameters of the model are  $q_2$  (in  $\text{MPa}^{-1}$ ) and  $\alpha$  (dimensionless) while the others are fixed, with the following values:  $n = 0.1$ ,  $\lambda_0 = 1$  day, and  $m = 0.5$ . Variable  $\gamma(t)$  is the viscoelastic strain in the cement gel, which is fully recoverable upon unloading. On the other hand,  $\varepsilon^v$  is only partially recoverable, which is caused by  $v(t)$ .

The purely viscous strain (flow strain) is the completely irrecoverable part of the creep strain, which is modeled by the viscous dashpot in Fig. 1 and is formulated as

$$\dot{\varepsilon}^f(t) = \frac{\sigma(t)}{\eta(S)} \quad (4)$$

where  $\eta(S)$  = viscosity parameter which, for reason given in Bažant et al. (1997a,b), is defined as a power function of the so-called microstress  $S$ ,  $1/\eta(S) = c_b S^{b-1}$ . The microstress  $S$  is a variable characterizing the average of normal stresses acting across the slip planes represented by the hindered adsorbed water layers in the microstructure of the cement paste. According to Bažant et al. (1997a,b), the evolution of microstress can be assumed to obey a Maxwell-type rheological model with nonlinear viscosity,  $\eta$ , characterized by the differential equation

$$\frac{\dot{S}(t)}{C_S} + \frac{S(t)}{\eta(S)} = \frac{\dot{s}(t)}{C_S} \quad (5)$$

where  $\dot{s}(t)/C_S$  = time rate of the Maxwell model strain due to pore humidity and temperature changes. This first-order differential equation can be solved in a closed form in the case of basic creep. With the notation  $c_0 = C_S c_b$ , the solution is

$$S(t) = \{S_0^{1-b} + c_0(b-1)(t-t_0)\}^{1/(1-b)} \quad (6)$$

where  $S(t_0) = S_0$  gives the initial condition. In this case,

$$\dot{\varepsilon}^f(t) = \frac{bc}{S_0^{1-b} + c_0(b-1)(t-t_0)} \sigma(t) \quad (7)$$

It is fairly well established that the flow part of the creep strain has a logarithmic evolution. From Eq. (7), we can deduce this evolution if we set  $S_0^{1-b} = (b-1)c_0 t_0$ . Having set it, we obtain  $S(t) = S_0(t_0/t)^{1/b-1}$  and  $\dot{\varepsilon}^f(t) = q_4 \sigma(t)/t$ , where  $q_4 = bc/[c_0(b-1)]$ . There are reasons (Bažant et al. 1997a,b) to assume that  $b = 2$ , and then  $S_0 c_0 t_0 = 1$ ,  $S(t) = S_0 t_0/t$ ,  $q_4 = 2c/c_0$ . The free parameters of the model are  $c$  (in  $\text{MPa}^{-2} \text{day}^{-1}$  for  $b = 2$ ) and  $c_0$  (in  $\text{MPa}^{-1} \text{day}^{-1}$  for  $b = 2$ ).

Note that while the general formulation depends separately on  $c$  and  $c_0$ , only their ratio  $c/c_0$  governs the basic creep; it does so through parameter  $q_4$ .

## Temperature and Humidity Effects

The effect of temperature on concrete creep is twofold, generated by two different mechanisms:

1. A temperature increase accelerates the bond breakages and restorations causing creep, and thus increases the creep rate.
2. The higher the temperature, the faster is the chemical process of cement hydration and thus the aging of concrete, which reduces the creep rate.

Usually the former effect prevails and then the overall effect of temperature rise is an increase of creep. Nevertheless, for very young concretes in which the hydration rapidly progresses at elevated temperatures, heating can have the opposite effect. The

pore humidity plays a similar role in aging; in fact, both the hydration and creep processes are slowed down by a decrease of pore humidity.

These two effects can be described by introducing two different time variables: (1) The equivalent time  $t_e$  (equivalent hydration period, or "maturity"), which indirectly characterizes the degree of hydration, and (2) the reduced time  $t_r$ , characterizing the changes in the rate of bond breakages and restorations on the microstructural level. Following Bažant (1995), we introduce

$$t_e(t) = \int_0^t \beta(\tau) d\tau, \quad t_r(t) = \int_0^t \psi(\tau) d\tau \quad (8)$$

$$\beta(t) = \beta_T(t) \beta_h(t), \quad \beta_h(t) = \{1 + [a_h - a_h h(t)]^4\}^{-1}$$

$$\beta_T(t) = \exp\left\{\frac{Q_h}{R} \left(\frac{1}{T_0} - \frac{1}{T(t)}\right)\right\} \quad (9)$$

and

$$\psi(t) = \psi_T(t) \psi_h(t), \quad \psi_h(t) = \alpha_h + (1 - \alpha_h) h(t)^2$$

$$\psi_T(t) = \exp\left\{\frac{Q_v}{R} \left(\frac{1}{T_0} - \frac{1}{T(t)}\right)\right\} \quad (10)$$

where  $T$  = absolute temperature;  $T_0$  = reference temperature;  $h$  = humidity (vapor pressure) in the capillary pores of cement paste;  $R$  = gas constant; and  $Q_h$  and  $Q_v$  = activation energies for the hydration and viscous processes, respectively. We can set  $T_0 = 296$  K,  $Q_h/R \approx 2700$  K,  $Q_v/R \approx 5000$  K,  $a_h = 5$ , and  $\alpha_h = 0.1$  (Bažant 1995).

