ALGEBRAIC METHODS FOR CREEP ANALYSIS OF CONTINUOUS COMPOSITE BEAMS[a]

Discussion by Zdeněk P. Bažant,[4] Fellow, ASCE

This interesting study is similar to the study by Bažant and Najjar (1973) [extensively reviewed in Section 5.5 of Bažant (1975)], which apparently escaped the attention of the authors. However, the conclusion in which the authors recommend the effective modulus method, is different. It appears unwarranted and conflicts with more extensive evidence.

The authors dismiss the age-adjusted effective modulus method on the basis that it requires evaluation of the relaxation function. However, it must be pointed out that this function can be easily and quite accurately evaluated from an approximate formula by Bažant and Kim (1979), also reported in Bažant (1988) and other review works. Contrary to what the authors say, solution of the integral equation need not be carried out. (An accurate calculation of the relaxation function by a computer solution of the integral equation is nevertheless an almost trivial matter.)

In a broad range of examples worked out by Bažant and Najjar (1973), including stress redistributions due to changes of the structural system, stress redistributions in composite cross sections, differential creep of a structure with concretes of different age, shrinkage stresses, effect of settlement of supports, creep buckling, etc., the age-adjusted effective modulus method gave results overall far closer to the exact solution according to the principle of superposition than other algebraic methods.

The differences between various solutions happen to be for the problem considered by the authors, considerably less than in some other creep and shrinkage problems, e.g., those analyzed by Bažant and Najjar (1973). This means that the examples they considered are not the cases most sensitive to creep, and therefore not the cases most suitable for comparison of various methods.

Furthermore, the accuracy of the application of the age-adjusted effective modulus method to shrinkage stresses in a concrete slab strongly depends on the shrinkage function. The function given by "CEB FIP Model Code 1990" (1988) is not quite realistic, showing a much higher overall deviation from the bulk of available test data than the B3 model (Bažant and Baweja 1995). The accuracy of the age-adjusted effective modulus method for shrinkage is high only when the average rate of shrinkage, compared to the long-term value, is roughly the same as the rate of creep, in which case the assumption that the shrinkage curve should be approximately a linear function of the compliance function for creep is a good approximation. Anyway, the authors would obtain very different conclusions if they studied a range of possible shrinkage curves, as predicted by the B3 model for concretes of different diffusivities and various thicknesses. It makes little sense to draw a general conclusion from an isolated example, especially if an unrealistic shrinkage function is used.

Regarding the mean stress method, its apparently good performance for shrinkage problem must be due to the high rate of shrinkage, in their chosen shrinkage function. This rate varies greatly with the thickness of the cross section and the type of concrete. Overall, the mean stress method is not as good as the age-adjusted effective modulus method, even if the aging coefficient is taken as a constant equal to 0.8. Fixing this coefficient as constant gives the same simplicity as the mean stress method but usually a better accuracy.

APPENDIX. REFERENCES

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Closure by Luigino Dezi,[5] Graziano Leoni,[6] and Angelo Marcello Tarantino[7]

The writers wish to thank the discusser for his interest in their paper and for having stimulated an extension of the analysis to the B3 model, thus allowing a further generalization of their results. The closure is organized under the following three points.

The paper by Bažant and Najjar (1973) is not similar to the writers' paper. In their work, Bažant and Najjar evaluated the effectiveness of the algebraic methods on a broad set of elementary problems that can be encountered in the viscoelastic theory. A very marginal space was dedicated to composite structures (only 10 lines on page 1864), examining the simple case of a steel-concrete cross section subjected to an external bending moment constant in time. On the contrary, the analysis developed by the writers, which is far more complete and exhaustive, focuses attention exclusively on the class of composite structures. A global analysis was in fact performed for statically indeterminate composite structures, taking into account the deformability of the shear connection with the final aim of estimating the validity of the algebraic methods for each type of external action considered separately (namely, for the following four types of actions: static, geometrical, shrinkage, and prestressing of the slab). Such a separation (which is very important in structures such as bridges, where each single action may assume a fundamental role) permits substantially to better understand the approximations induced by application of the algebraic methods to composite structures.

