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Minimizing Statistical Bias to Identify Size Effect from Beam Shear Database

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The existing database for size effect on shear strength of reinforced concrete beams without stirrups has a bias of two types: 1) Most data points are crowded in the small size range; and 2) the means of the subsidiary influencing parameters, such as the steel ratio and shear-span ratio are very different within different intervals of beam size (or beam depth). To minimize Type 2 bias, the database must be properly filtered. To this end, the size range is first subdivided into intervals of constant size ratio. Then, in each size interval, a computer program progressively restricts the range of influencing parameters both from above and from below, until the mean of the influencing parameter values remaining in that interval attains about the same value in all the size intervals. The centroids of the filtered shear strength data within the individual size interval are found to exhibit a rather systematic trend. Giving equal weight to each interval centroid overcomes the Type 1 bias. The centroids can be closely matched by bivariate least-square regression using Bažant's (energetic) size effect law which was proposed for beam shear in 1984 and in detailed form in 2005. This purely statistical inference of minimized bias also supports the previous fracture-mechanics-based conclusion that, for large sizes, the bi-logarithmic size effect plot must terminate with the asymptotic slope of –1/2. Similar filtering of the database gives further evidence for the previous empirical observation that the shear strength of beams is approximately proportional to the 3/8-power of the longitudinal reinforcement ratio.

Keywords: failure probability; fracture mechanics; scaling of failure; size effect; shear strength; statistical analysis;

INTRODUCTION AND NATURE OF PROBLEM

Sound arguments for a realistic design formula capturing the size effect on shear strength of beams have to be based on fracture mechanics, verified by properly designed experiments, and statistically calibrated by a broad database. To some engineers, though, a purely statistical evidence, with no use of mathematics and mechanics, is most convincing. For many aspects of concrete design where experiments are easy through the entire range of all parameters, such statistical evidence can be, and has been, readily provided.

In the case of size effect, however, it is financially prohibitive to conduct experiments through the entire range of beam depths of interest, which spans from 2 in. (0.05 m) to perhaps 551 in. (14 m) (which was the depth of the box girder in Palau, whose compression-shear collapse is a type of brittle failure notorious for size effect). Obtaining statistics, and covering by experiments the full range of influencing parameters other than the size, has been easy for small beams, but is almost impossible for very large ones. Thus it is not surprising that the existing database has major gaps and a strong subjective statistical bias caused by crowding of the test data in the small-size range, scant data in the large-size range, and no data at all for the largest sizes of practical interest (depths >118 in. [3 m]). Consequently, simple bivariate statistical regression of all the points of the database yields a misleading trend.[1] Eliminating the bias is important for a realistic update of the code provisions currently under consideration for the design codes of many countries.

The shear strength of beams is generally characterized by $v_c = V/bd$, which coincides with what is in the size effect theory known as the nominal strength; d is the effective depth equal to the distance from the top face to the longitudinal reinforcement centroid; and b is the beam width. The ACI-445F database[2] for shear strength of longitudinally reinforced concrete beams with no stirrups (ACI Committee 445), obtained mostly under three- or four-point bending (beams under distributed load are excluded), has a bias of two types: 1) crowding of the data in the small size range: 86% of the 398 data points pertain to beam depths less than 20 in. (0.5 m) and 99% to depths less than 43 in. (1.1 m), whereas only 1% of data pertains to depths from 48 to 79 in. (1.2 to 2 m); and 2) strongly dissimilar distributions, among different size intervals, of the subsidiary influencing parameters, particularly the longitudinal steel ratio ρ_w, shear span ratio (a/d), and the maximum aggregate size d_a (a is the shear span). To reach any meaningful statistical conclusion, both types of bias must be eliminated.

If the entire database on size effect in beam shear were to be obtained in one testing program in one laboratory, a sound statistical design of experiment would have dictated choosing the same number of tests in each size interval and maintaining within each size interval the same distribution of parameters f_c' (specified compressive strength of concrete), ρ_w, a/d, and d_a throughout the entire size range. There is no choice, however, but to use the database that exists. So the question is how to minimize its statistical bias.

RESEARCH SIGNIFICANCE

Minimizing the statistical bias in evaluation of a database on beam shear failure is important for improving the design provisions and ACI standard. The experimental support of many formulas in concrete design codes suffers from nonuniform sampling of the main variable throughout its range and from nonuniform means of subsidiary influencing parameters throughout the range of the main variable. The statistical procedure presented next can be used to improve the calibration of these formulas.

