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營建技術與材料之創新發展

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在混凝土設計規範及實務中忽視或誤判尺寸效應之影響

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混凝土設計規範小小的誤判，將會有嚴重的影響。（圖：陳振川提供）

除了很巨大的無鋼筋結構外，一般混凝土構造物所顯現的韋布統計尺寸效應不明顯，基本上，在混凝土構造物承受最大載重前，由於大破裂進展區或大裂縫之形成，使構造物內之應力重新分佈，成為尺寸效應活躍性的來源，由無箍筋之鋼筋混凝土梁承受剪力試驗所得現有資料庫數據之無偏差統計分析結果證實活躍性尺寸效應定理之存在，同時，尺寸效應對含箍筋梁也會造成影響，但效果較輕微，雖然對此種形式的尺寸效應之瞭解已超過二十年，但在設計規範或實務中大都被忽略，會有何種後果呢？對小構造物過度設計，但更嚴重的是對大構造物產生無法接受的風險，一般而言，對於工程結構物可被接受的破壞風險為使用年限的百萬分之一，由崩塌統計分析中顯示這個數字的確符合小構造物情況，以大量統計資料庫數據所率定之機率分析證實：對於承受剪力且梁深小於 0.2 m 之無箍筋鋼筋混凝土梁也具有此項同等程度的破壞機率，但如將尺寸效應予以忽略（如 ACI 規範），對於梁深達 1 m 之混凝土梁，此破壞機率證實會劇烈地增加至千分之一（CEB 及 fib 規範幾乎同樣地使用不合實際之公式，JSCE 則低估尺寸效應），此項發現大致符合數項統計結果所顯示：已崩毀大型結構物的發生率約為小結構物的一千倍，此種現象是無法接受的，因此，至今，不再是只有少數理論學家，甚至整個科學界及混凝土斷裂委員會（IA-FraMCoS、ASCE-EMD, ACI 446 委員會）也都信服：在混凝土構造物會由不可避免的活躍性尺寸效應造成脆性破壞，工程界或許甚至於顧問機構在設計規範中忽略或嚴重地低估尺寸效應而造成另一次的結構崩塌時，將會面對法律上的風險。 c

（譯者：國立臺灣科技大學 營建系 張大鵬教授）

Consequences of Ignoring or Mis-Judging the Size Effect in Concrete Design Codes and Practice

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Except for huge unreinforced structures, Weibull's statistical size effect is weak in concrete structures. The size effect source is principally energetic, caused by stress redistribution due to a large fracture process zone size or large cracks formed before maximum load. It is shown that an unbiased statistical analysis of the existing database for shear of R.C. beams without stirrups supports the energetic size effect theory, and that the size effect, albeit milder, afflicts beams with stirrups, too. Known though has this type of size effect been for two decades, it has been mostly ignored in design codes as well as practice. What are the consequences? — overdesign of many small structures but, more seriously, unacceptable risk for large ones. A tolerable failure probability of engineering structures is 10^{-6} per lifetime, and collapse statistics indicate that this has indeed been true for small structures. Probabilistic analysis calibrated by a large statistical database confirms this level of failure probability for shear of reinforced concrete beams without stirrups < 0.2 m deep. However, if the size effect is ignored (as in ACI code), the failure probability is shown to increase drastically — to 10^{-3} for beams 1 m deep (and nearly as much for the unrealistic formulae of CEB, fib and JSCE underestimating the size effect). This finding roughly matches several statistics showing that very large structures have been collapsing with a frequency roughly 10^3-times greater than small ones. This is unacceptable. Now that no longer just a handful of theoreticians, but entire scientific societies and concrete fracture committees (IA-FraMCoS, ASCE-EMD, ACI Comm. 446), are convinced of the inevitability of energetic size effect in brittle failures of concrete structures, the engineering societies ignoring or severely underestimating the size effect in their design codes, and perhaps even design

firms, are exposed to legal risk when another collapse occurs.

Introduction

Concrete structures much larger than the specimens tested in laboratories are being built in ever increasing numbers. For example, the box girder of the record-span Koror-Babeldaob Bridge in Palau, which collapsed in a brittle shear-compression mode, was 14.2 m deep. The outriggers of the Trump Tower under construction in Chicago are 6 m deep. However, the experimental databases collected to establish design code specifications consist mostly of small-size laboratory tests. The mean beam depth in the ACI-445 database [1] on which the shear design in the current ACI standard 318 still rests is only 0.34 m, and in the latest ACI-445 database [2] it is 0.345 m. In the latter, 86% of the 398 data points pertain to beam depths < 0.5 m and 99% to depths < 1.1 m, and only 1% to depths from 1.2 to 2 m. Since the code-making committees prefer to rely on experiments only, it is thus no surprise that the size effect is not correctly represented.

Concrete is an archetypical quasi-brittle material whose fracture propagation is characterized by a rather large fracture process zone (FPZ), typically 0.5 m long. This causes that small structures (cross section ≤ FPZ length) fail in a quasi-ductile manner (*i.e.*, with a plastic yield plateau) and exhibit almost no size effect, while very large structures (cross section >> FPZ length) failing in concrete rather than steel behave in an almost perfectly brittle manner, with a steep load drop right after the peak load, and exhibit the strongest possible size effect;[3-6] Fig. 1. The size effect has been studied mainly for shear of longitudinally reinforced beams without stirrups,[7,8] but it also occurs in shear of beams with stirrups, in torsion, punching of slabs, failure of columns, arches and prestressed girders failing due to crushing of concrete, failure of anchors and splices, and bar pullout.[8,9] Accumulating

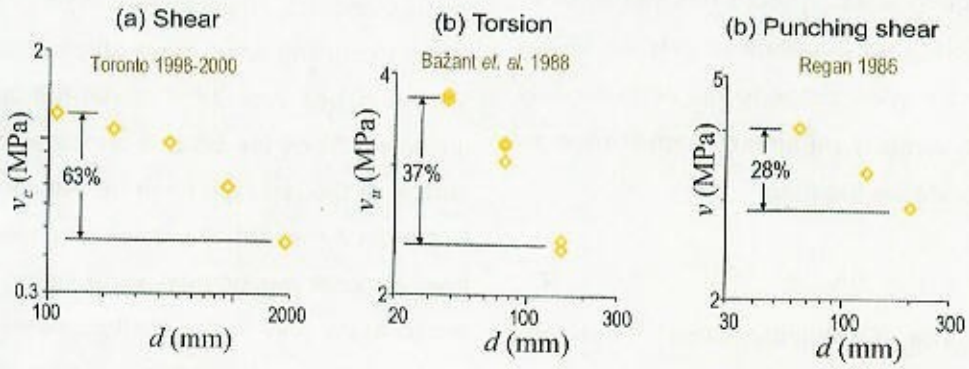


Fig. 1 Size Effect Tests of (a) Shear Failure; (b) Torsional Failure; and (c) Punching Shear Failure.

experimental evidence 10 ~ 20 shows that the size effect causes beams about 2 m deep to fail at loads much lower than that calculated from the design code using the understrength factor ϕ = 0.75 and the specified concrete strength (about 30% less than the mean strength). For instance, the largest beam in Toronto tests [3,4] had a shear strength almost 50% lower than what is calculated from ACI design code; [21] see Fig. 1(a). This strength reduction due to size effect now causes serious concern about the safety of current design codes and engineering practice.