With reference to the age-adjusted effective modulus (AAEM) method, in many parts of the paper, it is clearly declared that such a method provides excellent results, except for the shrinkage problem. It is also stated that the principal technical codes for practical purposes tend to recommend the modular ratio method, with time-independent coefficients depending on the type of external action, following the simpler applicative philosophy of the effective modulus (EM) method. In the paper it is pointed out that even if the EM method gives

[a]April 1996, Vol. 122, No. 4, by Luigino Dezi, Graziano Leoni, and Angelo Marcello Tarantino (Paper 7747).

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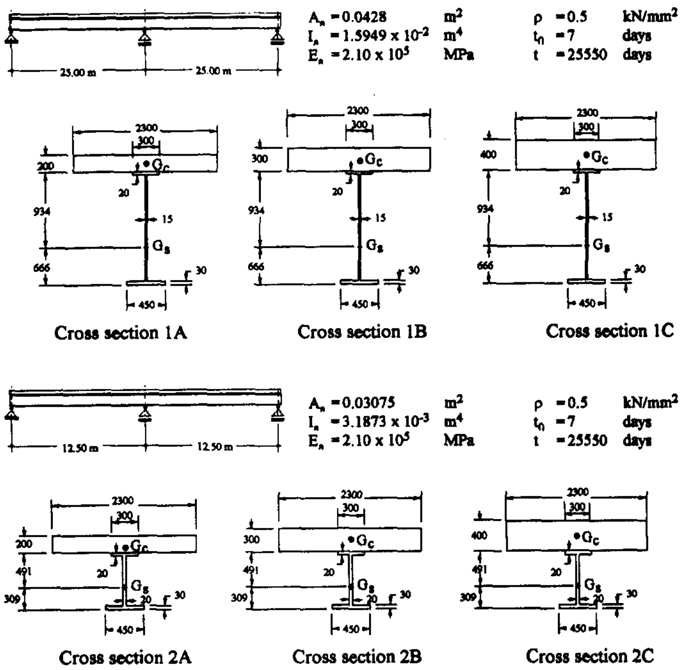
less accurate results than the AAEM method it still maintains a level of accuracy that is acceptable for practical applications.

Finally, the crucial question of the discussion, concerning the validity of the algebraic methods for the shrinkage problem, is examined. The discusser contests the writers' conclusions, which establish that the mean stress (MS) method is the best algebraic method in evaluating the effects due to shrinkage of the concrete slab, conjecturing that such a good performance of the MS method (shown in the writers' paper) depends on the particular creep and shrinkage models used (CEB model). Moreover, he states that when the B3 model is adopted, the AAEM method would always be the best algebraic method, even in its simplified form with $\chi = 0.8$.

To check this assertion, the writers, who had previously tested the validity of the algebraic methods by using the CEB model for a large class of composite structures (as clearly stated at the end of section 5), have extended the numerical



Concrete characteristics
water-cementitious material ratio by weight w/c= 0.5
concrete density cd =2300 kg/m³
cement content of concrete c=300 kg/m³

FIG. 9. Data of Structures Adopted in Numerical Analysis

TABLE 2. Absolute Values and Relative Errors of Middle Support Reaction Obtained with Different Algebraic Methods [Influence of Cross-Section Type (CEB Model—f_{ck} = 30 MPa; RH = 80%)]

Type of cross section (1)	Method G (2)	EM (3)	MS (4)	AAEM (5)	$\chi = 0.8$ (6)
1A	149.07	112.58 (−24.5%)	148.53 (−0.4%)	128.89 (−13.5%)	123.37 (−17.2%)
1B	193.50	148.99 (−23.0%)	186.81 (−3.5%)	168.30 (−13.0%)	160.72 (−16.9%)
1C	223.89	176.84 (−21.0%)	214.03 (−4.4%)	197.39 (−1.8%)	188.62 (−15.8%)
2A	135.38	107.60 (−20.5%)	134.80 (−0.4%)	120.29 (−11.1%)	116.06 (−14.3%)
2B	171.92	140.40 (−18.3%)	167.35 (−2.7%)	154.49 (−10.1%)	149.04 (−13.3%)
2C	198.12	165.97 (−16.2%)	191.65 (−3.3%)	180.45 (−8.9%)	174.38 (−12.0%)

Note: Relative errors are in parentheses.