SIZE EFFECT REGRESSION

The size effect trend needs to be isolated from a database exhibiting statistical trends with respect to several other

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variables. The standard way to do that is to carry out multivariate least-square nonlinear regression in which all the parameters are optimized simultaneously.

This is the approach pursued in previous work.[1,3] There is another way, though. It does not lead to multivariate regression, yet makes the statistical trend conspicuous without any use of mechanics. To this end, the database must be filtered to minimize statistical bias.

The range of beam depths d of the existing test data may be subdivided into five size intervals (vertical strips in Fig. 1(a) to (c). They range from 3 to 6, 6 to 12, 12 to 24, 24 to 48, and 48 to 96 in. (0.075 to 0.15, 0.15 to 0.3, 0.3 to 0.6, 0.6 to 1.2, and 1.2 to 2.4 m), respectively. In the ACI-445F database, these intervals contain 26, 251, 80, 38, and 3 data points, respectively (refer to Fig. 1 (a) to (c)). Note that the borders between the size intervals are chosen to form a geometric (rather than arithmetic) progression because what matters for size effect is the ratio of sizes, not their difference. For example, the size effect is strong if d = 4 in. (0.1 m) is increased to 4 + 40 in. (0.1 + 1 m), but doubtless negligible if 394 in. (10 m) is increased to 394 + 40 in. (10 + 1 m). Thus, the chosen intervals are constant in the scale of log d. This is also needed for another reason[3]—in the plot of $v_c/\sqrt{f'_c}$ versus d, the database is heteroscedastic (variance decreasing with size),[2] but transformation to the plot of $\log(v_c/\sqrt{f'_c})$ versus $\log d$ renders the database almost homoscedastic (uniform variance), which is what is necessary for good regression statistics.[4-11] It has been checked that the results would not change substantially if four or six intervals were selected. For the present database, five intervals are preferred because, for more than five, some intervals would have less than three data points; and for less than five, the size effect within each interval would not be as close to negligible and the interval centroids would be too sparse to reveal the size effect transition clearly.

The problem with the distribution of subsidiary influencing parameters in the full database is graphically documented by Fig. 2(a) and (b), in which the diamonds show the mean of each parameter in each individual size interval, and the vertical error bars show the span from the minimum to the maximum retained value.

To filter out the effect of influencing parameters other than d, each interval of d must include only the data within a certain restricted range of ρ_w-values such that the average ρ_w would be almost the same for each interval of d. Similarly, the range of a/d and d_a in each interval must be restricted so that the averages $\overline{a/d}$ and $\overline{d}_a$ of the a/d and d_a values would also be about the same for each interval of d. The filtering of data must be done in an objective manner, that is, without any subjective human choice. This is achieved by formulating a general data filtering algorithm, which is presented in the Appendix. This algorithm progressively examines the data points currently located at the upper and lower margins of the data strip in each interval. It deletes them, one by one, if and only if the deletion causes the mean of that interval to