To make the risk of structural failure much smaller than various inevitable risks that people face, the maximum tolerable failure probability is about 1 in a million.[22] This value roughly agrees with the frequency of failures experienced for small beams. But for large ones, it has been about 1 in a thousand [23,24] and could become 1 in a hundred or higher as ever larger beams are being built. Whether or not such unacceptable risk will have to be tolerated depends largely on taking the size effect properly into account. This is an issue of paramount significance for concrete engineering.

Risk of Failure in Shear Design

The ACI Building Code [21] currently specifies the contribution of concrete to the cross-section shear strength of reinforced concrete members by the formula

$$V_c = 2\sqrt{f_c'}\, b_w\, d \qquad (1)$$

(which is valid only in psi, lb. and inches). Here f_c' is the specified compressive strength of concrete, d is the beam depth measured from the top face to the longitudinal reinforcement centroid, and b_w is the web width. The code formula gives a size-independent average concrete shear strength, $v_c = V_c / b_w d$ (identical to the 'nominal strength' in mechanics terminology). However, ignoring the size effect in Eq. (1) would lead to statistically dangerous designs with insufficient safety margins for large shear-critical concrete beams. We evidence it next.

Statistical Strength Distribution of Small Beams

While the probability density distribution (pdf) of strength scatter due to material randomness has recently been theoretically established for quasibrittle failures at crack initiation (type 1),[25-27] for those occurring after large stable crack growth (types 2 or 3)[18,28] it still remains unknown. Since the latter is our case, our choice of the pdf type must be empirical. But even if the pdf of scatter originating from material randomness were known, it would apply only to the scatter observed in carefully controlled laboratory test

series such as those conducted at the University of Toronto [3,4] and Northwestern University [13] for which the coefficients of variation (C.o.V.) of errors (*i.e.*, standard error of regression normalized by data centroid) are only about 6.9% and 12%, respectively.

The errors of the current code formula $v_c = 2\sqrt{f_c'}$ are approximately characterized by the scatter seen in the ACI-445F database [2] (Fig. 2), which originates from material randomness only to a minor extent. Because this formula must apply to a broad variety of beams used in practice, the database covers a wide range of secondary characteristics such as the steel ratio, shear-span ratio and concrete type (which includes concrete strength, curing environment, water-cement ratio, aggregate-cement ratio and other mix proportions, *etc.*). While the scatter of these secondary characteristics is the result of human choices, it roughly reflects the range of characteristics occurring in practice (even though the distributions of these characteristics in design practice might not be exactly the same as in the database, there exists no better information anyway).

Even if we considered the recently proposed refinement in which the effects of the secondary characteristics such as the steel ratio, shear-span ratio and concrete type are incorporated into the formula for v_c,[29,30] their representation would be only approximate, with a high degree of uncertainty. So, the scatter due exclusively to material randomness, exemplified roughly by the aforementioned laboratory tests in Toronto and

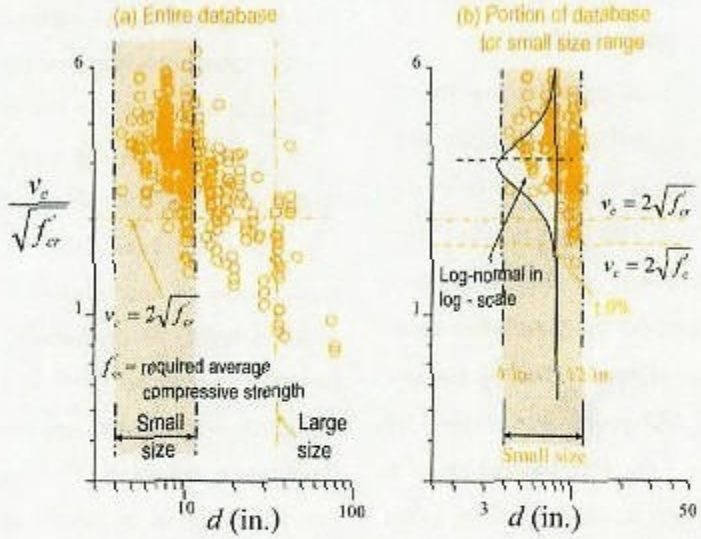


Fig. 2 (a) ACI-445F database of 398 data points; (b) Portion of the database for beams from 10 to 30 cm deep (v_c, f_{cr} and f_c are in psi).

Northwestern, would still be only a minor part of the overall scatter. This is revealed by the width of the scatter band in previous work [30] where the regression does take the secondary characteristics into account. The C.o.V. of regression errors in that scatter band is of the order of $\omega_1 \approx 20\%$, while the C.o.V. due to material randomness per se is of the order of $\omega_2 \approx 10\%$.

To make this argument precise, note that if the points of a database whose C.o.V. = ω_1 are perturbed by independent random scatter whose C.o.V. = ω_2, then the resulting scatter of the perturbed database will have the C.o.V. of $\omega_3 = \sqrt{\omega_1^2 + \omega_2^2}$. In the present case, $\omega_3 \approx 20\%$ and $\omega_2 \approx 10\%$, which gives $\omega_1 \approx 17.3\%$. This is only 13% less than ω_3. Obviously, ω_2, ensuing from material randomness, has only a minor effect on the overall ω_3, and so its pdf type cannot matter much.

To decide which data to use for an empirical basis of pdf choice, note also that the scatter band in the ACI-445F database (Fig. 2) with 398 data points [2] has a downward trend with respect to depth d (this is also confirmed by the earlier databases of 296 points assembled by Bažant and Kim [7], and 461 points assembled by Bažant and Sun [12]). The existence of a marked size effect trend becomes even clearer if the influences of shear span, steel ratio and concrete strength are taken into account as subsidiary parameters in the regression. [30] Therefore, the entire ACI-445F database cannot be treated as a statistical population from which the pdf of shear strength could be identified.

However, if we isolate from the database in Fig. 2(a) the data in the small size range of depths d ranging from 100 mm to 300 mm (4 in. to 12 in.), centered at 200 mm (8 in.), as shown in Fig. 2(b), then the size effect trend is weak enough for treating the data as a population with no statistical trend (indeed, within this range, the size effect in the Toronto tests [3,4] causes a strength reduction of only about 10%). The mean and coefficient of variation (C.o.V.) of this population of data are found to be $\bar{y} = \overline{v_c / f_{cr}'^{1/2}} = 3.2$ and $\omega = 27\%$, where f_{cr}' is the required average compressive strength of concrete. The relatively high value of ω is a consequence of variability of the secondary characteristics which have a non-negligible influence on the shear strength.

To determine the appropriate pdf of shear strength for the small size beams, we plot the data points from the small size range as cumulative histograms on various types of probability paper. While several methods [31,32] to calculate the cumulative histogram are used in practice, Gumbel's method [33] is adopted here due to clarity of its justification as well as simplicity; the plotting positions are $m/(n+1)$(where m denotes the mth point among

the data arranged in the increasing order of normalized shear strength $v_c / \sqrt{f_{cr}}$, and n is the total number of points in the isolated database).

Figures 3(a) and 3(b) shows the cumulative histograms and their fits by cumulative distribution functions (cdf) in the normal and log-normal probability papers. Now note that the data points fit a straight line on the log-normal probability paper significantly better than they do on the normal probability paper (for the former, the mean and standard deviation are 3.22 and 0.895, and for the latter they are 3.22 and 0.885). Also note that if the Weibull probability paper were used, the fit of a straight line would be still worse. Hence, based on the information that exists, a log-normal pdf appears to be the best choice.