TABLE 3. Absolute Values and Relative Errors of Stress at the Cross Section over the Middle Support Obtained with Different Algebraic Methods [Influence of Cross-Section Type (CEB Model—f_{ck} = 30 MPa; RH = 80%)]

Method (1)	Slab σ_{Top} [MPa (%)] (2)	Slab σ_{Bottom} [MPa (%)] (3)	Beam σ_{Top} [MPa (%)] (4)	Beam σ_{Bottom} [MPa (%)] (5)
		(*a*) Section 1A		
G	3.36	3.06	−15.01	−48.44
EM	2.46 (−27.0)	2.29 (−25.0)	−9.35 (−37.7)	−37.06 (−23.5)
MS	3.35 (−0.3)	3.05 (−0.5)	−14.96 (−0.4)	−48.26 (−0.4)
AAEM	2.86 (−15.1)	2.64 (−13.8)	−11.82 (−21.2)	−42.16 (−13.0)
$\chi = 0.8$	2.72 (−19.1)	2.52 (−17.6)	−10.97 (−26.9)	−40.44 (−16.5)
		(*b*) Section 1B		
G	3.00	2.43	−21.61	−59.56
EM	2.20 (−26.8)	1.91 (−21.3)	−14.29 (−33.9)	−46.53 (−21.9)
MS	2.88 (−4.2)	2.35 (−3.1)	−20.49 (−5.2)	−57.61 (−3.3)
AAEM	2.54 (−15.5)	2.14 (−11.7)	−17.40 (−19.5)	−52.21 (−12.3)
$\chi = 0.8$	2.40 (−20.0)	2.05 (−15.4)	−16.17 (−25.2)	−49.98 (−16.1)
		(*c*) Section 1C		
G	2.70	1.86	−26.00	−65.44
EM	1.98 (−26.6)	1.57 (−15.4)	−18.10 (−30.4)	−52.55 (−19.7)
MS	2.54 (−6.1)	1.81 (−2.5)	−24.30 (−6.5)	−62.77 (−4.1)
AAEM	2.28 (−15.7)	1.71 (−7.7)	−21.48 (−17.4)	−58.23 (−11.0)
$\chi = 0.8$	2.15 (−20.5)	1.66 (−10.8)	−20.0 (−23.0)	−55.81 (−14.7)
		(*d*) Section 2A		
G	2.89	2.23	−16.10	−52.34
EM	2.19 (−24.3)	1.83 (−18.1)	−11.06 (−31.3)	−42.03 (−19.7)
MS	2.88 (−0.2)	2.22 (−0.7)	−16.01 (−0.5)	−52.11 (−0.4)
AAEM	2.50 (−13.4)	2.02 (−9.5)	−13.32 (−17.2)	−46.75 (−10.7)
$\chi = 0.8$	2.40 (−17.1)	1.96 (−12.3)	−12.56 (−22.0)	−45.18 (−13.7)
		(*e*) Section 2B		
G	2.62	1.44	−21.65	−60.66
EM	1.96 (−25.4)	1.34 (−7.3)	−15.58 (−28.0)	−50.39 (−16.9)
MS	2.52 (−4.0)	1.43 (−0.5)	−20.74 (−4.2)	−59.20 (−2.4)
AAEM	2.23 (−14.8)	1.41 (−2.5)	−18.22 (−15.9)	−55.04 (−9.3)
$\chi = 0.8$	2.12 (−19.1)	1.38 (−4.1)	−17.18 (−20.6)	−53.25 (−12.2)
		(*f*) Section 2C		
G	2.44	0.83	−25.99	−63.39
EM	1.79 (−26.5)	0.94 (13.4)	−19.20 (−26.1)	−54.63 (−13.8)
MS	2.29 (−6.2)	0.88 (5.4)	−24.53 (−5.6)	−61.73 (−2.6)
AAEM	2.05 (−15.8)	0.93 (11.4)	−22.11 (−14.9)	−58.73 (−7.3)
$\chi = 0.8$	1.94 (−20.5)	0.94 (12.9)	−20.87 (−19.7)	−57.04 (−10.0)

Note: Relative errors are in parentheses.