Eqs. (2) and (3) are valid for  $h(t) = 1$  and  $T(t) = T_0$ . For general temperature and humidity histories, these equations must be generalized as

$$\dot{\epsilon}^{ev}(t) = \frac{\dot{\gamma}(t)}{\nu[t_e(t)]} \quad (11)$$

$$\dot{\gamma}(t) = \int_0^t \Phi[t_r(t) - t_r(\tau)] \dot{\sigma}(\tau) d\tau \quad (12)$$

The temperature and humidity influence the rate of both the viscous and viscoelastic strains. However, by virtue of separating the effect of solidification (hydration), only the reduced time  $t_r$  is needed. So, when temperature and humidity variations occur, Eqs. (4) and (5) should be rewritten as

$$\frac{d\epsilon^f}{dt_r} = \frac{\sigma(t)}{\eta(S)} \quad (13)$$

$$\frac{1}{C_S} \frac{dS}{dt_{r,S}} + \frac{S}{\eta(S)} = \frac{1}{C_S} \frac{dS}{dt_{r,S}} \quad (14)$$

Since, from Eq. (8),  $dt_r = \psi(t) dt$ , we can set  $dt_{r,S} = \psi_S(t) dt$ . Also taking into account the expression for the viscosity  $\eta$  as a function of the microstress  $S$ , Eqs. (13) and (14) become

$$\dot{\epsilon}^f(t) = \frac{\psi(t)}{\eta(S)} \sigma(t) \quad (15)$$

$$\dot{S}(t) + \psi_S(t) c_0 S(t)^2 = \dot{s}(t) \quad (16)$$

The reduced time coefficient  $\psi_S$  defining the rate of microstress evolution may be expressed similarly to Eq. (10) except that  $\alpha_h$  is now replaced by  $\alpha_S$  and  $Q_v$  by  $Q_S$ . An analysis of experimental test data shows that the values  $\alpha_S = 0.1$  and  $Q_v/R \approx 3000$  K are quite appropriate.

In Eq. (16), the right-hand side term  $\dot{s}$  depends on capillary tension, surface tension, crystal growth pressure, and disjoining pressure (Bažant et al. 1997a,b), all of which are sensitive to temperature and humidity changes. Since the distances between adjacent micropores and capillary pores are extremely short (of the order of micrometers), all the phases of water can be assumed to be in thermodynamic equilibrium (locally, from the point of view of macroscopic continuum). Under this assumption, it follows from Kelvin's equation of capillarity and the equality of chemical potentials at thermodynamic equilibrium (Bažant 1972a, 1975) that all the aforementioned quantities must vary with  $h$  as  $f(t) = f_0 - C_0 RT(t) \ln[h(t)]/M$ , where  $f_0$  is a constant (usually of a large negative value). So, for constant  $S$ , we must have

$$s(t) = -C_1 \frac{RT(t)}{M} \ln[h(t)] + s_1 \quad (17)$$

where  $s_1$  = value of  $s$  at  $h = 1$ . Since this relation is satisfied initially, it will suffice to impose its rate form:

$$\dot{s}(t) = -k_1 \left( \dot{T} \ln h + T \frac{\dot{h}}{h} \right) \quad (18)$$

where  $k_1 = C_1 R/M$  (in MPa K<sup>-1</sup>). Upon substitution into Eq. (15), the governing equation of the microstress at simultaneous humidity and temperature variation becomes

$$\dot{S} + \psi_S c_0 S^2 = -k_1 \left( \dot{T} \ln h + T \frac{\dot{h}}{h} \right) \quad (19)$$

Eq. (19) gives results that depend on the sign of  $\dot{T}$  and  $\dot{h}$ . In particular, negative increments of  $h$  (drying) and positive increments of  $T$  (heating) lead to an increase of the microstress (i.e., of the magnitude of stress peaks in the microstructure), and thus to an increase of the slip rate and the rate of creep flow, counteracting (and usually overpowering) the effect of  $\psi_h$ . When drying switches to wetting, or heating to cooling, i.e.,  $\dot{h}$  or  $\dot{T}$  changes its sign. So, the microstress rate  $\dot{S}$  also changes sign, which must reduce the creep rate at least temporarily.

After a while, this must deactivate the current creep sites, i.e., the sites of the highest microstress at which shear slip must take place. However, new creep sites will form at other locations in the microstructure at which the microstress is extreme in the opposite sense. At the new sites, creep will again be promoted by the change in  $\dot{h}$  or  $\dot{T}$ . To model such effects of the reversal of  $\dot{h}$  or  $\dot{T}$  in a simplified manner, we may simply introduce the absolute value sign into Eq. (19), as follows (Cusatis 1998):

$$\dot{S} + \psi_S c_0 S^2 = k_1 \left| \dot{T} \ln(h) + T \frac{\dot{h}}{h} \right| \quad (20)$$

Although this simple formulation does not capture the expected temporary decrease of creep rate right after a change of sign of  $\dot{h}$  or  $\dot{T}$ , it does reflect in the simplest manner the overall increase of creep rate likely to occur with some delay. Experimental data (Pickett 1942; Gamble and Parrot 1978) indeed suggest an increase of the creep rate for both drying and wetting. As for heating and cooling, the evidence is ambiguous; some data (Wallo et al. 1965; Fahmi et al. 1972) suggest that the creep rate increases under cooling but others (Hansen and Eriksson 1966; Illston and Sanders 1973) suggest the opposite, and so no experimental verification is possible.