The type of pdf for small beams may alternatively be examined by the goodness-of-fit tests. The widely used Kolmogorov-Smirnov or K-S test [34] com-

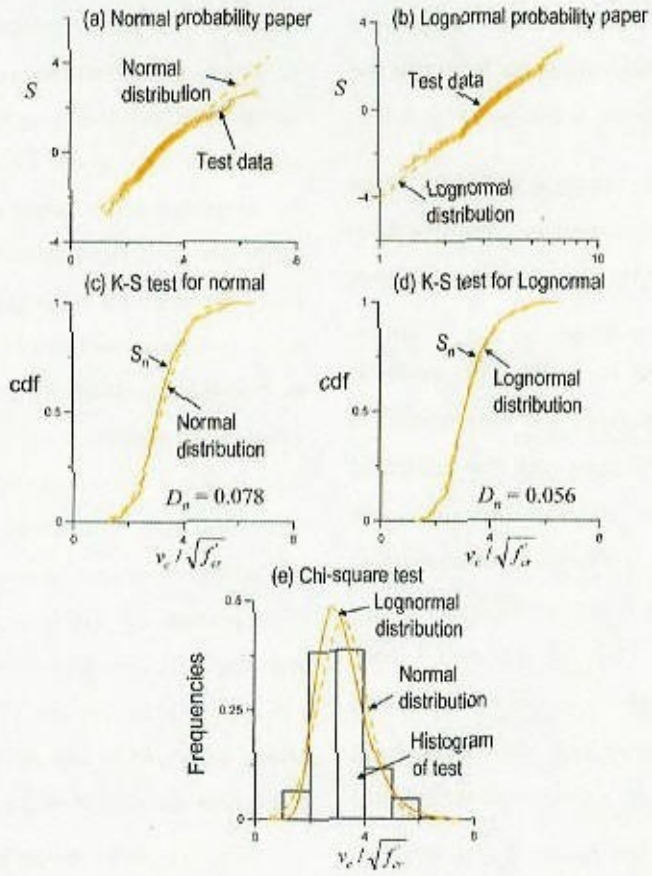


Fig. 3 (a) Cumulative histogram of data on normalized beam shear strength for small beams extracted from the ACI-445F database, plotted on normal probability paper, and their straight-line fit; (b) ditto on log-normal probability paper; (c) K-S test for normal distribution; (d) K-S test for log-normal distribution; (e) Chi-square test for goodness-of-fit

pares the observed cumulative probability S_n (solid curve) with the assumed normal distribution obtained by optimal fit (dashed curve), and generates the maximum discrepancy of $D_n = D_{277} = 0.078$; see Fig. 3(c). This value satisfies the critical value for the 5% significance level ($D_{277}^{05} = 0.081$) but exceeds the critical value for the 10% significance level ($D_{277}^{10} = 0.073$). By contrast, the maximum discrepancy for log-normal distribution is $D_{277} = 0.056$, which is much less than that observed in the K-S test for normal distribution and satisfies the critical values for both the 5% and 10% significance level; see Fig. 3(d).

Furthermore, the type of pdf for small beams may be examined by the chi-square test.[35] In this test, one subdivides the range of coordinate $v_c / \sqrt{f_{cr}}$, which spans from 1.32 to 6.56, into several intervals and compares the frequencies n_i of the small beam data with the assumed frequencies e_i for all the intervals in the histogram. Here, 6 intervals, limited by 1, 2, 3, 4, 5, 6 and 7, are considered. They contain 18, 106, 107, 32, 13, and 1 data points, respectively; see the histogram in Fig. 3(e). Compared with the frequencies corresponding to normal distribution (dashed curve), we have $\sum (n_i - e_i)^2 / e_i = 20.95$, which cannot satisfy the critical value $c_{0.95,3} = 7.81$ for 5% significance level. On the other hand, we obtain $\sum (n_i - e_i)^2 / e_i = 3.45$ for log-normal distribution (solid curve), which satisfies the critical value for 5% significance level.

The foregoing comparisons demonstrate that, among simple distributions, the log-normal pdf is the best choice for the small beam data in the ACI-445F database.

Statistical Strength Distribution of Large Beams

Again, theoretical deductions based on the scatter in one and the same material [25,26] are inapplicable because this scatter is overwhelmed by the scatter due to random variability of steel ratio, shear span ratio, *etc.*, in the ACI-445F database. As emphasized in previous work,[29,30] the database is heteroscedastic in the plot of normalized shear strength $v_c / \sqrt{f_{cr}}$ versus size, but becomes nearly homoscedastic in the doubly logarithmic plot; in other words, the variance or C.o.V. of the data becomes almost independent of the structure size.[30] Furthermore, in view of the aforementioned origin of scatter, there is no reason for the type of pdf to change with the structure size. Therefore, it is logical to assume the pdf of the normalized shear strength in the ACI-445F database to be log-normal for all the sizes.

Figure 4(a) shows the same pdf (log-normal, with the same coefficient of variation) superposed on the series of individual tests of beams of various sizes

made at the University of Toronto.[3,4] Now it should be noted that, for the type of concrete, steel ratio, shear span ratio, *etc.*, used in the Toronto tests, the shear strength value in these tests lies (in the logarithmic scale) at certain distance a below the mean of the pdf (Fig. 4(a)). Since the width of the scatter band in Fig. 2(a) in logarithmic scale does not vary appreciably with the beam size, the same pdf and the same distance a between the pdf mean and the Toronto data must be expected for every beam size *d*, including the size of *d* = 925 mm (36.4 in.), for which there is only one data point, and also the size of 1.89 m (74.4 in.). In other words, if the Toronto test for *d* = 925 mm (36.4 in.) were repeated for many different types of concrete, steel ratios, shear span ratios, humidity and temperature conditions, *etc.*, one would have to expect a pdf shifted downwards in the logarithmic scale as shown in Fig. 4(a).

Lest it be thought that distance a should be treated as random, it must be emphasized that a represents a certain percentage cut-off on the pdf of shear strength, and thus it is a property of pdf. It is the basic tenet of the theory of probability that the pdf per se, including any of its properties, is not random. It is a deterministic descriptor of random variability. Assuming pdf, including any of its properties, to be random, would wreck the whole edifice of the existing theory of probability. By assuming the value of a to be the same for the small and large size ranges, we simply imply that the probabil-

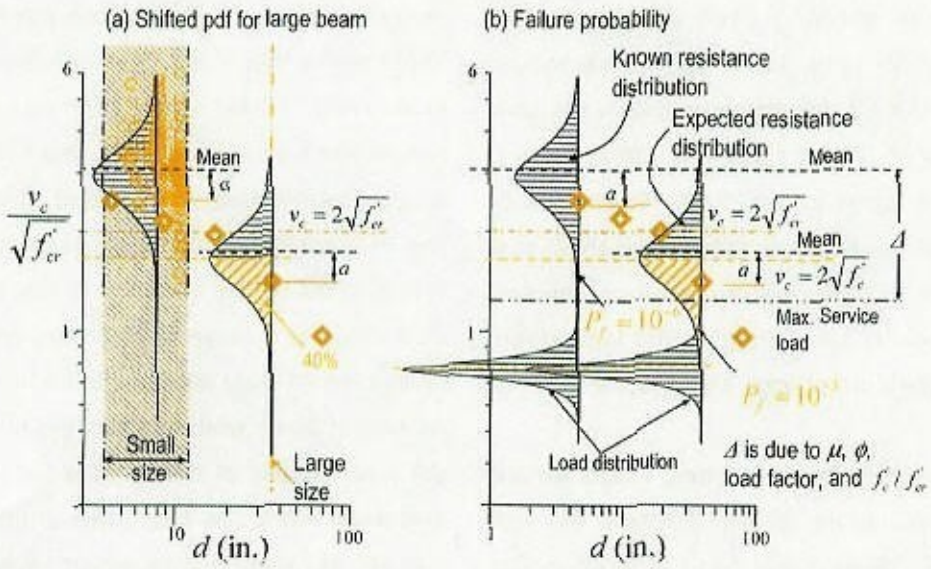


Fig. 4 (a) Probability Distribution of Shear Strength of Beams from 10 to 30 cm Deep, Based on the ACI-445F database, Compared to Toronto Data; (b) Failure Probability for Small Beam and 1 m Deep Beam.

ity, or frequency, of beams having shear strength below the value characterized by a will be the same for these size ranges.

