Why the Observed Motion History of World Trade Center Towers Is Smooth

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Abstract: The collapse of the World Trade Center towers was initiated by the impact of the upper falling part onto the underlying intact story. At the moment of impact, the velocity of the upper part must have decreased. The fact that no velocity decrease can be discerned in the videos of the early motion of the tower top has been recently exploited to claim that the collapse explanation generally accepted within the structural mechanics community was invalid. This claim is here shown to be groundless. Calculations show that the velocity drop is far too small to be perceptible in amateur video records and is much smaller than the inevitable error of such video records.

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Introduction

The collapse of the World Trade Center (WTC) towers has been explained as a gravity-driven process triggered by the collapse of a critical story heated by fire (Bažant and Zhou 2002; Bažant and Verdure 2007; Bažant et al. 2008; Bažant and Le 2008). All the objections of the proponents of the controlled demolition hypothesis have been shown invalid. Recently, though, a new objection, pertaining to the smoothness of the observed motion history of the tower top, has been raised and disseminated on the internet. This objection is based on the intuition that, if the collapse of WTC towers were gravity driven, then the existing amateur video of collapse would have to show a pronounced velocity drop at the moment at which the upper falling part impacted the lower intact story (the Naudet video was used for the collapse of WTC 1 , and the WNBC live video was used for WTC 2).

Here it is shown that the velocity drop must have been three orders of magnitudes smaller than the error of an amateur video, and thus undetectable. An upper bound on the velocity drop is first obtained by simple hand calculations, and then the magnitude of velocity drop is determined accurately from the previously developed computer program (Bažant et al. 2008; Bažant and Le 2008).

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Simple Calculation of an Upper Bound on Velocity Drop

Although NIST determined that some columns in the fire zone underwent slow (i.e., viscoplastic) buckling spanning several floors, for the purpose on an upper bound we may assume that the progressive collapse of each tower was triggered by the collapse of a single story. Were the trigger provided by a multistory collapse, the underlying intact story would be impacted at a higher velocity, causing the velocity drop upon impact to be a smaller percentage of the impact velocity.

Consider first only the North Tower. The downward velocity of the upper part tower at the moment of impact on the concrete floor slab is

$$v_0 = \sqrt{2\alpha g h} = \sqrt{\alpha} \times 8.52 \ \text{m/s} = \sqrt{\alpha} \times 19.06 \ \text{mi/h} \quad (1)$$

where $g=9.81\ \text{m/s}^2$=gravity acceleration; $h=3.7$ m=clear height of the columns; and α=uncertainty parameter ≤ 1 and probably very close to 1, which characterizes the resisting upward force of the heated columns, expressed as $(1-\alpha)mg$ where m =mass of the top part of the tower. An accurate calculation of α would require knowledge of the temperature history of all columns, which is unavailable. However, α must clearly be larger than $\alpha_{\min}=0.794$ which corresponds to the average resistance of cold columns calculated by Bažant and Verdure (2007; Fig. 3) and Bažant and Zhou (2002; Eq. (8)).

The concrete floor slab, of mass m_s, may be considered to behave upon impact as one rigid body. The impact is inelastic, with restitution coefficient=0 (which means that the upper part does not rebound from the slab). Conservation of momentum requires that $mv_0=(m+m_s)v_0'$, where v_0'=velocity of the upper part with the underlying slab after impact. Hence

$$v_0' = \frac{v_0}{1 + m_s/m} = 0.989 v_0 \approx v_0 \quad (2)$$

The input numbers are taken from Bažant et al. (2008).

The subsequent motion of the top part is slowed down by the resistance of the steel columns of the underlying floor. After the displacement of only 3.2 mm, these columns reach their axial yield capacity, which is

$$F_p = A\sigma_0 = 1.513 \times 10^9 \text{ N} \approx 2.84mg \tag{3}$$

where $A = 6.05\ \text{m}^2$ = combined cross section area of all the columns of the underlying floor. After plastic shortening of only 5.7 mm, the vertical resisting force F_b of all columns provided by a buckling collapse mechanism with three plastic hinges, described in Bažant and Zhou (2002), becomes smaller than F_p, and so the columns must buckle plastically (Bažant and Cedolin 1991, Chapter 8). Force F_b may be calculated from the free body equilibrium diagram in Fig. 5 of Bažant and Zhou (2002), which gives $(F_b/n)(\theta_1 h/2) = 2M_p$, or

$$\theta = 4nM_P/F_b h \tag{4}$$

where $M_p = 0.32$ MNm = average yield bending moment of one column; $n = 287$ = number of columns (approximately considered as identical); and θ = rotation of the plastic hinges at column ends. If θ is expressed from Eq. (4), the shortening of each column due to plastic buckling is found to be

$$u_c = (1 - \cos\theta)h \approx \frac{\theta^2}{2}h = \frac{8n^2M_P^2}{F_b^2 h} \tag{5}$$

The resisting force F_b rapidly decreases as θ and u_c grow. The displacement u_{eq} at which F_b becomes equal to the weight mg of the upper part of tower is obtained by substituting $F_b = (m + m_s)g \approx mg$

$$u_{eq} = = \frac{8n^2M_P^2}{m^2g^2h} = 64.56 \text{ mm} \tag{6}$$

For displacements $u_c > u_{eq}$, $F_b < mg$ and so the motion of the upper part of tower accelerates. For $0 < u_c < u_{eq}$, the resisting force F_b of columns exceeds the weight mg of the upper part, and so the motion of the upper part decelerates.

