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Creep Design Aid: Open-Source Website Program for Concrete Creep and Shrinkage Prediction

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To facilitate the calculation of creep and shrinkage properties needed for analyzing the creep and shrinkage effects in structures, an open-source creep design aid program has been developed and is made freely available on a website. It is based on Model B3, but can be easily adapted to other creep and shrinkage prediction models. Based on the given strength and the main characteristics of the given concrete, the program calculates, plots, and tabulates the compliance function, relaxation function, creep coefficient, aging coefficient, and creep rate function. For ease of use, there is a user-friendly graphical user interface. The program also uses Latin hypercube sampling to calculate 95, 90, or 85% confidence limits of the response curves from given coefficients variations of input data, and performs updating of long-term prediction based on given short-time test data, using either linear regression or Bayesian estimation.

Keywords: aging coefficient; Bayesian updating; compliance function; Latin hypercube sampling; relaxation function; statistical analysis; viscoelasticity.

INTRODUCTION

Structural analysis for creep and shrinkage normally proceeds in two stages. In the first, the mathematical characteristics of creep and shrinkage are estimated from the selected concrete strength and, if available, the basic mixture parameters of the concrete. A simplified quasi-elastic one-step structural analysis is then performed, typically using the age-adjusted effective modulus method.[1-4] This stage is normally sufficient for prestressed bridges of smaller spans (less than 40 m [131.2 ft]). For larger spans, however, and generally for structures of high creep sensitivity—such as long-span prestressed box girders, super-tall buildings, large-span roofs, and nuclear reactor containments or vessels—the first stage expounded herein is intended only for approximate preliminary design. It should be followed by a second stage featuring a detailed three-dimensional nonlinear step-by-step creep analysis, such as exemplified in References 5, 6, and 7, or else a multi-decade serviceability could be compromised. The second stage should involve recalibration of the prediction model for the given concrete based on short-time creep, shrinkage, and weight-loss tests, and statistical estimates of 100-year creep effects.

The present article concerns only the first stage, and presents a freely usable open-source website program to serve as a design aid. The prediction of creep is based on Model B3,[8] which was adopted as an international RILEM Recommendation.[9] This model, which is the last of three published models progressively developed over the span of approximately 20 years and will be explained later in this paper, was shown to give superior agreement with an extensive statistical creep and shrinkage database of laboratory test data with filtered-out bias,[10] and also with the measurements of multi-decade deflections on 69 large-span bridges.[11] Unlike other models,[1,12,13] Model B3 rests on a strong theoretical foundation in the theories of solidification, nanoscale thermal activation, thermodynamics, diffusion, and asymptotic matching.[4,14,15]

Despite using Model B3, the present creep design aid (CDA) can be easily adapted to other prediction models—for example, to those of ACI, CEB, or JSCE, or to the improved Model B4 that is currently near completion. It suffices to replace the definition of the compliance and shrinkage functions by simply updating the subroutine files.

The CDA is an open-source program. The users can thus insert new lines of code into the subroutine files or revise them. Graphics are available to help engineers visualize the long-term prediction. Built-in statistical tools can automatically improve the prediction if short-time laboratory creep and shrinkage tests of the given concrete are carried out. Detailed information about how the model is implemented is given in the Appendixes.[*]

A website program for the compliance and shrinkage functions of Model B3 was previously made available by Křístek et al.[16] The present program adds a number of the following useful features. It calculates, tabulates, and plots not only the compliance function, but also the relaxation function, creep coefficient, aging coefficient, and creep rate function of the given concrete. For ease of use, there is a user-friendly graphical user interface (GUI). The user can further obtain tables of the calculated functions, which can be readily exported. The program also calculates the confidence limits on the response curves from given coefficients of variation of input parameters. To facilitate statistical updating of Model B3 based on short-time test data, the users can choose an adaptation of Model B3 to either linear regression or Bayesian analysis.

[*]The Appendix is available at www.concrete.org in PDF format as an addendum to the published paper. It is also available in hard copy from ACI headquarters for a fee equal to the cost of reproduction plus handling at the time of the request.

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RESEARCH SIGNIFICANCE

Recent studies revealed excessive or nearly excessive multi-decade deflections of 69 large-span prestressed concrete segmental box-girder bridges,[5,11,17] most of them with spans from 90 to 241 m (295.2 to 790.5 ft), and a few with spans from 60 to 90 m. (196.8 to 295.2 ft). This finding documents the necessity of using a realistic concrete creep and shrinkage model.[5,11,17] The present work facilitates this task.

BACKGROUND OF MODEL B3

The compliance function $J(t, t')$ represents the strain at time t caused by a unit sustained uniaxial stress applied at age t'. In Model B3, it is defined as follows

$$J(t, t') = q_1 + C_0(t, t') + C_d(t, t', t_0) \tag{1}$$

where

$$C_0(t, t') = q_2Q(t, t') + q_3\ln[1 + (t - t')] + q_4\ln(t/t') \tag{2}$$

$$C_d(t, t', t_0) = q_5\left[e^{-8H(t)} - e^{-8H(t_0')}\right]^{1/2} \tag{3}$$

$$\frac{\partial Q(t,t')}{\partial t} = \frac{n(q_2 t^{-m} + q_3)}{(t-t') + (t-t')^{1-n}} \tag{4}$$

Herein, m and n are empirical constants; $H(t)$ is a function of environmental humidity h defined in Appendix I and time t in days; q_1, q_2, q_3, q_4, and q_5 are empirical magnitude factors of five terms of different physical meanings; q_1 gives the instantaneous deformation asymptotically extrapolated to infinitely short load duration (for which the strain is much smaller than even the 1-second creep, and has the advantage of being approximately age-independent); q_2, q_3, and q_4 control the magnitudes of the aging viscoelastic compliance, the non-aging viscoelastic compliance, and the flow compliance, respectively; q_5 controls the magnitude drying creep; $J(t, t')$ is the compliance (total); $C_0(t, t')$ is the basic creep compliance; $C_d(t, t', t_0)$ is the drying creep compliance; and t_0 denotes the time at the start of drying.