## Hygrometric, Thermal, and Cracking Strain

Temperature and humidity changes cause two kinds of strain: (1) Free hygrometric strain, and (2) thermal strain. The free hygrometric strain (i.e., free shrinkage or swelling) is caused by changes in capillary tension, surface tension, and disjoining pressure reduced by the movement of moisture into and out of the pores of the cement gel (the disjoining pressure is a concept due to Deryagin 1962; see also Schmidt-Döhl and Rostásy 1995). The thermal strain is caused by temperature changes. Approximately

$$\varepsilon^{\text{sh}}(t) = k_{\text{sh}} \dot{h}(t), \quad \varepsilon^T(t) = \alpha_T \dot{T}(t) \quad (21)$$

Coefficient  $k_{\text{sh}}$  depends on  $h$  and  $\alpha_T$  on  $T$  but only weakly and, as an approximation, can be assumed to be constant. The coefficient of thermal expansion of concrete  $\alpha_T$  can be computed from the coefficients of thermal expansion of the constituents, cement paste, and aggregate. The coefficient of thermal expansion of cement paste depends on many factors. Although there is no significant difference between  $\alpha_T$  for saturated cement paste and  $\alpha_T$  for dry cement paste, measurements show  $\alpha_T$  to have higher values for partial saturation. The hydration process tends to diminish  $\alpha_T$ . The thermal expansion coefficient of aggregates depends mainly on their chemical composition but, in general, can be assumed constant. For concrete,  $\alpha_T$  is bounded by the formulas for composites based on parallel and series coupling of cement and aggregates. Both bounds show that the influence of the cement paste is quite small, and so  $\alpha_T$  must be governed chiefly by the aggregates (Bažant and Kaplan 1996). Thus we can consider  $\alpha_T$  for concrete at moderate temperature to be constant, with the usual value  $10^{-5}/^\circ\text{C}$ .

Environmental temperature and humidity variations lead to a nonuniform stress distribution which causes diffuse cracking. It may be most simply described according to the crack band model (Bažant and Oh 1983), in which the cracking strain  $\varepsilon^{\text{cr}}$  may be characterized by  $\varepsilon^{\text{cr}} = \sigma / C^{\text{cr}}(\varepsilon^{\text{cr}})$  where  $C^{\text{cr}}$  is the secant cracking modulus of concrete (Bažant and Chern 1985a,b);  $C^{\text{cr}} = E_s \exp\{-c_s(\varepsilon^{\text{cr}})^s\}$  with parameter  $c_s = s^{-1} \xi_p^{-s}$ , and  $E_s = f'_t \xi_p^{-1} \exp(c_s(\xi_p)^s)$ ;  $\xi_p$  is the cracking strain at the peak stress,  $f'_t$  is the tensile strength, and  $s$  is a constant.

Differentiating the relation  $\sigma = C^{\text{cr}}(\varepsilon^{\text{cr}})$ , we get a differential equation that governs the evolution of cracking strain;

$$\dot{\sigma} = C^{\text{cr}} \dot{\varepsilon}^{\text{cr}} + \dot{C}^{\text{cr}} \varepsilon^{\text{cr}} \quad (22)$$

The unloading–reloading behavior of cracked concrete lies between those of plasticity theory (elastic unloading) and of damage theory (unloading line passing through the origin). The present calculations used the following simple unloading–reloading rule which does not require any additional material parameters:

$$\dot{\sigma} = C^{\text{cr}}(\varepsilon^{\text{cr}}) \dot{\varepsilon}^{\text{cr}} \quad \text{if } \varepsilon^{\text{cr}} < \varepsilon_{\text{max}}^{\text{cr}} \quad (23)$$

where  $\varepsilon_{\text{max}}^{\text{cr}}$  is the maximum value of  $\varepsilon^{\text{cr}}$  reached during the analysis (Bažant and Chern 1985a,b).

## Numerical Implementation

For the sake of step-by-step finite element analysis, an incremental stress–strain relation needs to be formulated. To this end, it is convenient to convert the integral in Eq. (12), which governs, together with Eq. (11), the evolution of the viscoelastic strain, into a system of first-order differential equations. This can be done by expanding the non-aging microcompliance function  $\Phi(t-t')$  into a Dirichlet series, which is equivalent to modeling

the viscoelastic strain of cement gel in terms of a non-aging Kelvin chain. Following Bažant and Prasannan (1989a), we may set

$$\Phi(t-t') = A_0 + \sum_1^N A_\mu \left\{ 1 - \exp\left(-\frac{t-t'}{\tau_\mu}\right) \right\} \quad (24)$$