For fracture specimens, information on the scatter in size effect is much more abundant than for beams. Much of this information tends to show a decrease of scatter band width as the size increases, but is obtained from specimens with the same geometry in which parameters other than the size are not varied. As for the random scatter in tests of size effect in beam, the only information appears to be the reduced-scale tests 13 conducted at Northwestern University on geometrically similar specimens with the size range of 1 : 16. These tests show the coefficients of variation to be almost the same for all the 5 sizes tested (C.o.V. = 6%, 7%, 8%, 6% and 8%).

For the ACI-445F database, the C.o.V. for large sizes may be estimated from the 22 test points falling in the size range of 760 to 1,000 mm (30 to 40 in.), and is found to be 27.9%. This value is almost the same as that for the small size range. It confirms our assumption that the scatter band width in the logarithmic plot does not change significantly with the size.

A question now arises: Could we not directly use the 22 test points in the size range 760 to 1,000 mm (30 to 40 in.) to determine the distance a? We could not, because these 22 points cover only a portion of the entire range of the influencing parameters of interest and the distribution of these parameters is very different from that in the small size range. For example, the steel ratios in the small size range of the ACI-445F database vary from 0.25% to 6.64%, with the mean of 2.55%, while the aforementioned 22 points correspond on the average to much lighter reinforcement, with the steel ratios varying from 0.14% to 2.1%, and the mean of 0.96%. A similar discrepancy exists for *a/d*. So, using the data points in this size range would be misleading (yielding *a* distance a as only 0.07 instead of 0.45).

Now it is inescapable to recognize that this shifted pdf for d ≈ 1 m (40 in.) reaches well below the line of required nominal strength $v_c/2\sqrt{f_c'}$ of $y = v_c/\sqrt{f_c'}$ = 2 (while the pdf for the small beam range lies almost entirely above this line). This means that if the type of concrete, steel ratio, shear span, humidity and temperature conditions, *etc.*, used in the single Toronto test were varied through the entire range occurring in practice (exemplified by the variation in the small size range), a large percentage of the beams would likely be found to be unsafe. According to our assumption of log-normal pdf and equality of distances a for small and large sizes, the proportion of unsafe 1 m (40 in.) deep beams would be about 40%, while for small beams 100 to 300 mm (4 to 12 in.) deep it is only 1.0%. This

is not acceptable. A design code known to have such a dangerous property cannot be sanctioned.

Failure Probability for Large and Small Beams

To determine precisely the consequences for failure probability P_f of the beam, we need to consider also the pdf of the extreme loads expected to be applied on the structure, which is denoted as $f(y)$. To calculate P_f, we need to consider a certain value of the load factor. We will consider only the load factor of 1.6, which is applicable to the cases where the live load dominates, as is the case for bridge beams up to 1 m (40 in.) deep (for load combinations with a significant dead load component, for which the blended load factor is less than 1.6, the failure probabilities for both small and large beams would be higher than those obtained in what follows, but their ratio, which is of main interest, would be about the same).

The distribution of the applied extreme loads will be considered as log-normal (it is debatable whether the Gumbel distribution might be more realistic [36,37], but it would make little difference for the ratio of probabilities and would make the calculation more tedious). The C.o.V. of the applied extreme loads will be considered as 10%. Under the foregoing assumptions, and based on the understrength factor of = 0.75, the mean of the pdf of extreme applied loads and function $f(y)$ representing this pdf will be positioned as shown in Fig. 4(b). The failure probability may now be calculated from the well-known reliability integral [38-40]

$$P_F = \int_0^\infty f(y)\, R(y)\, dy \qquad (2)$$

where $R(y)$ is the cumulative distribution function (cdf) of structural resistance, which is obtained by integrating the log-normal pdf in Figs. 4(a) and 4(b).

When this integral is evaluated for small beams within the range of depths d from 100 mm (4 in.) to 300 mm (12 in.), centered at d = 200 mm (8 in.), and also for the large beams of 1 m (40 in.) depth, one obtains the following failure probabilities:

$$\begin{aligned} &\text{for beams 0.2 m deep:} \quad P_f \approx 10^{-6} \\ &\text{for beams 1.0 m deep:} \quad P_f \approx 10^{-3} \end{aligned} \qquad (3)$$

The failure probability of 10^{-6}, *i.e.*, one in a million, obtained for small beams, corresponds to what the risk analysis experts generally consider as the maximum acceptable for engineering structures in general,[22-24] because it does not appreciably add to the inevitable risks that people face anyway.

So, if the size effect in beam shear were ignored for beams without stirrups up to 1 m deep, the probability of failure for 1 m (40 in.) depth would be about 1000-times

greater than for 200 mm (8 in.) depth. This would be unacceptable. If there should be any difference, it should be in the opposite sense because, for large beams, the failure consequences are usually more serious than for small ones.

Statistical Analysis Overcoming Bias in the Database

Sound arguments for a realistic design formula capturing the size effect on shear strength of beams must be based on fracture mechanics, verified by properly designed experiments, and statistically calibrated by a broad database. For many engineers, though, a purely statistical evidence, with no use of mathematics and mechanics, is most convincing. Such evidence can be, and has been, readily provided for many design problems where experiments are easy to perform through the entire range of all parameters. But the problem of size effect is different.

In the case of size effect, it is financially prohibitive to conduct experiments through the entire range of beam depths of practical interest, which spans from 0.05 m to perhaps 14 m (the latter being the depth of the record-setting box girder in Palau, whose compression-shear collapse must be partly attributed to size effect). Obtaining statistics and covering by experiments the full range of influencing parameters other than the size (or beam depth) has been easy for small beams, but is almost impossible for very large ones. Thus it is not surprising that the existing ACI database [1,2] has major gaps and a strong subjective statistical bias caused by crowding of the test data in the small-size range, scant data in the large size range, and no data at all for the largest sizes of practical interest (depths > 2 m). Consequently, simple bivariate statistical regression of all the points of the ACI-445F database yields a misleading trend.[29,30] Eliminating the bias is important for a realistic update of the code provisions currently under consideration for the design codes of many countries.

The size effect is defined as the size dependence of the nominal strength of structure when geometrical similarity is maintained and all the parameters other than the size are kept constant. In the case of beam shear, the size may be measured by the beam depth d, the nominal strength of structure may be taken as the average concrete shear strength in the cross section, v_c, and the parameters that must be kept constant comprise all the concrete

properties (including the maximum aggregate size d_a), the longitudinal reinforcement ratio ρ_w, and the shear span ratio a/d (here a = distance of the load from the support).