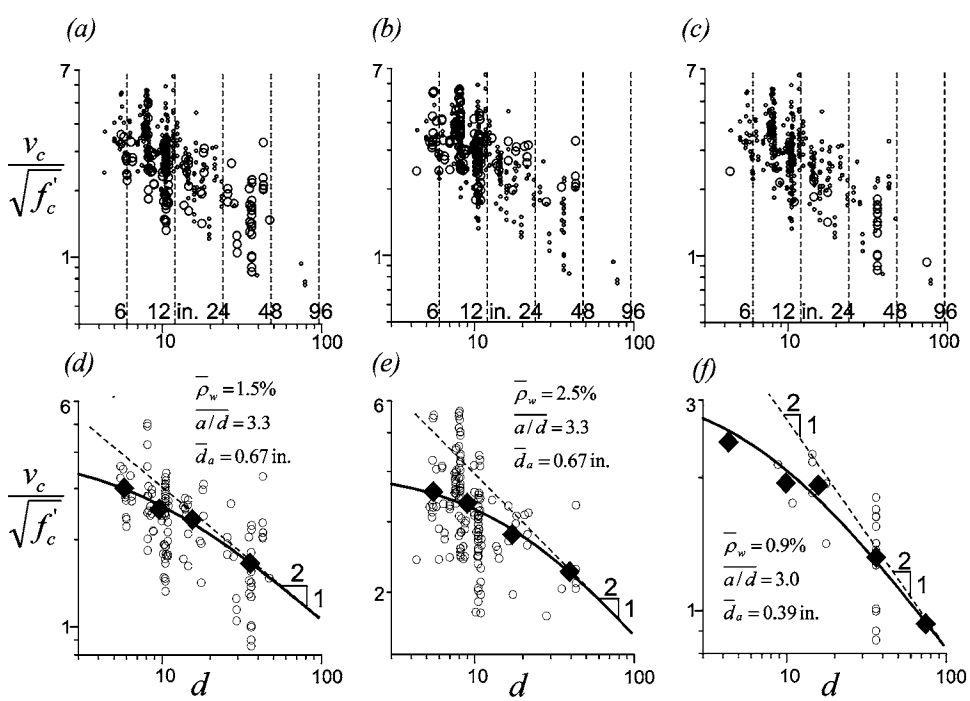

Fig. 1—ACI 445F database and statistical regression of centroids of test data subdivided into intervals of equal size ratio: (a) to (c) full database (data retained are shown by larger circles, and those filtered out in various cases by tiny circles); and (d) to (f) filtered restricted data giving the indicated combinations of uniform mean values of subsidiary parameters, their centroids, and regression curves.

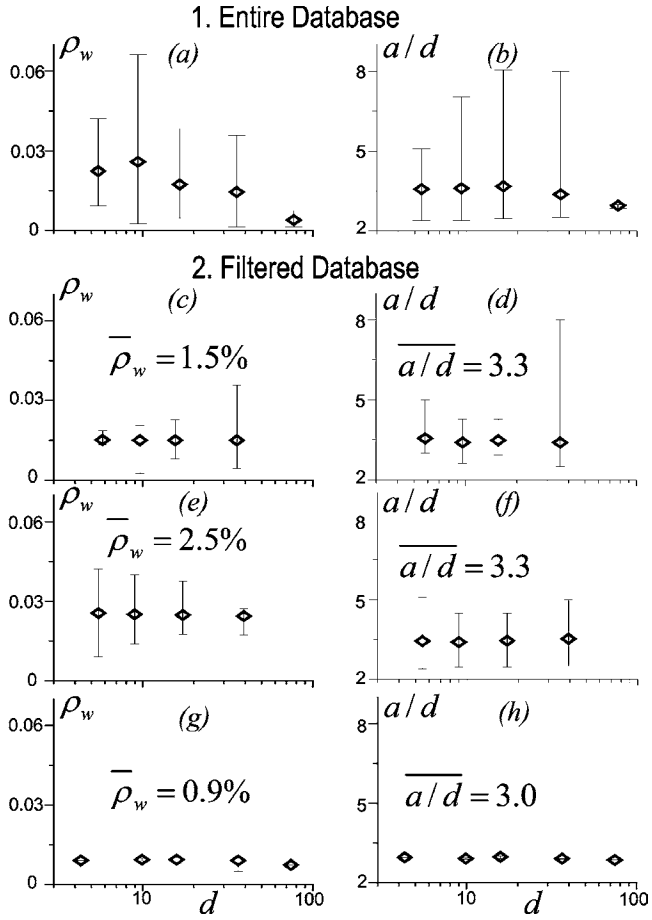

*Fig. 2—Interval centroids and spread between the maximum and minimum values of reinforcement ratio ρ_w and shear span ratio (*a/d*) for: (a) and (b) full ACI-445F database; (c) and (d) for restricted database with mean $\rho_w \approx 1.5\%$,* a/d *≈ 3.3; (e) and (f) for restricted database with mean $\rho_w \approx 2.5\%$,* a/d *≈3.3; and (g) and (h) for restricted database with mean $\rho_w \approx 0.9\%$,* a/d *≈3.0.*

move closer to the overall mean for all the intervals, until uniformity of each influencing parameter through all the intervals is optimally approached. The filtering has led to the data plots with minimized bias shown in Fig. 2(c) to (f).

According to ACI 318 and many studies,[1] the effect of the specified concrete compressive strength f_c' is adequately captured by assuming the shear strength of cross section v_c to be approximately proportional to $\sqrt{f_c'}$. Thus, fortunately, f_c' need not be considered as an independent parameter and the ordinate y of data centroid in each interval can be obtained by averaging, within that interval, not the v_c-values but the values of $y = v_c/\sqrt{f_c'}$ that fall into the aforementioned restricted ranges of ρ_w, a/d, and d_a. This is a useful simplification because, if approximate uniformity of f_c' were to be also required, the filtered data would be so few that no trend could be observed and a far larger database would be needed.