Empirical equations that predict material parameters q_1, ..., q_5, ε_{sh_∞}, and k_t according to Model B3 are built in. As a deviation from the original Model B3,[8,9] however, the prediction formula for parameter q_4, which controls the long-time asymptotic slope of compliance curves in log-time, has been increased by a factor of 1.6. This factor has been identified by matching multi-decade deflection curves of 69 prestressed segmental bridges (as part of the work on the forthcoming Model B4). Previously, all the parameters were calibrated by the laboratory database, which has insufficient data for multi-decade creep controlled mainly by parameter q_4.

Function $Q(t, t')$, representing the aging viscoelastic compliance, is obtained by integrating Eq. (4). The integration leads to a binomial integral that cannot be integrated analytically in a closed form. In practice, $Q(t, t')$ has usually been evaluated from an approximate formula, whose error, within the ranges $1 \leq t' \leq 10,000$ days and $0.01 \leq t - t' \leq 10,000$ days, is generally within ± 0.5%; refer to Eq. (17) in Reference 14. For numerical integration, the binomial integral can be transformed by substitution of a new variable $\xi = \ln(t - t')$ to the integral

$$Q(t,t') = \int_{-\infty}^{\ln(t-t')} \left(\frac{\lambda_0}{t' + e^{\xi}}\right)^m \frac{ne^{(n-1)\xi}}{1+e^{n\xi}} e^{\xi} d\xi \tag{5}$$

which may be evaluated with high accuracy by the trapezoidal rule with constant steps $\Delta\xi$ (eight steps per decade suffice); n and λ_0 are material constants. The first integration step from $-\infty$ to the ξ value corresponding to a sufficiently short duration $t_1 - t'$ (taken as 0.001 day), should be obtained by analytical integration made possible by the fact that, for $t - t' << t'$, $1/\tau$ can be replaced with $1/t'$; refer to Eq. (19) in Reference 14. While the approximate formula for $Q(t, t')$ is useful for manual calculations, in a computer program, its use makes no sense because even a virtually exact numerical integration is immediate (it should be noted that in finite element creep analysis, integral (5) never needs to be evaluated because the rate equation (4) is all that is used in each time step).

The shrinkage function $\varepsilon_{sh}(t, t_0)$, giving the shrinkage strain at time t for a concrete specimen exposed to drying environment at age t_0, is defined in Model B3 as follows

$$\varepsilon_{sh}(t, t_0) = \varepsilon_{sh_\infty} k_h S(t) \tag{6}$$

where

$$S(t) = \tanh\sqrt{\frac{t - t_0}{\tau_{sh}}} \tag{7}$$

in which $k(h) = 1 - h^3$, where h is the environmental relative humidity and τ_{sh} is size dependence factor $k_t(k_sD)$.[2] The final shrinkage ε_{sh_∞}, rate parameter k_t, and shape parameter k_s are defined in References 8 and 9.

CREEP DESIGN AID (CDA) AND ITS USE

The CDA is a MATLAB program for computing and visualizing concrete creep compliance and shrinkage strains. Numerical solution, matrix computations, and graphics are integrated into a single package. The program uses the GUI, which allows the users to manipulate, perform, and execute various tasks by a mouse click, in a simple and intuitive way. For detailed information on GUIs and MATLAB, refer to Reference 18. The basic structure of the CDA is explained in the flow chart in Fig. 1. The major features of the CDA include:

1. Plotting graphs of compliance $J(t, t')$, creep coefficient $\varphi(t, t')$, relaxation function $R(t, t')$, aging coefficient $\chi(t, t')$, age-adjusted effective modulus $E''(t, t')$, compliance rate $J(t, t') = \partial J(t, t')/\partial t$, and shrinkage strain $\varepsilon_{sh}(t, t_0)$.

2. Performing the statistical Latin hypercube sampling based on specified parameter uncertainties.

3. Performing statistical analysis providing 95, 90, and 85% (two-sided) upper and lower bounds of confidence

intervals based on assuming the Gaussian (normal) distribution (the 95, 90, and 85% cutoffs are two-sided; that is, the probabilities of creep or shrinkage being above the upper cutoff are 2.5, 5, and 7.5%, respectively, and the same for being below the lower cutoff; these cutoffs correspond to 1.97, 1.65, and 1.44 standard deviations, respectively).

4. Updating the prediction of creep and shrinkage based on short-time test data for the given concrete, using either least-square regression or Bayesian analysis.

Starting CDA program

1. Open the MATLAB program.

2. On the "Command Window," type "*ModelB3*" (case sensitive), or double-click the file named "ModelB3.m" in the "Current Folder."