If the entire database on size effect

in beam shear were to be obtained in one testing program in one laboratory, a sound statistical design of size effect experiments would dictate choosing the same number of tests in equally relevant size intervals and maintaining within all the size intervals the same means and distributions of parameters ρ_w, a/d, d_a, over their entire practical range. This condition is far from satisfied by the existing database. But there is no other choice. So the question is how to minimize the statistical bias in regard to the size effect. From the size effect viewpoint, this database has a bias of two kinds:

- **Kind 1.** Crowding of the data in the small size range — 86% of the 398 data points pertain to three-point-loaded beams of depths less than 0.5 m, and 99% to depths less than 1.1 m, while only 1% of data pertain to depths from 1.2 to 2 m.
- **Kind 2.** Strongly dissimilar means and distributions, among different size intervals, of the subsidiary influencing parameters, particularly the steel ratio ρ_w, shear span ratio a/d, and the maximum aggregate size d_a.

To reach any meaningful statistical conclusion on the size effect, both kinds of bias must be filtered out.

Statistical Regression of Size Effect

We want to isolate the trend of size effect from a database governed by multiple variables. The standard way to do that is to carry out multivariate least-square nonlinear regression in which all the parameters are optimized simultaneously. This is the approach which was pursued in previous work.[29,30] There is another way, though. It does not lead to multivariate regression, yet makes the statistical trend conspicuous without any mathematics. To this end, an unbiased (*i.e.*, objective) procedure of data filtering is required.

Let us subdivide the range of beam depths d of the existing test data into 5 size intervals (vertical strips in Figs. 5(a) to 5(c)). They range from 0.075 to 0.15 m, from 0.15 to 0.3 m, 0.3 to 0.6 m, from 0.6 to 1.2 m, and from 1.2 to 2.4 m. In the ACI database, these intervals contain 26, 251, 80, 38, and 3 data points, respectively; see Figs. 5(a) to 5(c). Note that the borders between the size intervals are chosen to form a geometric (rather than arithmetic) progression because what matters for size effect is the ratio of sizes, not their difference (to wit, from d = 0.1 to 0.1 + 1 m, the size effect is strong, from 10 to 10 + 1 m negligible). The chosen intervals are constant in the scale of log d, and this is also needed for another reason — in the plot of $y = v_c / \sqrt{f'_c}$ versus d, the database is heteroscedastic (*i.e.*, has a variance density decreasing with size), but

transformation to the plot of $\log(v_c / \sqrt{f'_c})$ versus log d renders the database almost homoscedastic (*i.e.*, of uniform variance density), which is necessary for meaningful regression analysis.[41] Figures 5(a) to 5(c) shows the restricted (filtered) data points by bigger circles, and those filtered out by the tiny circles.

The problem with the distribution of subsidiary influencing parameters in the full database is graphically documented by Figs. 6(a) and 6(b), in which the diamonds show their means in the individual size intervals, and the error bars show the span from the minimum to the maximum retained value (Figs. 6(c) to 6(f) shows the same plots achieved by filtering the database). In Fig. 6(a), the mean of ρ_w is in the second interval nearly 7-times larger than in the last interval (and almost 2-times larger than in the fourth interval). In Fig. 6(b), the mean of a/d is in the third interval 30% larger than in the last interval (and 10% larger than in the fourth interval). Obviously, such differences among size intervals must completely distort size effect statistics.

To filter out the effect of influencing parameters other than d, each interval of d must include only the data within a certain restricted range of ρ_w-values such that the average $\overline{\rho_w}$ would be almost the same for each interval of d. Similarly, the range of a/d and d_a in each interval must be restricted so that the average $\overline{a/d}$ and $\overline{d_a}$ would also be about the same for each interval of d. The filtering of data must be done in an objective manner (*i.e.*, with no human preference). To this end, a computer optimization algorithm has been formulated. It progressively deletes from each interval, one by one, the data points in each size interval that lie at the top and bottom margins of the ranges of ρ_w, a/d and d_a, until uniformity of each subsidiary influencing parameter throughout all the intervals is optimally approached.

Because, as generally agreed, the effect of the specified concrete strength f'_c is adequately captured by assuming the shear strength of cross section, v_c, to be proportional to $\sqrt{f'_c}$, we do not need to restrict the range of f'_c and may obtain the ordinate $\overline{y}$ of data centroid in each interval by averaging, within that interval, not the v_c-values but the values of $y = v_c / \sqrt{f'_c}$ that fall into the aforementioned restricted ranges of ρ_w, d/a and d_a.

As seen in Fig. 5, there are only three test data in the size interval spanning 1.2 to 2.4 m. The first has the longitudinal steel ratio of ρ_w = 0.14%, the second 0.28% and the third 0.74%. The extremely low ρ_w of the first two makes it impossible to find similar data in other intervals of d. For example, the minimum ρ_w is 0.91% within the first interval of d, and 0.46% within the third interval. Therefore, one must consider the size range from 0.075 to 1.2 m. Formulating a statistical opti-

mization algorithm for database filtering (to be presented in a forthcoming journal article), one finds 7, 68, 17, and 36 data points within the admissible ranges for each interval of d (ideally, of course, the number of data in each interval should be the same, and the fact that it is not shows that complete elimination of statistical bias is impossible; nevertheless for obtaining reliable means, 7 data certainly suffice).

After filtering, the mean values of ρ_w for the restricted ranges are 1.51%, 1.5%, 1.5%, and 1.5%, the mean values of a/d are 3.45, 3.33, 3.33, and 3.23, respectively, and the mean values of d_a are 16.8, 17.0, 16.8, and 16.5 mm. This provides data samples with minimum bias in terms of ρ_w, a/d and d_a. The data centroids for each interval are plotted as the diamond points in the plot of $\log(v_c/\sqrt{f_c'})$ versus $\log d$ (Fig. 5(d)). We see that, despite enormous scatter in the database (Fig. 5(d)), the trend of these centroids is quite systematic.

Under the assumption that the statistical weight of each size interval centroid in Fig. 5 is the same, the foregoing procedure is used to obtain the optimum least-square fit of these 4 centroids with the classical size effect law (type 2 energetic size effect law [42]), which was proposed for beam shear in 1984 [7] and recalibrated in 2005,[29,30] and is written here as $v_c/\sqrt{f_c'} = C(1+d/d_0)^{-1/2}$ where C, d_0 = free constants to be found by the fitting algorithm (for reasons of

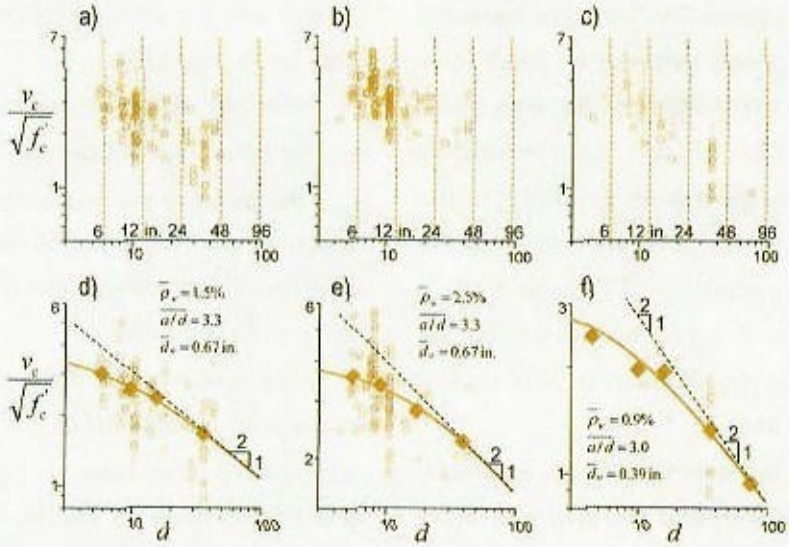


Fig. 5 ACI-445F Database and Statistical Regression of Centroids of Test Data Subdivided into Intervals of Equal Size Ratio. (a-c) Full Database (the data retained are shown by larger circles and those filtered out in various cases by tiny circles); (d-f) Filtered Restricted Data Giving the Indicated Combinations of Uniform Mean Values of Subsidiary Parameters, Their Centroids and Regression Curves.