Then, as it can be easily checked (Bažant and Prasannan 1989b; Carol and Bažant 1992), Eq. (12) is equivalent to the evolution equation

$$\dot{\sigma}(t) = A_\mu^{-1} \dot{\gamma}_\mu(t) + \frac{\tau_\mu}{\psi(t)} A_\mu^{-1} \dot{\gamma}_\mu(t) \quad (\mu = 1 \dots N) \quad (25)$$

where  $\gamma_\mu$  = the strain in the individual Kelvin unit. This differential equation may be integrated over the time step  $(t_n, t_{n+1})$  ( $n = 1, 2, \dots$ ) under the assumption that  $\sigma$  is constant within time step  $\Delta = t_{n+1} - t_n$  (varying only by jumps at  $t_n$ ). This yields

$$\gamma_{\mu,n+1} = k_{\mu,n} \gamma_{\mu,n} + A_\mu (1 - k_{\mu,n}) \sigma_n + A_\mu (1 - \lambda_{\mu,n}) \Delta \sigma_n \quad (26)$$

where  $\Delta \sigma_n = \sigma_{n+1} - \sigma_n$ ,  $\gamma_{\mu,n+1} = \gamma_\mu(t_{n+1})$ ;  $\gamma_{\mu,n} = \gamma_\mu(t_n)$ ,  $\psi_m = \psi(t_{n+1/2})$ ;  $t_{n+1/2} = t_n + \Delta t/2$ ;  $\Delta y_{\mu,n} = \psi_m \Delta t_n / \tau_\mu$ ,  $k_{\mu,n} = \exp(-\Delta y_{\mu,n})$ ; and  $\lambda_{\mu,n} = 1 - (1 - k_{\mu,n}) / \Delta y_{\mu,n}$ . Finally, from Eqs. (12) and (11) in an incremental form, i.e.,  $\Delta \varepsilon_n^{\text{ev}} = \Delta \gamma_n / \nu_{e,m}$ ,  $\nu_{e,m} = \nu[t_e(t_{n+1/2})]$ , we obtain

$$\Delta \varepsilon_n^{\text{ev}} = A_n^{\text{ev}} \Delta \sigma_n + \Delta \varepsilon_n^{\text{ev}''} \quad (27)$$

in which

$$A_n^{\text{ev}} = \frac{1}{\nu_{e,m}} \left\{ A_0 + \sum_1^N A_\mu (1 - \lambda_{\mu,n}) \right\} \quad (28)$$

$$\Delta \varepsilon_n^{\text{ev}''} = \frac{1}{\nu_{e,m}} \left\{ \sum_1^N (1 - k_{\mu,n}) (A_\mu \sigma_n - \gamma_{\mu,n}) \right\} \quad (29)$$

The most effective way to determine the terms of the series expansion (24) is to choose  $\tau_\mu = 10\tau_{\mu-1}$  ( $\mu = 1, 2, \dots$ ) and to assume  $A_\mu = \ln(10)L(\tau_\mu)$ , where  $L(\tau_\mu)$  is the continuous relaxation spectrum (Bažant and Xi 1995). But an extra term,  $A_0$ , is needed to take into account the instantaneous tail of the continuous spectrum  $L(\tau)$  for  $\log(\tau) \rightarrow -\infty$  (or  $\tau \rightarrow 0$ ). Theoretically,  $A_0 = \int_{-\infty}^{\tau_0} L(\tau) d\tau$  where  $\tau_0 = \tau_{1-1/2} = \tau_1 / \sqrt{10}$ , but it is preferable to compute  $A_0$  by minimizing the averaged error of series expansion (24) compared to the theoretical expression of  $\Phi(\xi)$ . The error, averaged over the period  $\tilde{\xi}$ , is

$$R(A_0) = \int_0^{\tilde{\xi}} \left\{ \Phi(\xi) - A_0 - \sum_1^N A_\mu A_\mu \left\{ 1 - \exp\left(-\frac{\xi}{\tau_\mu}\right) \right\} \right\} d\xi \quad (30)$$

and by imposing  $R(A_0) = 0$ , we get

$$A_0 = \frac{1}{\tilde{\xi}} \int_0^{\tilde{\xi}} \left\{ \Phi(\xi) - \sum_1^N A_\mu \left\{ 1 - \exp\left(-\frac{\xi}{\tau_\mu}\right) \right\} \right\} d\xi. \quad (31)$$

which can be easily and accurately computed (if we assume  $\tilde{\xi} = 365$  days,  $N = 10$ , and  $\tau_1 = 10^{-4}$ , then we get  $A_0 = 0.2794q_2$ , Cusatis 1998).

The purely viscous strain component (flow) can be integrated by the central difference approximation;  $\Delta \varepsilon_n^f = \psi_m \Delta t_n \sigma_m / \eta_m$ , where  $\psi_m = \psi(t_{n+1/2})$ ,  $\eta_m = \eta(S_m)$ ,  $S_m = S(t_{n+1/2})$ ,  $\sigma_m = \sigma(t_{n+1/2})$ . If we assume  $\sigma(t_{n+1/2}) = \sigma_n + \Delta \sigma_n / 2$ , we can write the following incremental relation:

$$\Delta \varepsilon_n^f = A_n^f \Delta \sigma_n + \Delta \varepsilon_n^{f''} \quad (32)$$

in which  $A_n^f = \psi_m \Delta t_n / (2\eta_m)$ ,  $\Delta \varepsilon_n^{f''} = \psi_m \sigma_n / \eta_m$ .