As seen in Fig. 1(a) to (c), there are only three test data in the size interval 48 to 96 in. (1.2 to 2.4 m). The first has the longitudinal steel ratio of $\rho_w = 0.14\%$, the second 0.28%, and the third 0.74%. The extremely low ρ_w of the first two makes it impossible to find similar data in other intervals of d. For example, the minimum ρ_w is 0.91% within the first interval of d and 0.46% within the third interval. Therefore, the size range from 3 to only 48 in. (0.075 to 1.2 m) needs to be considered. The statistical optimization algorithm, presented in the Appendix, is then applied to the database to filter 7, 68, 17, and 36 data points within the admissible ranges in each interval of d (ideally, of course, the number of data in each interval should be the same, and the fact that it is not shows that complete elimination of statistical bias is impossible; nevertheless, for obtaining reliable means, seven data certainly suffice).

For the restricted ranges, the mean values of ρ_w are 1.51, 1.50, 1.50, and 1.50%; the mean values of a/d are 3.45, 3.33, 3.33, and 3.23, respectively; and the mean values of d_a are 0.66, 0.67, 0.66, and 0.65 in. (16.8, 17.0, 16.8 and 16.5 mm). This provides data samples with minimum bias in terms of ρ_w, a/d, and d_a (refer to Fig. 3(a)). The data centroids for each interval are plotted as the diamond points in the plot of $\log(v_c/\sqrt{f_c'})$ versus $\log d$ (Fig. 1(d)). It can be seen that, despite enormous scatter in the database (Fig. 1(d)), the trend of these centroids is quite systematic.

Under the assumption that the statistical weight of each size interval centroid in Fig. 1 is the same, the foregoing procedure is employed to obtain the optimum least-square fit of these four centroids with Bažant's [12] size effect law (or Type 2 energetic size effect law[12-14]), which has been applied to beam shear in previous studies [1,15-18] and was unanimously endorsed in 2006 by a vote of ACI Committee 446, Fracture Mechanics.[19] This law is written as

$$v_c/\sqrt{f_c'} = C(1 + d/d_0)^{-1/2} \tag{1}$$

where C and d_0 are free constants to be found by standard least-square regression. For reasons of proper weighting (as well as homoscedasticity), it is preferable to conduct nonlinear regression with a nonlinear optimization subroutine; but a linear regression in transformed variables is generally good enough.[14]

The resulting fit of the centroids (the solid curve) is seen to be quite close, as characterized by the coefficient of variation (CoV) of regression errors, ω (that is, the standard error of regression[4-11] divided by the data mean). For predicting the mean strength, the CoV is very small, namely, $\omega = 2.5\%$, although for individual beams, ω is, of course, larger. The negative curvature of the curve connecting the centroids describes the transition from plastic behavior for small sizes to brittle failure for large sizes. The trend of the last two centroids conforms to the slope of the asymptote $v_c \propto d^{-1/2}$ of Bažant's size effect law[1,3,12,20] (ACI Committee 446, Fracture Mechanics, unanimously endorsed this property as fundamental by a vote in Vancouver, BC, Canada, in 2004).

The same statistical algorithm has further been applied to generate a filtered database for which the average steel ratio for each interval is 2.5% (refer to the larger circles in Fig. 3(b)). The fitting of the centroids is shown in Fig. 1(e). The asymptotic slope of –1/2 is again in agreement with the fit and the negative curvature is obvious.

To increase the size range, one may further include one point from the largest size interval spanning 48 to 96 in. (1.2 to 2.4 m), namely, the Toronto beam[21] with $\rho_w = 0.74\%$ (refer to Fig. 1(c)). Admittedly, one data point is too little, but nothing more exists because of the cost of testing very large beams. Then the same procedure as above is followed; and for the other four intervals of d, one finds 1, 2, 5, and 15 data points, and the means of ρ_w in the interval of d become 0.91%, 0.94%, 0.94%, 0.91% and 0.74%, respectively, while