An accurate calculation of the displacement history $u_c(t)$ during the deceleration and acceleration periods of time t requires numerical integration of the equation of motion, presented in Bažant and Verdure (2007) and Bažant et al. (2008). However, an upper bound $\Delta v_{\max}\sqrt{a^2+b^2}$ on the drop of velocity $c = \dot{u}_c$ during the deceleration period can be easily obtained by hand calculations, based on the assumption that the resisting force for $u_c \in (0, u_{eq})$ is constant and equal to its maximum F_p given by Eq. (3). For this upper bound, the equation of motion of the upper part of mass $m + m_s \approx m$ reads

$$(m + m_s)\ddot{u} = -[F_p - (m + m_s)g] = \text{constant} \tag{7}$$

Integration from the moment of impact on the floor slab ($t = 0$) to time $t = t_{\text{eq}}$ corresponding to the end of deceleration yields

$$u_{\text{eq}} = -\left[\frac{F_p - (m + m_s)g}{m + m_s}\right]\frac{t_{\text{eq}}^2}{2} + v_0' t_{\text{eq}} \tag{8}$$

from which one may solve

$$t_{\text{eq}} = \frac{(m + m_s)v_0'}{F_p - (m + m_s)g} - \sqrt{\frac{(m + m_s)^2 v_0'^2}{[F_p - (m + m_s)g]^2} - \frac{2(m + m_s)u_{\text{eq}}}{F_p - (m + m_s)g}}$$
$$= 7.72 \times 10^{-3} \text{ s} \tag{9}$$

For comparison, if the motion continued at constant velocity v_0' given by Eq. (2) for $\alpha = \alpha_{\min} = 0.794$, the displacement increase would be $v_0' t_{\text{eq}} = 65.24$ mm. The maximum displacement difference $\Delta u_{\max}$ caused by deceleration from $t = 0$ (the moment of impact) to $t = t_{\text{eq}}$ is obtained by substituting Eq. (9) into Eq. (8)

$$\Delta u_{\max} = v_0' t_{\text{eq}} - u_{\text{eq}} = 0.68 \text{ mm} \tag{10}$$

This upper bound should be compared to the displacement uncertainty in the amateur video record, which was shown by the error bars in Fig. 7 of Bažant and Le (2008) and is about ± 500 mm. Obviously even this upper bound on the effect of deceleration is far too small for being discerned in the video. It is thus no surprise that no drop of velocity can be detected.

For the South Tower, the upper impacting part is heavier but the columns of the impacted story underlying the fire zone are stronger. Analogous calculations yield $\Delta u_{\max} = 0.84$ mm.

For the collapse of the subsequent stories, the initial crush-down velocity v_0' becomes much larger while the maximum deceleration due to the column resisting force will not change much ($\approx -2g$) and the deformation of columns u_{eq} at which the resisting force becomes smaller than the falling weight is about the same. Therefore, one will expect that velocity drop during the collapse of the subsequent stories will become smaller and its duration will be shorter. This explains why there is no discernable velocity change in the observed motion history of the tower top (Fig. 7 in Bažant et al. 2008).

For the collapse of lower stories of the tower, due to the dominance of other resisting forces, the deformation at which the deceleration ends is expected to be much larger than the deformation at which the upper falling weight exceeds the column resisting force. However, at the same time, the crush-down velocity during the collapse of these stories is also much higher, and is in the order of 40 m/s. Hence, the velocity drop will also not be perceptible from the motion of the tower top.

All the preceding analysis is based on the simplifying assumption of one dimensional motion. In reality, the top part of each tower was tilting during the collapse, which implies that the impact of the top part onto the floor slab was not simultaneous. This caused the motion history to be smoother than predicted by one-dimensional analysis, and thus any sudden velocity decrease to be even smaller and less detectable. Another simplifying assumption has been the neglect of the resisting force due to the comminution of concrete and ejection of air and debris (Bažant et al. 2008), which is, however, very small for the first few collapsing stories.

Motion during Two-Way Crushing of Upper and Lower Parts of Tower

The most accurate picture can be obtained by numerical solution allowing for a possible combination of crush-down and crush-up, i.e., for possible two-way crushing. The basic mode of gravity-driven collapse is crush-down followed by crush-up (Bažant and Verdure 2007). However, right after the impact of the upper part of tower onto the underlying floor slab, crush-up must occur simultaneously with crush-down, though only for a very short period (Bažant et al. 2008; Bažant and Le 2008).