To obtain the increment of viscous strain, we need, in advance, the value of the microprestress at time  $t = t_{n+1/2}$ . We can get it by setting  $S(t_{n+1/2}) = S_n + \Delta S_n/2$  and, based on the central difference approximation of Eq. (20),

$$\Delta S_n = -c_0 + \psi_{s,m} \Delta t_n S_m^2 + k_1 \left[ \Delta T_n \ln(h_m) + T_m \frac{\Delta h_n}{h_m} \right] \quad (33)$$

It is computationally convenient to integrate Eq. (22) also in the sense of exponential algorithm. Following Bažant and Chern (1985a,b), we write  $\varepsilon^{cr} = \sigma / C^{cr}$  and set  $-C^{cr} / \dot{C}^{cr} = \beta$ . Thus, we obtain from Eq. (22) the evolution equation

$$\dot{\sigma} + \frac{\sigma}{\beta} = C^{cr} \varepsilon^{cr} \quad (34)$$

So, assuming  $\beta$ ,  $C^{cr}$ ,  $\varepsilon^{cr}$ , and  $\sigma$  to be constant within each step, we finally obtain

$$\Delta \varepsilon_n^{cr} = A_n^{cr} \Delta \sigma_n + \Delta \varepsilon_n^{cr''} \quad (35)$$

where  $1/A_n^{cr} = [1 - \exp(-\Delta z_n)] C_m^{cr} / \Delta z_n$ ,  $\Delta \varepsilon_n^{cr''} = \Delta z_n / C_m^{cr} \sigma_n$ ,  $\Delta z_n = \Delta t_n / \beta$ , and  $C_m^{cr} = C^{cr}(\varepsilon_{n+1/2}^{cr})$ . The numerical form of the unloading-reloading rule is  $\Delta \varepsilon_n^{cr''} = 0$  when  $\varepsilon_m^{cr} < \varepsilon_{max}^{cr}$  where  $\varepsilon_m^{cr} = \varepsilon_{n+1/2}^{cr}$ ,  $\varepsilon_{max}^{cr} = \max[\varepsilon^{cr}(t)]$ .

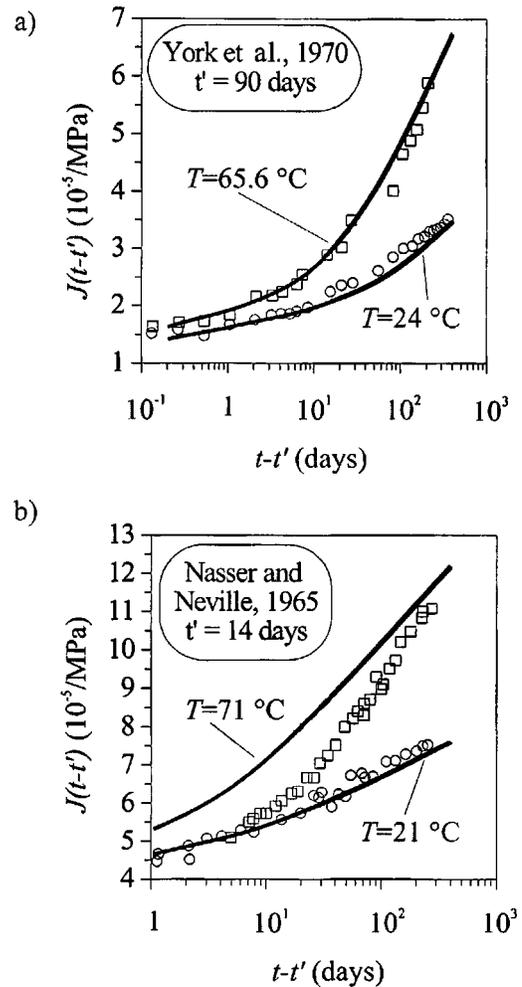
Summing up all the strain components, we eventually get the incremental stress-strain relation  $\Delta \varepsilon_n = A_n \Delta \sigma_n + \Delta \varepsilon_n^0$  or  $\Delta \sigma_n = E_n (\Delta \varepsilon_n - \Delta \varepsilon_n^0)$ , where  $E_n = 1/A_n$  and  $A_n = q_1 + A_n^{ev} + A_n^f + A_n^{cr}$ ,  $\Delta \varepsilon_n^0 = \Delta \varepsilon_n^{ev''} + \Delta \varepsilon_n^{f''} + \Delta \varepsilon_n^{cr''} + k_{sh} \Delta h_n + \alpha \Delta T_n$ . This quasi-elastic relation has been introduced into a finite element program for step-by-step loading. If cracking does not occur, this relation is fully explicit in each step and no iterations are required.