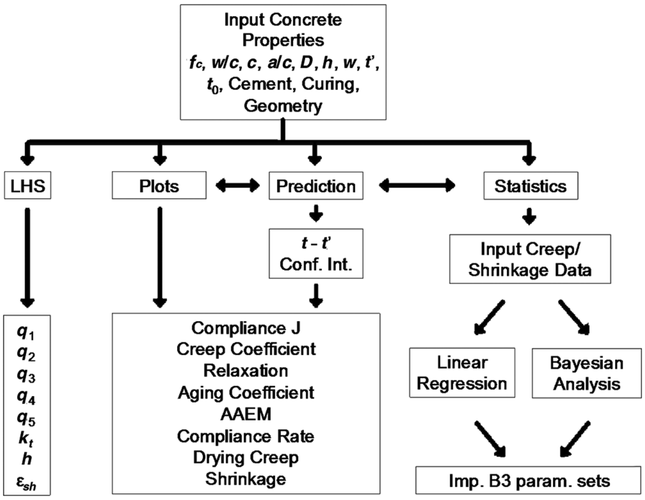

Fig. 1—Flow chart of CDA.

3. Click "Debug" from the top menu, and click "Run ModelB3.m".

4. A new graphical window will appear on the screen (Fig. 2(a)).

Input plotting functions

The concrete strength and basic mixture composition parameters are specified, and Model B3 formulae are used to evaluate the parameters of the compliance and shrinkage functions stated in Eq. (1) through (6).

1. Input concrete properties in the windows of Fig. 2(b), the concrete properties and other required parameters for which the input should be made. CDA accepts input files in the Excel format, named "properties.xls".

2. Open "properties.xls" and type the concrete properties, environmental conditions, and values of t', t_0. Make sure the values are in U.S. units (Appendix I).

3. Save "properties.xls" and exit.

4. Click the "Load Data" button in the left column (Fig. 2(c)).

5. The "Select Data File" window will appear. Then, select "properties.xls" in the folder. This file should be in the same work directory.

6. Click "Open."

7. Select "2 - Plot options" in Fig. 2(c), upon which the curves of compliance function (in both logarithmic and linear scales), creep coefficient, relaxation function, aging coefficient, AAEM, compliance rate, drying creep, and shrinkage will be generated.

8. Check the box "Compare" in Fig. 2(c) to see multiple plots, in which a set of curves for various times at loading, or in the case of shrinkage for various humidity h and effective thicknesses D, is added to the plot. The plot returns to a single specified curve if this box is unchecked.

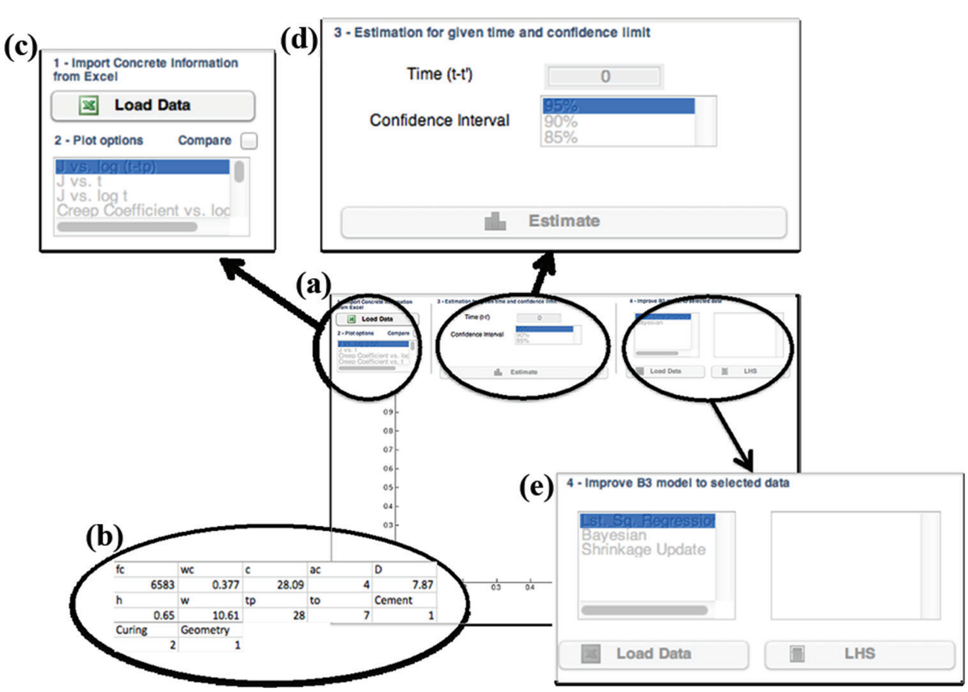

Fig. 2—Configurations of CDA: (a) main screen; (b) load data and plot selection; (c) estimation of function at specific time; and (d) statistical improvement; (e) input screen of properties.xls.