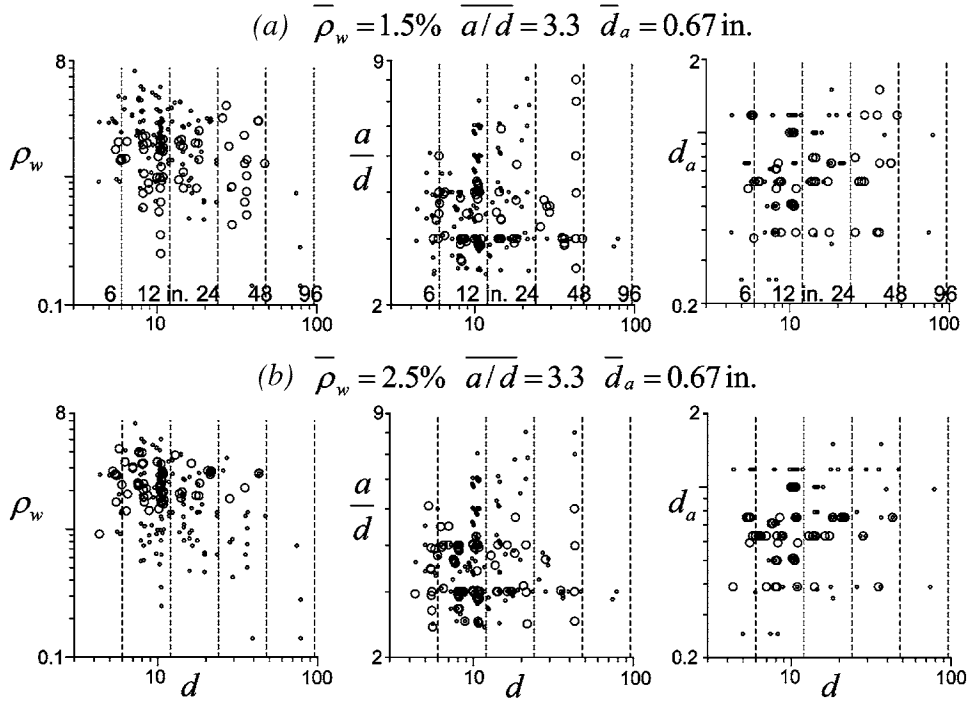

Fig. 3—The data remained (larger circles) compared with those filtered out (tiny circles) with regard to parameters ρ_w, a/d, and d_a: (a) for $\overline{\rho}_w$ = 1.5%, $\overline{a/d}$ = 3.3 and $\overline{d}_a$ = 0.67 in. (17 mm); and (b) for $\overline{\rho}_w$ = 2.5%, $\overline{a/d}$ = 3.3, and $\overline{d}_a$ = 0.67 in. (17 mm).

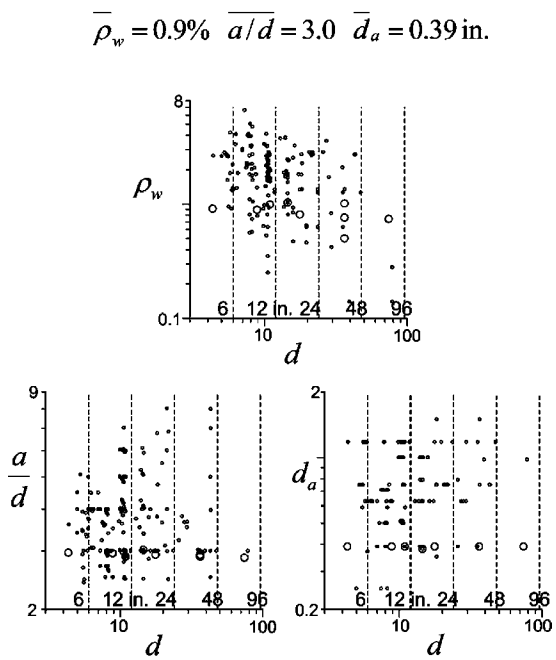

Fig. 4—Data that remain after filtering (larger circles) and data filtered out (tiny circles) for the case of $\overline{\rho}_w$ = 0.90%, $\overline{a/d}$ = 3.0 and $\overline{d}_a$ = 0.39 in. (10 mm).

the mean of a/d (= 2.9) and the mean maximum aggregate size d_a (= 0.39 in. [10 mm]) are the same for each interval (refer to Fig. 4). From regression statistics, the CoV of regression errors now is ω = 5%, and the size effect trend is very clear. Again, the trend agrees well with the asymptotic slope of –1/2 and with Bažant's size effect law (solid curve, Fig. 1(f)).

The foregoing bivariate regression with minimized statistical bias lends no support for the previously proposed power laws $v_c/\sqrt{f'_c} = Cd^{-1/4}$ based on Weibull's statistical theory.[22] Neither does it lend any support to an alternative model based on the Modified Compression Field Theory (MCFT) introduced by Vecchio and Collins[23] and Collins et al.,[24] especially not to the asymptotic size effect $v_c/\sqrt{f'_c} = Cd^{-1}$ implied by that theory.[1] This exponent magnitude, or any magnitude >1/2, is energetically impossible.[13,14,25] The reason, briefly, is that for geometrically similar structures of different sizes, the energy release rate $\mathcal{G}$ from a propagating crack is maximized when the fracture process zone size becomes negligible compared with the cross section dimensions. This happens as the size → ∞, and then the size effect is $v_c \propto d^{-1/2}$. For all other situations, $\mathcal{G}$ is less, which implies a size effect weaker than $d^{-1/2}$ and excludes d^{-1}. As for the statistical size effect theory, it can never yield a size effect stronger than $d^{-1/2}$.