The three-dimensional tensorial generalization of Eqs. (24), (26), (29), (32), (34), and (35), which is required for finite element programming, is made easy by material isotropy and the fact that the Poisson ratio for basic creep can be taken approximately equal to its elastic value, about 0.18 (RILEM 1988). The aforementioned equations are considered separately for the volumetric and the deviatoric components of tensors  $\boldsymbol{\varepsilon}$ ,  $\boldsymbol{\sigma}$ ,  $\boldsymbol{\gamma}$ ,  $\boldsymbol{\varepsilon}^{ev}$ ,  $\boldsymbol{\varepsilon}^{ev''}$ ,  $\boldsymbol{\varepsilon}^f$ , and  $\boldsymbol{\varepsilon}^{f''}$ . The tensorial relations are composed of these volumetric and deviatoric relations, and the fourth-rank material stiffness tensor  $\mathbf{E}$  is composed of the volumetric and deviatoric material moduli according to the condition of isotropy. For programming, all these tensors are represented by  $6 \times 1$  and  $6 \times 6$  matrices. The procedure is similar to that in (Bažant 1975, 1982, 1995; Bažant and Chern 1985b; RILEM 1988; Bažant et al. 1997b; Bažant and Xi 1994; Bažant and Jirásek 2002; Bažant and Zi 2003; Bažant et al. 2003).

### Analysis of Experimental Data

To validate the model and to calibrate the unknown material parameters, the typical experiments reported in literature have been numerically simulated. For each tested concrete, the parameters  $q_1$ ,  $q_2$ ,  $\alpha$ , and  $q_4$  are identified by fitting the experimental data for basic creep at room temperature. Parameters  $c$  and  $k_1$ , which govern the evolution of microprestresses, must be identified from transient analysis of tests at variable temperature or humidity.

The effect of temperature on basic creep is analyzed first. The specimens, after casting, are protected against water loss. There-

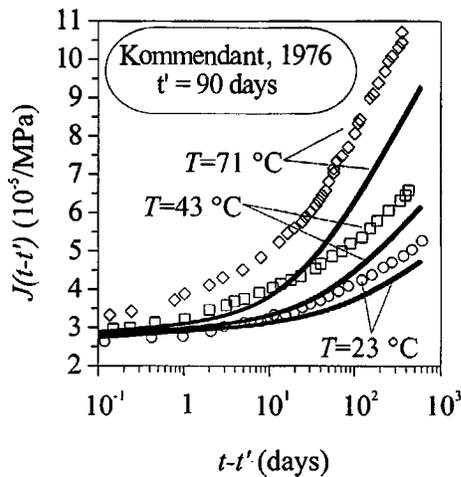
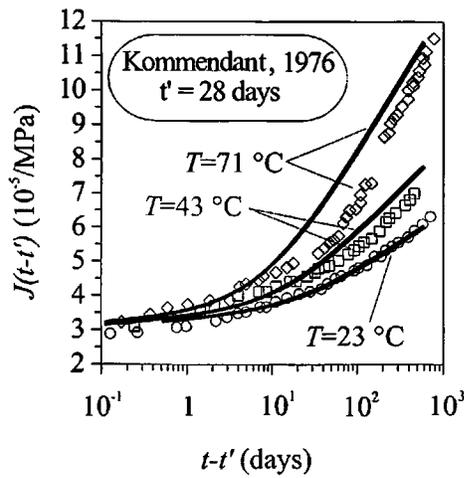


**Fig. 2.** Fits of the effect of temperature on basic creep; data of (a) York et al. (1970) and (b) Nasser and Neville (1965)

fore, it is reasonable to consider  $h \approx 1$  during the analysis ( $h = 0.98$  is assumed). The specimens are heated to the test temperature before loading and then the evolution of the strain under the load at constant temperature is recorded. Fig. 2 shows the compliance function  $J(t, t')$  for two different sets of data. For York's data (York et al. 1970) [Fig. 2(a)], the fits have been obtained for the following optimum values of the parameters:  $q_1 = 4.0 \times 10^{-7}/\text{MPa}$ ,  $q_2 = 5.0 \times 10^{-5}/\text{MPa}$ ,  $\alpha = 0.27$ ,  $q_4 = 6.0 \times 10^{-6}/\text{MPa}$ ,  $c = 1.0 \times 10^{-8}/(\text{MPa day})$ , and  $k_1 = 3 \text{ MPa/K}$ , while for Nasser's data (Nasser and Neville 1965) [Fig. 2(b)],  $q_1 = 1.5 \times 10^{-5}/\text{MPa}$ ,  $q_2 = 8.0 \times 10^{-5}/\text{MPa}$ ,  $\alpha = 0.3$ ,  $q_4 = 5.0 \times 10^{-6}/\text{MPa}$ ,  $c = 1.0 \times 10^{-8}/(\text{MPa day})$ , and  $k_1 = 3 \text{ MPa/K}$ . The agreement is seen to be satisfactory.