TABLE 4. Absolute Values and Relative Errors of the Middle Support Reaction Obtained with Different Algebraic Methods [Influence of Relative Humidity (CEB Model—f_{ck} = 30 MPa)]

Relative humidity (%) (1)	Method G (2)	EM (3)	MS (4)	AAEM (5)	$\chi = 0.8$ (6)
			(*a*) Cross section 1A		
70	193.10	143.31 (−25.8%)	192.65 (−0.2%)	162.79 (−15.7%)	158.12 (−18.1%)
80	149.07	112.58 (−24.5%)	148.53 (−0.4%)	128.89 (−13.5%)	123.37 (−17.2%)
90	86.05	66.54 (−22.7%)	85.91 (−0.2%)	78.63 (−8.6%)	72.35 (−15.9%)
			(*b*) Cross section 2A		
70	176.77	138.23 (−21.8%)	176.19 (−0.3%)	153.71 (−13.0%)	150.05 (−15.1%)
80	135.38	107.60 (−20.5%)	134.80 (−0.4%)	120.29 (−11.1%)	116.06 (−14.3%)
90	77.57	62.93 (−18.9%)	77.30 (−0.3%)	72.06 (−7.1%)	67.39 (−13.1%)

Note: Relative errors are in parentheses.

comparisons to the B3 model. Two steel beams combined with three different slabs have been considered. These are illustrated in detail in Fig. 9. The numerical results are collected in Tables 2–9. Tables 2–5 report the numerical computations already performed by the writers using the CEB model, whereas Tables 6–9 are related to the new results obtained by using the B3 model. In each table, the numerical values determined with the different algebraic methods (EM, MS, and AAEM methods) are directly compared with those provided by the general method (G method—accurate numerical solution). The numerical values obtained by applying the AAEM method with $\chi = 0.8$ are also reported. For an immediate comparison, the relative errors are specified in parentheses. The influence of the cross-section type is shown in Tables 2, 3, 6, and 7, the influence of the relative humidity in Tables 4 and 8, and the influence of the concrete strength in Tables 5 and 9.

As is clearly evidenced by Tables 6–9, the MS method gives the most accurate values even when the B3 model is used. Then the writers' conclusions, based on the CEB model (see Tables 2–5), are now also confirmed for the B3 model. The MS method, in evaluating the effects due to shrinkage of concrete slab, is preferable to the AAEM method not only for its better numerical precision, but also for its simpler use. It

TABLE 5. Absolute Values and Relative Errors of the Middle Support Reaction Obtained with Different Algebraic Methods [Influence of Concrete Strength (CEB Model—RH = 80%)]

Concrete strength (1)	Method G (2)	EM (3)	MS (4)	AAEM (5)	$\chi = 0.8$ (6)
(a) Cross section 1A					
25	150.42	112.02 (−25.5%)	150.15 (−0.2%)	128.61 (−14.5%)	123.42 (−17.9%)
35	146.00	111.62 (−23.5%)	145.26 (−0.5%)	127.42 (−12.7%)	121.75 (−16.6%)
40	141.55	109.40 (−22.7%)	140.67 (−0.6%)	124.54 (−12.0%)	118.83 (−16.1%)
(b) Cross section 2A					
25	138.01	108.16 (−21.6%)	137.59 (−0.3%)	121.35 (−12.1%)	117.28 (−15.0%)
35	131.44	105.73 (−19.6%)	130.75 (−0.5%)	117.80 (−10.4%)	113.54 (−13.6%)
40	126.45	102.80 (−18.7%)	125.70 (−0.6%)	114.17 (−9.7%)	109.95 (−13.0%)

Note: Relative errors are in parentheses.