The initial conditions for the two-way crush phase are obtained from the condition of conservation of momentum and energy during the impact (Eqs. 30 and 31 in Bažant et al. 2008). To calculate the velocity history accurately, the entire load-deflection curve of the plastically buckling columns must be considered (see Fig. 3 in Bažant and Verdure 2007, based on Bažant and Cedolin 1991, Sections 8.1, 8.2, and 8.6). Solution of the equations of motion (Eqs. 32 and 33 in Bažant et al. 2008) thus led to the velocity histories of crush-down and crush-up fronts shown in

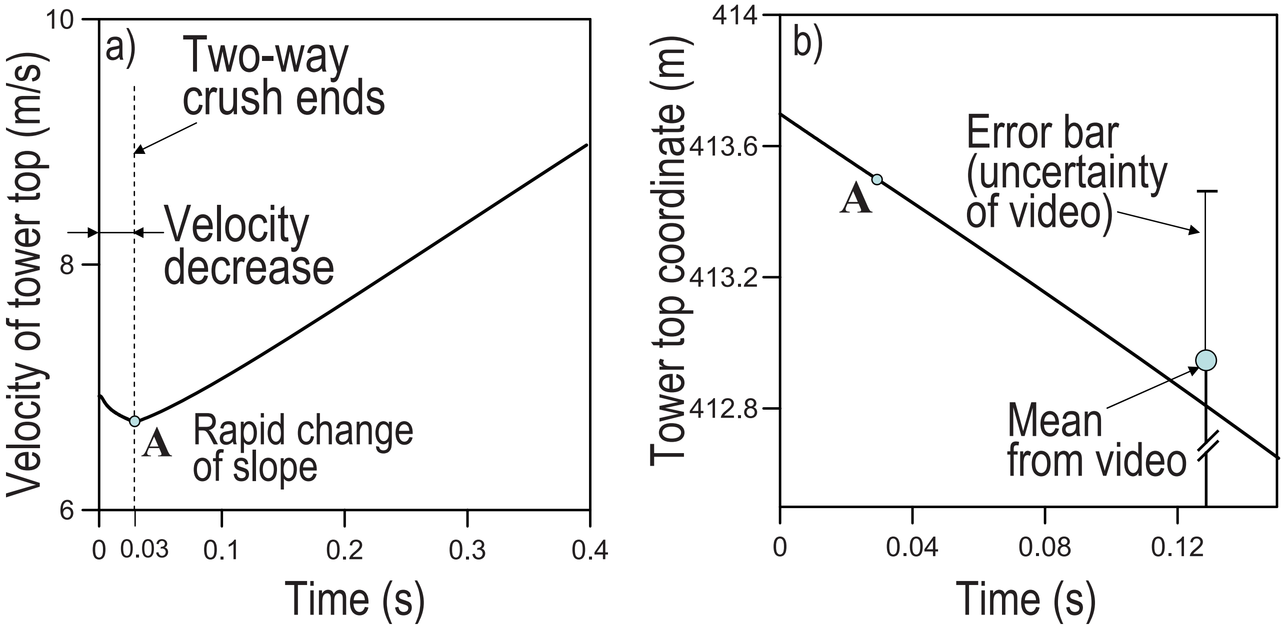

Fig. 1. Motion history during two-way crush phase

Fig. 9 in Bažant et al. (2008), which were supported by all the known observations.

During the two-way collapse phase, the velocity of the tower top can be calculated as

$$v_{top} = (1-\lambda)(v_{cd} + \langle v_{cu} \rangle) \quad (11)$$

where $\langle x \rangle = \max(x, 0)$; λ=compaction ratio; and v_{cd}, v_{cu} =velocities of crush-down and crush-up fronts. Fig. 1(a) presents the velocity history of the tower top. As seen, the velocity of the tower top decreases during the two-way crush phase by only 3%, which lasts for only about 0.03 s. After that, the collapse proceeds in the one-way, crush-down, mode. During the crush-down phase, the velocity of the tower top depends solely on the velocity of the crush-down front, which is accelerating at the rate of about 6.2 m/s^2. Calculations show an almost sudden decrease of the slope of the velocity profile, at 0.03 s, due to the sudden transition from the two-way crush phase to the one-way crush phase.

Fig. 1(b) shows the motion of the tower top during the first 0.16 s of the collapse of the story underlying the critical story. Based on Fig. 1(a), the tower top decelerates during the two-way crush phase, whose duration is 0.03 s, and accelerates afterwards. It can be seen that, compared to the observed data and the uncertainties of observation, the velocity drop during the two-way crush phase is not discernable from the observed motion of the tower top. The reason is that the velocity drops by only 3% within only 0.03s, and increases again afterward, which is the start of one-way crush.

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