VARIANCE OF INDIVIDUAL DATA VIA WEIGHTED REGRESSION

The bias of Types 1 or 2 afflicts not only the mean trend of the full database but also its scatter. The scatter may be measured by an unbiased CoV of the regression errors compared with the individual data points. This CoV provides the basis of safe design. It can be ascertained by one of two methods:

1. One method is a simple bivariate nonlinear regression of the aforementioned filtered restricted database, in which the Type 2 bias is already suppressed. To suppress the Type 1 bias, one needs to give the same weight to the data in each size interval i, regardless of the number m_i of the points that fall into that interval. This may be achieved by assigning to the data in each interval i the normalized weights $w_i = (1/m_i)/\Sigma_{k=1}^{N}\ (1/m_k)$, where N is the number of intervals and m_k is the number of remaining points in the k-th interval. Nonlinear regression, that is, the minimization of the

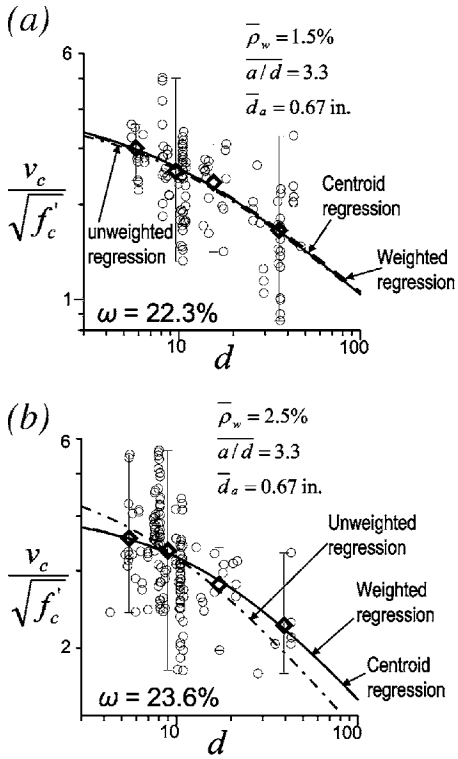

Fig. 5—Regression curves corresponding to weighted fitting (dashed curves), unweighted fitting (dash-dot curves), and fitting of centroids (solid curves) for filtered database of: (a) average steel ratio = 1.5%; and (b) average steel ratio = 2.5%.

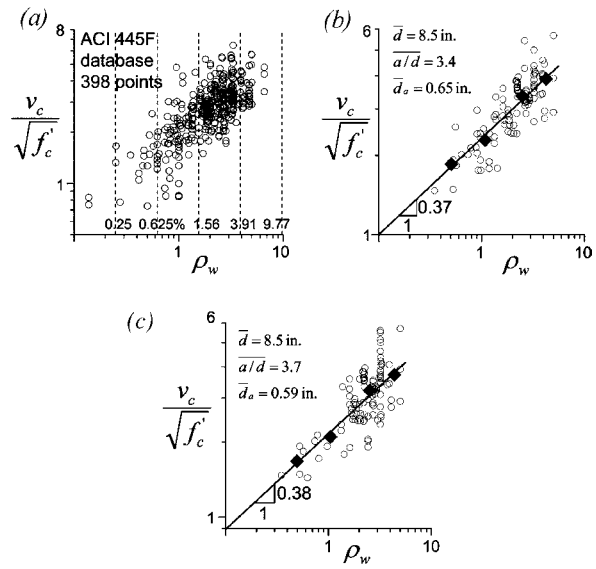

Fig. 6—ACI-445F database and statistical regression of centroids of test data for steel ratio with intervals of equal ratio: (a) data points and steel ratio intervals; (b) and (c) centroids of the last four intervals with data ranges restricted to the means indicated all centroids used.

weighted sum of squared deviations from Bažant's size effect law, then yields the CoV of regression errors; its value is 22.3% for the filtered database with $\overline{\rho}_w$ =1.5% and 23.6% for that with $\overline{\rho}_w$ = 2.5% (Fig. 5).