proper weighting, it is best to conduct nonlinear regression with a nonlinear optimization subroutine, although a linear regression in transformed variables is possible and acceptable [6]). The resulting fit of the centroids (the solid curve) is seen to be quite close; it gives, for predicting the mean strength, a very small coefficient of variation of errors, namely ω = 2.5% = standard deviation of the optimum fit curve from the centroids, divided by the data mean (for individual beams, ω is, of course, much larger). This coefficient of variation characterizes the uncertainty in the mean strength of many structures of a given size rather than in the strength of an individual structure, which is of main interest for design. The negative curvature of the trend of the centroids confirms the theoretically predicted [8] gradual transition from quasi-plastic behavior for small sizes to perfectly brittle behavior for large sizes. The trend of the last two centroids roughly matches the theoretical prediction of the slope −1/2 of the final asymptote of the size effect curve,[7,13,29,30,42] given by $v_c \sim d^{-1/2}$ (which is a property unanimously endorsed as fundamental in 2004 by ACI Committee 446).

Using the same statistical algorithm, let us now increase the average steel ratio for each interval to 2.5%. The fitting of the centroids is shown in Fig. 5(e). The asymptotic slope of −1/2 is confirmed and the negative curvature is obvious.

To increase the size range, one may further include one point from the largest size interval spanning 1.2 to 2.4 m, namely the Toronto beam with ρ_w = 0.74%; see Fig. 5(f). Admittedly, one data point is too little, but nothing more exists because of the cost of testing very large beams. Then the same procedure as above is followed and, for the other 4 intervals of d, one finds 1, 2, 5, and 15 data points for which the means of ρ_w in the interval of d are 0.91%, 0.94%, 0.94%, 0.91%, and 0.74%, while the mean of a/d (= 2.9) and the mean maximum aggregate size d_a (= 10 mm) are the same for each interval. The coefficient of variation of errors of mean prediction now is ω = 5%, and the size effect trend is very clear. Again, the trend agrees well with the asymptotic slope of −1/2 and with the energetic size effect law (solid curve, Fig. 5(f)).

Now, an important point to note is that, for different averages $\overline{\rho_w}$, $\overline{a/d}$ and $\overline{d_a}$, the trend of the interval centroids is the same, and closely matches the size effect law. This demonstrates objectivity of the data filtering approach.

Also note that the present statistical results lend no support to the previously proposed power laws $v_c/\sqrt{f'_c} = Cd^{-1/4}$ based on Weibull's statistical theory.[43] Neither do they lend any support to the asymptotic size effect $v_c/\sqrt{f'_c} = Cd^{-1}$ which is implied by an alternative model [44,45] based on MCFT (Modified Compres-

sion Field Theory) (besides, an exponent of magnitude > 1/2 is impossible thermodynamically as well as from the viewpoint of material strength randomness [8,9,28]).

Variance of Individual Data Via Weighted Regression

Kinds 1 and 2 of bias afflict not only the mean trend of the full database, but also its scatter. The scatter may be measured by an unbiased coefficient of variation of the errors of the optimum fit curve compared to the individual data points. This is the error that must be considered for safe design. It can be ascertained by one of two methods:

1) One method is a simple bivariate nonlinear regression of our filtered restricted database, in which the kind 2 bias is already suppressed. To suppress the kind 1 bias, one needs to give the same weight to the data in each size interval i, regardless of the number mi of the points that fall into that interval. This may be achieved by assigning to the data in each interval i the normalized weight $w_i = (1/m_i)/\sum_k (1/m_k)$. Nonlinear regression, *i.e.*, the minimization of the weighted sum of square deviations from the size effect law, then yields the coefficient of variation of 22.3% for the filtered database with $\bar{\rho}_w$ =1.5%, and 23.6% for that with $\bar{\rho}_w$ = 2.5% (Fig. 7).

2) The other method, which is the standard one, is a multivariate weighted nonlinear regression of the entire database. Compared to the first method, there is the complication that, instead of filtering the database, one must judiciously select the mathematical functions describing the dependence of the parameters C and d_0 of the size effect law for shear strength on the subsidiary influencing parameters ρ_w, a/d and d_a, and then optimize simultaneously the coefficients of all these functions by minimizing the variance of errors. Proper choice of these functions suppresses the kind 2 bias. The kind 1 bias is in Ref. 30 minimized by weighting the data points in inverse proportion to the value of a smoothed histogram of the number of tests versus size. The result is quite similar to the first method — the coefficient of variation is 19.0%, after transformation to the variable $y = v_c / \sqrt{f'_c}$. However, the range from the minimum to the maximum value of each subsidiary parameter (Fig. 6) fluctuates, from one size interval to the next, more than in the first method (ideally, the range should be the same for all the intervals, and the fact that it is not introduces some extra measure of bias, which cannot be removed although it prob-

ably is small).

The effect of data weighting can further be clarified by Figs. 7(a) and 7(b) where the solid curves are the bivariate nonlinear regression curves of the interval centroids, with the same weight on each centroid. As one can see, almost undistinguishable curves (dashed ones) are obtained by the weighted nonlinear bivariate statistical regression of all the data points in the restricted (filtered) database. An unweighted regression of the same data points is shown in Fig. 7 by the dash-dot curves, and, as we can see, the dash-dot curve is again hardly distinguishable from the regression curve of the centroids in Fig. 7(a), but is very different in Fig. 7(b). One reason for this difference is that the vertical ranges of the restricted data in the individual size intervals, marked by vertical bars, are in Fig. 7(a) nearly symmetric with respect to the centroid curve, but not in Fig. 5(b). Another reason is that the restricted database in Fig. 7(a)

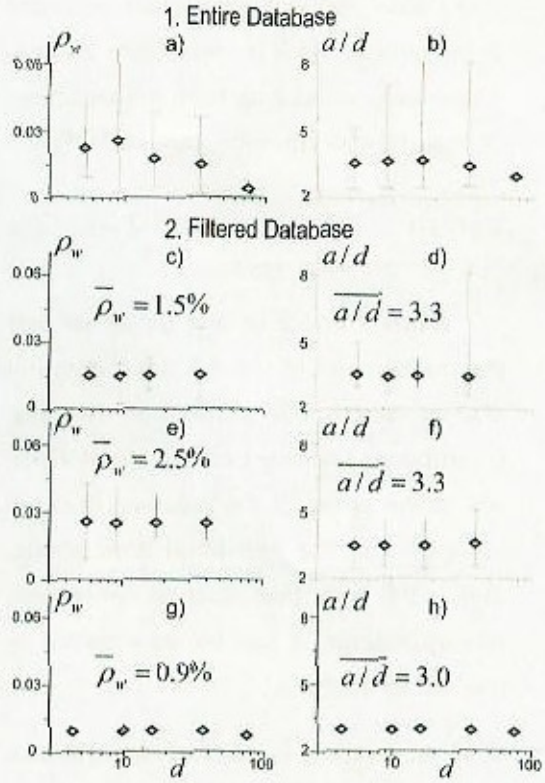


Fig. 6 Interval Centroids and Spread between the Maximum and Minimum Values of Reinforcement Ratio ρ_w and Shear Span Ratio a/d ; (a,b) for Full ACI-445F Database; (c,d) for Restricted Database with Mean $\rho_w \approx 1.5\%$, $a/d \approx 3.3$; (e,f) Ditto but $\rho_w \approx 2.5\%$, $a/d \approx 3.3$; (g,h) Ditto but $\rho_w \approx 0.9\%$, $a/d \approx 3.0$.