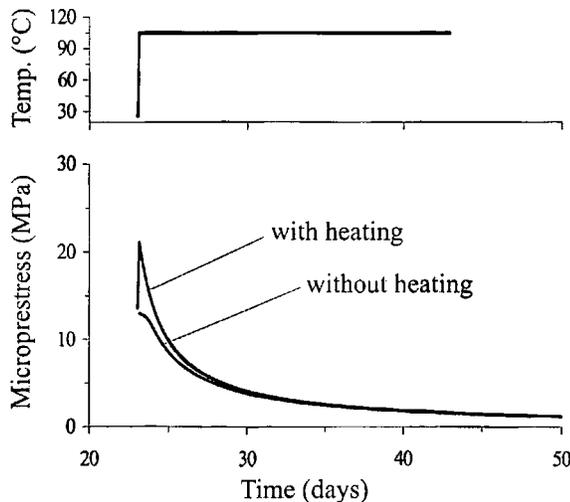
Fig. 3 shows the results of simulation of the experiments of Kommendant et al. (1976), in which two different ages of loading and three different test temperatures were used. The optimum parameters are found to be:  $q_1 = 2.0 \times 10^{-5}/\text{MPa}$ ,  $q_2 = 7.0 \times 10^{-5}/\text{MPa}$ ,  $\alpha = 0.08$ ,  $q_4 = 7.0 \times 10^{-6}/\text{MPa}$ ,  $c = 1.0 \times 10^{-8}/(\text{MPa day})$ , and  $k_1 = 3 \text{ MPa/K}$ . The prediction is seen to be quite accurate for the first age at loading [Fig. 3(a)], but the accuracy is poorer for the second age at loading.

In the foregoing cases, parameters  $k_1$  and  $c$  have a negligible effect, and so they cannot actually be identified by these tests. In these simulations, humidity variations do not occur and then Eq. (33) becomes  $\Delta S_n = -c_0 \psi_{s,m} \Delta t_n S_m^2 + k_1 [\Delta T_n \ln(h_m)]$ . When tem-

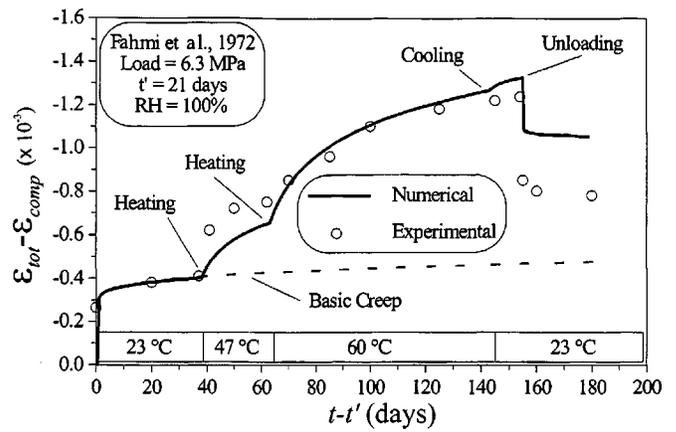


**Fig. 3.** Fits of the effect of temperature on basic creep; data of Kommendant et al. (1976)

perature changes, the microprestress increases but, since heating is usually quite fast, the microprestress has already relaxed when the load is applied and has approached the values it would have without heating. Fig. 4 shows the evolutions of microprestress for a given temperature history and for room temperature, calculated for the values  $k_1 = 3 \text{ MPa/K}$  and  $c_0 = (1/3) \times 10^{-3}/(\text{MPa day})$ .



**Fig. 4.** Evolutions of microprestress with and without heating

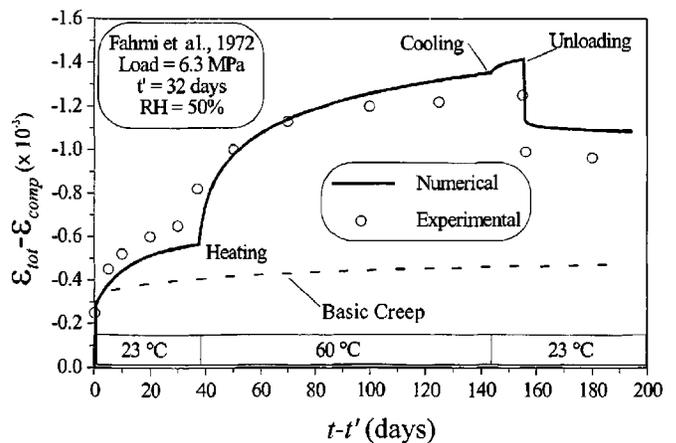


**Fig. 5.** Transitional thermal creep at constant water content

Obviously, a different behavior would be obtained during transient tests in which the load is applied during heating.

Consider now transitional thermal creep tests in which the temperature is changed during the load application. In this regard, the data by Fahmi et al. (1972) have been analyzed. Two different test procedures have been used. The first involves temperature variations while the specimen is sealed ( $h \approx 0.98$ ). The second involves also a moisture loss due to exposing the specimens to an environmental relative humidity equal to 50%. In these cases, the parameters  $c$  and  $k_1$  have an important effect. Their values have been optimized to get the best fit of the experimental data. The optimized values of all the parameters are:  $q_1 = 2.5 \times 10^{-5}/\text{MPa}$ ,  $q_2 = 1.0 \times 10^{-4}/\text{MPa}$ ,  $\alpha = 0.015$ ,  $q_4 = 6.0 \times 10^{-6}/\text{MPa}$ ,  $c = 3.0 \times 10^{-8}/(\text{MPa day})$ , and  $k_1 = 3 \text{ MPa/K}$ .