2. The other method, which is the standard one, is a multivariate weighted nonlinear regression of the entire database. Compared with the first method, there is the complication that, instead of filtering the database, one must judiciously select the mathematical functions describing the dependence of the parameters C and d_0 of the size effect law for shear strength on the subsidiary influencing parameters ρ_w, a/d, and d_a and then optimize simultaneously the coefficients of all these functions by minimizing the variance of regression errors. The Type 2 bias is suppressed by the proper choice of these functions. The Type 1 bias is in previous work [3] minimized by weighting the data points in inverse proportion to the value of a smoothed histogram of the number of tests versus size. The result is quite similar to the first method—the CoV of regression errors is 19.0%, after transformation to the variable $y = v_c/\sqrt{f_c'}$. The range from the minimum to the maximum value of each subsidiary parameter (Fig. 2), however, fluctuates from one size interval to the next, more than in the first method (ideally, the range should be the same for all the intervals, and the fact that it is not introduces some extra measure of bias, which cannot be removed although it probably is small).

The effect of data weighting can further be clarified by Fig. 5(a) and (b) where the solid curves are the bivariate nonlinear regression curves of the interval centroids, with the same weight of each centroid. As one can see, almost indistinguishable curves (dashed ones) are obtained by the weighted nonlinear bivariate statistical regression of all the data points in the restricted (filtered) database. An unweighted regression of the same data points is shown in Fig. 5 by the dash-dot curves, and, again, the dash-dot curve is barely distinguishable from the regression curve of the centroids in Fig. 5(a) while it is very different in Fig. 5(b). One reason for this difference is that the vertical ranges of the restricted data in the individual size intervals, marked by vertical bars, are in Fig. 5(a) nearly symmetric with respect to the centroid curve, but not in Fig. 5 (b). Another reason is that the restricted database in Fig. 5(a) is roughly homoscedastic, whereas that in Fig. 5(b) is not.

For comparison, the CoV of errors of the multivariate nonlinear regression conducted on the entire database[2] is 15% if the data are weighted and 17% if unweighted. When only the 11 beams deeper than 3.281 ft (1 m) are considered, the CoV is 14% if these data are weighted and 16% if unweighted. In other words, the weighted regression gives a better prediction for the scatter of shear strength of large beams.

REINFORCEMENT EFFECT

A similar statistical procedure can be used to identify the effect of longitudinal reinforcement ratio ρ_w on the shear strength of concrete beams without stirrups. Fig. 6 (a) shows the plot of $\log(v_c/\sqrt{f_c'})$ versus $\log \rho_w$ according to the ACI-445F database. The range of ρ_w was subdivided by the values ρ_w = 0.1, 0.25, 0.625, 1.56, 3.91, and 9.77 into five intervals. Then, using the same algorithm as before, the data in each interval of ρ_w was filtered by restricting their range to attain approximately the same values of parameters other than ρ_w, that is, d, a/d, and d_a.

For the filtered data with restricted ranges $\overline{d}$ = 8.5 in. (216 mm), $\overline{a/d}$ = 3.4, and $\overline{d}_a$ = 0.65 in. (16.5 mm), linear regression yields a straight line of slope 0.37, which corresponds to the power law

$$v_c/\sqrt{f_c'} = C\rho_w^{0.37} \tag{2}$$

where C is constant. The correlation coefficient is r = 98%, and now the CoV of errors of mean prediction is 2.9% (Fig. 6(b)). Note that there is no centroid in the first interval of ρ_w because it contains only four data points for which the corresponding d, a/d, and d_a cannot match the averages of filtered data in the other intervals of ρ_w.

If the restricted range is changed into d = 8.5 in. (216 mm), a/d = 3.7, and d_a = 0.59 in. (15 mm), the linear regression yields a straight line of slope 0.38, with CoV of errors of mean prediction ω = 3.6% (Fig. 6(c)). Now it should be noted that exponents 0.37 and 0.38 are nearly the same as the exponent of 3/8 identified by Bažant and Yu[3] in a more complex multivariate nonlinear regression analysis, in which all the parameters were optimized simultaneously.