TABLE 6. Absolute Values and Relative Errors of Middle Support Reaction Obtained with Different Algebraic Methods [Influence of Cross-Section Type (B3 Model—f_{ck} = 30 MPa; RH = 80%)]

Type of cross section (1)	Method G (2)	EM (3)	MS (4)	AAEM (5)	$\chi = 0.8$ (6)
1A	100.84	75.82 (−24.8%)	107.90 (−7.0%)	90.94 (−9.8%)	83.30 (−17.4%)
1B	141.56	104.43 (−26.2%)	141.40 (−0.1%)	123.04 (−13.1%)	113.39 (−19.9%)
1C	173.61	128.91 (−25.7%)	168.09 (−3.2%)	149.50 (−13.9%)	138.68 (−20.1%)
2A	96.50	75.07 (−22.2%)	101.01 (4.7%)	87.66 (−9.2%)	81.39 (−15.7%)
2B	131.54	102.47 (−22.1%)	130.76 (−0.6%)	117.12 (−11.0%)	109.63 (−16.7%)
2C	159.43	126.15 (−20.9%)	155.17 (−2.7%)	141.82 (−11.0%)	133.70 (−16.1%)

Note: Relative errors are in parentheses.

TABLE 7. Absolute Values and Relative Errors of Stress at the Cross Section over the Middle Support Obtained with Different Algebraic Methods [Influence of Cross-Section Type (B3 Model —f_{ck} = 30 MPa; RH = 80%)]

Method (1)	Slab σ_{Top} [MPa (%)] (2)	Slab σ_{Bottom} [MPa (%)] (3)	Beam σ_{Top} [MPa (%)] (4)	Beam σ_{Bottom} [MPa (%)] (5)
(a) Section 1A				
G	2.19	2.07	−8.53	−33.17
EM	1.61 (−26.6)	1.53 (−26.1)	−5.15 (−39.6)	−25.24 (−23.9)
MS	2.39 (8.8)	2.21 (6.7)	−9.78 (14.71)	−35.32 (6.5)
AAEM	1.97 (−10.2)	1.85 (−10.6)	−7.25 (−15.0)	−30.01 (−9.5)
$\chi = 0.8$	1.79 (−18.5)	1.69 (−18.4)	−6.17 (−27.7)	−27.61 (−6.8)
(b) Section 1B				
G	2.10	1.83	−14.26	−44.04
EM	1.49 (−28.9)	1.34 (−26.6)	−8.61 (−39.67)	−32.99 (−25.1)
MS	2.12 (0.91)	1.81 (−1.2)	−14.33 (0.5)	−43.94 (−0.2)
AAEM	1.80 (−14.2)	1.58 (−13.6)	−11.41 (−20.0)	−38.53 (−12.5)
$\chi = 0.8$	1.64 (−21.9)	1.46 (−20.3)	−9.94 (−30.3)	−35.66 (−19.0)
(c) Section 1C				
G	1.98	1.54	−18.75	−51.30
EM	1.39 (−29.6)	1.17 (−24.1)	−11.67 (−37.8)	−38.77 (−24.4)
MS	1.92 (−2.8)	1.47 (−4.11)	−17.91 (−4.5)	−49.72 (−3.1)
AAEM	1.66 9−16.0)	1.34 (−13.1)	−14.89 (−20.6)	−44.56 (−13.1)
$\chi = 0.8$	1.52 (−23.2)	1.25 (−18.7)	−13.18 (−29.7)	−41.53 (−19.1)
(d) Section 2A				
G	1.95	1.66	−10.17	−37.66
EM	1.48 (−24.2)	1.29 (−22.5)	−6.63 (−34.8)	−29.57 (−21.5)
MS	2.10 (7.4)	1.70 (2.2)	−11.09 (9.1)	−39.28 (4.3)
AAEM	1.77 (−9.3)	1.50 (−10.1)	−8.73 (−14.1)	−34.30 (−8.9)
$\chi = 0.8$	1.62 (−16.8)	1.39 (−16.2)	−7.67 (−24.6)	−31.95 (−15.2)
(e) Section 2B				
G	1.84	1.25	−15.18	−47.06
EM	1.36 (−26.4)	1.02 (−18.3)	−10.09.(−33.5)	−37.19 (−21.0)
MS	1.87 (1.3)	1.21 (−3.6)	−15.17 (−0.1)	−46.69 (−0.8)
AAEM	1.61 (−12.7)	1.13 (−9.7)	−12.66 (−16.6)	−42.15 (−10.4)
$\chi = 0.8$	1.48 (−19.8)	1.08 (−13.9)	−11.33 (−25.4)	−39.62 (−15.8)
(f) Section 2C				
G	1.77	0.86	−19.32	−52.1
EM	1.27 (−28.0)	0.80 (−8.0)	−13.14 (−32.0)	−42.19 (−19.0)
MS	1.73 (−2.3)	0.83 (−3.9)	−18.63 (−3.6)	−50.72 (−2.6)
AAEM	1.51 (−15.1)	0.83 (−3.8)	−16.01 (−17.1)	−46.88 (−10.0)
$\chi = 0.8$	1.38 (−22.0)	0.82 (−5.5)	−14.50 (−25.0)	−44.47 (−14.7)