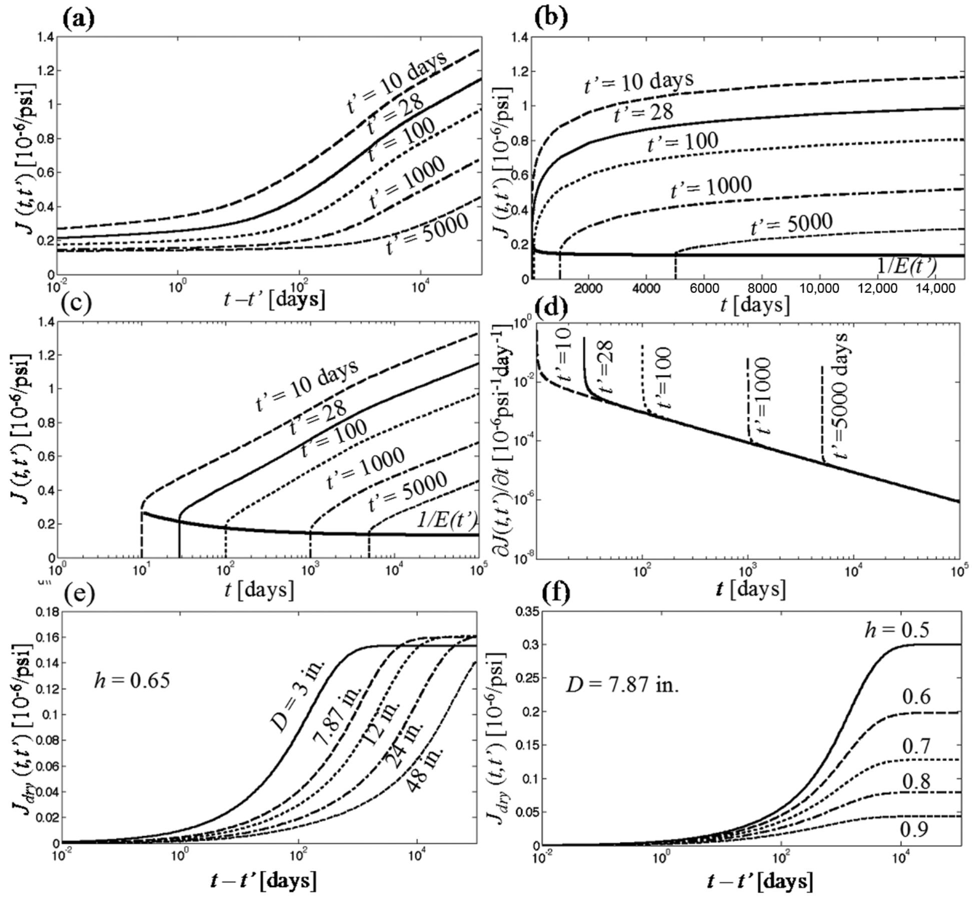

Fig. 3—Different curves of compliance function of Model B3: (a) total compliance versus t – t′ *in log scale; (b) total compliance versus* t*; (c) total compliance versus* t *in log scale; (d) total compliance versus* t *in bi-logarithmic scale; (e) drying compliance versus* t – t′ *for different thickness; and (f) drying compliance versus* t – t′ *for different humidity. Bold lines in (b) and (c) represent compliance at loading age* t′.

9. In the case of drying creep, there is an option to generate, for comparison, the logarithmic scale curves for various humidity h and for various thicknesses D compared with the curve for the specified t_0, h, and D. Drying creep plot options can be found below "Compliance rate" inside of the "2 - Plot options" menu. Example curves are shown in Fig. 3(e) and (f).

10. Similar to drying creep plots, shrinkage curves are generated for various humidity h and thickness D in logarithmic scale. Plot options can be found below "Drying creep for different humidity." Example curves are shown in Fig. 4(a) and (b).

It should be noted that the effective thickness D is equal to the actual thickness of an infinite concrete slab. For flanged cross sections (T, I, or box), D may be approximately taken equal to the thickness of the wall. More generally, D is defined as $2V/S$ times a shape correction factor, whose values for various cross section shapes were solved from the nonlinear diffusion equation for concrete drying, checked by experimental data on drying, and are stated in Model B3[4,8,9] (V/S is the volume-surface ratio of the cross section).

Evaluating functions for specific times

1. In "2. Plot options," select a plot that has been evaluated.

2. In Fig. 2(d), type loading duration $t - t'$ in the box "Time ($t - t'$)."

3. Select the confidence interval (two-sided).

4. Click "Estimate."

5. The corresponding values at (t, t') will be presented below the "Confidence interval" box. The values inside the parentheses represent the lower and upper confidence interval bounds.

6. By clicking "Estimate," the output files are automatically exported and saved as text files: "B3.txt," "Phi.txt," "R.txt," "Chi.txt," "ChiPhi.txt," "AAEM.txt," "dJdt.txt," and "shrinkage.txt." These files can be found in the same work directory. Examples are shown in Tables 1 through 8.

Statistical improvement of compliance function $J(t, t')$ and $\varepsilon_{sh}(t, t', t_0)$

To update compliance function $J(t, t')$ or shrinkage strain ε_{sh} versus $(t - t')$, experimental data should be supplemented. The measured values of $J(t, t')$ or $\varepsilon_{sh}(t, t', t_0)$ can be conveniently imported from the Excel format, in which the file names are "creep data.xls" and "shrinkage data.xls." In the Excel files, the first column represents the duration, $t - t'$ or $t - t_0$, and the second column represents the measured values $J(t, t')$ or $\varepsilon_{sh}(t, t', t_0)$.