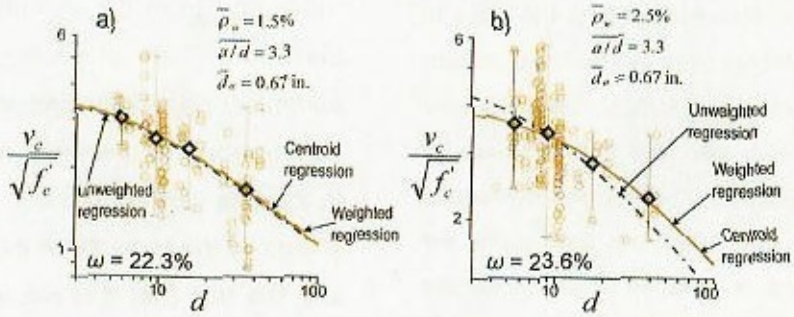


Fig. 7 Regression Curves Corresponding to Weighted Fitting (Dashed Curves), Unweighted Fitting (Dash-dot Curves) and Fitting on Centroids (Solid Curves) for Filtered Database of (a) Average Steel Ratio = 1.5%; and (b) Average Steel Ratio = 2.5%.

is roughly homoscedastic, while that in Fig. 7(b) is not.

For comparison, the coefficient of variation of the multivariate nonlinear regression conducted on the entire database 2 is 15% if the data are weighted, and 17% if unweighted. When only the 11 beams deeper than 1 m are considered, the coefficient of variation is 14% if these data are weighted and 16% if unweighted. As we see, the weighted regression gives a better prediction for the scatter of shear strength of large beams.

Size Effect for Concrete Beams with Stirrups

Although much information exists on the size effect on reinforced concrete beams without shear reinforcement, there is little information on the size effect in shear failure of beams with minimum or heavier shear reinforcement (stirrups). Many engineers are of the opinion that beams with minimum or heavier stirrups exhibit no size effect. However, this opinion is incorrect and would lead to unsafe designs for large structures. Computational simulations, and even the limited experimental evidence that exists, reveal that stirrups do not eliminate the size effect. They only mitigate it. According to the analysis by Bažant,[48] the energetic size effect law [42] remains valid and the effect of stirrups is to increase the transitional size d_0.[9] Avoidance of size effect would require elimination of post-peak softening on the load-deflection diagram, and this could be achieved only if the concrete were subjected to triaxial confinement with all negative principal stresses exceeding in magnitude several times the uniaxial compression strength.

The test series conducted by Walraven *et al.*[19] clearly show that there is a strong size effect for deep beams with $a/d < 2$ (Fig. 8(a)) to which the strut-and-tie model is applicable. As is well known, if the failure is triggered by the compression crushing of concrete strut, it typically exhibits size effect.[8] For slender beams with $a/d > 2$, two test series are found in the literature:

- tests conducted by Bhal [11] in 1968 in Stuttgart, in which the shear span ratio is $a/d = 3$, the shear reinforcement is heavier than the minimum requirement, and the size range is almost 1 : 4.
- tests conducted by Kong and Rangan [20] in 1998 in Perth, in which the shear span ratio is $a/d = 2.4$, the shear reinforcement is heavier than the minimum requirement, and the size range is 1 : 3.

When plotted in the logarithmic scale (Figs. 8(b) and 8(c)), it can be seen clearly that, in both data sets, the shear strength

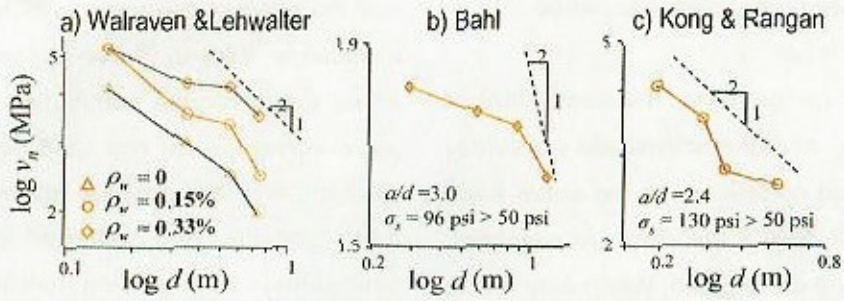


Fig. 8 Size Effect Test of Concrete Beams with Stirrups. (a) Deep Beam with $a/d = 1$; (b) Slender Beam with $a/d = 3$; (c) Slender Beam with $a/d = 2.4$.

markedly decreases with increasing beam depth. The asymptotic size effect trend of slope −1/2 does not contradict these test results.

Extensive finite element simulations based on the crack band model and micro-plane model have also been carried out to investigate whether the shear failure of beams with stirrups exhibits a size effect. The beam geometry considered in these simulations is the same as in the Toronto tests. [3,4,18] Computations are run for geometrically similar beams of depths 0.47 m, 1.89 m, which is the size of Toronto test, and 7.56 m. The stirrups and longitudinal bars are assumed not to slip (although the bond slip was found to play only a minor role and tend to intensify the size effect).

The mesh and the computed cracking pattern at maximum load are shown in Fig. 9(a), and the simulated load-deflection diagrams are shown in Fig. 9(b), for all the sizes. The diagram for d = 1.89 m (the size tested in Toronto) shows the peak load of 283 kips. This is very close to the value measured in Toronto. The yield plateau observed in this test is also well reproduced by the simulation. However, for the largest beam simulated, the yield plateau disappears and the load descends steeply right after the peak. Fig. 9(c) shows the dependence of the average beam shear strength $v_n = V/b_w d$ on beam depth d, and Fig. 9(d) shows the same for the average shear strength $v_c = v_c/b_w d$ contributed by concrete ($= V - V_s$, $V_s = A_s f_y d/s$; A_s, s = stirrup area and spacing). Compared with the concrete beams without stirrups tested at same laboratory, the transitional size d_0 shown in Figs. 9(c) and 9(d) is significantly increased. These plots document that a strong size effect exists also in the beams with stirrups, although it is pushed into larger sizes. The asymptotic slope of −1/2 is seen to remain.