CONCLUSIONS

1. Statistical bias in the size effect database for beam shear can and should be minimized by introducing intervals of equal size ratio and filtering the data within each interval in such a way that the average of influencing variables other than the beam size (or depth) be approximately the same in all the size intervals;

2. Replacing the filtered data in each size interval by their centroid eliminates the statistical bias due to data crowding in the small size range;

3. The centroids of filtered data in the individual size intervals yield a clear size effect trend, and their bivariate regression yields an optimum fit of minimized bias, characterized by a realistic CoV of regression errors;

4. Weights need to be introduced in statistical bivariate regression to minimize the statistical bias in the CoV of individual tests;

5. The results support the fracture-mechanics-based design formula[3] that terminates, in a bi-logarithmic size effect plot, with an asymptote of slope –1/2, and was previously calibrated by multivariate nonlinear regression analysis of the database. The results also provide additional evidence against the size effect formulae used in the JSCE, CEB (*fib*), and Canadian design codes; and

6. The results of the present bias-minimizing algorithm also support the previous conclusion [3] on the effect of longitudinal reinforcement ratio.

ACKNOWLEDGMENTS

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APPENDIX: OBJECTIVE ALGORITHM SUPPRESSING BIAS BY FILTERING THE DATABASE

Let the range of each parameter X_k (k = 1, 2, 3), corresponding to ρ_w, a/d, and d_a, be divided into five size interval D_v (v = 1,2,...5). Subsequently, the range of remaining data in each interval is progressively restricted from above and from below in such a way that the means $\overline{X}_k$ of the data that remain within each interval be rendered approximately the same for all the intervals. In this way, the effect of random variations of X_k in the database gets suppressed and the trend of $y = v_c/\sqrt{f_c'}$ versus $x = d$ gets isolated.

All the data points in the database are numbered as i = 1, 2, …, N, where N is the total number of data points (N = 398 for the ACI-445F database). Let y_i be the value of y for data point number i; J_{ikv} is the integer matrix in which element ikv gives the data point i in variable X_k in interval v; N_v is the total number of data points in interval v in the admissible range of variable X_k (k = 1, 2, 3); A_{vk}, B_{vk} is the lower (or upper) limit of variable X_k in interval v; $\overline{X}_k$ is the mean value of variable X_k selected for the whole size range; $V_v(X_k) = \omega_k^z$ is used to evaluate the overall variance of variables X_k

for interval v, where ω_k is the coefficient of deviation with respect to $\overline{X}_k$. The algorithm of data filtering may be formulated as follows.

Subroutine A:

INPUT: Three chosen initial values of all $\overline{X}_k$ and chosen weights w_k.

1. Loop over intervals $v = 1, 2, ...5$ (that is, over all the size intervals D_v of variable X_k).
2. Loop $k = 1, 2, 3$ over all the lower and upper bounds (range limits) A_{vk} and B_{vk} of subsidiary influencing variables to be filtered (that is, to be restricted in their range).
3. Initialize $N_v = 0$ and run loop $i = 1, 2, ..., N$ over all the data points in the database.
4. If $J_{ikv} = v$ and $A_{vk} \leq J_{ikv} < B_{vk}$, reset $N_v \leftarrow N_v + 1$.
5. End of loop over $i = 1, 2, ..., N$.
6. For each influencing variable X_k, calculate variance $V_v(X_k)$ of each filtered influencing variable X_k, using only the data points, of number $n = N_v$, that remain between the lower and upper current limits of interval v.
7. If the number of remaining data points $N_v \leq 2$, go to 2.
8. End of loop over k.
9. End of loop over v.

OUTPUT: The values of N_v, $V_v(X_k)$, A_{vk}, and B_{vk}. END.

To suppress the effect of unequal values of influencing variables X_k in various size intervals, the sum of variances, $\sum_v \sum_k V_k(X_k)$, must be minimized. Although, ideally, the minimum attainable value of each $V_v(X_k)$ is 0, there would be no data left in each interval if $V_v(X_k)$ were to be minimized to 0. Therefore, the objective of data selection must be to reduce $\sum_v \sum_k V_k(X_k)$ to a certain tolerably small but finite value (corresponding to the previously indicated deviations of the optimized data centroids from their mean), while keeping the number of remaining data points sufficiently large. This can be achieved by using a suitable optimization algorithm to minimize the following objective function which employs Subroutine A

$$\Phi(A_{vk}, B_{vk}) = \sum_{k=1}^{3} w_k V_v(X_k) + w\langle pN - N_v\rangle^2 \qquad \text{(A-1)}$$

Here, the given empirical weight w governs the penalty for the number of remaining data points being too small (p is the fraction of N to be specified) and $\langle x\rangle = \max(x, 0)$. In the present calculations, 128 and 157 data points fall into the admissible range for $\overline{\rho}_w = 1.5\%$ and $\overline{\rho}_w = 2.5\%$, respectively. Therefore, one may choose $w = 0$ and $w_k = 1$.