Table 1—Compliance function*

	$t - t'$							
t'	48,300	0.01	0.1	1	10	100	1000	10,000
28	1.080	0.2140	0.2312	0.2555	0.3077	0.4526	0.7021	1.091
1	1.882	0.5610	0.6507	0.7832	0.9985	1.237	1.502	1.893
10	1.259	0.2684	0.2975	0.3392	0.4310	0.6202	0.8800	1.270
100	0.900	0.1754	0.1852	0.1986	0.2252	0.3118	0.5285	0.9111
1000	0.613	0.1461	0.1498	0.1554	0.1663	0.1970	0.3020	0.6241
5000	0.393	0.1384	0.1403	0.1428	0.1471	0.1573	0.1908	0.4027

*$J(t, t')$, 10^{-6} psi (1/6894 MPa).

Table 2—Creep coefficient*

	$t - t'$							
t'	48,300	0.01	0.1	1	10	100	1000	10,000
28	1.279	0.094	0.186	0.313	0.626	1.366	2.430	4.489
1	1.800	0.20	0.41	0.750	1.25	1.864	2.635	4.178
10	1.545	0.127	0.253	0.440	0.879	1.626	2.604	4.502
100	0.870	0.061	0.120	0.19	0.360	0.954	2.078	4.301
1000	0.274	0.02	0.049	0.078	0.123	0.312	1.231	3.609
5000	0.112	0.014	0.028	0.043	0.064	0.124	0.651	2.997

*$\varphi(t, t')$.

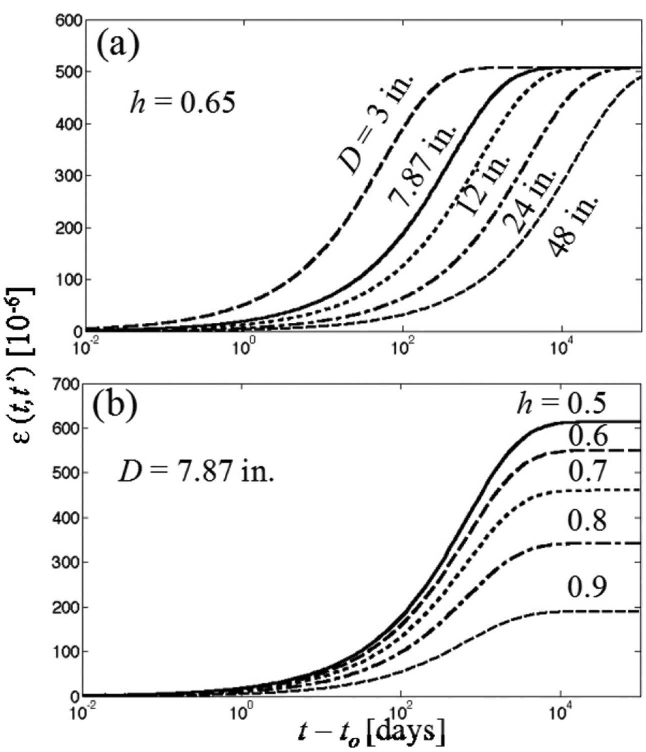

Fig. 4—Shrinkage plots of Model B3 ($t - t_0$ *in logarithmic scale): (a) for different thicknesses; and (b) for different humidity.*

For shrinkage update, it requires the weight loss information. In the same file, "shrinkage data.xls," weight loss versus $t - t_0$ can be inserted. The third column represents $t - t_0$, and the fourth column is the weight loss in percentage. The final weight loss can then be linearly extrapolated from these data.

Update of $J(t, t')$

1. Open "creep data.xls." The first column is for the time duration $t - t'$ in days, and the second is for the measured $J(t, t')$ values in 10^{-6} psi. Then save "creep data.xls" and exit.
2. Click "J versus log (t-t$_0$)."
3. The box "Load Data" is highlighted in Fig. 2(d).
4. Click "Load Data."
5. When the window "Select data file" appears, select "creep data.xls" in the folder. This file should be in the same work directory.
6. Click "Open." A solid curve for the prior and a dotted curve for the posterior, along with the data points, will then automatically be generated in Fig. 2.
7. For least-square regression analysis, select "Lst. Sq. Regression" in the box.
8. Select the desired number of data points to be used for updating (in practice, use them all). As an exercise, one may check that as the number of data points to be used is changed, the graph will automatically adjust the updated curve of Model B3.
9. For Bayesian updating, select "Bayesian" in the box, and repeat Step 7.
10. For shrinkage update, select "Shrinkage Update" in the box and repeat Step 7.
11. The sets of improved B3 parameters are automatically exported and saved as "Improve creep.txt."

Table 3—Relaxation function*

t'	$t - t'$							
	48,300	0.01	0.1	1	10	100	1000	10,000
28	1.778	4.587	4.23	3.81	3.004	1.657	0.670	0.045
1	0.512	1.702	1.44	1.13	0.78	0.486	0.231	0.017
10	1.215	3.625	3.261	2.828	2.042	1.140	0.494	0.035
100	2.850	5.629	5.333	4.995	4.364	2.650	0.972	0.060
1000	5.434	6.817	6.660	6.481	6.223	5.251	2.129	0.089
5000	6.565	7.206	7.111	7.003	6.866	6.492	3.860	0.071

*$R(t, t')$, 10^6 psi (6894 MPa).