Figs. 5 and 6 show the comparisons between the experimental and numerical results. For both, as usual, the difference between the total strain measured under load ( $\epsilon_{\text{tot}}$ ) and the total strain measured on a load-free companion specimen ( $\epsilon_{\text{comp}}$ ) is reported as a measure of creep strain, although it actually includes also the shrinkage reduction in the companion specimen due to diffuse cracking caused by the nonuniformity of temperature and humidity over the specimen. To analyze this effect, cracking simulation (similar to Bažant and Xi (1994) and Bažant et al. (1997b)) has been introduced into the model. The optimized fits, shown in Figs. 5 and 6, correspond to  $f'_i = 2.5 \text{ MPa}$ ,  $\xi_p = 0.6 \times 10^{-5}$  and  $s = 0.25$ , while it has been also assumed that  $k_{\text{sh}} = 1.1 \times 10^{-3}$  and



**Fig. 6.** Transitional thermal creep at variable water content

$\alpha_T$   
=  $1.0 \times 10^{-5}/^\circ\text{C}$ . For these particular tests, unfortunately, the effect of cracking is negligible because the specimen dimensions are very small.

### Possible Effect of Long-Term Polymerization in Calcium Silicate Hydrates

In Bažant and Prasanna (1989a,b), it was speculated that some sort of polymerization, i.e., a gradual long-term increase in the number of bonds in solidified cement gel, might explain the long-term decrease of creep with the age at loading (mechanism 1b). The latest microscopic observations (Hamlin M. Jennings, private communication, 2002) now confirm the existence of polymerization. This suggests a slight reinterpretation of the concept of microprestress. It might be appropriate to consider not the absolute level of microprestress  $S$  but only its difference ( $S - S^*$ ) from some kind of "microstructure resistance  $S^*$ ," representing a certain microprestress threshold that depends on the number of bonds per unit volume of cement gel (or, indirectly, on the effective stiffness of the cement gel microstructure). However, the present scope of deformation data from laboratory specimens does not suffice for verifying and calibrating this kind of microstructure resistance. The existing data can be fitted without, as well as with, this hypothesis.

### Conclusion

The previously developed microprestress-solidification theory, already capable of realistic modeling of phenomena of long-term aging and drying creep, has been extended to simulate also the effect of temperature changes. The new generalized theory can also capture the transitional thermal creep. Both phenomena are automatically predicted through the concept of microprestress, particularly its dependence on the chemical potential of pore water which is sensitive to both the pore humidity and temperature. A further simple modification of the theory allows simulating also the transient creep increase due to wetting and cooling.

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### Appendix. Basic Information on Test Data

*Kommendant et al. (1976)*. Cylinders 6 in.  $\times$  6 in. (152 mm  $\times$  305 mm) sealed with butyl rubber and cured at 73°F (23°C), until 5 days before the age of loading. The specimens were then heated to test temperatures 100 and 160°F (43 and 71°C) at the rate of 24°F per day (13.3°C per day). 28-day cylinder strengths = 6590 psi (45.4 MPa) and 6700 psi (46.2 MPa); water:cement:sand:coarse aggregate ratios = 0.38:1:1.73:2.61 and 0.38:1:1.65:2.38, respectively; cement content = 419 kg  $\text{cm}^{-3}$ ; maximum aggregate size = 1.5 in. (38.1 mm). Axial compressive

stress = 32% of 28-day strength.

*Fahmi et al. (1972)*. Hollow microconcrete cylinders 6 in. outer diameter = (15.24 cm), inner diameter = 5 in. (12.70 cm) and length = 40 in. (101.6 cm). Water:cement = 0.58. Maximum aggregate size = 2.38 mm. 21-day compressive strength = 5840 psi (40.26 MPa). The specimens exposed to 50% relative humidity (RH) were moist cured at 73°F (23°C) for 18 days and then stored at 50% RH and 73°F (23°C) for 14 days before loading. The specimens exposed to 100% RH were moist cured for 21 days before loading. About 37 days after loading, the temperature was raised to 140°F. The heating process took about 3 h. Axial compressive stress = 910 psi (6.3 MPa)

*Nasser and Neville (1965)*. Cylinders 3 in.  $\times$  9.25 in. (76 cm  $\times$  235 cm) were sealed in propylene jackets, stored in water bath at a desired temperature for 14 days before loading. Water:cement:aggregate ratio = 0.6:1:7.15; cement content = 320 kg  $\text{cm}^{-3}$ ; maximum aggregate size = 3.4 in. (19 mm). 14-day cylinder strength = 5660 psi (39.0 MPa). Axial compressive stress = 1950 psi (13.4 MPa).

*York et al. (1970)*. Cylinders 6 in.  $\times$  16 in. (152 mm  $\times$  406 cm) sealed 48 h after casting and the cured for 81 days at 75°F (24°C). At the age of 83 days, the specimens were sealed in neoprene jackets and exposed to test temperature. Loaded at 90 days. Water:cement:sand:coarse aggregate ratio = 0.425:1:2.03:2.62; cement content = 404 kg  $\text{m}^{-3}$ ; maximum aggregate size = 0.75 in. (19 mm). 28-day cylinder strength = 6460 psi (44.6 MPa). Axial compressive stress = 2400 psi (16.6 MPa).

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