Together with the experimental evidence, the finite element simulations clearly demonstrate that the shear rein-

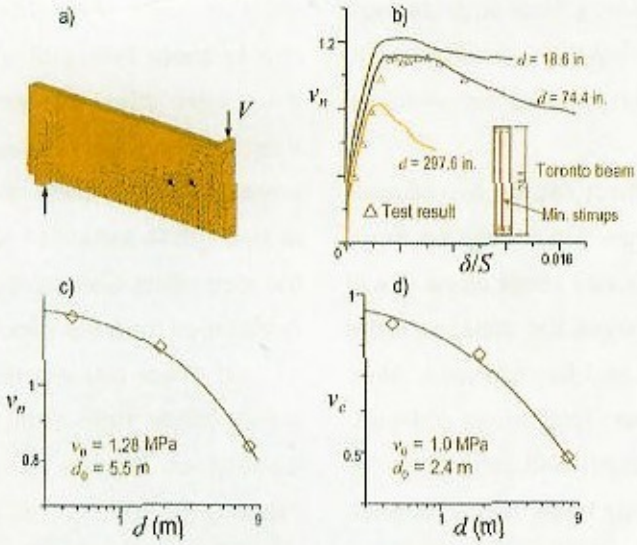


Fig. 9 Computational Simulations of Toronto Beam with Minimum Stirrups. (a) Mesh and Cracking pattern at failure; (b) Load-deflection Curves Generated by Simulations; (c) Size Effect Fitting of the Total Shear Strength; (d) Size Effect Fitting of the Concrete Shear Strength

forcement, whether minimum or heavier than minimum, is unable to suppress the size effect. It mitigates the size effect in the larger size range, but not enough by far to make it negligible.

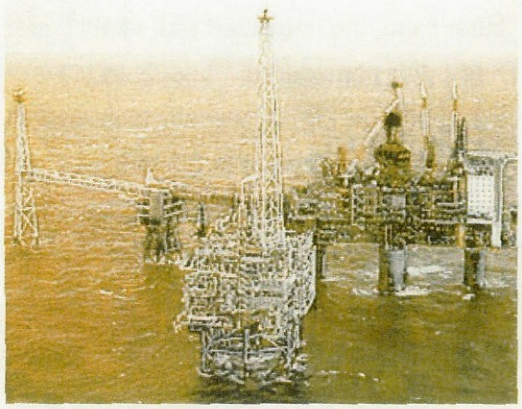
Some Catastrophic Collapses with a Role of Size Effect

The overall safety factor μ, although not used in the current codes, is defined as the mean of failure test data divided by the mean (or unfactored) design load. The part of μ of concern here is the understrength factor. Besides the overt understrength factor ϕ characterizing the brittleness of failure mode, there also exists a covert understrength factors ϕ_r due to the design formula error and ϕ_m due to the material randomness.[47] Consequently, for shear failure of longitudinally reinforced concrete beams without stirrups, the overall safety factor currently is $\mu \approx 3.8$ for small sizes and $\mu \approx 1.7$ for large sizes. The former is totally dominated by the live load, and the latter is totally dominated by the self weight. In the latter case, the neglected size effect factor has been considered [38] as 2. In view of the scatter in Fig. 2, the individual overall safety factors vary within 2.3 to 6 for small sizes, and 1.05 to 2.8 for large sizes. The very large values of these safety factors are doubtless one reason why, despite the neglect of size effect, there have not been many more structural collapses than actually experienced. These large values also reveal that, in concrete engineering

(by contrast to aeronautical engineering), a single error in design or construction is usually not enough to bring the structure down.

The size effect factor for normal concrete structures can hardly be more than 2, and so the size effect alone would rarely suffice to cause the collapse if the material strength and formula error have nearly mean values. To produce collapse, the material strength and formula error must simultaneously have values of small probability, far from the mean. Thus, at least two, and typically three, simultaneous mistakes or lapses of quality control are needed to make a concrete structure collapse. This makes it easy for an investigating committee to blame collapse entirely on the other factors and ignore the theoretically more difficult size effect. For example, in the case of catastrophic sinking of Sleipner oil platform in a Norwegian fjord in 1991 (Fig. 10(a)), which was due to shear failure of a thick tricell wall, there were three simultaneous mistakes. Besides two mistakes recognized by government forensic committee, the necessity of strength reduction of about 34% due to the size effect was pointed out by Bažant but omitted from the conclusions.

Of major interest for the size effect theory is the 1996 collapse of the Koror-Babeldaob Bridge in the Republic of Palau (Fig. 10(b)). This prestressed box girder had the world record span of 241 m when it was built in 1976. In addition to the erroneous initial prediction of creep and shrinkage deflections and apparently inappropriate remedial prestressing, one would have to expect a major strength reduction due to size effect on the compression-shear failure seen in the photograph. Analysis of this collapse would offer a unique opportunity to check



(a) Sleipner Oil Platform, 1991.

(b) Koror-Babeldaob Box Girder in Republic of Palau, 1996.

Fig. 10 Examples of Catastrophic Failures of Concrete Structures.

and calibrate the size effect theory but, incredibly, all the technical information was after litigation sealed by a court verdict. Scientific ethics demands this verdict to be reversed, in the interest of progress (imagine, *e.g.*, that all the technical information on the collapse of Tacoma Narrows Bridge were suppressed).

Another reason why structural collapses have not been more numerous is that most codes, unwittingly, hide a partial (thought imperfect) protection against size effect in an excessive value of the load factor for self-weight, which is 1.4 for the self-weight acting alone, according to the current ACI code. In small structures, the self-weight is a negligible part of the load, and so the value of self-weight load factor does not matter. But in a very large bridge, self-weight alone is the decisive loading case. Now, how could the self-weight be 40% larger than assumed in design? This is inconceivable (except as a sabotage). At most it could differ by a few percent. So very large structures are penalized by almost 40% compared to small ones. This way most codes give a covert protection against the neglect of size effect.[48] But such covert protection is insufficient, by far, for very large structures. It also exhibits an incorrect trend from the viewpoint of size effect,[48] as well as other wrong features. *E.g.*, it gives greater protection to unprestressed or normal concretes compared to prestressed or high-strength concretes, because they lead to heavier structures (although the opposite should be the case because they are much more brittle); it gives too little protection to columns compared to beams; *etc.* This covert size effect should be eliminated and replaced by introducing the proper size effect in the code formulae.

Question of Concern to Concrete Societies: Legal Exposure

In the face of ever increasing diversification of science, it is nowadays impossible for the code-making committees, typically composed of the best and most renowned engineers, to follow in detail all the recently solidified scientific advances relevant to the building code article or recommended practice that they are developing. Nevertheless, keep informed they must. A quarter century ago, when the experimental data were scant and scattered, and only a handful of scientists espoused a coherent scientific theory, it was entirely plausible and defensible for concrete societies to ignore the size effect. When a failure attributable to size effect occurred, they could not be held liable. Not any more. The experimental evidence has become undeniable and the theoretical basis solid. Virtually all the researchers in fracture mechanics

of concrete and entire research-oriented societies and committees in this field (*e.g.*, IA-FraMCoS, ASCE-EMD, ACI Committee 446) have no doubt that a significant non-statistical size effect exists in all the brittle failures of concrete structures. Consequently, ignoring the size effect for the sake of simplicity, or even sanctioning a simplistic or partial consideration of size effect that is now known to imply a significantly increased risk of failure of large structures, is no longer acceptable. It might expose concrete engineering societies to legal liability when another catastrophe occurs.

Conclusion

At the dawn of this century, the size effect in brittle failures of concrete structure has become an established fact. It is time to introduce it into the design codes and practice. Ignoring it will cause large structures to be failing with the frequency of about one per thousand or more, instead of less than one per million as generally considered tolerable for engineering structures. The human society must not be knowingly exposed to such a risk.

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