Table 4—Aging coefficient*

t'	$t - t'$							
	48,300	0.01	0.1	1	10	100	1000	10,000
28	0.766	0.869	0.953	0.968	0.889	0.760	0.742	0.786
1	0.777	0.881	0.955	0.913	0.822	0.773	0.747	0.769
10	0.775	0.874	0.955	0.962	0.857	0.771	0.753	0.787
100	0.763	0.868	0.948	0.967	0.931	0.749	0.713	0.778
1000	0.843	0.830&	0.932&	0.956&	0.956	0.815	0.625	0.736
5000	0.905	0.791	0.913	0.940	0.944	0.890	0.583	0.676

*$\chi(t, t')$.

Table 5—Product of aging and creep coefficient*

t'	$t - t'$							
	48,300	0.01	0.1	1	10	100	1000	10,000
28	0.979	0.082	0.177	0.303	0.557	1.037	1.803	3.530
1	1.398	0.177	0.392	0.685	1.032	1.441	1.968	3.213
10	1.197	0.111	0.242	0.423	0.754	1.254	1.961	3.541
100	0.664	0.052	0.113	0.189	0.335	0.715	1.481	3.345
1000	0.231	0.021	0.046	0.075	0.117	0.254	0.769	2.656
5000	0.101	0.011	0.025	0.041	0.060	0.110	0.379	2.027

*$\varphi(t, t')\chi(t, t')$.

Table 6—Age-adjusted effective modulus*

t'	$t - t'$							
	48,300	0.01	0.1	1	10	100	1000	10,000
28	2.539	4.644	4.268	3.857	3.228	2.467	1.793	1.109
1	0.856	1.744	1.475	1.218	1.010	0.841	0.691	0.487
10	1.864	3.684	3.298	2.877	2.335	1.817	1.383	0.902
100	3.592	5.677	5.366	5.027	4.475	3.484	2.408	1.375
1000	5.679	6.847	6.683	6.504	6.256	5.573	3.951	1.912
5000	6.635	7.227	7.128	7.024	6.891	6.582	5.298	2.414

*$E''(t, t')$, 10^6 psi (6894 MPa).

Update of $\varepsilon_{sh}(t, t', t_0)$

1. For shrinkage, use a different input file named "shrinkage data.xls."

2. Click "Shrinkage versus log $(t - t_0)$" in the box.

3. As soon as the box "Load Data" is highlighted, click "Load data."

Table 7—Shrinkage*

	$t - t_0$							
D	48,300	0.01	0.1	1	10	100	1000	10,000
3	126.4	1.507	4.764	15.04	46.70	125.9	189.8	192.5
6	71.32	0.7458	2.358	7.455	23.47	71.00	161.0	190
7.87	55.39	0.5675	1.795	5.674	17.89	55.13	140.2	189.3
12	36.90	0.3719	1.176	3.719	11.75	36.73	104.6	182.6
24	18.62	0.1858	0.5877	1.858	5.875	18.53	56.96	142.9

*$\varepsilon(t, t_0)$, $h = 65\%$.

Table 8—Shrinkage*

	$t - t_0$							
h	48,300	0.01	0.1	1	10	100	1000	10,000
50	178.8	1.832	5.795	18.32	57.78	178.0	452.6	611.2
60	160.2	1.642	5.192	16.41	51.77	159.5	405.5	547.6
70	134.3	1.376	4.351	13.76	43.38	133.7	339.8	458.9
80	99.74	1.022	3.232	10.22	32.22	99.27	252.4	340.9
90	55.39	0.5675	1.795	5.674	17.89	55.13	140.2	189.3

*$\varepsilon(t, t_0)$, $D = 7.87$ in. = 20 cm.

4. When the window "Select data file" appears, select "shrinkage data.xls" in the folder. This file should be in the same work directory.
5. Click "Open." A solid curve for the prior and a dotted curve for the posterior, along with the data points, will then automatically be generated in Fig. 2.
6. Select "Shrinkage Update" in the box.
7. Similar to other update measures, select the different number of data points. One can visualize the update progress of shrinkage curves.
8. The sets of improved shrinkage parameters are automatically exported and saved as "Improve shrinkage.txt."

Latin hypercube sampling

1. Input concrete properties in "properties.xls."
2. Click "LHS." The randomly generated parameters (q_1, q_2, q_3, q_4, q_5, k_t, h, and ε_{sh}) are automatically exported and saved as "LHS.txt."

Main file and functions

The CDA package contains more than one primary file, "ModelB3.m". The additional 11 functions are the subroutine files. They are called frequently within the primary file. Their roles include:

1. B3parameter.m: It calculates Model B3 parameters from the concrete properties.
2. bay.m: For Bayesian analysis, it calculates random variables p_1 and p_2 for each time step θ_i.
3. dJdt.m: It evaluates the time derivative of the basic creep compliance function $J(t, t')$.
4. Jbasic.m: It calculates the compliance function for basic creep.
5. Jdry.m: It calculates the compliance function for drying creep.
6. lhs.m: It samples a random variable from a stratum of inverse cumulative density function (CDF).
7. Qf.m: It accurately integrates the binomial integral in Eq. (5) for the aging viscoelastic compliance component $Q(t, t')$.
8. Relaxation.m: It calculates $\varphi(t, t')$, $\chi(t, t')$, $R(t, t')$, $E''(t, t')$, and $\varepsilon_{sh}(t, t_0)$.
9. shorttime.m: It calculates random variables p_1 and p_2 for the least-square regression.
10. shrinkage.m: It calculates the shrinkage strains.
11. sh_update.m: It calculates the shrinkage update parameter p_6.