Note: Relative errors are in parentheses.

TABLE 8. Absolute Values and Relative Errors of the Middle Support Reaction Obtained with Different Algebraic Methods [Influence of Relative Humidity (B3 Model—f_{ck} = 30 MPa)]

Relative humidity (%) (1)	Method G (2)	EM (3)	MS (4)	AAEM (5)	$\chi = 0.8$ (6)
(a) Cross section 1A					
70	124.01	96.08 (−22.5%)	138.99 (12.1%)	113.55 (−8.4%)	106.29 (−14.3%)
80	100.84	75.82 (−24.8%)	107.90 (7.0%)	90.94 (−9.8%)	83.30 (−17.4%)
90	60.25	44.17 (−26.7%)	62.04 (3.0%)	53.84 (−10.6%)	48.26 (−19.9%)
(b) Cross section 2A					
70	120.58	95.89 (−20.5%)	131.15 (8.8%)	110.73 (−8.2%)	104.65 (−13.2%)
80	96.50	75.07 (−22.2%)	101.01 (4.7%)	87.66 (−9.2%)	81.39 (−15.7%)
90	56.87	43.44 (−23.6%)	57.70 (−1.5%)	51.36 (−9.7%)	46.85 (−17.6%)

Note: Relative errors are in parentheses.

is also evident that the AAEM method with $\chi = 0.8$ cannot be recommended.

The cross sections and the creep parameters chosen in the numerical analysis can be considered as representative of standard situations, namely, those that can be more frequently encountered in practice. However, for some limit situations, as in the case of small slab thickness and low relative humidity (RH), the B3 model produces a stress history with the maximum value at an intermediate time (see Figs. 10 and 11) and not at the final time, as always occurs with the CEB model. In these circumstances, the use of the algebraic methods is not advised, since the time at which the stress reaches the maximum value is not known a priori, so that only the general method can be applied. With reference to the examples of Figs. 10 and 11, after having computed these special intermediate times by means of a step-by-step analysis, the algebraic methods have nonetheless been applied to perform a numerical comparison. Figs. 10 and 11 show that the MS method, once again, gives the most accurate solution.

Given the results obtained, it is possible to conclude that, in the shrinkage problem for composite structures, the MS method gives a better accuracy with respect to the AAEM method. Contrary to what the discusser writes, this conclusion holds for both the CEB and B3 models.

TABLE 9. Absolute Values and Relative Errors of the Middle Support Reaction Obtained with Different Algebraic Methods [Influence of Concrete Strength (B3 Model—RH = 80%)]

Concrete strength (1)	Method G (2)	EM (3)	MS (4)	AAEM (5)	$\chi = 0.8$ (6)
(*a*) Cross section 1A					
25	97.67	72.16 (−26.1%)	103.95 (−6.4%)	86.99 (−10.9%)	79.35 (−18.8%)
35	103.25	78.73 (−23.7%)	110.96 (−7.5%)	94.02 (−8.9%)	86.45 (−16.3%)
40	105.12	81.10 (−22.9%)	113.39 (−7.9%)	96.46 (−8.2%)	89.02 (−15.3%)
(*b*) Cross section 2A					
25	94.33	72.16 (−23.5%)	98.42 (−4.3%)	84.77 (−10.1%)	78.36 (−16.9%)
35	98.07	77.32 (−21.2%)	102.92 (−4.9%)	89.83 (−8.4%)	83.72 (−14.6%)
40	99.23	79.09 (−20.3%)	104.36 (−5.2%)	91.48 (−7.8%)	85.56 (−13.8%)

Note: Relative errors are in parentheses.