Changing subroutine files for compliance and shrinkage functions

Although the CDA is set up to use only Model B3, the user could easily substitute another creep prediction model, such as that of ACI or CEB-*fib*, or the improved Model B4 once its development is complete. For all models, the load duration $t - t'$, age t' at loading and age t_0 at the start of drying should be given; that is,

basic compliance = Jbasic(tdur, t_p, p1, p2, ...)

drying compliance = Jdrying(tdur, t_p, t0, p1, p2, ...)

shrinkage strains = shrinkage(tdur, t0, p1, p2, ...)

1. tdur: $t - t'$, duration of loading
2. t_p: t', age at initial loading
3. p: Parameters of the model, which are q_2, q_3, and q_4 for basic compliance; q_5, k_t, k_s, D, and h for drying compliance; and ε_{sh_∞}, k_h, and τ_{sh} for shrinkage.

Table 9—Concrete properties used in example

f_c, psi	6583
w/c	0.377
W, lb/ft^3	10.61
C, lb/ft^3	28.09
A, lb/ft^3	112.4
H	0.65
a/c	4
D, in.	7.87
Cement	Type I
Curing	Air dry with initial protection
Geometry	Infinite slab
t', days	28
t_0, days	7

tdur and t_p are initially set by the program. Therefore, these two variables should be introduced in every subroutine file. If some parameters are not needed on input, they can be ignored and left blank.

EXAMPLE PROBLEMS

Consider a concrete mixture with average compressive strength f_c = 6583 psi (45.4 MPa), cement content c = 28.09 lb/ft^3 (450 kg/m^3), water content w = 10.61 lb/ft^3 (170 kg/m^3), and aggregate content a = 112.4 lb/ft^3 (1800 kg/m^3). As an approximation of a long and wide slab, consider an infinite slab of thickness D = 7.87 in. (0.2 m). The slab is cured under protection from drying until age t_0 = 7 days, and is then exposed to drying at constant humidity h = 65% (refer to summary in Table 9).

Based on formulae,[8,9] Model B3 can be evaluated from the effective thickness D; environmental humidity h; parameters ε_{sh_∞}, k_t, and k_s; the average (rather than the specified) concrete strength f_c; and the basic mixture parameters. These formulae were calibrated by statistical analysis of the data in a large computerized database.[10]

B3 parameters:

$$E_{28} = 57{,}000\sqrt{f_c} = 4{,}624{,}700 \text{ psi } = 31{,}910 \text{ MPa} \quad (8)$$

$$q_1 = 0.6 \times 10^6/E_{28} = 0.1297\ [\times 10^6/\text{psi}] \quad (9)$$

$$q_2 = 451.1c^{0.5}f_c^{-0.9} = 0.8749\ [\times 10^6/\text{psi}] \quad (10)$$

$$q_3 = 0.29(w/c)^4/q_2 = 0.0051\ [\times 10^6/\text{psi}] \quad (11)$$

$$q_4 = 1.6 \times 0.14(a/c)^{-0.7} = 0.0850\ [\times 10^6/\text{psi}] \quad (12)$$

$$\tau_{sh} = k_t(k_sD)^2 = 1124\ [\text{day/in.}^2] \quad (13)$$

$$\varepsilon_{s\infty} = \alpha_1\alpha_2(26w^{2.1}f_c^{-0.28} + 270 = 703.3 \quad (14)$$

$$\varepsilon_{sh\infty} = \varepsilon_{s\infty}\frac{E(607)}{E(t_0+\tau_{sh})} = 702.1$$
$$\text{where } E(t) = E(28)\sqrt{\frac{t}{4+0.85t}}\ \ [\text{psi}] \quad (15)$$

$$q_5 = 7.57\times 10^5 f_c^{-1}\left|\varepsilon_{sh\infty}\right|^{-0.6} = 2.256\ \ [\times 10^6/\text{psi}] \quad (16)$$

As an example, the CDA is used to estimate the compliance value for sustained load duration $t - t'$ = 120 years (43,800 days), age at loading t' = 28 days, environmental humidity h = 65%, effective thickness D = 7.87 in., and the default material model parameters. n = 0.1 and m = 0.5 can be used for the basic creep part. Function $Q(t, t')$ is accurately integrated using Eq. (5), which gives $Q(t, t')$ = 0.1816. Then, J(43828, 28) becomes

$$J_{basic}(t,t') = q_2Q(t,t') + q_3\ln[1 + (t - t')] + q_4\ln(t / t') = 0.7902 \times 10^{-6}/\text{psi} \quad (17)$$

Finally, $J = q_1 + J_{basic} = 0.9200 \times 10^{-6}$ psi.

Compliance for drying creep: obtain $t_0' = \max(t', t_0)$ = 28 days, $H(t) = 1 - (1 - h)S(t)$, H(43,828) = 0.65, and H(28) = 0.9524. $S(t)$ is given by Eq. (7), and S(43,828) = 1, S(28) = 0.1359. Finally

$$J_{drying}(t,t',t_0) = q_5\left[e^{-8H(t)} - e^{-8H(t_0')}\right]^{1/2} = 0.1598\times 10^{-6}/\text{psi} \quad (18)$$

Shrinkage strains: get $\varepsilon_{sh\infty}$ = 702.1, $k_h = 1 - h^3$ = 0.7254, S(43,828) = 1, and then

$$\varepsilon(t, t', \tau_0) = \varepsilon_{sh\infty}k_hS(t) = 509.3 \quad (19)$$

The CDA also gives two-sided 95, 90, and 85% confidence intervals (CIs) of normal distribution, for which the cutoff z-values are 1.96, 1.64 and 1.44, respectively. The selection is made by clicking CIs options in the list in Fig. 2(b). Once the selection is made, the values in the parenthesis reflecting CIs change automatically.

$$\begin{aligned} J &= 1.080 \pm \text{z-value} \times \left(\sigma / \sqrt{n_s}\right) \\ &= (1.029,\ 1.146) \text{ for } 95\% \\ &= (1.038,\ 1.137) \text{ for } 90\% \\ &= (1.049,\ 1.126) \text{ for } 85\% \end{aligned} \quad (20)$$

where $\sigma/\sqrt{ns} \approx 0.06$ according to data of Kommendant et al.[19]; ns = number of observations or data; and z = a dimensionless number indicating how far the observation is from the mean in terms of the standard deviation of normal distribution. The difference between the credible interval and the confidence interval should be noted (the 95, 90, or 85% confidence interval means that if the user performs independent tests of concrete specimens, each independent interval would capture the true population parameters 95, 90, or 85%

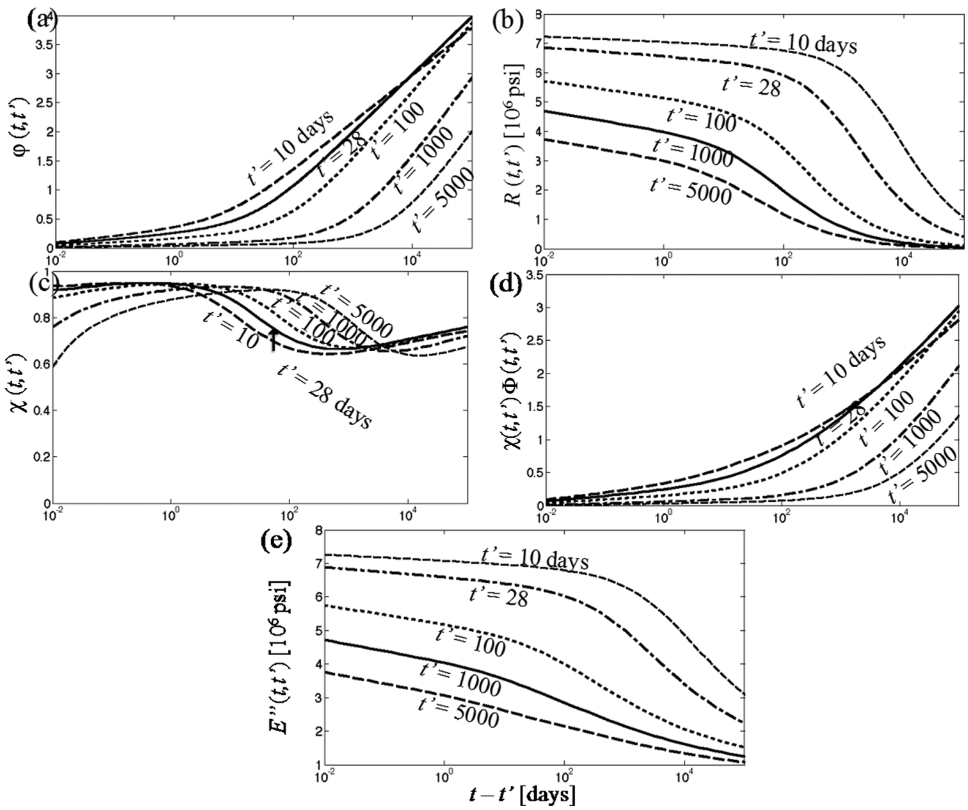

Fig. 5—Multiple plots generated by CDA: (a) creep coefficient versus t – t′; (b) relaxation versus t – t′; (c) aging coefficient versus t – t′; (d) product of aging and creep coefficients versus t – t′; and (e) AAEM versus t – t′ in logarithmic scale.

of the time; it does not mean that the parameters would have a 95, 90, or 85% credibility.) Suppose one is also interested, for comparison, in the response of the same concrete but at different ages at loading; for example, $t' = 10$, 100, 1000, and 5000 days. These multiple responses can be plotted on the same graph as shown in Fig. 3. Other plots, including relaxation, creep coefficient, aging coefficient, age-adjusted effective modulus, compliance rate, and shrinkage, are shown in Fig. 3, 4, and 5.

CLOSING COMMENTS

The method of analysis presented herein is not new, but the programming of the calculations is more tedious than it is for other creep and shrinkage prediction models recommended by engineering societies. The present program takes away the burden of complexity, and also evaluates other deterministic and statistical responses. This facilitates a more realistic prediction of creep and shrinkage—an important goal for structures of high creep sensitivity.

The website program will soon be updated by including the compliance and shrinkage functions of Model B4, soon to be completed. Model B4 has the same mathematical form as B3, except that the shrinkage function is represented as a sum of drying shrinkage and autogenous shrinkage. The applicability is extended to high-strength concretes. The formulas for predicting Model B4 parameters from concrete composition and strength are greatly improved by calibration with a new laboratory database, which is more than doubled in size, and the long-term trend is calibrated by inverse analysis of the multi-decade deflection histories of 69 bridges.

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