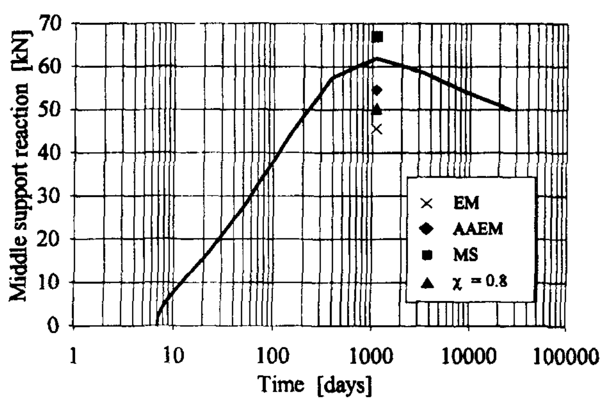


FIG. 10. Time Evolution of Middle Support Reaction (Steel Beam Type 1; Slab Thickness 100 mm; RH = 80%; f_{ck} = 30 MPa)

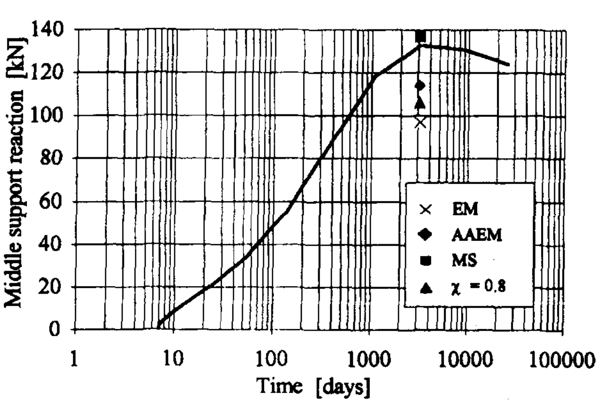


FIG. 11. Time Evolution of Middle Support Reaction (Steel Beam Type 1; Slab Thickness 200 mm; RH = 70%; f_{ck} = 30 MPa)

DESIGN PROVISIONS FOR STAIR SLABS IN THE BANGLADESH BUILDING CODE[a]

Discussion by Hakan Kılıç[4] and Ergin Çıtıpıtıoğlu,[5] Member, ASCE

The authors presented results of the analysis of staircases having various boundary conditions based on the Bangladesh National Building Code. They must be commended for bringing this subject to the attention of the civil engineering community. Basically four different cases are presented in the paper. In practice, it is not likely to have the same boundary conditions for the midstory landing and the story level landing because the story level landing is continuously cast with the floor slab of the building. The discussers developed a finite-element model, shown in Fig. 6, to represent continuous flights by introducing symmetrical boundary conditions at the mid-landing section, which allows vertical displacements. A number of analysis are performed by *SAP 90* (Wilson and Mabibullah 1992), a computer program using four-node shell elements to verify the results presented in the paper, and an alternative approach, which is widely used in Turkey (Köseoğlu 1992).

Discussers analyzed the four cases presented in Fig. 4 under a unit load. Two important deviations are observed. First, the sum of support and span moments of case I is not equal to the span moment of case II in the figure. They must be equal due to simple statics. Second, a discontinuity exists at the transition region of the case I moment diagram in Fig. 4. The discussers' analyses, based on a finite-element model (shown in Fig. 6) indicated correct results for cases I and II and a smooth transition region in case I. The boundary conditions and the finite-element model used are not given in the paper. Additionally appreciable axial forces are obtained by the discussers for cases II and III. Axial forces must be considered in the determination of steel reinforcement. Furthermore, it is observed that hinge and roller type of end boundary conditions result in large changes in moments and axial forces.

Practical analysis of staircases with various boundary conditions can be performed on a two-dimensional beam model representing a strip of unit width (Köseoğlu 1992). The cases

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